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Research Article

Effect of Cavity Losses on Atomic-squeezed States Produced via Time-varying Cavity Frequency

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Abstract. We investigate the effect of cavity losses on squeezed-spin states generated via two-photon character of the field for a Bose-Einstein Condensate embedded within the Fabry-Perot optical cavity with a moving mirror for the initial vacuum cavity field. We further show how cavity decay acts as an additional factor in controlling the spin-squeezing.

Keywords. Bose-Einstein condensates, Optomechanical cavity, Squeezed states

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1. Introduction

Spin squeezing [27, 53, 54] grabbed much attention both theoretically and experimentally for over a decade. The spin-squeezed states are important quantum resources in improving the precision of measurements in experiments [8, 43, 53, 54] and in studying the particle correlations and entanglement [3, 15, 49]. Also, there has been a great surge of interest in the phenomenon of spin squeezing in collective spin system not only because of fundamental physical interests [16, 27, 32, 35, 38, 45, 48, 50, 53, 54], but also for its application in atomic clocks for reducing quantum noise [35, 38, 50, 53, 54] and quantum information [29, 30, 49, 52, 55]. The definition of atomic-squeezing is not unique and the most widely studied spin-squeezing parameters were proposed by Kitagawa and Ueda in [27] and by Wineland *et al*. in [53, 54]. Squeezed states of electromagnetic field have also grabbed considerable attention [4, 26, 28]. The generation of squeezed states of the cavity field via two-photon process has also been discussed [11, 20, 46]. Moore was the first one to discuss the quantization of electromagnetic field in an optical cavity with perfectly reflecting movable cavityboundaries [39]. Dodonov and co-workers [9, 10] further generalized histheory by including the effects of a time-varying refractive index of the medium inside the optical cavity. The major interest in this kind of system is the creation of photons [6] from the vacuum state via two-photon character of the field. The spin squeezing in atomic ensembles can be producedusing light-atom interactions. It involves the transferring of squeezing from light to atoms [2, 16, 23, 32, 51, 53]. The production of atomic-squeezed states in a two-component Bose-Einstein Condensates (BECs) vianonlinear interaction between them has been investigated [42, 44, 49]. It has been theoretically proposed thatthe atomic-squeezed states in BEC can be used in the detection of weak forces [19] and in performing sub-shot-noise measurements [13, 34]. Experimental realizations of spin-squeezed states in BECs [5, 7, 14, 17, 21, 22, 31, 36, 42] were also reported.

Motivated by these interesting developments in this field, we propose a non-stationary cavity quantum electrodynamical (QED) system composed of an elongated cigar-shaped gas of two-level BEC atoms interacting with asingle mode of an optical cavity, with a moving mirror, whose frequency is rapidly modulated in time. The generation of correlated-particle states or atomic-squeezed states for initial vacuum cavity field has already been investigated [1]. Here, we mainly discuss how the cavity dissipation into the system plays a vital role in controlling the spin-squeezingfor initial vacuum cavity field.

2. The System Model

In this section, we introduce the basic model and Hamiltonian for our system. The system involves a Fabry-Perot optical cavity with one mirror fixed and another mirror movable, with an additional elongated cigar-shaped gas of N BEC atoms of ^{87}Rb having two different hyperfine levels $|F=2, m_f=-1\rangle$ and $|F=2, m_f=1\rangle$ with transition frequency ω_a and mass M [37]. Each atomic mode is associated with an annihilation operator c_j $(j=1,2)$ in the two-mode approximation. The cloud of BEC is strongly coupled to a single quantized cavity mode of the optical cavity. The single-mode quantized optical cavity field has sinusoidally time-modulated frequency $\omega_c(t)=\omega_c(1+\varepsilon\sin(\Omega t))$ with unperturbed frequency ω_c. Here, ε is the modulation amplitude and Ω represents the modulation frequency. The harmonic motion of the movable mirror is responsible for such a form of time-dependent cavity frequency. The simplest modelof such system is provided by the following Hamiltonian [12]:

$$H=\hbar\omega_c(1+\varepsilon\sin(\Omega t))a^\dagger a+\hbar\omega_a J_z+\hbar\frac{g_0}{\sqrt{N}}(a+a^\dagger)J_x+i\hbar\xi(t)(a^{\dagger 2}-a^2)-i\hbar\frac{\kappa}{2}a^\dagger a-i\hbar\frac{\gamma}{2}J_+J_-. \tag{2.1}$$

Here, the ensemble of N atoms is described using the picture of a collective spin operators as: $J_x=(c_1^\dagger c_2+c_2^\dagger c_1)/2$, $J_y=(c_1^\dagger c_2-c_2^\dagger c_1)/2i$ and $J_z=(c_1^\dagger c_1-c_2^\dagger c_2)/2$. The spin operators satisfy the commutation relations $[J_+,J_-]=2J_z$ and $[J_\pm,J_z]=\mp J_\pm$. The optical cavityphoton annihilation and creation operators are denoted by a and $a^\dagger$respectively satisfying the commutationrelation $[a,a^\dagger]=1$. The parameter g_0 is the atom-field coupling. Moreover, the two-photon character of the field is responsible for the fourth term in the Hamiltonian that arises due to the time-varying cavity frequency and is responsiblefor generating the squeezed states of the cavity field [47]. $\xi(t)$ is the effective frequencywhich is related to $\omega_c(t)$ as [33]:

$$\xi(t) = \frac{1}{4\omega_c(t)} \frac{d\omega_c(t)}{dt}.$$

Considering the realistic case of a small-amplitude time modulation $|\varepsilon| \ll 1$, we shall use the approximation $\xi(t) \approx (\varepsilon\Omega/4)\cos(\Omega t) \approx 2\xi_0 \cos(\Omega t)$, with $\xi_0 = (\varepsilon\Omega/8) \ll 1$. The cavity and atomic decay rates are denoted by κ and Υ, respectively. The above Hamiltonian in the small atom-field coupling regime can be rewritten as [37] (we are now considering $\hbar = 1$ for simplicity):

$$H = \omega_a J_z + \omega_c(t) a^\dagger a + \frac{g_0}{2\sqrt{N}}(aJ_+ + a^\dagger J_-) + \phi\left(a^\dagger a + \frac{1}{2}\right) J_z + i\xi(t)(a^{\dagger 2} - a^2) - i\frac{\kappa}{2} a^\dagger a - i\frac{\gamma}{2} J_+ J_-, \tag{2.2}$$

where $\phi = \frac{g_0^2}{4N(\omega_c + 2\omega_a)}$. In the next section, we study the effect of decay of the cavity mode on the atomic-squeezed states when an ensemble of BEC atoms interacts with the single-mode quantized cavity field whose frequency is rapidly modulated in time.

3. Effect of Cavity Dissipation on Atomic-Squeezed States

In this section, we investigate the effect of cavity dissipation on squeezed-spin statesby numerically solving the Schrodinger's equation for the Hamiltonian given by eqn. (2.2). Using the definition of Kitagawa and Ueda, squeezing parameter can be defined as [27]:

$$\zeta_S = \sqrt{\frac{\min(\Delta J_{\vec{n}_\perp})^2}{J/2}} = \sqrt{\frac{4\min(\Delta J_{\vec{n}_\perp})^2}{N}}. \tag{3.1}$$

Here, $\vec{n}_\perp$ denotes the axis perpendicular to the mean-spin direction (MSD)$\vec{n}_0 = \langle \vec{J} \rangle / |\langle \vec{J} \rangle|$ with the minimization over all directions $\vec{n}_\perp$. The atomic-squeezing condition in terms of this parameter is given as $\zeta_S < 1$ where the fluctuation in one direction is reduced. The total wave function for the complete system can be written as:

$$\Psi_s(t) = \sum_{n,m} C_{n,m}(t) |n\rangle\, |J, m\rangle, \tag{3.2}$$

where $|n\rangle$ represent the cavity field eigen states such that $a|n\rangle = \sqrt{n}|n-1\rangle$, $a^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle$. We also assume that the cavity field and BEC atoms are uncorrelated for the initial wave function such that the wavefunction canbe written as a direct product:

$$\Psi_s(0) = \left(\sum_n C_n(0)|n\rangle\right) \psi(0),$$

where $\psi(0) = |J = N/2, M = -N/2\rangle$ and $C_n(0)$ are the initial harmonic oscillator wave function coefficients [18]. The equation of motion for the Hamiltonian can be evaluated using Schrodinger's equation and is given as:

$$\begin{aligned} i\dot{C}_{n,m}(t) = {} & \left[\omega_c(1+\varepsilon\sin(\Omega t)) + \omega_a m + \phi m(n+1/2) - i\kappa n/2 - i\gamma(J(J+1) - m(m-1))/2\right] C_{n,m}(t) \\ & + \frac{g_0}{2\sqrt{N}}[\sqrt{n}\sqrt{J(J+1) - m(m+1)}]C_{n-1,m+1}(t) \\ & + \frac{g_0}{2\sqrt{N}}[\sqrt{n+1}\sqrt{J(J+1) - m(m-1)}]C_{n+1,m-1}(t) \end{aligned}$$

$$+2i\xi_0\cos(\Omega t)[\sqrt{n+1}\sqrt{n+2}]C_{n+2,m}(t)-2i\xi_0\cos(\Omega t)[\sqrt{n}\sqrt{n-1}]C_{n-2,m}(t). \quad (3.3)$$

The time-dependent wave function for the complete system can be written as a sum over all the possible eigen states. In order toobtain the time evolution of the wave function, the time dependent wave-function coefficients are evaluated using MATHEMATICA 9.0. Using this wave function, the time-evolution of spin-squeezing parameter can be evaluated, which has already been investigated [1]. This production of squeezed-spin state can be realized practically with the help of schemesproposed in [24, 25].

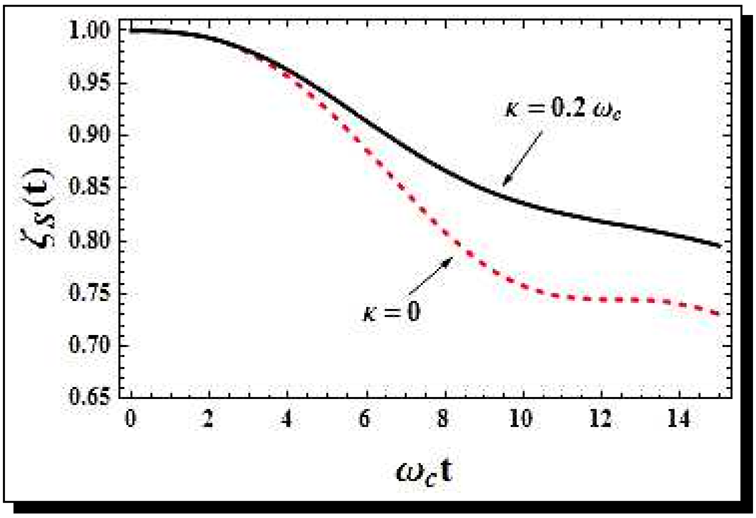

Figure 1. Plot of $\zeta_S(t)$ as a function of time for $\varepsilon = 0.12$ in the absence of cavity decay (dashed line) and in the presence of cavity decay (solid line) with $\kappa = 0.2\omega_c$. The other parameters used are $g_0 = 0.6\omega_c$, $\Omega = 2$, $\omega_a = \omega_c$ and $\gamma = 10^{-4}$. We assume $\psi(0) = |J, -J\rangle$ $(J = 1)$ and the optical field is initially prepared in the vacuum state

Figure 1 shows the time evolution of $\zeta_S(t)$ in the absence (dashed line) and presence (solid line) of cavity damping for modulation amplitude with $\varepsilon = 0.12$ for $\psi(0) = |J, -J\rangle$ and the harmonic oscillator initially in the vacuum state. It shows that the decay of cavity mode in any realistic quantum cavity system deteriorates the squeezing of the harmonic oscillator and consequently the atomic spin-squeezing. However, substantial amount of atomic-squeezing can still be achieved by using a high-finesse optical cavity. Therefore, we can say that cavity losses act as an additional factor in controlling the squeezing of spins.For example, the threshold condition for the production of squeezed photons can be possibly achieved by using a semiconductor plasma mirror [40] having quality factor of 10^3 [24]. Hence, we can expect that a significant amount of spin-squeezing can be achieved for a beam of condensate atoms interacting with the cavity field mode. Such atomic-squeezed states have applications in entanglement detection which plays an important role in both the foundations of quantum physics and quantum-information processing [41].

4. Conclusion

In conclusion, we have observed that the cavity dissipation into the system deteriorates the atomic-squeezing produced via periodic time modulation of cavity frequency for a BEC confined in an optomechanical cavity for the initial cavity field in the vacuum state but an appreciable amount of spin-squeezing can still be obtained by using a large enough cavity finesse. Hence, cavity decay mode acts as an additional factor in controlling the squeezing of spins.

Competing Interests

The author declares that she has no competing interests.

Authors' Contributions

The author wrote, read and approved the final manuscript.

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