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Dynamics of Periodically Modulated Cavity Frequency of A Microwave Cavity Consisting of Cold Atoms

Research Article

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Abstract. We investigate the possibility to enhance the atomic fluorescence and using it to measure the 'second' by means of a quantum device formed by two-level cold atoms confined in a microwave cavity with harmonically modulated cavity frequency. Besides the harmonically modulated cavity frequency, we have also studied the modification in the atomic fluorescence due to the addition of the squeezing term in the Hamiltonian of the system. The periodic modulation of cavity frequency give rise to non adiabatic process characterized as two-photon process. It has been observed that the two-photon process can be used as a new handle to enhance the atomic fluorescence which helps in improving the accuracy of the atomic fountain clocks.

Keywords. Cold atoms; Microwave cavity; Two-photon process

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1. Introduction

The field of atomic physics has been remarkably developed due to the achievement of Bose-Einstein condensation [1–3]. Ultracold atoms have found significant applications in high precision measurements of physical constants [4] and metrology [5]. They can be cooled to quantum degeneracy and can be used in condensed-matter systems [6, 7]. Nowadays, cold atoms are used in atomic fountain clocks which are being developed at various laboratories across the world [8–18]. The atomic clock is called the atomic fountain clock as the atoms move in a controlled manner against gravity, resembling the water in a pulsed fountain. The motion of ultracold atoms is very slow. For example, cesium atoms cooled using Sisyphus cooling have r.m.s. velocity of 1 cm/s with an effective temperature on the order of 1μK. This allow them to spend longer duration in an observation zone which thereby helps in improving the stability of atomic clocks [19, 20]. The SI definition of the 'second' is the duration of 9,192,631,770 oscillations of the radiation corresponding to the transition between the two-hyperfine levels of the cesium-133 atom [21]. In addition to, being used as a time standard, the atomic fountain clock can be used as a frequency standard which is the basis of modern day communication and navigation system. Moreover, a large variety of experiments in physics ranging from quantum metrology, quantum information science and basic physics experiments require quantum-state preparation. It is basically preparation of an ensemble of quantum systems in a given quantum state. For example, in cesium atomic clocks, a pure quantum-state is prepared due to the clock stability. The principle of operation of cold-atom interferometers is very similar to atomic clocks. In the interferometers, quantum-state preparation results in increase signal-to-noise ratio and thereby increased sensor sensitivity. Other examples are masers, lasers, and all quantum information science experiments where quantum-state preparation plays a significant role.

A moving boundary with non-uniform motion [22,23] or a fast changes in the refractive index of the cavity medium [24,25] can generate real photons from the vacuum state. Yablonovitch [24] and Schwinger [26] have apparently introduced this phenomenon which is nowadays termed as dynamical Casimir effect. An effective Hamiltonian was derived for this type of system composed of radiation inside a cavity with moving boundary and a time-varying dielectric medium in the cavity [27]. It was presumed that a nonadiabatic process characterized by two-photon process helps in creating photon pairs from the vacuum state. This two-photon character of the optical field is related to the squeezing phenomenon [28,29].

Given the necessity to improve the accuracy of the atomic fountain clocks and the promising developments in the field of two-level cold atoms, we propose in this paper a model in which cold atoms are coupled to a harmonically modulated microwave cavity mode. Such model helps in producing enhanced atomic fluorescence which can be used to define the second. Further, the system is examined by adding the squeezing term in the Hamiltonian.

2. Model Hamiltonian

The system investigated here consists of two-level cold atoms in a cylindrical microwave cavity as shown in Figure 1.

The source for the microwave interrogation is a synthesizer which produces the required interrogation frequency. In our model, we have a cloud of N two-level cold atoms of ^{133}Cs in the $|F=3\rangle$ state having mass m with atomic transition $|F=3\rangle \rightarrow |F'=4\rangle$. The cold atoms have transition frequency ω_a. The cloud of atoms strongly interacts with a single quantized cavity mode with frequency ω_c. Such type of interaction is similar to Ramsey interaction. This basic cylindrical microwave cavity setup is analogous to the microwave cavity inside the atomic fountain clocks to measure 'second' [30]. The cylindrical microwave cavities have been used in the fountain clocks to minimize the effect of transverse phase variations on the atomic transition

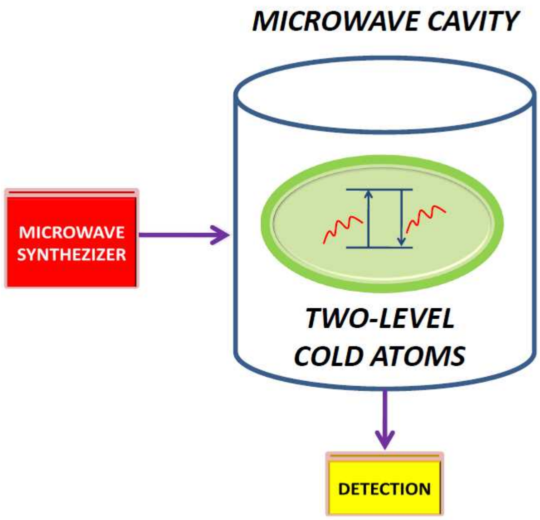

Figure 1. (color online) Schematic figure of the system. Figure shows two-level cold atoms confined in a cylindrical microwave cavity. Here the cavity mode is driven by the microwave synthesizer. The fluorescence produced by the atoms with the emission of photons is detected.

frequency [10, 31–34]. The most significant part of the fountain clock is the microwave cavity for the Ramsey interaction. The atoms pass twice through the cavity, once their way up and again once their way down, during which they interacts with the cavity. In this paper, we study the dynamics of the system by considering time dependent cavity frequency. The time varying cavity frequency can be produced using the microwave synthesizer. This results in harmonically varying one-dimensional single quantized cavity mode i.e. $\omega_c(t) = \omega_c(1+\epsilon\sin(\Omega t))$. Here, ϵ is the modulation amplitude and Ω is the modulation frequency of the sinusoidally modulated cavity frequency. The time-dependent Hamiltonian of the system in the dipole and rotating-wave approximation is given by

$$H_I = \hbar\omega_c(t)a^\dagger a + \frac{\hbar\omega_a\sigma_z}{2} + \hbar g_0(t)\left(\sigma_+ a + \sigma_- a^\dagger\right). \tag{1}$$

Here a and $a^\dagger$ are the cavity mode annihilation and creation operator respectively satisfying the commutation relation $\left[a, a^\dagger\right] = 1$. σ_-, σ_+ and σ_z are the standard Pauli operators which obey the commutation relations $[\sigma_+, \sigma_-] = \sigma_z$ and $[\sigma_z, \sigma_\pm] = \pm 2\sigma_\pm$. The first term in above Hamiltonian (H_I) represents the free energy of the cavity mode. The second term gives the free energy of the two-level atom inside the cavity. The third term depicts the interaction energy in which the single quantized cavity mode and two-level cold atoms interacts through the dipole interaction. In this term, $g_0(t)$ is the coupling parameter which modifies with time as $g_0(t) = g_0(1+\epsilon\sin(\Omega t))$ where g_0 is the unperturbed coupling constant. The last term is widely referred to as the Jaynes-Cummings model [35]. Under the RWA, the non-energy conserving terms are basically dropped which modifies the coupling term of the Hamiltonian.

The detection of atoms in a fountain clock is performed via the fluorescence light of the atoms. To measure the atomic fluorescence, firstly, we find the Heisenberg equations of motion for all the operators of the system as follows:

$$\dot{a} = -i\omega_c(t)a - ig_0(t)\sigma_-, \tag{2}$$

$$\dot{\sigma_z} = 2ig_0(t)\left(a^\dagger\sigma_- - a\sigma_+\right), \tag{3}$$

$$\dot{\sigma_+} = i\omega_a\sigma_+ - ig_0(t)a^\dagger\sigma_z. \tag{4}$$

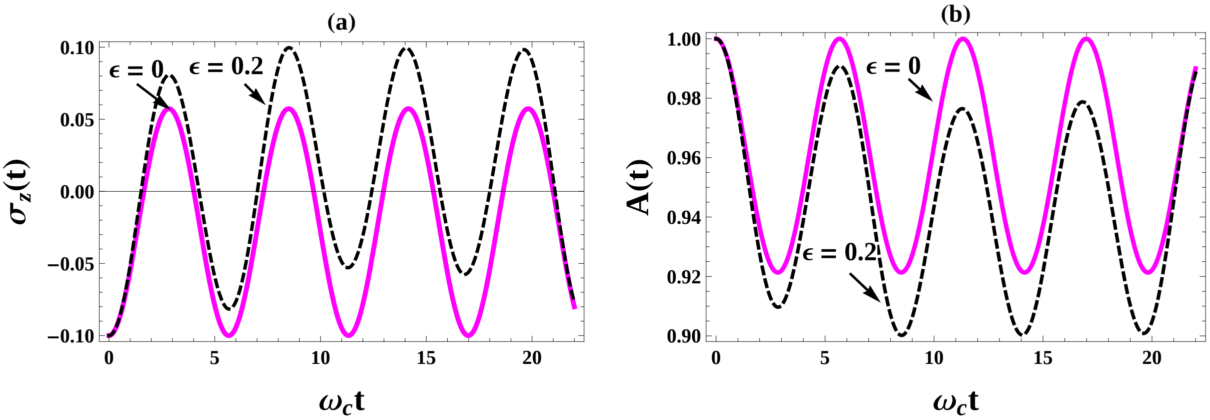

Figure 2. (Color online) Plot (a) Atomic inversion ($\sigma_z(t)$) and plot (b) corresponding intracavity Photon number $\left(A(t) = \langle a^\dagger a\rangle\right)$ with scaled time ($\omega_c t$) using time modulated cavity frequency and coupling parameter at resonant modulating frequency ($\Omega = \omega_c$) for $\epsilon = 0$ (solid line) and $\epsilon = 0.2$ (dashed line). Parameters used are $g_0 = 2\omega_c$ and $\omega_a = 1.5\omega_c$.

Now we solve the above coupled differential equations of motion using MATHEMATICA 9.0 to study the atomic fluorescence. We will particularly see how the time varying cavity frequency and time-dependent coupling parameter can be used to enhance the atomic fluorescence. Figure 2(a) depicts the atomic inversion ($\sigma_z(t)$) with scaled time ($\omega_c t$) at resonant frequency ($\Omega = \omega_c$) in the absence of the modulation ($\epsilon = 0$) and in the presence of the modulation with $\epsilon = 0.2$ at resonant modulating frequency $\Omega = \omega_c$. The plot for $\sigma_z(t)$ shows sinusoidal variation with time. It depicts higher amplitude of atomic inversion $\sigma_z(t)$ in the presence of time modulating cavity frequency and coupling parameter. This means that more number of atoms are in the excited state in the presence of modulation. However, exact opposite behaviour is observed for the photon number as shown in Figure 2(b). It represents the corresponding photon number inside the cavity $\left(A(t) = \langle a^\dagger a\rangle\right)$ with scaled time ($\omega_c t$) in the absence of the modulation ($\epsilon = 0$) and in the presence of the modulation with $\epsilon = 0.2$ at resonant modulating frequency $\Omega = \omega_c$. The plot illustrates lower photon number amplitude in the presence of modulation. This is because more intracavity photons are absorbed by the atoms to reach the excited sate in the presence of modulation. In an atomic fountain clock, the atoms reach an excited state depending on the microwave cavity frequency. These atoms will produce fluorescence by emitting photons upon passing through a laser beam. The microwave cavity frequency at which maximum fluorescence is produced, is basically used to define the 'second'. Within a set of experimentally achievable parameters, in an caesium atomic fountain clock, the microwave signal used for interrogation is 9.2 GHz. Also, the unperturbed caesium transition frequency is 9192631770 Hz [30].

3. Modified Hamiltonian with Additional Two-Photon Process

In this section, the current system is investigated with an additional two-photon process [27] in the Hamiltonian (eqn. (1)). Therefore, the modified Hamiltonian of the system becomes

$$H_{II} = \hbar\omega_c(t)a^\dagger a + \frac{\hbar\omega_a\sigma_z}{2} + \hbar g_0(t)\left(\sigma_+ a + \sigma_- a^\dagger\right) + i\chi(t)\hbar\left(a^{\dagger 2} - a^2\right). \tag{5}$$

The last term in the above Hamiltonian is responsible for the non-adiabatic effect. As described in [27], there are basically two types of non-adiabatic processes in a system with an optomechanical cavity. The first process is characterized by $\left(a^\dagger a\right)$ terms in the Hamiltonian. In this kind of process, the total number of photons remain constant inside the cavity. The second process is characterized by $\left(a^2\right)$ or $\left(a^{\dagger 2}\right)$ terms. In this kind of process, the total number of intracavity photons are not constant. This kind of process is termed as two-photon process. It leads to annihilation or creation of photon pairs inside the cavity. Since the two-photon process helps in creating photons from the vacuum state, therefore, it is also used to study the Dynamical Casimir effect in various systems [29, 36, 37]. The relation between the functions $\omega_c(t)$ and $\chi(t)$ is given as follows [27]

$$\chi(t) \equiv \frac{1}{4\omega_c(t)}\frac{d\omega_c(t)}{dt}. \tag{6}$$

The realistic case with small modulation amplitude $\epsilon \ll 1$ is considered. This leads to the approximation equivalent to

$$\chi(t) \approx \frac{\epsilon\Omega}{4}\cos(\Omega t) \approx 2\chi_0\cos(\Omega t), \tag{7}$$

where,

$$\chi_0 = \frac{\epsilon\Omega}{8}. \tag{8}$$

This additional term in the Hamiltonian further modifies the Heisenberg equation of motion of the operator a as

$$\dot{a} = -i\omega_c(t)a - ig_0(t)\sigma_- + 2\chi(t)a^\dagger. \tag{9}$$

The additional term does not alter the Heisenberg equations of motion for rest of the operators of the system. Now we study how the two-photon process alters the atomic fluorescence and intracavity photon number. The coupled differential equations (eqns. (3), (4) and (9)) are solved using MATHEMATICA 9.0. Figure 3(a) shows the atomic inversion ($\sigma_z(t)$) with scaled time ($\omega_c t$) at resonant frequency ($\Omega = \omega_c$) in the absence of the periodic modulation of cavity frequency ($\epsilon = 0$) and in the presence of the periodic modulation of cavity frequency with $\epsilon = 0.2$. It shows the sinusoidal oscillations with time. The amplitude of the atomic inversion is more in the presence of periodic modulation of cavity frequency. This clearly means that more number of atoms are present in the excited state with the periodic modulation of cavity frequency. The corresponding intracavity photon number $\left(A(t) = \langle a^\dagger a\rangle\right)$ is plotted with scaled time ($\omega_c t$) in Figure 3(b). The plot clearly illustrates the enhancement in the photon number in the presence

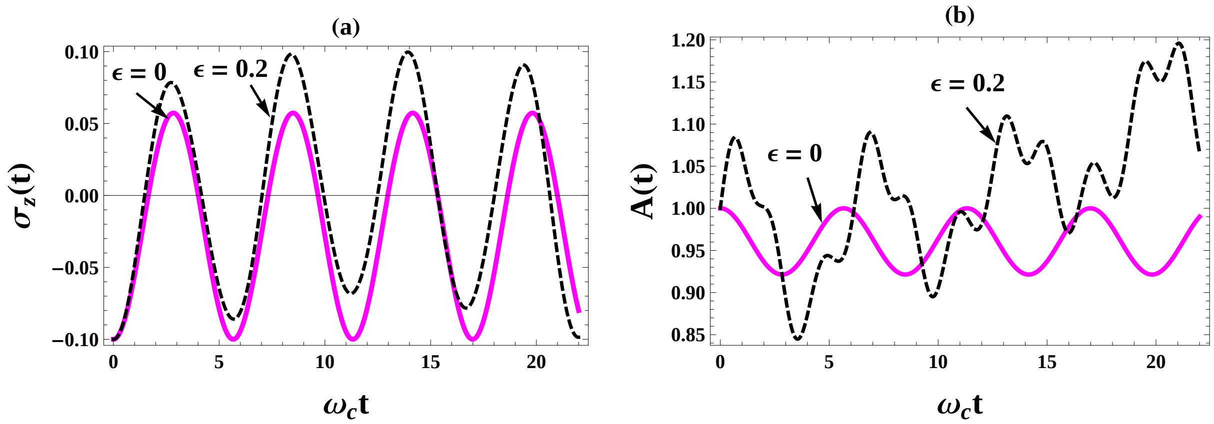

Figure 3. (Color online) Plot (a) Atomic inversion ($\sigma_z(t)$) and plot (b) corresponding intracavity Photon number $\left(A(t) = \langle a^\dagger a\rangle\right)$ with scaled time ($\omega_c t$) using time modulated cavity frequency and time-dependent coupling parameter with squeezing term at resonant modulating frequency ($\Omega = \omega_c$) for $\epsilon = 0$ (solid line) and $\epsilon = 0.2$ (dashed line). Parameters used are $g_0 = 0.5\omega_c$ and $\omega_a = 1.5\omega_c$.

of periodic modulation of cavity frequency. Due to the presence of periodic modulation of cavity frequency and atoms being in the excited state, large number of photons are produced inside the cavity. This produces the fluorescence as the photons are emitted due to the atomic transition. As we compare it with the previous case, here we observe enhancement in the fluorescence of emitted light in the presence of periodic modulation of cavity frequency. This would help in improving the detection efficiency of the atomic clock.

4. Conclusion

In conclusion, we have studied how a gas of two-level cold atoms confined in a harmonically modulated microwave cavity can be used to measure a 'second' in an atomic fountain clock. We have investigated the system with and without the modulation of the cavity frequency. The system has also been examined in the presence of two-photon process. It is observed that the amplitude of atomic inversion is more in the presence of modulation. This means that more number of atoms are present in the excited state than the ground state in the presence of modulation. Therefore, more atoms will fluoresce by emitting the photons. This helps in measuring the 'second' and improving the detection efficiency of the system. In the two-photon process, the enhancement is observed in the intracavity photon number. The cavity frequency at which maximum fluorescence is produced is used to define the second. Therefore, the two-photon process can be used as a new handle in atomic fountain clocks to measure the second and to improve their detection efficiency.

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References

[1] M. H. Anderson *et al.*, Science **269**, 198 (1995).

[2] C. C. Bradley *et al.*, Phys. Rev. Lett. **75**, 1687 (1995).

[3] K. B. Davis *et al.*, Phys. Rev. Lett. **75**, 3969 (1995).

[4] F. Biraben, Eur. Phys. J.-Special Topics **172**, 109 (2009).

[5] J. Guena *et al.*, IEEE Trans. Ultrason. Ferroelectr. Freq. Control **59**, 391 (2012).

[6] I. Bloch, J. Dalibard and W. Zwerger, Rev. Mod. Phys. **80**, 885 (2008).

[7] S. Giorgini, L. P. Pitaevskii and S. Stringari, Rev. Mod. Phys. **80**, 1215 (2008).

[8] R. Wynands and S. Weyers, Metrologia **42**, 64 (2005).

[9] D. M. Meekhof *et al.*, IEEE Trans. Instrum. Meas. **50**, 507 (2001).

[10] S. Weyers *et al.*, Metrologia **38**, 343 (2001).

[11] L. Marmet *et al.*, Proc. IEEE International Frequency Control Symposium, p. 386 (2008).

[12] K. Szymaniec, W. Chalupezak and D. Henderson, Proc. IEEE International Frequency Control Symposium and PDA Exhibition, p. 112 (2003).

[13] M. Kumagai *et al.*, Proc. IEEE International Frequency Control Symposium and PDA Exhibition, p. 120 (2003).

[14] M. Kumagai *et al.*, Proc. European Frequency and Time Forum 07 (EETF07), Geneva, Switzerland, p. 602 (2007).

[15] Y. T. Kwon *et al.*, IEEE Trans. Instrum. Meas. **52**, 263 (2003).

[16] F. Levi *et al.*, Proc. IEEE International Frequency Control Symposium and PDA Exhibition, p. 199 (2003).

[17] A. Clairon *et al.*, IEEE Trans. Instrum. Meas. **IM44**, 128 (1995).

[18] A. Sen Gupta *et al.*, CPEM 2010 Technical Digest (2010).

[19] K. Gibble and S. Chu, Metrologia **29**, 201 (1992).

[20] S. N. Lea *et al.*, Physica Scripta **T51**, 78 (1994).

[21] Ashish Agarwal and A. Sen Gupta, MAPAN **27**, 169 (2012).

[22] B. S. Dewitt, Phys. Rep. **19**, 295 (1975).

[23] S. A. Fulling and P. C. W. Davies, Proc. R. Soc. London, Ser. A **348**, 393 (1976).

[24] E. Yablonovitch, Phys. Rev. Lett. **62**, 1742 (1989).

[25] V. V. Hizhnyakov, Quantum Opt. **4**, 277 (1992).

[26] J. Schwinger, Proc. Natl Acad. Sci. USA **89**, 4091 (1992).

[27] C. K. Law, Phys. Re. A **49**, 433 (1994).

[28] S. Sarkar, Quantum Opt. **4**, 345 (1992).

[29] A. V. Dodonov *et al.*, J. Phys. B: At. Mol. Opt. Phys. **44**, 225502 (2011).

[30] R. Wynands and S. Weyers, Metrologia **42**, S64 (2005).

[31] F. Levi *et al.*, IEEE Trans. Ultrason. Ferroelectrics Freq. Control **51**, 1216 (2004).

[32] K. Szymaniec *et al.*, Metrologia **42**, 49 (2005).

[33] S. R. Jefferts *et al.*, Metrologia **39**, 321 (2002).

[34] S. Bize *et al.*, C. R. Physique **5**, 829 (2004).

[35] E. T. Jaynes and F. W. Cummings, Proc. IEEE **51**, 89 (1963).

[36] A. V. Dodonov, V. V. Dodonov, Phys. Rev. A **85**, 015805 (2012).

[37] A. V. Dodonov, V. V. Dodonov, Phys. Rev. A **85**, 055805 (2012).