Communications in Mathematics and Applications
Vol. 7, No. 2, pp. 151–157, 2016
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications

http://www.rgnpublications.com

On the Monty Hall Dilemma and Some Related Variations

Case Study

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Abstract. The current paper refers to the probabilistic problem posed by Monty Hall dilemma. The classical variation was considered along with two proposed variations on this two round chance game. Random number generators were used to obtain samples for individual games in order to assess the outcome, depending on the strategy employed by the player. An assumption was made and confirmed numerically, that if the decision in the second round is made randomly, the odds become $1/2$ both ways whereas if the player uses an "always switch" approach, they increase their winning chances - confirming vos Savant's hypothesis. Hence the main conclusions are that there is no mandatory chronological order for the two decision stages and that "stick-or-switch" choice refers to the choice to play the original or the converse game. The dilemma has a wide range of applications from social psychology to other fields of science such as quantum mechanics.

Keywords. Game theory; Monty Hall problem; Random number; Decision making

MSC. 03B20; 03B65; 97K80

Received: July 14, 2016 **Accepted:** July 23, 2016

1. Introduction

In its current form, the Monty-Hall Dilemma (M.H.D.) was introduced in 1975 by Selvin [1] and presents a two stage chance game:

A player is asked to select between three "doors" the only winning one. After a selection has been made, the game presenter (G.P) eliminates one non-winning options, leaving only two "doors" left for picking, one of which is the winner and one of which is the original selection of the player. Note that only if the player selected the winning "door", the two coincide. A second round begins and the player is allowed to either stick to their original choice or to switch to the other remaining choice. Following this second round, the "doors" are "opened" and the result is revealed.

This problem from the game theory and applied statistics has ramifications in economy [2], psychology [3, 4] and in other theoretical and applied fields [5] that deal with probability and chance.

The dilemma surrounding this game is whether or not the choice to stick or to switch has any influence on the player's chance of winning. A vivid debate followed the a popularized science article published by vos Savant, republished in her book [6] in which a number of Ph.D. scientists contradicted vos Savant's arguments in letters to the editor [7]. Her claim was that switching in the second stage of the game actually improves the player's chance of winning from $1/3$ to $2/3$, essentially reversing the odds of losing to the odds of winning. Critics however claim that, since there are two options left, the chances should be $1/2$ either way. A brief description of vos Savant's arguments is presented in Figure 1.

Table 1. vos Savant's arguments for switching in the second round of the game

Possible states	Option 1	Option 2	Option 3	Result after round 1 if choosing Option 1	Result if switch in round 2
State 1	correct			win	lose
State 2		correct		lose	win
State 3			correct	lose	win

2. Interpretation of the Problem

A first comment is that the G.P. is, at all times, aware of the winning "door" and adjusts their second stage setup accordingly, in such a way that there is exactly one winning and one losing choice, regardless of the initial choice of the player. This aspect of the game provides the basis of two more variations which will be presented further in the paper.

The theory of probability states that in the second stage of the game chances are $1/2$ both ways. However, the true question in solving this dilemma is in which way does the first round affect the second. One interpretation, which will be explored in this paper, is that the existence of the first round transforms the second round form a pure guessing game to one of probabilistic guessing. In other words the choice to stick with the original choice is essentially a bet on it whereas the choice to switch is a bet against the original choice. This important distinction leads to the notion that the randomness of the decision process in the second round will be the one influencing the results rather than the number of options presented - which is two.

A sensible player can recognize that their chances in the first round of the problem are $2/3$ against them and hence their safer bet would be against their original choice (i.e. to switch their choice). This reasoning is an alternative to the explanation given by vos Savant. In contrast, the arguments of the critics assume no information about the first round is usable by the player in the second round, which would make the second round a guessing game.

A number of variations of this dilemma have been analyzed and published [8], however in this article we will be focusing on the following:

(1) The choice and its truth value in the first round are truly random

(2) In the second round only two choices remain, containing exactly one correct and one incorrect choice.

(3) The game does not require a chronological sequence (i.e. using random number generators all choices and their truth values can be determined at the same time).

The following hypotheses which will be tested through numerical simulations:

(A) The second round is not a true choice between the two options presented but rather

(B) If the second round decision is not conscientiously guided by an interpretation of the probability to win in the first round, and is random in nature, the odds become $1/2$ for either outcome.

3. The Numerical Simulations

In order to test the afore mentioned hypotheses, numerical simulations were carried out on the original version of the game as well as two other variations with significantly lower odds of winning in the first round.

The model of the game requires a number to be assigned to the:

(i) winning choice

(ii) the original choice of the player in the first round

(iii) the decision to either "stick" or "switch" in the second round

Randomization algorithms [9] were used to generate the set of parameters for a number of 10 series of 1000 samples each.

In order to test vos Savant's arguments, for the parameter (iii) an "always switch" decision has been set in parallel. Therefore the game outcome can be monitored both in random switch/stick and in accordance with vos Savant's strategy. Note that the three variables can be generated simultaneously.

3.1 The Original Monty-Hall Game

This subsection presents the generic outline of the model used for the simulations which is fundamentally the same in all three variations of the Monty-Hall problem. Table 1 displays a number of sample games with the random choices: winner number, player initial choice number and the decision to switch or stick. Also the final outcome of the game, after round 2, is presented for both the random switch and the "always switch" strategies.

In Figure 1 the results of the winning statistics for each of the ten sample groups, each group containing 1000 samples.

It can be observed that when the decision to switch is made randomly, the chances of winning are approaching $1/2$ - as expected, whereas when the "always switch" strategy is employed, the winning chances approach $2/3$ - as predicted by vos Savant's arguments.

Table 2. Example of random parameter generation and the game outcome for each sample

Sample	Winning choice	Player's initial choice	Switch* (y/n)	random switch	always switch	outcome round 1	outcome round 2	outcome round 2 always switch
1	2	3	0	0	1	0	0	1
2	3	3	0	1	1	1	0	0
3	2	3	1	1	1	1	0	0
4	1	1	0	1	1	0	1	1
5	2	1	0	0	1	0	0	1
6	1	2	0	1	1	0	1	1
7	1	3	1	0	1	1	1	0
8	1	3	1	1	1	0	1	1
9	2	1	0	0	1	1	1	0
10	3	1	1	0	1	0	0	1

*: 0 = No; 1 = Yes

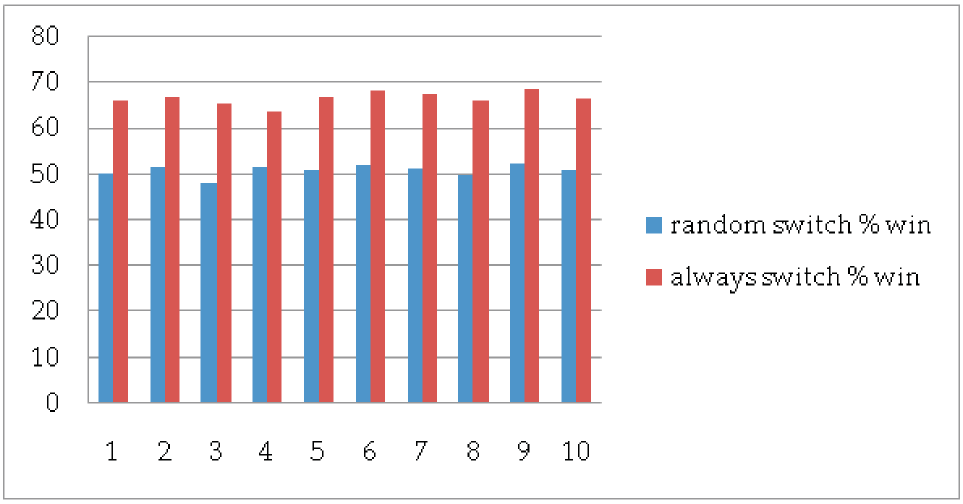

Figure 1. The outcome statistics for the original Monty-Hall game

3.2 The 1/10 variation of the Monty Hall Game

Since the percentages are arguably close 2/3 vs 1/2, a new variation was devised in order to provide a higher difference in winning odds.

This variation starts with 10 initial options in the first round, whereas in the second round 8 of the incorrect options are eliminated, leaving only 2 options: one correct and one incorrect. As with the original game, in this case a number of 10 groups of 1000 samples were tested.

Figure 2 depicts the winning statistics for the random switch and for the "always switch" strategies for this variation.

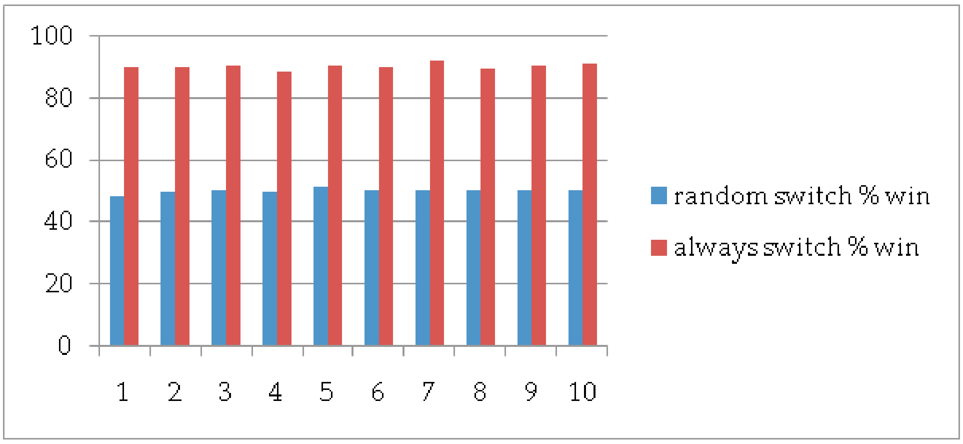

Figure 2. The outcome statistics for the 1/10 Monty-Hall game variation

3.3 The 1/100 variation of the Monty Hall Game

A final variation of the game was devised, having 100 initial options in the first round and two options in the second round. As in the cases presented before, the simulations confirm the assumption that there is indeed an influence on the winning odds if an "always switch" strategy is adopted. Figure 3 presents the winning statistics for both the "always switch" and the random switch strategies.

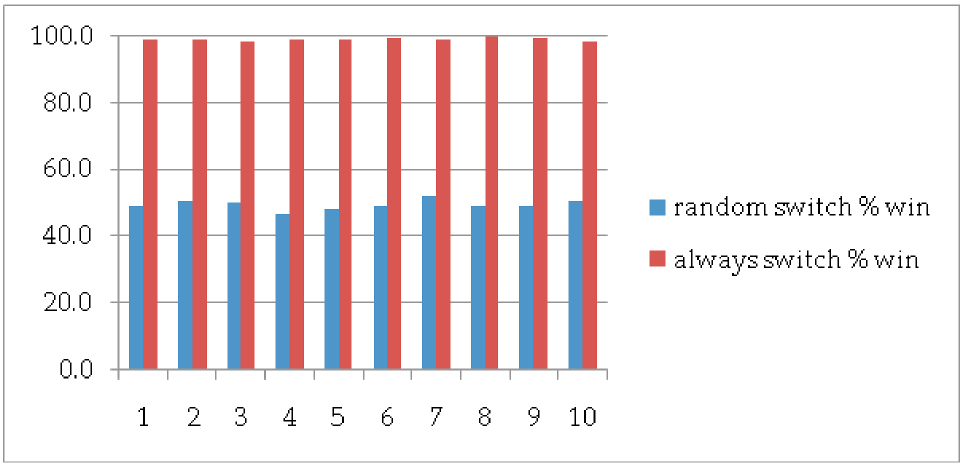

Figure 3. The outcome statistics for the 1/100 Monty-Hall game variation

4. Conclusions

Perhaps the most important realization of this paper is that the outcome is dependent on the way the second round choice is made, however the chronology of the rounds is false, meaning that due to the way the entire game is played, the choices in both rounds can be made simultaneously. This is because, irrespective of the choice of the final round, in the so-called second round exactly one wrong and one correct choice are also presented to the player.

A more correct formulation of the problem should insist on this realization, re-designating the rounds in such a way so that it does not suggest a chronological order or dependency.

As seen in the charts in the previous section, when the decision to switch was random, in all variations of the M-H game the probability of winning approached $1/2$. On the other hand, when an "always switch" strategy was employed, the odds of winning the second round were equal to those of losing (i.e. the odds of choosing the incorrect option) in the first round.

This confirms that the existence of a first round changes the nature of the second round. Therefore, if the player can calculate their odds of winning in the first round, the choice to switch in the second round is in fact a bet against their original choice.

However, not considering the information given by the previous round would lead the player to a random choice to either stick to their original choice or switch. Thus, ignoring the first round completely gives the player a $1/2$ chance both ways.

It is therefore the conclusion of this paper that, indeed if the player employs an "always switch' strategy in the second round, they increase the chances of winning to

$$\eta = 1 - \frac{1}{n'}$$

where n is the number of initial choices presented in the first round.

Table 3 presents a synthesis of all the tests carried out numerically in this paper.

Table 3. The probability of winning for the combined $10 \cdot 1000$ samples of the variations of the M.H.D.

	Variation 1/3 (original MHD)	Variation 1/10	Variation 1/100
random switch % win	50.76	49.99	49.37
always switch % win	66.51	90.44	98.94

Competing Interests

The author declares that he has no competing interests.

Authors' Contributions

The author wrote, read and approved the final manuscript.

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