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An Extension of Fuzzy WV Control Chart based on α-Level Fuzzy Midrange

Research Article

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Abstract. Control chart is one of the most important tools in statistical process control (SPC) that leads to improve quality processes and ensure the required quality levels. The usual assumption for designing a control chart is that the data or measurement should have a normal distribution. However, this assumption may not be true for some processes, there are some factors that cause an uncertainty data such as human, measurement device or environmental conditions. Therefore, the purposes of this research are to study, develop and compare the efficiency of fuzzy weighted variance (*FWV*) control charts which the data has non-normal distribution as Weibull, gamma and Chi-squared. FWV control charts use fuzzy set theory to help in solving the uncertainty data. The random variable for the experiment will be transformed to fuzzy control chart by using triangular membership function. Finally, the performance and comparative efficiency of the FWV control charts are measured in term of average run length (*ARL*) by Monte Carlo simulation technique.

Keywords. Fuzzy; α-cut; α-level fuzzy midrange

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1. Introduction

We use SPC to evaluate the output of a process and to determine if it is statistically acceptable. The seven basic quality tools are often referred, namely, flowcharts, check sheet, histogram, Pareto chart, scatter diagram, control charts and cause-and-effect diagram.

Exactly, the most important tool in SPC is control chart invented by Dr. Walter Andrew Shewhart in 1924. It is a time-ordered plot of representative sample e.g. sample obtained from an ongoing process. The main objectives of using control chart are to examine and improve the analysis in order to reduce random variation in manufacturing process. Control chart has two categories, variables and attributes control charts, used in SPC, which can detect changes at large data.

Normally, we study on the data in normal distribution. However, data can also be measured in several uncommon distributions such as Weibull, gamma, or lognormal distributions. For this reason, several control charts based on data were proposed in order to implement the optimized results of data distribution. M.B. Vermaat *et al.* [8] compared the effectiveness between individual control chart, empirical quantiled (EQ) control chart, extreme value (EV) theory control chart and kernel control chart. Several data distributions; normal, T, uniform, exponential, Laplace and logistical distributions which size of samples are 250, 500, 1000, 2500, 5000 and 10,000 shift equal to 0σ, 0.25σ, 3.5σ, 4σ and 5σ. Average run length, $ARL$and standard deviation of the run Length, SDRL are used for an effectiveness criteria. His research concluded that EQ has the best efficiency. A. Pongpullponsak *et al.* [5] compared the effectiveness of WV, scaled weighted variance (SWV), EQ and EV control charts, which have data distributions as Weibull, lognormal and Burr's. ARL is also used for the effectiveness criteria. In their study, the problem may be uncertainty data as human, measurement devices or environmental conditions. From these problems, results from the control chart have different distributions. Senturk and Erginel [6] applied fuzzy set theory to $\overline{X}$, range and standard deviation control charts. A. Pongpullponsak *et al.* [3] compared FWV, Fuzzy scaled weighted variance (FSWV), fuzzy empirical quantiles (FEQ) and fuzzy extreme value (FEV) control charts, these control charts are developed from WV, SWV, EQ and EV, respectively. Several data distributions were considered in this study such as Weibull, lognormal and Burr's. ARL is used for effectiveness criteria. This appears that FEQ has Weibull distribution and be the most efficient control chart. However, this study did not conclude that the control charts is the most effective based on several factors namely, the distribution of the data of sampling, coefficient of skewness of the distribution etc. According to the mentioned studies, we would like to study and compare the previous researches with the others data distribution, in order to reach the best efficiency of the control charts.

In this research, we develop and compare the efficiency of FWV control chart, that has data distributions as Weibull, gamma and chi-squared. ARL is used for an effectiveness criteria and calculated by Monte Carlo (MC) simulation method under mean process shift 0σ, 0.5σ, 1σ, 1.5σ, 2σ, 2.5σ and 3σ.

This paper is organized as follows: non-normal distributions as Weibull, gamma and Chi-squared distributions, ARL and α-level fuzzy midrange are introduced in the second section. FWV control chart based on range is described in section 3. The efficiency of FWV control chart is examined and discussed in section 4. The conclusion and the future research are presented in the final section.

2. Method

2.1 Weibull Distribution

Weibull is a continuous distribution that is used widely. Let X be Weibull random variables where $\theta > 0$ and $\beta > 0$, the probability density function is

$$f(x;\theta,\beta) = \frac{\beta}{\theta^\beta} x^{\beta-1} e^{-(\frac{x}{\theta})^\beta}; \quad x > 0$$

where θ is scale parameter, β is shape parameter.

2.2 Chi-squared Distribution

Chi-squared is a continuous distribution that is used widely. Let X be chi-squared random variables where $r > 0$, the probability density function is defined as:

$$f(x) = \frac{1}{\Gamma\left(\frac{r}{2}\right) 2^{\frac{r}{2}}}\, x^{\left(\frac{r}{2}\right)-1} e^{-\frac{x}{2}}; \;\; x > 0$$

where r is degree of freedom, $\Gamma(\alpha)$ is gamma function, which $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$, $\alpha > 0$.

2.3 Gamma Distribution

Gamma is a continuous distribution that is used widely. Let X be gamma random variables where $\alpha > 0$ and $\beta > 0$, the probability density function is:

$$f(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} \exp\left(-\frac{x}{\beta}\right); \;\; x > 0$$

where α is shape parameter, β is scale parameter.
$\Gamma(\alpha)$ is gamma function, which $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$, $\alpha > 0$.

2.4 Average Run Length

The performance of a control chart is measured by the ARL that is the number of points that, on average, will be plotted on a control chart before an out of control condition is indicated.
If the process is in control, ARL is calculated from

$$ARL_0 = \frac{1}{\alpha}.$$

If the process is out of control, ARL is calculated from

$$ARL_1 = \frac{1}{1-\beta},$$

where α is the probability of a Type I error that is a probability to produce an alarm signal when there is no real change (false alarm).
β is the probability of a Type II error that is a probability to have no signal when there is real change.

2.5 α-Level Fuzzy Midrange

There are four types of fuzzy transformation techniques including fuzzy mode, fuzzy median, fuzzy average and α-level fuzzy midrange. In this study, we mention the last techniques called α-level fuzzy midrange which is defined as the midpoint of the α-level cut and further used for FWV control chart.

To determine its value, we consider the midpoint of crisp interval that divide triangular fuzzy set into two subsets which one subset contains all values larger than or equal to α in the original set. Exactly, another one subset contains all values that less than α.

Let f^{α}_{mr} be denoted as the α-level midrange and an α-level cuts, A^{α} be a non-fuzzy sets that consists of any elements whose membership is greater than or equal to α. If a^{α} and b^{α} are the end points of A^{α}, then the α-level midrange is calculated by the following equation:

$$f^{\alpha}_{mr} = \frac{1}{2}(a^{\alpha} + c^{\alpha}). \tag{2.1}$$

In fact the fuzzy mode is a special case of α-level fuzzy midrange when $\alpha = 1$.
α-level fuzzy midrange of sample j, $S^{\alpha}_{mr,j}$ is determined by

$$S^{\alpha}_{mr,j} = \frac{(a_j + c_j) + \alpha\left[(b_j - a_j) - (c_j - b_j)\right]}{2}. \tag{2.2}$$

Then, the condition of process control for each sample can be defined as:

$$\text{Process control} = \left\{ \begin{array}{ll} \text{in-control} & \text{for } LCL^{\alpha}_{mr} \le S^{\alpha}_{mr,j} \le UCL^{\alpha}_{mr} \\ \text{out-of control} & \text{for otherwise} \end{array} \right\}. \tag{2.3}$$

In Figure 1, α-level cuts of samples by triangular fuzzy number (a,b,c) is represented; also α-level fuzzy midrange of sample j, $S^{\alpha}_{mr,j}$ is determined by Eq. (2.2).

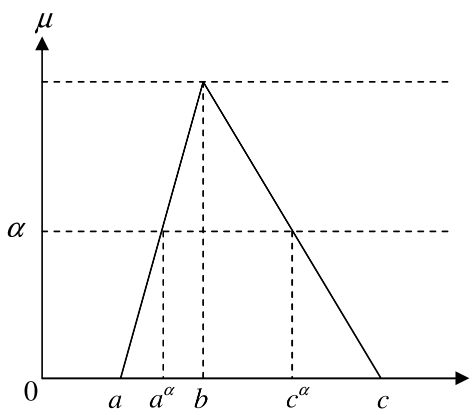

Figure 1. α-level cuts of samples by triangular fuzzy number.

3. FWV Control Chart

Control charts are used for monitoring the SPC. Fuzzy control charts are used when consider the uncertainty data, for providing flexibility on control limits to prevent false alarms. FWV is developed from WV control charts which introduced by Bai and Choi [1]. The WV control charts adjusts the control limits according to skewness population. The probability of the quality variable X, will be less than or equal to its mean μ_x is $P_X = P(X \le \mu_x)$.
If the parameter of the process are known, WV control chart are:

$$UCL = \mu_x + 3\frac{\sigma_x}{\sqrt{n}}\sqrt{2P_x}$$

and

$$LCL = \mu_x - 3\frac{\sigma_x}{\sqrt{n}}\sqrt{2(1-P_x)}$$

where σ_x is the standard deviation of X, X is the observation, n is the sample size and P_x is the process parameters that are not known. In this case, we must estimate. The probability, P_x is estimated by using the observation of sample which less than or equal to $\overline{\overline{X}}$ that is $P_X = P(X \le \mu_x)$ approximated by the following equation.

$$\widehat{P}_X = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}\delta(\overline{\overline{X}} - X_{k_{ij}})}{n \times m},$$

where

$$\delta(\overline{\overline{X}} - X_{k_{ij}}) = \begin{cases} 1, & X_{k_{ij}} \le \overline{\overline{X}} \\ 0, & X_{k_{ij}} > \overline{\overline{X}} \end{cases}$$

m is the number of sample, X_{ij} is the observation of sample size i, $i = 1,2,\dots,n$ of fuzzy sample j, $j = 1,2,\dots,m$.

If the parameter of the process are unknown, WV control chart is:

$$UCL = \overline{\overline{X}} + \frac{3\sqrt{2\widehat{P}_x}}{d_2'\sqrt{n}} = \overline{\overline{X}} + W_U\overline{R} \tag{3.1}$$

$$LCL = \overline{\overline{X}} - \frac{3\sqrt{2(1-\widehat{P}_x)}}{d_2'\sqrt{n}}\overline{R} = \overline{\overline{X}} - W_L\overline{R} \tag{3.2}$$

where $\overline{\overline{X}} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m} X_{ij}}{n\times m}$, $\overline{\overline{X}}$ is the overall mean, X_{ij} is the observation of fuzzy sample size i, $i = 1,2,\dots,n$ of fuzzy sample j, $j = 1,2,\dots,m$.

$\overline{R} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m} R_{ij}}{n\times m}$, $\overline{R}$ is the average of the R_{ij}, while R_{ij} is the range for sample size i, $i = 1,2,\dots,n$ of fuzzy number of subgroup, j, $j = 1,2,\dots,m$, d_2' is the constant based on WV control chart.

3.1 FWV Control Charts

According to the study on WV control chart, it has to deal with the uncertain data, so we use fuzzy theory to solve this problem. The studies of Senturk and Erginel [6] used fuzzy theory to improve WV control chart to FWV control chart, which used the membership represented by a triangular fuzzy number (a,b,c). FWV control chart is shown as follow:

$$\begin{aligned} U\widetilde{C}L_{FWV} &= C\widetilde{L} + W_{Uk}\overline{R} \\ &= (\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c) + W_{Uk}(\overline{R}_a, \overline{R}_b, \overline{R}_c) \\ &= (\overline{\overline{X}}_a + W_{Ua}\overline{R}_a, \overline{\overline{X}}_b + W_{Ub}\overline{R}_b, \overline{\overline{X}}_c + W_{Uc}\overline{R}_c) \\ &= (U\widetilde{C}L_1, U\widetilde{C}L_2, U\widetilde{C}L_3), \end{aligned} \tag{3.3}$$

$$CL_{FWV} = (\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c) = \left(\frac{\sum_{j=1}^{m} \overline{X}_{a_j}}{m}, \frac{\sum_{j=1}^{m} \overline{X}_{b_j}}{m}, \frac{\sum_{j=1}^{m} \overline{X}_{c_j}}{m}\right), \tag{3.4}$$

$$\begin{aligned} L\widetilde{C}L_{FWV} &= C\widetilde{L} - W_{Lk}\overline{R} \\ &= \left(\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c\right) - W_{Lk}(\overline{R}_a, \overline{R}_b, \overline{R}_c) \\ &= (\overline{\overline{X}}_a - W_{Lk}\overline{R}_a, \overline{\overline{X}}_b - W_{Lk}\overline{R}_b, \overline{\overline{X}}_c - W_{Lk}\overline{R}_c) \\ &= (L\widetilde{C}L_1, L\widetilde{C}L_2, L\widetilde{C}L_3), \end{aligned} \tag{3.5}$$

where $(\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c)$ is the overall mean of fuzzy number (a, b, c), respectively.
$\overline{R}_a$, $\overline{R}_b$ and $\overline{R}_c$ are the arithmetic means of maximum and minimum fuzzy number in the sample, respectively. These are calculated as follows:

$$\overline{R}_a = \frac{\sum_{j=1}^{m} R_{a_j}}{m}, \quad \overline{R}_b = \frac{\sum_{j=1}^{m} R_{b_j}}{m} \text{ and } \overline{R}_c = \frac{\sum_{j=1}^{m} R_{c_j}}{m}; \quad j = 1, 2, \dots, m$$

where $R_{a_j} = X_{\max,a_j} - X_{\min,c_j}$, $R_{b_j} = X_{\max,b_j} - X_{\min,b_j}$ and $R_{c_j} = X_{\max,c_j} - X_{\min,a_j}$; $(X_{\max,a_j}, X_{\max,b_j}, X_{\max,c_j})$ and $(X_{\min,a_j}, X_{\min,b_j}, X_{\min,c_j})$ are fuzzy random variable of maximum and minimum fuzzy number (a, b, c).

3.2 α-Cut FWV Control Chart

In this paper, we develop FWV control chart by using α-cut theory. The interpretation FWV control chart is the same as mentioned in the previous section. By applying α-cuts on fuzzy sets, therefore, the α-cut FWV control chart is:

$$\begin{aligned} U\widetilde{C}L^{\alpha}_{FWV} &= (\overline{\overline{X}}^{\alpha}_a, \overline{\overline{X}}_b, \overline{\overline{X}}^{\alpha}_c) + W_{Uk}(\overline{R}^{\alpha}_a, \overline{R}_b, \overline{R}^{\alpha}_c) \\ &= (\overline{\overline{X}}^{\alpha}_a + W_{Ua}\overline{R}^{\alpha}_a, \overline{\overline{X}}_b + W_{Ub}\overline{R}_b, \overline{\overline{X}}^{\alpha}_c + W_{Uc}\overline{R}^{\alpha}_c) \\ &= (U\widetilde{C}L^{\alpha}_1, U\widetilde{C}L_2, U\widetilde{C}L^{\alpha}_3), \end{aligned} \tag{3.6}$$

$$\begin{aligned} C\widetilde{L} &= (\overline{\overline{X}}^{\alpha}_a, \overline{\overline{X}}_b, \overline{\overline{X}}^{\alpha}_c) \\ &= (C\widetilde{L}^{\alpha}_1, C\widetilde{L}_2, C\widetilde{L}^{\alpha}_3), \end{aligned} \tag{3.7}$$

$$\begin{aligned} L\widetilde{C}L^{\alpha}_{FWV} &= (\overline{\overline{X}}^{\alpha}_a, \overline{\overline{X}}_b, \overline{\overline{X}}^{\alpha}_c) - W_{Lk}(\overline{R}^{\alpha}_a, \overline{R}_b, \overline{R}^{\alpha}_c) \\ &= (\overline{\overline{X}}^{\alpha}_a - W_{La}\overline{R}^{\alpha}_a, \overline{\overline{X}}_b - W_{Lb}\overline{R}_b, \overline{\overline{X}}^{\alpha}_c - W_{Lc}\overline{R}^{\alpha}_c) \\ &= (L\widetilde{C}L^{\alpha}_1, L\widetilde{C}L_2, L\widetilde{C}L^{\alpha}_3). \end{aligned} \tag{3.8}$$

Let $\alpha = 0.65$, where in $\overline{\overline{X}}^{\alpha}_a$, $\overline{\overline{X}}^{\alpha}_c$, $\overline{R}^{\alpha}_a$ and $\overline{R}^{\alpha}_a$ are as follows:

$$\overline{\overline{X}}^{\alpha}_a = \overline{\overline{X}}_a + \alpha(\overline{\overline{X}}_b - \overline{\overline{X}}_a), \tag{3.9}$$

$$\overline{\overline{X}}^{\alpha}_c = \overline{\overline{X}}_c - \alpha(\overline{\overline{X}}_c - \overline{\overline{X}}_b), \tag{3.10}$$

$$\overline{R}_a^{\alpha} = \overline{R}_a + \alpha(\overline{R}_b - \overline{R}_a), \tag{3.11}$$

$$\overline{R}_c^{\alpha} = \overline{R}_c - \alpha(\overline{R}_c - \overline{R}_b). \tag{3.12}$$

3.3 α-Level Fuzzy Midrange for α-Cut FWV Control Chart

The α-level fuzzy midrange is one of four transformation techniques used to determine the FWV control charts. In this study, α-level fuzzy midrange is used as the fuzzy transformation techniques while calculating α-level fuzzy midrange forα-cut FWV control chart is:

$$U\widetilde{C}L_{mr-FWV}^{\alpha} = CL_{mr-FWV}^{\alpha} + \frac{1}{2}(W_{Ua}\overline{R}_a^{\alpha} + W_{Uc}\overline{R}_c^{\alpha}), \tag{3.13}$$

$$CL_{mr-FWV}^{\alpha} = f_{mr-FWV}^{\alpha} = \frac{C\widetilde{L}_1^{\alpha} + C\widetilde{L}_3^{\alpha}}{2}, \tag{3.14}$$

$$L\widetilde{C}L_{mr-FWV}^{\alpha} = CL_{mr-FWV}^{\alpha} - \frac{1}{2}(W_{La}\overline{R}_a^{\alpha} + W_{Lc}\overline{R}_c^{\alpha}). \tag{3.15}$$

For a sample j, the value of α-level fuzzy midrange is calculated as follows:

$$S_{mr-FWV,j}^{\alpha} = \frac{(\overline{x}_{a_j} + \overline{x}_{c_j}) + \alpha\left[(\overline{x}_{b_j} - \overline{x}_{a_j}) - (\overline{x}_{c_j} - \overline{x}_{b_j})\right]}{2}. \tag{3.16}$$

Then, the condition of process control for each sample can be defined as:

$$\text{Process control} = \left\{ \begin{array}{ll} \text{in-control} & \text{for } LCL_{mr-FWV}^{\alpha} \le S_{mr-FWV,j}^{\alpha} \le UCL_{mr-FWV}^{\alpha} \\ \text{out-of control} & \text{for otherwise} \end{array} \right\}. \tag{3.17}$$

4. Results and Discussion

The purpose of this research is to study, develop and compare the efficiency of FWV control charts, which are calculated from equations (3.13), (3.14) and (3.15). In this research, data has skewed populations: Weibull, gamma and Chi-squared distributions in order to show that the proposed approach has better performance for detecting the shift process.

Let, $ARL_0 = 300$, the fuzzy sample size ($n = 10$), the number of fuzzy samples ($m = 300$) are randomly generated from Weibull, gamma and Chi-squared distributions with parameters $\theta = 1$, $\beta = 1$, $\alpha = 1$, $\beta = 1$ and $r = 1$, respectively. The, ARL_0 and ARL_1 of FWV control chart are calculated by using MATLAB, 7.6.0(R2009a) via Monte Carlo (MC) simulation method, which is calculated repeatedly up to 10,000 times for shift sizes of 0.5σ, 1.0σ, 2.0σ, 2.5σ and 3.0σ. We use ARL_1 to determine the efficiency of FWV control chart. We input data until the process is out of control, then we stop and count the number of samples that are in control. We use the ARL to decide which is the most effective. The results of ARL_1 of FWV control chart, which have Weibull, gamma and Chi-squared distributions are shown in Table 1. By considering Table 1, it can be concluded that if the data is shifted, the $ARL_1$is decreasing. According to FWV control chart, ARL_1 calculated from the data which has a chi-squared distribution is smaller than calculated from Weibull and gamma distributions under different shifts. Therefore, by considering the robust and performance of FWV control chart, data in case of chi-square distribution has been significantly improved to detect assignable causes with high performance and detect shifts in the process faster than Weibull and gamma distributions.

Table 1. A comparison of the ARL_1 of FWV control chart, the data of which has Weibull, gamma and chi-squared distributions under mean process shift 0σ, 0.5σ, 1σ, 1.5σ, 2σ, 2.5σ and 3σ.

δ	Weibull	Gamma	Chi-squared
0.5	259.13	1.137	1.132
1.0	216.13	1.115	1.110
1.5	170.14	1.111	1.106
2.0	128.78	1.089	1.084
2.5	94.10	1.039	1.032
3.0	62.65	1.020	1.015

5. Conclusions and Future Research

Control charts have extensive applications to find shifts in the process and indicate abnormal process condition. One of the most important SPC tools is control chart that monitors quality characteristics. Some causes as human, measurement devices or environmental conditions in quality characteristic lead to exist some level of uncertainty in attribute control chart, in these situations it is better to use fuzzy set theory applied in control charts. Thus, in this paper, we developed an FWV control chart to monitor attribute quality characteristic. Results of the comparison efficiency by using ARL criterion showed that FWV control chart which data has chi-squared distribution, has a high performance and could detect shifts in the process faster than Weibull and gamma distributions. For future research, considering trapezoidal fuzzy numbers and applying other control charts such as exponentially weighted moving average are recommended.

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Competing Interests

The author declare that he/she has no competing interests.

Authors' Contributions

The author wrote, read and approved the final manuscript.

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