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Fixed Point Theorems of *L*-Fuzzy Mappings Via a Rational Inequality

Research Article

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Abstract. In this paper some fixed point results for L-fuzzy mappings via a rational inequality are obtained. An example is also given which supports the proved result.

Keywords. Fixed point; Complete metric space; L-fuzzy mappings; Hausdorff metric space

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1. Introduction

In the existence of the solutions of various problems in the field of mathematics, medicines engineering and social sciences fixed point theory plays a fundamental role. After the first publication of Zadeh [13] about fuzzy sets a lot of work has been conducted on the generalizations of the concept of a fuzzy set. The idea about fuzzy mappings was investigated by Weiss [12] and Butnariu [2]. Later on Heilpern [4] proved a fixed point theorem for fuzzy mapping which was the generalization of Nadler's result [6]. Afterwards in 1967 Goguen [3] generalized the idea of fuzzy sets in form of another notion of L-fuzzy sets. The concept of fuzzy sets is a special case of L-fuzzy sets when $L = [0,1]$. Then, the several results were achieved by various authors for L-fuzzy mappings [7–9].

In this paper we obtained fixed point results for L-fuzzy mappings via a rational inequality. An example is also given which supports the proved result.

2. Preliminaries

Let (X,d) be a metric space and denote

$$P(X)=\{A : A \text{ is a subset of } X\}$$
$$C(X)=\{A\in 2^X : A \text{ is nonempty and compact}\}$$
$$CB(X)=\{A\in 2^X : A \text{ is nonempty closed and bounded}\}.$$

For $A,B\in CB(X)$

$$d(x,A)=\inf_{y\in A} d(x,y)$$
$$d(A,B)=\inf_{x\in A,\, y\in B} d(x,y)$$

Definition 2.1 ([9]). A partially ordered set $(L,\preceq_L)$ is called :

(i) a lattice, if $a\vee b\in L$, $a\wedge b\in L$, for any $a,b\in L$;

(ii) a complete lattice, if $\vee A\in L$, and $\wedge A\in L$, for any $A\subseteq L$;

(iii) distributive if $a\vee(b\wedge c)=(a\vee b)\wedge(a\vee c)$, $a\wedge(b\vee c)=(a\wedge b)\vee(a\wedge c)$, for any $a,b,c\in L$.

Definition 2.2 ([9]). Let L be a lattice with top element 1_L and bottom element 0_L for $a,b\in L$. Then, b is called a complement of a, if $a\vee b=1_L$, and $a\wedge b\in 0_L$. If $a\in L$, has a complement element then it is unique. It is denoted by a'.

Definition 2.3 ([9]). An L-fuzzy set A on a nonempty set X is a function $A : X\to L$, where L is complete distributive lattice with 1_L and 0_L.

Remark 2.4 ([9]). The class of L-fuzzy sets is larger than the class of fuzzy set. An L-fuzzy set is a fuzzy set if $L=[0,1]$, L^X is collection of all L-fuzzy sets in X. The α_L-level set of L-fuzzy set A is denoted and defined as

$$A_{\alpha_L}=\{x:\alpha_L\preceq_L A(x)\} \quad \text{if} \quad \alpha_L\in L\backslash\{0_L\}$$
$$A_{0_L}=cl(\{x:0_L\preceq_L A(x)\}).$$

Here, $cl(B)$ denotes the closure of the set B.

Definition 2.5 ([9]). Let X be an arbitrary set and Y be a metric space. A mapping T is called L-fuzzy mapping if T is a mapping from X into L^Y. An L-fuzzy mapping T is an L-fuzzy subset on $X\times Y$ with membership function $T(x)(y)$. The function $T(x)(y)$ is the grade of membership of y in $T(x)$.

Definition 2.6 ([7]). Let (X,d) be a metric space and A, B be any two nonempty subsets of X. Then the Hausdorff distance between the subsets A and B is defined as

$$H(A,B)=\max\left\{\sup_{a\in A} d(a,B), \sup_{b\in B} d(b,A)\right\}.$$

Definition 2.7 ([7]). Let (X,d) be a metric space and S,T be L-fuzzy mappings from X into L^X. A point $x \in X$ is called as an L-fuzzy fixed point of T if $x \in [Tx]_{\alpha_{L_T}(x)}$ where $\alpha_{L_T}(x) \in L \backslash \{0_L\}$. The point x is called as common L-fuzzy fixed point of S and T if $x \in [Sx]_{\alpha_{L_S}(x)} \cap [Tx]_{\alpha_{L_T}(x)}$.

Lemma 2.8 ([7]). *Let A and B be nonempty closed and bounded subsets of a metric space (X,d). If $a \in A$, then*

$$d(a,B) \le H(A,B).$$

Lemma 2.9 ([7]). *Let A and B be nonempty closed and bounded subsets of a metric space (X,d) and $0 < \varepsilon \in \mathbb{R}$. Then, for $a \in A$, there exists $b \in B$ such that*

$$d(a,b) \le H(A,B) + \varepsilon.$$

3. Main Results

Theorem 3.1. *Let $S,T \to L^X$ be two L-fuzzy mappings and for $x \in X$, there exists $\alpha_{L_S}(x), \alpha_{L_T}(x) \in L \backslash \{0_L\}$ such that $[Sx]_{\alpha_{L_S}(x)}, [Tx]_{\alpha_{L_T}(x)} \in CB(2^X)$. If for all $x,y \in X$*

$$H([Sx]_{\alpha_{L_S}(x)},[Ty]_{\alpha_{L_T}(y)}) \le \alpha d(x,y) + \beta d(x,[Sx]_{\alpha_{L_S}(x)}) + \gamma d(y,[Ty]_{\alpha_{L_T}(y)}) + \frac{\delta d(x,[Sx]_{\alpha_{L_S}(x)}) d(y,[Ty]_{\alpha_{L_T}(y)})}{1+d(x,y)} \tag{3.1}$$

and

$$\gamma + \frac{\delta d(x,[Sx]_{\alpha_{L_S}(x)})}{1+d(x,y)} < 1\,,\ \beta + \frac{\delta d(y,[Ty]_{\alpha_{L_T}(y)})}{1+d(x,y)} < 1 \tag{3.2}$$

where α, β, γ and δ are non negative real numbers with $\alpha+\beta+\gamma+\delta < 1$. Then, there exists $u \in X$ such that $u \in [Su]_{\alpha_{L_S}(u)} \cap [Tu]_{\alpha_{L_T}(u)}$.

Proof. We prove this theorem by considering the following three possible cases:

(i) $\alpha+\beta = 0$

(ii) $\alpha+\gamma = 0$

(iii) $\alpha+\beta \ne 0$, $\alpha+\gamma \ne 0$

Case I: If $\alpha+\beta=0$. Then for any $x \in X$, there exists $\alpha_{L_S}(x) \in L \backslash \{0\}$ such that $[Sx]_{\alpha_{L_S}(x)}$ is a nonempty closed and bounded subset of X. Take $y \in [Sx]_{\alpha_{L_S}(x)}$and in the same way $z \in [Ty]_{\alpha_{L_T}(y)}$. Then by above Lemma 2.8, we have

$$d(y,[Ty]_{\alpha_{L_T}(y)}) \le H([Sx]_{\alpha_{L_S}(x)},[Ty]_{\alpha_{L_T}(y)}).$$

Now by (3.1), we have

$$d(y,[Ty]_{\alpha_{L_T}(y)}) \le \alpha d(x,y) + \beta d(x,[Sx]_{\alpha_{L_S}(x)}) + \gamma d(y,[Ty]_{\alpha_{L_T}(y)}) + \frac{\delta d(x,[Sx]_{\alpha_{L_S}(x)}) d(y,[Ty]_{\alpha_{L_T}(y)})}{1+d(x,y)}$$

using $\alpha+\beta=0$, we have

$$\left[1-\gamma-\frac{\delta d(x,[Sx]_{\alpha_{L_S}(x)})}{1+d(x,y)}\right] d(y,[Ty]_{\alpha_{L_T}(y)}) \le 0.$$

Then by one of (3.2) yields

$$d(y,[Ty]_{\alpha_{L_T}(y)}) \le 0$$

it follows that

$$y \in [Ty]_{\alpha_{L_T}(y)}.$$

Again by (3.1), we have

$$(1-\beta)d(y,[Sy]_{\alpha_{L_S}(y)}) \le \gamma d(y,[Ty]_{\alpha_{L_T}(y)}) + \frac{\delta d(y,[Sy]_{\alpha_{L_S}(x)})d(y,[Ty]_{\alpha_{L_T}(y)})}{1+d(y,y)}$$

$$(1-\beta)d(y,[Sy]_{\alpha_{L_S}(y)}) \le 0$$

$$(1-\beta)d(y,[Sy]_{\alpha_{L_S}(y)}) = 0$$

which implies that

$$y \in [Sy]_{\alpha_{L_S}(y)}).$$

So, we get

$$y \in [Sy]_{\alpha_{L_S}(y)} \cap [Ty]_{\alpha_{L_T}(y)}.$$

Case II: If $\alpha+\gamma=0$. Then for any $x \in X$, as in case (i), take $y \in [Sx]_{\alpha_{L_S}(x)}$ and $z \in [Ty]_{\alpha_{L_T}(y)}$. Then by above Lemma 2.8, we have

$$d(z,[Sz]_{\alpha_{L_T}(z)}) = H([Ty]_{\alpha_{L_T}(y)},[Sz]_{\alpha_{L_T}(z)}).$$

Now by (3.1), we have

$$d(z,[Sz]_{\alpha_{L_T}(z)}) \le \alpha d(z,y) + \beta d(z,[Sz]_{\alpha_{L_S}(z)}) + \gamma d(y,[Ty]_{\alpha_{L_T}(y)}) + \frac{\delta d(z,[Sz]_{\alpha_{L_S}(z)})d(y,[Ty]_{\alpha_{L_T}(y)})}{1+d(z,y)}$$

using $\alpha+\gamma=0$, we have

$$\left[1-\beta-\frac{\delta d(y,[Ty]_{\alpha_{L_T}(y)})}{1+d(x,y)}\right] d(z,[Sz]_{\alpha_{L_S}(z)}) \le 0.$$

Then one of (3.2) yields

$$d(z,[Sz]_{\alpha_{L_S}(z)}) \le 0$$

it follows that

$$z \in [Sz]_{\alpha_{L_S}(z)}.$$

Again by (3.1), we have

$$(1-\gamma)d(z,[Tz]_{\alpha_{L_T}(z)}) \le \beta d(z,[Sz]_{\alpha_{L_S}(z)}) + \frac{\delta d(z,[Sz]_{\alpha_{L_S}(z)})d(z,[Tz]_{\alpha_{L_T}(z)})}{1+d(z,z)}$$

$$(1-\gamma)d(z,[Tz]_{\alpha_{L_T}(z)}) \le 0$$

$$(1-\gamma)d(z,[Tz]_{\alpha_{L_T}(z)}) = 0$$

which implies that

$$z \in [Tz]_{\alpha_{L_T}(z)}.$$

So, we get that

$$z \in [Sz]_{\alpha_{L_S}(z)} \cap [Tz]_{\alpha_{L_T}(z)}.$$

Case III: Let

$$\max\left\{\left(\frac{\alpha+\gamma}{1-\beta-\delta}\right),\left(\frac{\alpha+\beta}{1-\gamma-\delta}\right)\right\}=\lambda.$$

Then by $\alpha+\gamma$, $\alpha+\beta \neq 0$ and $\alpha+\beta+\gamma+\delta<1$, it follows that $0<\lambda<1$. Take $x_o \in X$. Then by hypotheses, there exists $\alpha_{L_S}(x_o) \in L\backslash\{0_L\}$ such that $[Sx_o]_{\alpha_{L_S}(x_o)}$ is a nonempty closed and bounded subset of X. For convenience, we denote $\alpha_{L_S}(x_o)$ by α_{L_1}. Let, $x_1 \in [Sx_o]_{\alpha_{L_1}}$, for this x_1 there exists $\alpha_{L_T}(x_1) \in L\backslash\{0_L\}$ such that $[Tx_1]_{\alpha_{L_T}(x_1)} \in CB(X)$. Denote $\alpha_{L_T}(x_1)$ by α_{L_2}. By above Lemma 2.9, there exists $x_2 \in [Tx_1]_{\alpha_{L_2}}$ such that

$$d(x_1,x_2) \le H([Sx_o]_{\alpha_{L_1}},[Tx_1]_{\alpha_{L_2}})+\lambda(1-\gamma-\delta). \tag{3.3}$$

By the same argument, we can find $\alpha_{L_3} \in L\backslash\{0_L\}$ and $x_3 \in [Sx_2]_{\alpha_{L_3}}$ such that

$$d(x_2,x_3) \le H([Sx_2]_{\alpha_{L_3}},[Tx_1]_{\alpha_{L_2}})+\lambda^2(1-\beta-\delta). \tag{3.4}$$

By induction we can get a sequence $\{x_n\}$ of points of X,

$$x_{2k+1} \in [Sx_{2k}]_{\alpha_{L_{2k+1}}}$$

$$x_{2k+2} \in [Tx_{2k+1}]_{\alpha_{L_{2k+2}}} \quad \text{where } k=0,1,2,\dots,$$

such as

$$d(x_{2k+1},x_{2k+2}) \le H([Sx_{2k}]_{\alpha_{L_{2k+1}}},[Tx_{2k+1}]_{\alpha_{L_{2k+2}}})+\lambda^{2k+1}(1-\gamma-\delta)$$

$$d(x_{2k+2},x_{2k+3}) \le H([Sx_{2k+2}]_{\alpha_{L_{2k+3}}},[Tx_{2k+1}]_{\alpha_{L_{2k+2}}})+\lambda^{2k+2}(1-\beta-\delta)$$

By (3.1) and (3.3), we get

$$d(x_1,x_2) \le \alpha d(x_o,x_1)+\beta d(x_o,[Sx_o]_{\alpha_{L_1}})+\gamma d(x_1,[Tx_1]_{\alpha_{L_2}}) + \frac{\delta d(x_o,[Sx_o]_{\alpha_{L_1}})d(x_1,[Tx_1]_{\alpha_{L_2}})}{1+d(x_o,x_1)}+\lambda(1-\gamma-\delta)$$

the above inequality implies that

$$d(x_1,x_2) \le \left(\frac{\alpha+\beta}{1-\gamma-\delta}\right)d(x_o,x_1)+\lambda.$$

Using inequalities (3.1) and (3.4), we get

$$d(x_2,x_3) \le \alpha d(x_2,x_1)+\beta d(x_2,[Sx_2]_{\alpha_{L_3}})+\gamma d(x_1,[Tx_1]_{\alpha_{L_2}}) + \frac{\delta d(x_2,[Sx_2]_{\alpha_{L_3}})d(x_1,[Tx_1]_{\alpha_{L_2}})}{1+d(x_2,x_1)}+\lambda^2(1-\beta-\delta)$$

thus,

$$d(x_2,x_3) \le \left(\frac{\alpha+\gamma}{1-\beta-\delta}\right)d(x_1,x_2)+\lambda^2$$

$$d(x_2,x_3) \le \lambda d(x_1,x_2)+\lambda^2.$$

This implies that

$$d(x_n,x_{n+1}) \le \lambda d(x_{n-1},x_n)+\lambda^n$$

$$\leq \lambda[\lambda d(x_{n-2},x_{n-1})+\lambda^{n-1}]+\lambda^n$$
$$\leq \lambda^2 d(x_{n-2},x_{n-1})+2\lambda^n$$
$$d(x_n,x_{n+1})\leq \lambda^3 d(x_{n-3},x_{n-2})+3\lambda^n.$$

It follows that for each $n=1,2,3,\dots$

$$d(x_n,x_{n+1})\leq \lambda^n d(x_o,x_1)+n\lambda^n.$$

Now, for each positive integer m,n with $n>m$, we have

$$\begin{aligned}d(x_m,x_n)&\leq d(x_m,x_{m+1})+d(x_{m+1},x_{m+2})+\dots d(x_{n-1},x_n)\\&\leq \lambda^m d(x_o,x_1)+m\lambda^m+\lambda^{m+1}d(x_o,x_1)+(m+1)\lambda^{m+1}\\&\quad+\dots+\lambda^{n-1}d(x_o,x_1)+(n-1)\lambda^{n-1}\\&\leq \sum_{i=m}^{n-1}\lambda^i d(x_o,x_1)+\sum_{i=m}^{n-1} i\lambda^i\\&\leq \frac{\lambda^m}{1-\lambda}d(x_o,x_1)+S_{n-1}-S_{m-1},\ \text{ where } S_n=\sum_{i=1}^{n} i\lambda^i.\end{aligned}$$

Since, $\lambda<1$ it follows from Cauchy's root test that $\sum i\lambda^i$ is convergent and hence $\{x_n\}$ is a Cauchy sequence in X. As X is complete, there exists $u\in X$ such that $x_n\to u$. Now by above lemma implies that

$$d(u,[Su]_{\alpha_{L_S}(u)})\leq d(u,x_{2n})+d(x_{2n},[Su]_{\alpha_{L_S}(u)})$$
$$d(u,[Su]_{\alpha_{L_S}(u)})\leq d(u,x_{2n})+H([Tx_{2n-1}]_{\alpha_{L_{2n}}},[Su]_{\alpha_{L_S}(u)}).$$

So, the above inequality implies that

$$d(u,[Su]_{\alpha_{L_S}(u)})\leq \left(1-\beta-\delta\frac{d(x_{2n-1},x_{2n})}{1+d(u,x_{2n-1})}\right)^{-1}\left(d(u,x_{2n})+\alpha d(u,x_{2n-1})+\gamma d(x_{2n-1},x_{2n})\right)$$

Letting $n\to\infty$, we have

$$d(u,[Su]_{\alpha_{L_S}(u)})\leq 0$$
$$d(u,[Su]_{\alpha_{L_S}(u)})=0.$$

This implies that

$$u\in[Su]_{\alpha_{L_S}(u)}.$$

Similarly, by using

$$d(u,[Tu]_{\alpha_{L_T}(u)})\leq d(u,x_{2n+1})+d(x_{2n+1},[Tu]_{\alpha_{L_T}(u)})$$

we can prove that

$$u\in[Tu]_{\alpha_{L_T}(u)}$$

which shows that

$$u\in[Su]_{\alpha_{L_S}(u)}\cap[Tu]_{\alpha_{L_T}(u)}.$$

□

Example 3.2. Let $X=[0,1]$ and $d(x,y)=|x-y|$, whenever $x,y\in X$, then (X,d) be a complete metric space. Let $L=\{\eta,\theta,\lambda,\mu\}$ with $\eta\preceq_L\theta\preceq_L\mu$ and $\eta\preceq_L\lambda\preceq_L\mu$, where θ and λ are not comparable, then $(L,\preceq_L)$ is a complete distributive lattice. Let S and T be the L-fuzzy mappings from X to L^X defined as:

$$S(x)(t)=\begin{cases}\theta & \text{if } 0\le t\le \frac{x}{14}\\ \eta & \text{if } \frac{x}{14}<t\le \frac{x}{10}\\ \mu & \text{if } \frac{x}{10}<t\le \frac{x}{3}\\ \lambda & \text{if } \frac{x}{3}<t\le 1\end{cases}$$

and

$$T(x)(t)=\begin{cases}\eta & \text{if } 0\le t\le \frac{x}{12}\\ \theta & \text{if } \frac{x}{12}<t\le \frac{x}{10}\\ \mu & \text{if } \frac{x}{10}<t\le \frac{x}{5}\\ \lambda & \text{if } \frac{x}{5}<t\le 1\end{cases}$$

For all $x\in X$, there exist $\alpha_{L_S}(x)=\theta$ and $\alpha_{L_T}(x)=\eta$, such that

$$[Sx]_\theta=\left[0,\frac{x}{14}\right] \quad\text{and}\quad [Tx]_\eta=\left[0,\frac{x}{12}\right].$$

Moreover for $\alpha=\frac{1}{5}$, $\beta=\frac{1}{10}$, $\gamma=\frac{1}{15}$ and $\delta=\frac{1}{20}$, we have

$$\gamma+\delta\frac{d(x,[Sx]_{\alpha_{L_S}(x)})}{1+d(x,y)}\le\frac{1}{15}+\frac{1}{20}\frac{\left|\frac{13x}{14}\right|}{1+|x-y|}<1.$$

Similarly, we have

$$\beta+\delta\frac{d(y,[Ty]_{\alpha_{L_T}(y)})}{1+d(x,y)}\le 1$$

and

$$H([Sx]_{\alpha_{L_S}(x)},[Ty]_{\alpha_{L_T}(y)})<\frac{1}{5}|x-y|+\frac{1}{10}\left|x-\frac{x}{14}\right|+\frac{1}{15}\left|y-\frac{y}{12}\right|+\frac{1}{20}\left[\frac{\left|x-\frac{x}{14}\right|\left|y-\frac{y}{12}\right|}{1+|x-y|}\right].$$

Since, S and T satisfy all the conditions of Theorem 3.1. So, $0\in X$ is a common fixed point of S and T.

Corollary 3.3. *Let $S,T\to F(X)$ be two fuzzy mappings and for $x\in X$, there exists $\alpha_S(x),\alpha_T(x)\in(0,1]$ such that $[Sx]_{\alpha_S(x)},[Tx]_{\alpha_T(x)}\in CB(2^X)$. If for all $x,y\in X$*

$$H([Sx]_{\alpha_S(x)},[Ty]_{\alpha_T(y)})\le\alpha d(x,y)+\beta d(x,[Sx]_{\alpha_S(x)})+\gamma d(y,[Tx]_{\alpha_T(x)})+\frac{\delta d(x,[Sx]_{\alpha_S(x)})d(y,[Tx]_{\alpha_T(x)})}{1+d(x,y)}$$

and

$$\gamma+\frac{\delta d(x,[Sx]_{\alpha_S(x)})}{1+d(x,y)}<1,\quad \beta+\frac{\delta d(y,[Tx]_{\alpha_T(x)})}{1+d(x,y)}<1$$

where α, β, γ and δ are non negative real numbers with $\alpha+\beta+\gamma+\delta<1$. Then, there exists $u\in X$ such that $u\in[Sx]_{\alpha_S(x)}\cap[Tx]_{\alpha_T(x)}$.

Theorem 3.4. *Let $S,T : X \to CB(X)$ be multivalued mappings and for all $x, y \in X$,*

$$H(Sx,Ty) \leq \alpha d(x,y) + \beta d(x,Sx) + \gamma d(y,Ty) + \frac{\delta d(x,Sx)d(y,Ty)}{1+d(x,y)} \tag{3.5}$$

and

$$\gamma + \frac{\delta d(x,Sx)}{1+d(x,y)} < 1, \quad \beta + \frac{\delta d(y,Ty)}{1+d(x,y)} < 1 \tag{3.6}$$

where α, β, γ and δ are non negative real numbers with $\alpha+\beta+\gamma+\delta<1$. Then, there exists $u \in X$ such that $u \in Su \cap Tu$.

Proof. Consider a pair of any mappings A, $B : X \to L\backslash\{0_L\}$ and a pair of L-fuzzy mappings G, $H : X \to L^X$ as

$$G(x)(t) = \begin{cases} Ax & t \in Sx \\ 0 & t \notin Sx \end{cases}$$

and

$$H(x)(t) = \begin{cases} Bx & t \in Tx \\ 0 & t \notin Tx \end{cases}$$

Then for $x \in X$, we have

$$[Gx]_{\alpha_{L_G}(x)} = \{t : G(x)(t) \geq \alpha_{L_G}(x)\} = Sx$$

and

$$[Hx]_{\alpha_{L_H}(x)} = \{t : H(x)(t) \geq \alpha_{L_H}(x)\} = Tx.$$

Thus, by applying Theorem 3.1, we get $z \in X$ such as

$$z \in [Gx]_{\alpha_{L_G}(x)} \cap [Hx]_{\alpha_{L_H}(x)} = Sz \cap Tz.$$

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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