Journal of Atomic, Molecular, Condensate & Nano Physics
Vol. 6, No. 2, pp. 69–79, 2019
ISSN 2349-2716 (online); 2349-6088 (print)
Published by RGN Publications

http://www.rgnpublications.com

DOI: 10.26713/jamcnp.v6i2.1268

Screening Constant by Unit Nuclear Charge Photoionization of Rb^{2+} Ions

Research Article

I. Sakho

Department of Experiential Sciences, UFR Sciences and Technologies, University of Thiès, Thiès, Senegal
aminafatima_sakho@yahoo.fr

Abstract. Photoionization data of the trans-Fe element $Rb^{2+}$are reported. Rydberg series $4s^24p^4(^1D_2)nd$ and $4s^24p^4(^3P_1)nd$ Rydberg series of $Rb^{2+}$from the $^2P°_{3/2}$ ground state and the $^2P°_{1/2}$ metastable state of Rb^{2+} converging respectively to the $4s^24p^4(^1D_2)$ $4s^24p^4(^3P_1)$ series limit in Rb^{3+} are considered. Calculations are performed in the framework of the Screening constant by unit nuclear charge (SCUNC) method. Accurate data are tabulated up to $n = 40$. It is shown that the SCUNC analytical formulas reproduce with an excellent precision, recent ALS measurements of Macaluso *et al.*, [*J. Phys. B: At. Mol. Opt. Phys.* **49** (2016), 235002; **50** (2017), 119501)]. The energy deviations with respect to the ALS data are equal to 0.001 eV. New data are tabulated for $n = 21-40$.

Keywords. Photoionization; Rydberg series; Ground state; Metastable state; SCUNC

PACS. 31.15.bu, 32.80.-t, 32.80.Ee, 32.80.Fb

Received: July 2, 2019 **Accepted:** August 1, 2019

1. Introduction

Photoionization studies of Rb ions (Sneden *et al.* [3]; Kilbane *et al.* [1]; Mueller *et al.* [2])remain an active field of investigations due to their importance for modeling astrophysical objects such those in the asymptotic giant branch (AGB) region. It is widely believed that, a major source of discrepancy is the quality of the atomic data used in the modelling (Mishenina *et al.* [4]; Roederer *et al.* [5, 6]; Frebel *et al.* [7]). Rb is one of the neutron-capture elements (Se, Cd, Ga, Ge, Rb, Kr, Br, Xe, Ba, Pb, etc.) (Pequignot and Baluteau [8]; Sharpee [9]; Sterling and Dinerstein [10]; Otsuka and Tajitsu [11]; García-Rojas *et al.* [12]; Sterling *et al.* [13]) produced by slow (s-process) or rapid (r-process) neutron-capture nucleosynthesis in ionized nebulae. Photoionization study of Rb^{2+} ions is especially crucial because it permits to provide benchmark data in connection with astrophysical applications. In addition such study permits to aid in the formulation of so-called "ionization correction factors" used in the modeling of planetary nebular emission lines of ions of Rb (Langanke and Wiescher [14]; Kwitter *et al.* [15]; Luridiana *et al.* [18, 19]). Recently, Macaluso *et al.* [21] performed high-resolution photoionization cross section measurements of the for Rb^{2+} over the photon energy range 37.31-44.08 eV using synchrotron radiation and the photo-ion, merged-beams technique at the Advanced Light Source at Lawrence Berkeley National Laboratory with a bandpass resolution of 13.5 ± 2.5 meV full width half maximum (FWHM). In tandem with the measurements, Breit-Pauli R-matrix calculations were performed in the intermediate coupling jK to facilitate the identification of several highly excited Auger Rydberg resonance states of the Rb^{2+} ions. Very recently, McLaughlin and Babb [17] used a fully relativistic approach within the Dirac-Coulomb R-matrix (DARC) approximation to calculate the cross sections for ground and metastable states. Although very good agreement are found between the DARC [17] and ALS measurements [19], calculations can be improved as maximum energy differences between theory and experiment at 0.008 eV are observed for the $4s^2 4p^4(^1D_2)nd$ and $4s^2 4p^4(^3P_1)nd$ Rydberg series. The motivation of this work is to use the Screening constant by unit nuclear charge (SCUNC) formalism (Sakho [20–22]; Ba *et al.* [23]; Badiane *et al.* [24]) to report precise high lying Photoionization data of the Rb^{2+} ions reducing energy deviations with respect to the ALS data at a maximum of 0.001 eV.

The layout of this work is as follows. Section 2 presents a brief outline of the theoretical part of the work. Section 3 presents a discussion of the results obtained compared with the available literature data. Finally, in Section 4 we summarize and conclude the present study.

2. Theory

In the framework of the Screening Constant by Unit Nuclear Charge formalism, the total energy of the $(Nl, nl')^{2S+1}L^{\pi}$ excited states is expressed in the form (in Rydberg)

$$E = -Z^2\left(\frac{1}{N^2} + \frac{1}{n^2}\left[1 - \beta\left(Nl, nl'; {}^{2S+1}L^{\pi}; Z\right)\right]^2\right). \tag{2.1}$$

In this equation, the principal quantum numbers N and n are respectively for the inner and the outer electron of the helium-isoelectronic series. The β-parameters are screening constants by unit nuclear charge expanded in inverse powers of Z and given by

$$\beta\left(Nl\,nl'; {}^{2S+1}L^{\pi}; Z\right) = \sum_{k=1}^{q} f_k \left(\frac{1}{Z}\right)^k \tag{2.2}$$

where $f_k = f_k\left(Nl\,nl'; {}^{2S+1}L^{\pi}\right)$ are parameters to be evaluated empirically.

For a given Rydberg series originating from a ${}^{2S+1}L_J$ state, we obtain

$$E_n = E_{\infty} - \frac{Z^2}{n^2}\left[1 - \beta(nl; s, \mu, \nu, {}^{2S+1}L^{\pi}.Z)\right]^2. \tag{2.3}$$

In this equation, ν and μ ($\mu > \nu$) denote the principal quantum numbers of the $(^{2S+1}L_J)\ nl$ Rydberg series used in the empirical determination of the f_k-screening constants, s represents the spin of the nl-electron (s = 1/2), E_∞ is the energy value of the series limit, E_n denotes the resonance energy and Z stands for the atomic number. The β-parameters are screening constants by unit nuclear charge expanded in inverse powers of Z and given by

$$\beta(Z, {}^{2S+1}L_J, n, s, \mu, \nu) = \sum_{k=1}^{q} f_k \left(\frac{1}{Z}\right)^k \tag{2.4}$$

where $f_k = f_k(^{2S+1}L_J, n, s, \mu, \nu)$ are screening constants to be evaluated empirically. In eq. (2.2), q stands for the number of terms in the expansion of the β-parameter. The resonance energy are the in the form

$$E_n = E_\infty - \frac{Z^2}{n^2}\left\{1 - \frac{f_1(^{2S+1}L_J^\pi)}{Z(n-1)} - \frac{f_2(^{2S+1}L_J^\pi)}{Z} \pm \sum_{k=1}^{q}\sum_{k'=1}^{q'} f_1^{k'} F(n,\mu,\nu,s) \times \left(\frac{1}{Z}\right)^k\right\}^2. \tag{2.5}$$

In this equation, $\pm \sum_{k=1}^{q}\sum_{k'=1}^{q'} f_1^{k'} F(n,\mu,\nu,s) \times \left(\frac{1}{Z}\right)^k$ is a corrective term introduce to stabilize the resonance energies with increasing the principal quantum number n. In general, resonance energies are analyzed from the standard quantum-defect expansion formula

$$E_n = E_\infty - \frac{RZ_{\text{core}}^2}{(n-\delta)^2}. \tag{2.6}$$

In this equation, R is the Rydberg constant, E_∞ denotes the converging limit, Z_{core} represents the electric charge of the core ion, and δ means the quantum defect. In addition, theoretical and measured energy positions can be analyzed by calculating the Z^*-effective charge in the framework of the SCUNC-procedure

$$E_n = E_\infty - \frac{Z^{*2}}{n^2}R. \tag{2.7}$$

The relationship between Z^* and δ is in the form

$$Z^* = \frac{Z_{\text{core}}}{\left(1 - \frac{\delta}{n}\right)}. \tag{2.8}$$

According to this equation, each Rydberg series must satisfy the following conditions

$$\begin{cases} Z^* \geq Z_{\text{core}} & \text{if } \delta \geq 0 \\ Z^* \leq Z_{\text{core}} & \text{if } \delta \leq 0 \\ \lim\limits_{n\to\infty} Z^* = Z_{\text{core}} & \end{cases} \tag{2.9}$$

Besides, comparing eq. (2.5) and eq. (2.7), the effective charge is in the form

$$Z* = Z\left\{1 - \frac{f_1(^{2S+1}L^\pi)}{Z(n-1)} - \frac{f_2(^{2S+1}L^\pi)}{Z} \pm \sum_{k=1}^{q}\sum_{k'=1}^{q'} f_1^{k'} F(n,\mu,\nu,s) \times \left(\frac{1}{Z}\right)^k\right\}. \tag{2.10}$$

Besides, the f_2-parameter in eq. (2.2) can be theoretically determined from eq. (2.10) by neglecting the corrective term with the condition

$$\lim_{n\to\infty} Z^* = Z\left(1 - \frac{f_2(^{2S+1}L^\pi)}{Z}\right) = Z_{\text{core}}. \tag{2.11}$$

We get then $f_2 = Z - Z_{\text{core}}$, where Z_{core} is directly obtain by the photoionization process from an atomic X^{p+} system $X^{p+} + h\nu \to X^{(p+1)+} + e^-$. We find then $Z_{\text{core}} = p+1$. Thus, for the Rb^{2+} ions, $Z_{\text{core}} = 3$ and $f_2 = (37-3) = 34.0$. The remaining f_1-parameter is to be evaluated empirically using the ALS data of Macaluso *et al*. [18, 19] for a given $(^{2S+1}L_J)\ \mu$l level with $\nu = 0$. The empirical procedure of the determination of the f_1-screening constant along with the corresponding uncertainty have been explained in details in our previous works (Sakho [20–22]; Ba *et al*. [23]; Badiane *et al*. [24]). In the present work, all the energy resonances are calculated using the following simple expression (in Rydberg units)

• for the $4s^24p^4(^3P_1)nd$ Rydberg series of Rb^{2+} converging to the $Rb^{3+}(3d^{10}4s^24p^{4\,3}P_1)$ threshold originating from the Rb^{2+} ground $4s^24p^{5\,2}P^\circ{}_{3/2}$ state and metastable $4s^24p^{5\,2}P^\circ{}_{1/2}$ state, we get

$$E_n = E_\infty - \frac{Z^2}{n^2}\left\{1 - \frac{f_1(^3P_1)}{Z(n-1)} - \frac{f_2(^3P_1)}{Z} - \frac{f_1(^3P_1)\times(n-\mu)}{Z^2(n-s-1)(n+\mu-s)} - \frac{f_1(^3P_1)\times(n-\mu)^2}{Z^3(n-s-1)(n+\mu-s)}\right\}^2. \quad (2.12)$$

• for the $4s^24p^4(^1D_2)nd$ Rydberg series of Rb^{2+} converging to the $Rb^{3+}(3d^{10}4s^24p^{4\,1}D_2)$ threshold originating from the Rb^{2+} ground $4s^24p^{5\,2}P^\circ{}_{3/2}$ state and from the Rb^{2+}metastable $4s^24p^{5\,2}P^\circ{}_{1/2}$ state, we obtain

$$E_n = E_\infty - \frac{Z^2}{n^2}\left\{1 - \frac{f_1(^1D_2)}{Z(n-1)} - \frac{f_2(^1D_2)}{Z} - \frac{f_1(^1D_2)\times(n-\mu)}{Z^2(n-s-2)(n-\mu+s)} - \frac{f_1(^1D_2)\times(n-\mu)^2}{Z^3(n-s-2)(n-\mu+s)}\right\}^2. \quad (2.13)$$

3. Results and Discussion

The present SCUNC calculations are listed in Tables 1-5. Comparisons are done with the Advanced Light Source (ALS) measurements of Macaluso *et al*. [18, 19] and with the fully relativistic approach within the Dirac-Coulomb R-matrix (DARC) calculations of McLaughlin and Babb [17]. Analysis of the values of the nuclear effective charge Z^* listed in the first entry of each table indicate that $Z^*_{max} > Z_{\text{core}} = 3.0$. This means that the quantum defect is positive according to the SCUNC's conditions analysis (2.9) in agreement with the sign of the theoretical and experimental quantum defects quoted in Tables 1-5. Besides, comparison of resonance energies indicate excellent agreements between theory and experiment. It should be mentioned that, the excellent SCUNC calculations with a maximum of energy deviation with respect to the ALS measurements at 0.001 eV. This allows one to expect the high lying data up to $n = 40$ to be useful benchmark data for astrophysical applications. In the work of McLaughlin and Babb [17], fully relativistic approach within the Dirac-Coulomb R-matrix calculations were performed in the intermediate coupling jK Breit-Pauli approximation from the DARC 687 level calculations. In the present work, very precise photoionization data are obtained within the very simple formalism of the SUCNC method. The possibility to provide accurate photoionization data using the SCUNC constant is due to the validity of the formalism to treat correctly photoionization properties of multi-charged atomic systems as demonstrated in our previous works (Sakho [20–22]; Ba *et al*. [23]; Badiane *et al*. [24]).

Table 1. Energy resonances (E_n, eV), quantum defect (δ) and effective nuclear charge Z^* of the $4s^24p^4(^3P_1)nd$ Rydberg series of Rb^{2+} converging to the $Rb^{3+}(3d^{10}4s^24p^{4\,3}P_1)$ threshold originating from the Rb^{2+}metastable $4s^24p^{5\,2}P^\circ{}_{1/2}$ state. $f_1(^3P_1)$= .797 ± 0.071; μ = 13. The present SCUNC calculations are compared to the DARC calculations (McLaughlin and Babb [17]) and the ALS measurements (Macaluso *et al*. [18, 19]). $|\Delta E|$ denotes the energy difference between the SCUNC calculations and the ALS measurements

	SCUNC	DARC	ALS		SCUNC	DARC	ALS	SCUNC
n		E_n		$\|\Delta E\|$		δ		Z^*
13	38.352	38.352	38.352	0.000	0.28	0.31	0.28	3.066
14	38.458	38.449	38.459	0.001	0.28	0.31	0.28	3.061
15	38.544	38.543	38.544	0.000	0.28	0.30	0.28	3.057
16	38.614	38.612	38.614	0.000	0.28	0.30	0.28	3.053
17	38.671	38.670	38.671	0.000	0.28	0.30	0.28	3.050
18	38.719	38.718	38.719	0.000	0.28	0.31	0.28	3.047
19	38.760	38.759	38.760	0.000	0.28	0.30	0.28	3.045
20	38.794	38.794	38.794	0.000	0.28	0.28	0.28	3.042
21	38.824				0.28			3.040
22	38.850				0.28			3.038
23	38.872				0.28			3.037
24	38.891				0.28			3.035
25	38.909				0.28			3.034
26	38.924				0.28			3.032
27	38.938				0.28			3.031
28	38.950				0.28			3.030
29	38.961				0.28			3.029
30	38.970				0.28			3.028
31	38.979				0.28			3.027
32	38.987				0.28			3.026
33	38.995				0.28			3.025
34	39.001				0.28			3.024
35	39.007				0.28			3.024
36	39.013				0.28			3.023
37	39.018				0.28			3.022
38	39.023				0.28			3.022
39	39.027				0.28			3.021
40	39.031				0.28			3.021
⋮	⋮				⋮			⋮
∞	39.109	39.109	39.109		...		...	3.000

Table 2. Energy resonances (E_n, eV), quantum defect (δ) and effective nuclear charge Z^* of the $4s^24p^4(^3P_1)nd$ Rydberg series of Rb^{2+} converging to the $Rb^{3+}(3d^{10}4s^24p^{4}\,{}^3P_1)$ threshold originating from the Rb^{2+} ground $4s^24p^{5}\,{}^2P°_{3/2}$ state. $f_1(^3P_1) = -0.748 \pm 0.071$; $\mu = 13$. The present SCUNC calculations are compared to the DARC calculations (McLaughlin and Babb [17]) and the ALS measurements (Macaluso *et al*. [18, 19]). $|\Delta E|$ denotes the energy difference between the SCUNC calculations and the ALS measurements

	SCUNC	DARC	ALS		SCUNC	DARC	ALS	SCUNC
n		E_n		$\|\Delta E\|$		δ		Z^*
13	39.268	39.263	39.268	0.000	0.26	0.27	0.31	3.062
14	39.374	39.379	39.374	0.000	0.26	0.27	0.32	3.058
15	39.459	39.457	39.459	0.000	0.26	0.27	0.32	3.054
16	39.529	39.526	39.528	0.001	0.26	0.27	0.30	3.050
17	39.586	39.584	39.586	0.000	0.26	0.27	0.30	3.047
18	39.634	39.632	39.634	0.000	0.26	0.27	0.32	3.044
19	39.674	39.673	39.674	0.000	0.26	0.27	0.30	3.042
20	39.709	39.707	39.709	0.000	0.26	0.27	0.30	3.040
21	39.738				0.26			3.038
22	39.764				0.26			3.036
23	39.786				0.26			3.034
24	39.806				0.26			3.033
25	39.823				0.26			3.032
26	39.838				0.26			3.030
27	39.852				0.26			3.029
28	39.864				0.26			3.028
29	39.875				0.26			3.027
30	39.885				0.26			3.026
31	39.893				0.26			3.025
32	39.901				0.26			3.025
33	39.909				0.26			3.024
34	39.915				0.26			3.023
35	39.922				0.26			3.022
36	39.927				0.26			3.022
37	39.932				0.26			3.021
38	39.937				0.26			3.021
39	39.941				0.26			3.020
40	39.945				0.26			3.020
⋮	⋮				⋮			⋮
∞	40.023	40.023	40.023		...		...	3.000

Table 3. Energy resonances (E_n, eV), quantum defect (δ) and effective nuclear charge Z^* of the $4s^2 4p^4(^1D_2)nd$ and Rydberg series of Rb^{2+} converging to the $Rb^{3+}(3d^{10}4s^2 4p^4\,^1D_2)$ threshold originating from the Rb^{2+}metastable $4s^2 4p^5\,^2P^\circ{}_{1/2}$state. $f_1(^1D_2) = -0.679 \pm 0.071$; $\mu = 8$. The present SCUNC calculations are compared to the DARC calculations (McLaughlin and Babb [17]) and the ALS measurements (Macaluso *et al*. [18, 19]). $|\Delta E|$ denotes the energy difference between the SCUNC calculations and the ALS measurements

	SCUNC	DARC	ALS		SCUNC	DARC	ALS	SCUNC
n		E_n		$\|\Delta E\|$		δ		Z^*
8	38.446	38.440	38.446	0.000	0.25	0.26		3.097
9	38.885	38.884	38.885	0.000	0.25	0.25		3.087
10	39.196	39.192	39.196	0.000	0.25	0.26		3.078
11	39.425	39.424	39.425	0.000	0.25	0.26		3.070
12	39.598	39.597	39.598	0.000	0.25	0.26		3.064
13	39.732	39.731	39.731	0.001	0.25	0.26		3.058
14	39.838	39.837	39.837	0.001	0.25	0.26		3.054
15	39.922	39.922	39.922	0.000	0.25	0.26		3.050
16	39.992	39.991	39.991	0.001	0.25	0.26		3.047
17	40.049	40.048	40.048	0.001	0.25	0.26		3.044
18	40.097	40.096	40.096	0.001	0.24	0.26		3.041
19	40.137				0.24			3.039
20	40.171				0.24			3.037
21	40.201				0.24			3.035
22	40.226				0.24			3.034
23	40.249				0.24			3.032
24	40.268				0.24			3.031
25	40.285				0.24			3.029
26	40.300				0.24			3.028
27	40.314				0.24			3.027
28	40.326				0.24			3.026
29	40.337				0.24			3.025
30	40.347				0.24			3.024
31	40.356				0.24			3.024
32	40.364				0.24			3.023
33	40.371				0.24			3.022
34	40.378				0.24			3.022
35	40.384				0.24			3.021
36	40.389				0.24			3.020
37	40.394				0.24			3.020
38	40.399				0.24			3.019
39	40.403				0.24			3.019
40	40.408				0.24			3.018
⋮	⋮				⋮			⋮
∞	40.485	40.485	40.485		…		…	3.000

Table 4. Energy resonances (E_n, eV), quantum defect (δ) and effective nuclear charge Z^* of the $4s^2 4p^4 (^1D_2) nd$ Rydberg series of Rb^{2+} converging to the $Rb^{3+}(3d^{10}4s^2 4p^4 {}^3P_1)$ threshold originating from the Rb^{2+} ground $4s^2 4p^5 {}^2P^\circ_{3/2}$ state. $f_1(^1D_2) = -0.748 \pm 0.071$; $\mu = 8$. The present SCUNC calculations are compared to the DARC calculations (McLaughlin and Babb [17]) and the ALS measurements (Macaluso *et al*. [18, 19]). $|\Delta E|$ denotes the energy difference between the SCUNC calculations and the ALS measurements

	SCUNC	DARC	ALS		SCUNC	DARC	ALS	SCUNC
n		E_n		$\lvert\Delta E\rvert$		δ		Z^*
8	39.347	39.355	39.347	0.000	0.28	0.28	0.26	3.107
9	39.789	39.797	39.790	0.001	0.28	0.28	0.26	3.096
10	40.104	40.109	40.104	0.000	0.28	0.28	0.26	3.085
11	40.334	40.339	40.334	0.000	0.28	0.28	0.26	3.077
12	40.508	40.511	40.508	0.000	0.27	0.28	0.26	3.070
13	40.643	40.645	40.643	0.001	0.27	0.28	0.26	3.064
14	40.749	40.751	40.749	0.000	0.27	0.28	0.26	3.059
15	40.835	40.836	40.834	0.001	0.27	0.28	0.26	3.055
16	40.904	40.905	40.904	0.000	0.27	0.28	0.26	3.052
17	40.962	40.962	40.961	0.001	0.27	0.28	0.26	3.048
18	41.009	41.010	41.009	0.000	0.27	0.28	0.26	3.046
19	41.050				0.27			3.043
20	41.084				0.27			3.041
21	41.114				0.27			3.039
22	41.140				0.27			3.037
23	41.162				0.27			3.035
24	41.182				0.27			3.034
25	41.199				0.27			3.032
26	41.214				0.27			3.031
27	41.228				0.27			3.030
28	41.240				0.27			3.029
29	41.251				0.27			3.028
30	41.260				0.27			3.027
31	41.269				0.27			3.026
32	41.277				0.27			3.025
33	41.285				0.27			3.024
34	41.291				0.27			3.024
35	41.297				0.27			3.023
36	41.303				0.27			3.022
37	41.308				0.27			3.022
38	41.313				0.27			3.021
39	41.317				0.27			3.021
40	41.321				0.27			3.020
⋮	⋮				⋮			⋮
∞	41.399	41.399	41.399		…		…	3.000

Table 5. Energy resonances (E_n, eV), quantum defect (δ) and effective nuclear charge Z^* of the $4s^24p^4(^1D_2)nd$ Rydberg series of Rb^{2+} converging to the $Rb^{3+}(3d^{10}4s^24p^{4\,3}P_1)$ threshold originating from the Rb^{2+} ground $4s^24p^{5\,2}P^\circ{}_{3/2}$ state. $f_1(^1D_2) = -0.748 \pm 0.071$; $\mu = 8$. The present SCUNC calculations are compared to the DARC calculations (McLaughlin and Babb [17]) and the ALS measurements (Macaluso *et al*. [18, 19]). $|\Delta E|$ denotes the energy difference between the SCUNC calculations and the ALS measurements

	SCUNC	DARC	ALS		SCUNC	DARC	ALS	SCUNC
n		E_n		$\|\Delta E\|$		δ		Z^*
8	39.347	39.355	39.347	0.000	0.28	0.28	0.26	3.107
9	39.789	39.797	39.790	0.001	0.28	0.28	0.26	3.096
10	40.104	40.109	40.104	0.000	0.28	0.28	0.26	3.085
11	40.334	40.339	40.334	0.000	0.28	0.28	0.26	3.077
12	40.508	40.511	40.508	0.000	0.27	0.28	0.26	3.070
13	40.643	40.645	40.643	0.001	0.27	0.28	0.26	3.064
14	40.749	40.751	40.749	0.000	0.27	0.28	0.26	3.059
15	40.835	40.836	40.834	0.001	0.27	0.28	0.26	3.055
16	40.904	40.905	40.904	0.000	0.27	0.28	0.26	3.052
17	40.962	40.962	40.961	0.001	0.27	0.28	0.26	3.048
18	41.009	41.010	41.009	0.000	0.27	0.28	0.26	3.046
19	41.050				0.27			3.043
20	41.084				0.27			3.041
21	41.114				0.27			3.039
22	41.140				0.27			3.037
23	41.162				0.27			3.035
24	41.182				0.27			3.034
25	41.199				0.27			3.032
26	41.214				0.27			3.031
27	41.228				0.27			3.030
28	41.240				0.27			3.029
29	41.251				0.27			3.028
30	41.260				0.27			3.027
31	41.269				0.27			3.026
32	41.277				0.27			3.025
33	41.285				0.27			3.024
34	41.291				0.27			3.024
35	41.297				0.27			3.023
36	41.303				0.27			3.022
37	41.308				0.27			3.022
38	41.313				0.27			3.021
39	41.317				0.27			3.021
40	41.321				0.27			3.020
⋮	⋮				⋮			⋮
∞	41.399	41.399	41.399		...		...	3.000

4. Conclusion

The screening constant by unit nuclear charge (SCUNC) is used to report accurate resonance energies belonging to the 4p→*n*d transitions from the $^2P°_{3/2}$ ground state and the $^2P°_{1/2}$ metastable state of Rb^{2+} converging to the $4s^24p^4$ ($^1D_2$) and $4s^24p^4$ ($^3P_1$) series limit in Rb^{3+}. It is seen that the SCUNC formula established reproduces with an excellent precision less than 0.002 eV high-resolution measurements of Macaluso *et al.*,(2017). New data $n = 21 - 40$ are tabulated as useful guidelines for the NIST data base and for future PI studies on Rb^{2+} focussed on high excited levels.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] D. Kilbane, F. Folkmann, J.-M. Bizau, C. Banahan, S. Scully, H. Kjeldsen, P. van Kampen, M. W. D. Mansfield, J. T. Costello and J. B. West, *Phys. Rev. A* **75** (2007), 032711, DOI: 10.1103/PhysRevA.75.032711.

[2] A. Mueller, D. Macaluso, N. Sterling, A. Juarez, I. Dumitriu, R. Bilodeau, E. Red, D. Hardy and A. Aguilar, *Bull. Am. Phys. Soc.* **58** (2013), Q1.00141, URL: `http://meetings.aps.org/Meeting/DAMOP13/Session/Q1.141`.

[3] C. Sneden, R. G. Gratton and D. A. Crocker, *Astron. & Astrophys.* **246** (1991), 354.

[4] T. V. Mishenina, V. V. Kovtyukh, C. Soubiran, C. Travaglio and M. Busso, *Astron. & Astrophys.* **396** (2002), 189, DOI: 10.1051/0004-6361:20021399.

[5] I. U. Roederer, C. Sneden, I. B. Thompson, G. W. Preston and S. A. Shectman, *Astrophys. J.* **711** (2010), 573, DOI: 10.1088/0004-637x/711/2/573.

[6] I. U. Roederer, A. F. Marino and C. Sneden. *Astrophys. J.* **742** (2011), 37, DOI: 10.1088/0004-637X/742/1/37.

[7] A. Frebel, J. D. Simon and E. N. Kirby, *Astrophys. J.* **786** (2014), 74, DOI: 10.1088/0004-637X/786/1/74.

[8] D. Pequignot and J. P. Baluteau, *Astron. & Astrophys.* **283** (2) (1994), 593–625.

[9] B. Sharpee, Y. Zhang, R. Williams, E. Pellegrini, K. Cavagnolo, J. A. Baldwin, M. Phillips and X.-W. Liu, *Astrophys. J*. **659** (2007) (preprint astro-ph/0612101), DOI: 10.1086/511665.

[10] N. C. Sterling and H. L. Dinerstein, *Astrophys. J. Suppl.* **174** (2008), DOI: 10.1086/520845.

[11] M. Otsuka and A. Tajitsu, *Astrophys. J.* **778** (2013) (preprint 1310.1151), DOI: 10.1088/0004-637X/778/2/146.

[12] J. García-Rojas, S. Madonna, V. Luridiana, N. C. Sterling, C. Morisset, G. Delgado-Inglada and L. Toribio San Cipriano, *Mon. Not. Roy. Astro. Soc.* **452** (2015) (preprint 1506.07079), DOI: 10.1093/mnras/stv1415.

[13] N. C. Sterling, H. L. Dinerstein, K. F. Kaplan and M. A. Bautista, *Astrophys. J.* **819** (2016), (preprint 1602.03188), DOI: 10.3847/2041-8205/819/1/L9.

[14] K. Langanke and M. Wiescher, *Rep. Prog. Phys.* **64** (2001) 1657, DOI: 10.1088/0034-4885/64/12/202.

[15] K. B. Kwitter, R. H. Méndez, M. Peña, L. Stanghellini, R. L. M. Corradi, O. De Marco, X. Fang, R. B. C. Henry, A. I. Karakas, X.-W. Liu, J. A. López, A. Manchado and Q. A. Parker, *Rev. Mex. Astron. Astro.* **50** (2014), 203 (preprint 1403.2246), `https://arxiv.org/abs/1403.2246v1`.

[16] V. Luridiana, C. Morisset and R. A. Shaw, *Astron. & Astrophys.* 573 (2015), A42, DOI: 10.1051/0004-6361/201323152.

[17] B. M. McLaughlin and J. F. Babb, *J. Phys. B: At. Mol. Opt. Phys*. **52** (2019), 125201, DOI: 10.1088/1361-6455/ab1e99.

[18] D. A. Macaluso, K. Bogolub, A. Johnson, A. Aguilar, A. L. D. Kilcoyne, R. C. Bilodeau, M. Bautista, A. B. Kerlin and N. C. Sterling, *J. Phys. B: At. Mol. Opt. Phys.* **49** (2016), 235002, DOI: 10.1088/0953-4075/49/23/235002.

[19] D. A. Macaluso, K. Bogolub, A. Johnson, A. Aguilar, A. L. D. Kilcoyne, R. C. Bilodeau, M. Bautista, A. B. Kerlin and N. C. Sterling, *J. Phys. B: At. Mol. Opt. Phys.* **50** (2017), 119501, DOI: 10.1088/1361-6455/aa6d1b.

[20] I. Sakho, *At. Data. Nuc. Data Tables* **117-118** (2017), 425, DOI: 10.1016/j.adt.2016.12.001.

[21] I. Sakho, *J. Electron Spectro & Related Phenomena* **222** (2018), 40, DOI: 10.1016/j.elspec.2017.10.001.

[22] I. Sakho, *The Screening Constant by Unit Nuclear Charge Method, Description & Application to the Photoionization of Atomic Systems*, ISTE Science Publishing Ltd., London, and John Wiley & Sons, Inc. USA (2018), ISBN: 978-1-119-47694-8.

[23] M. D. Ba, A. Diallo, J. K. Badiane, M. T. Gning, M. Sow and I. Sakho, *Rad. Phys. Chem.* **153** (2018), 111, DOI: 10.1016/j.radphyschem.2018.09.010.

[24] J. K. Badiane, A. Diallo, M. D. Ba, M. T. Gning, M. Sow and I. Sakho, *Rad. Phys. Chem.* **158** (2019), 17, DOI: 10.1016/j.radphyschem.2019.01.008.