Communications in Mathematics and Applications
Vol. 7, No. 3, pp. 179–187, 2016
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications

http://www.rgnpublications.com

Special Issue on
Advances in Fixed Point Theory towards Real World Optimization Problems
Proceedings of The 9th Asian Conference on Fixed Point Theory and Optimization
May 18 – 20, 2016, King Mongkut's University of Technology Thonburi, Bangkok, Thailand

Fuzzy Fixed Point Theorems for Multivalued Fuzzy F-Contraction Mappings in b-Metric Spaces

Research Article

Darunee Hunwisai[1] and Poom Kumam[1,2,*]

[1] *Department of Mathematics, Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), 126 Pracha-Uthit Road, Bang Mod, Thung Khru, Bangkok 10140, Thailand*
E-mail: darunee.3790@mail.kmutt.ac.th

[2] *Theoretical and Computational Science Center (TaCS), Science Laboratory Building, Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), 126 Pracha-Uthit Road, Bang Mod, Thung Khru, Bangkok 10140, Thailand*

***Corresponding author:** poom.kum@mail.kmutt.ac.th

Abstract. In this work, we introduce and suggest the new concept of multivalued fuzzy F-contraction mappings in b-metric spaces. We also establish and prove the existence of an α-fuzzy fixed point theorem in b-metric spaces.

Keywords. b-metric spaces; Fuzzy mappings; Fuzzy fixed point; F-contraction

MSC. 47H10

Received: June 4, 2016 **Revised:** June 9, 2016 **Accepted:** June 25, 2016

Copyright © 2016 Darunee Hunwisai and Poom Kumam. *This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.*

1. Introduction

The contraction is important tools for proving the existence and uniqueness of a fixed point in fixed point theory. Banach contraction principle [6] is one of most useful tools in the study of nonlinear equations. Many authors were motivate to extend and generalizations of Banach's contraction mapping principle in the literature (see in [2, 7, 10, 11, 16, 17, 21]). Nadler [21] studies multi-valued contraction mappings, he proves some fixed point theorem for multivalued contraction mappings by combined the ideas of set-valued mapping and Lipschitz mapping. The concept of fuzzy sets was introduced by Zadeh [27] in 1965.

In 1981, Heilpern [17] achieve a fixed point theorem for fuzzy contraction mappings, he also proved the existence of a fuzzy fixed point theorem which is generalization of Nadler's fixed point theorem for multivalued mapping. Phiangsungnnoen and Kumam [23] studied fuzzy fixed point theorems for multivalued fuzzy contractions in b-metric spaces. In addition, many author studied about fixed point results of fuzzy mappings is referred to [1,4,15,24]. Bakhtin [5] introduced the concept of b-metric space, which is a generalization of metric spaces. On the other hand, in 2012, Wardowski [25] suggested the concept of contraction and prove a fixed point theorem which generalizations Banach. Since then, many authors investigated fixed point theorem for F-contraction mappings [18–20,22,26].

In this paper, we suggest the new concept of multivalued fuzzy F-contraction mappings in b-metric spaces. We prove the existence of an α-fuzzy fixed point theorem in b-metric spaces. Our results improve and extend some fixed point results in original multivalued mappings and also in b-metric spaces.

2. Preliminaries

Firstly, we recall some basic definitions and results which will be used in the sequel. Throughout this paper, N, R and R^+ denote the set of natural numbers, real numbers and positive real numbers, respectively.

Definition 2.1 ([5]). Let X be a nonempty set and let $s \geq 1$ be a real number. A function $d : X \times X \to [0,\infty)$ is said to be a b-metric on X if it satisfies for all $x,y,z \in X$, the following conditions:

(i) $d(x,y) = 0$ if and only if $x = y$;

(ii) $d(x,y) = d(y,x)$ for all $x,y \in X$;

(iii) $d(x,z) \leq s[d(x,y)+d(y,z)]$.

A pair (X,d) is called a b-metric space.

Remark 2.2. From the definition of b-metric spaces if we set $s = 1$, it turns into normal metric spaces. Therefore, b-metric spaces are the extension of metric spaces.

Example 2.3 ([8]). The space l_p with $0 < p < 1$, define $l_p = \left\{\{x_n\} \subset R : \sum_{n=1}^{\infty} |x_n|^p < \infty\right\}$, together with the function $d : l_p \times l_p \to R$,

$$d(x,y) = \left(\sum_{n=1}^{\infty} |x_n - y_n|^p\right)^{\frac{1}{p}},$$

where $x = \{x_n\}$, $y = \{y_n\} \in l_p$ is a b-metric space with coefficient $s = \frac{1}{2^p} = 2^{\frac{1}{p}} > 1$. By an primary calculation we obtain that

$$d(x,z) \leq 2^{\frac{1}{p}}[d(x,y)+d(y,z)].$$

Example 2.4 ([8]). Let $t \in [0,1]$, the L_p $(0 < p < 1)$ of all real function $x(t)$ such that $\int_0^1 | x(t) |^p \, dt < \infty$, is b-metric space if we take

$$d(x,y) = \left[\int_0^1 | x(t) - y(t) |^p \, dt\right]^{\frac{1}{p}}$$

for all $x, y \in L_p$.

Definition 2.5 ([9]). Let (X,d) be a b-metric space.

(i) The sequence $\{x_n\}$ in X is called convergent to $x \in X$ if and only if $d(x_n, x) \to 0$ as $n \to \infty$.

(ii) The sequence $\{x_n\}$ in X is called Cauchy sequence if and only if $d(x_n, x_m) \to 0$ as $m, n \to \infty$.

(iii) The sequence $\{x_n\}$ in X is called complete if and only if every Cauchy sequence is convergent.

Let (X,d) be a b-metric space, denote $CL(X)$ be the class of all nonempty closed subset of X. $CB(X)$ be the collection of nonempty, closed and bounded subsets of X. And $K(X)$ be the family of all nonempty compact subsets of X. For $A, B \in CL(X)$ and $x \in X$, we define

$$d(x,A) = \inf\{d(x,a) : a \in A\},$$
$$\gamma(A,B) = \sup_{a \in A} d(a,B).$$

The generalized Hausdorff b-metric H on $CL(X)$ inducted by d is defined as

$$H(A,B) = \begin{cases} \max\{\gamma(B,A), \gamma(A,B)\} & \text{if the maximum exists;} \\ +\infty & \text{otherwise,} \end{cases}$$

for every $A, B \in CL(X)$.

Lemma 2.6 ([12–14]). *Let (X,d) be a b-metric space. For all $x, y \in X$ and for all $A, B, C \in CL(X)$, we have the following:*

(i) $d(x,b) \geq d(x,B)$ *for every* $b \in B$;

(ii) $H(A,B) \geq d(x,B)$ *for every* $x \in A$;

(iii) $H(A,B) \geq \gamma(A,B)$;

(iv) $H(A,A) = 0$;

(v) $H(B,A) = H(A,B)$;

(vi) $s(H(A,B) + H(B,C)) \geq H(A,C)$;

(vii) $s(d(x,y) + d(y,A)) \geq d(x,A)$.

Let (X,d) be a b-metric space. A fuzzy set D in X is a function from X into $[0,1]$. If $x \in X$, then the function value $D(x)$ is called the grade of membership of $x \in D$. $\mathscr{F}(X)$ is the collection of all fuzzy sets in X.

For $\alpha \in [0,1]$ and $D \in \mathscr{F}(X)$. The notation $[D]_\alpha$ is called α-level set (or α-cut set) of D and is defined as follows:

$$[D]_\alpha = \{x : D(x) \geq \alpha\} \text{ if } \alpha \in (0,1],$$

and

$$[D]_0 = \overline{\{x : D(x) > 0\}},$$

whenever $\bar{B}$ denotes the closure of the set B in X.

Let A and B are fuzzy set in X. A fuzzy set A is said to be more accurate than fuzzy set B, denote by $A \subset B$ if and only if $A(x) \leq B(x)$ for all x in X where the membership function of A and B denote by $A(x)$ and $B(x)$, respectively.

For $A, B \in \mathscr{F}(X)$, $x \in X$, $\alpha \in [0,1]$ and $[A]_\alpha, [B]_\alpha \in CB(X)$, define

$$d(x,A) = \inf_{a \in A} d(x,a),$$
$$p_\alpha(x,A) = \inf_{a \in [A]_\alpha} d(x,a),$$
$$p_\alpha(A,B) = \inf_{a \in [A]_\alpha, b \in [B]_\alpha} d(a,b),$$
$$p(A,B) = \sup_\alpha p_\alpha(A,B),$$
$$H([A]_\alpha,[B]_\alpha) = \max\left\{ \sup_{a \in [A]_\alpha} d(a,[B]_\alpha), \sup_{b \in [B]_\alpha} d(b,[A]_\alpha) \right\}.$$

Definition 2.7. Let X be an arbitrary set and Y be a b-metric space. A mapping $T : X \to \mathscr{F}(Y)$ is called a fuzzy mapping over the set Y.

Definition 2.8. Let (X,d) be a b-metric space and $T : X \to \mathscr{F}(X)$ be a fuzzy mapping. A point c in X is called an α-fuzzy fixed point of T if $c \in [Tc]_{\alpha(c)}$.

Next, we consider the following conditions for a mapping $F : R^+ \to R$.

Let F^* be the set of all functions $F : R^+ \to R$ satisfying the following conditions:

(F1) F is strictly increasing, that is, for all $\alpha, \beta \in R^+$ such that $\alpha < \beta$ implies $F(\alpha) < F(\beta)$;

(F2) for each sequence $\{\alpha_n\}_{n \in N}$ of positive numbers $\lim_{n\to\infty} \alpha_n = 0$ if and only if $\lim_{n\to\infty} F(\alpha_n) = -\infty$;

(F3) there exists $k \in (0,1)$ such that $\lim_{\alpha\to 0^+} \alpha^k F(\alpha) = 0$.

Definition 2.9 ([25]). Let (X,d) be a metric space and a mapping $T : X \to X$ is said to be an F-contraction on X if $F \in F^*$ and there exists $\tau > 0$ such that

$$\forall\, x, y \in X, [d(Tx,Ty) > 0 \quad \Rightarrow \quad \tau + F(d(Tx,Ty)) \leq F(d(x,y))].$$

Example 2.10 ([25]). The following function $F : R^+ \to R$ in F^* :

(1) $F_1(t_1) = \ln t_1$, with $t_1 > 0$,

$$\forall\, x, y \in X, \quad d(Tx,Ty) \leq e^{-\tau} d(x,y)$$

(2) $F_2(t_2) = \ln t_2 + t_2$, with $t_2 > 0$,

$$\forall\, x, y \in X, \quad \frac{d(Tx,Ty)}{d(x,y)} e^{d(Tx,Ty) - d(x,y)} \leq e^{-\tau}$$

(3) $F_3(t_3) = \dfrac{-1}{\sqrt{t_3}}$, with $t_3 > 0$,

$$\forall\, x, y \in X, \quad d(Tx,Ty) \leq \frac{1}{\left(1 + \tau\sqrt{d(x,y)}\right)^2} d(x,y).$$

Remark 2.11. Warodowski [25] concluded that every F-contraction T is a contractive mapping, i.e., $d(Tx,Ty) < d(x,y)$, for all $x,y \in X$, $Tx \neq Ty$. Hence, every F-contraction is a continuous mapping.

3. Main Results

In this part, in the framework of a b-metric space, we state and prove the existence result of an α-fuzzy fixed point theorem for multivalued fuzzy F-contraction mappings as follows:

Theorem 3.1. *Let (X,d) be a complete b-metric space and coefficient $s \geq 1$, let $T : X \to \mathscr{F}(X)$ be a fuzzy mapping and $\alpha : X \to (0,1]$ such that $[Tu]_{\alpha(u)}$ is a nonempty closed subset of X for all $u \in X$ and $F \in F^*$ if there exists $\tau > 0$ such that for all $u,v \in X, H\left([Tu]_{\alpha(u)},[Tv]_{\alpha(v)}\right) > 0$ implies*

$$\tau + F\left(H\left([Tu]_{\alpha(u)},[Tv]_{\alpha(v)}\right)\right) \leq F(d(u,v)), \tag{3.1}$$

then T has an α-fuzzy fixed point.

Proof. Let $u_0 \in X$ and $u_1 \in [Tu_0]_{\alpha(u_0)}$. Since $[Tu_1]_{\alpha(u_1)}$ is a nonempty closed subset of X. Clearly, if $u_0 = u_1$ or $u_1 \in [Tu_1]_{\alpha(u_1)}$, so u_1 is an α-fuzzy fixed point of T. So the proof is complete. Suppose that $u_0 \neq u_1$ and $u_1 \notin [Tu_1]_{\alpha(u_1)}$. Then, since $[Tu_1]_{\alpha(u_1)}$ is closed, $d\left(u_1,[Tu_1]_{\alpha(u_1)}\right) > 0$. On the other hand, from

$$d\left(u_1,[Tu_1]_{\alpha(u_1)}\right) \leq H\left([Tu_0]_{\alpha(u_0)},[Tu_1]_{\alpha(u_1)}\right)$$

and by (F1), we have

$$F\left(d(u_1,[Tu_1]_{\alpha(u_1)})\right) \leq F\left(H\left([Tu_0]_{\alpha(u_0)},[Tu_1]_{\alpha(u_1)}\right)\right).$$

From (3.1), we can write that

$$F\left(d(u_1,[Tu_1]_{\alpha(u_1)})\right) \leq F\left(H\left([Tu_0]_{\alpha(u_0)},[Tu_1]_{\alpha(u_1)}\right)\right) \leq F(d(u_1,u_0)) - \tau. \tag{3.2}$$

Since, $[Tu_1]_{\alpha(u_1)}$ is a nonempty closed subset of X. We obtain that there exists $u_2 \in [Tu_1]_{\alpha(u_1)}$ and $u_1 \neq u_2$ such that

$$d(u_1,u_2) = d\left(u_1,[Tu_1]_{\alpha(u_1)}\right).$$

Then, from (3.2), we get

$$F(d(u_1,u_2) \leq F\left(H\left([Tu_0]_{\alpha(u_0)},[Tu_1]_{\alpha(u_1)}\right)\right) \leq F(d(u_1,u_0)) - \tau. \tag{3.3}$$

By induction, we obtain a sequence $\{u_n\}$ in X such that $u_{n+1} \in [Tu_n]_{\alpha(u_n)}$ and

$$F(d(u_n,u_{n+1})) \leq F(d(u_n,u_{n-1})) - \tau \tag{3.4}$$

for all $n = 0,1,2,\dots$ If there exists $n_0 \in N$ for which $\{u_{n_0}\} \in [Tu_{n_0}]_{\alpha(u_{n_0})}$, then $\{u_{n_0}\}$ is an α-fuzzy fixed point of T and so the proof is complete. Thus, suppose that for every $n \in N$, $\{u_n\} \notin [Tu_n]_{\alpha(u_n)}$. Let $c_n := d(u_n,u_{n+1})$ for $n = 0,1,2,\dots$ then $c_n > 0$ for all $n \in N$ and using (3.4), the following hold:

$$\begin{aligned} F(c_n) &\leq F(c_{n-1}) - \tau \\ &\leq F(c_{n-2}) - 2\tau \\ &\ \vdots \\ &\leq F(c_0) - n\tau. \end{aligned} \tag{3.5}$$

Since, $F \in F^*$, from (3.5), we obtain $\lim_{n\to\infty} F(c_n) = -\infty$. Thus, from (F2) we have

$$\lim_{n\to\infty} c_n = 0.$$

From (F3), there exists $k \in (0,1)$ such that

$$\lim_{n\to\infty} c_n^k F(c_n) = 0.$$

By (3.5), the following holds for all $n \in N$,

$$c_n^k F(c_n) - c_n^k F(c_0) \le -c_n^k n\tau \le 0. \tag{3.6}$$

By taking lim as $n \to \infty$ in (3.6), we obtain

$$\lim_{n\to\infty} n c_n^k = 0. \tag{3.7}$$

From (3.7), there exists $n_1 \in N$ such that

$$n c_n^k \le 1 \tag{3.8}$$

for all $n \ge n_1$. This implies that

$$c_n \le \frac{1}{n^{\frac{1}{k}}} \tag{3.9}$$

for all $n \ge n_1$.

Next, we show that $\{u_n\}_{n=1}^{\infty}$ is a Cauchy sequence.

Let $m,n \in N$ with $m > n$, we have

$$\begin{aligned} d(u_n,u_m) &\le s[d(u_n,u_{n+1}) + d(u_{n+1},u_{n+2}) + \cdots + d(u_{m-1},u_m)] \\ &= s(c_n) + s(c_{n+1}) + \cdots s(c_{m-1}) \\ &= s\sum_{i=n}^{m-1} c_i \\ &\le s\sum_{i=n}^{\infty} c_i \\ &\le s\sum_{i=1}^{\infty} \frac{1}{i^{\frac{1}{k}}}. \end{aligned}$$

By the convergence of the series $\sum_{i=1}^{\infty} \frac{1}{i^{\frac{1}{k}}}$ passing to lim $n \to \infty$, we get

$$d(u_n,u_m) \to 0 \ \text{ as } \ n \to \infty.$$

Hence, $\{u_n\}_{n=1}^{\infty}$ is a Cauchy sequence in X. Since (X,d) be a complete b-metric space, the sequence $\{u_n\}$ converge to some point $u^* \in X$ that is, from (F1), for all $u,v \in X$ with

$$F\left(H\left([Tu]_{\alpha(u)},[Tv]_{\alpha(v)}\right)\right) < F(d(u,v))$$

and so

$$H\left([Tu]_{\alpha(u)},[Tv]_{\alpha(v)}\right) \le d(u,v)$$

for all $u,v \in X$. Then

$$\begin{aligned} d\left(u_{n+1},[Tu^*]_{\alpha(u^*)}\right) &\le H\left([Tu_n]_{\alpha(u_n)},[Tu^*]_{\alpha(u^*)}\right) \\ &\le d\left(u_n,u^*\right). \end{aligned}$$

On the other hand, we have

$$\begin{aligned} d\left(u^*,[Tu^*]_{\alpha(u^*)}\right) &\le sd\left(u^*,[Tu_n]_{\alpha(u_n)}\right)+sd\left([Tu_n]_{\alpha(u_n)},[Tu^*]_{\alpha(u^*)}\right)\\ &= sd\left(u^*,u_{n+1}\right)+sd\left(u_{n+1},[Tu^*]_{\alpha(u^*)}\right)\\ &\le sd\left(u^*,u_{n+1}\right)+sd\left(u_n,u^*\right). \end{aligned}$$

Passing to lim $n\to\infty$, we have

$$d\left(u^*,[Tu^*]_{\alpha(u^*)}\right)=0.$$

Thus, we get $u^*\in[Tu^*]_{\alpha(u^*)}$, that is, u^* is an α-fuzzy fixed point of T. This complete the proof. □

Corollary 3.2. *Let (X,d) be a complete metric space, $T:X\to\mathscr{F}(X)$ be a fuzzy mapping and $\alpha:X\to(0,1]$ such that $[Tu]_{\alpha(u)}$ is a nonempty closed bounded subset of X for all $u\in X$ and $F\in F^*$ if there exists $\tau>0$ such that for all $u,v\in X, H\left([Tu]_{\alpha(u)},[Tv]_{\alpha(v)}\right)>0$ implies*

$$\tau+F\left(H\left([Tu]_{\alpha(u)},[Tv]_{\alpha(v)}\right)\right)\le F(d(u,v)),$$

then T has an α-fuzzy fixed point.

Remark 3.3. In Corollary 3.2, we set $s=1$, so b-metric spaces it is turns into complete metric spaces.

Corollary 3.4. *Let (X,d) be a complete b-metric space, let coefficient $s\ge1$ and $T:X\to K(X)$ be a multivalued mapping such that Tu is a nonempty closed subset of X for all $u\in X$ and $F\in F^*$ if there exists $\tau>0$ such that*

$$\tau+F(H(Tu,Tv))\le F(d(u,v)),$$

for all $u,v\in X$, then T has a fixed point in X.

Remark 3.5. In Corollary 3.4, if we set $s=1$, we find theorem of Altun *et al.* [3]. Therefore, Corollary 3.4 is and extension the result of Altun *et al.* [3].

4. Conclusion

In this work, we first suggest the new concept of multivalued fuzzy F-contraction mappings. We also prove the existence of an α-fuzzy fixed point theorem in b-metric spaces. Our results improve and extend some fixed point results for multivalued mappings in b-metric spaces and also extension the result of Altun *et al.* [3].

Acknowledgements

This work was supported by the Higher Education Research Promotion and National Research University Project of Thailand (NRU). The first author was supported by the Petchra Pra Jom Klao Doctoral Scholarship for NRU Research Cluster. The second author was supported by the Theoretical and Computational Science (TaCS) Center (Project Grant No.Tacs2559-2).

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] H.M. Abu-Donia, Common fixed point theorems for fuzzy mappings in metrics space under ϕ-contraction condition, *Chaos Solitons Fractals* **44** (2007), 538–543.

[2] R.P. Agarwal, D.O. O'Regan and N. Shahzad, Fixed point theorem for generalized contractive maps of Meir-Keeler type, *Mathematische Nachrichten* **276** (2004), 3–22.

[3] I. Altun, G. Minak and H. Dag, Multivalued F-contractions on complete metric spaces, *Journal of Nonlinear and Convex Analysis* **16** (4) (2015), 659–666.

[4] A. Azam, M. Arshad and I. Beg, Fixed point of fuzzy contractive and fuzzy locally contractive maps, *Chaos Solitons Fractals* **42** (2009), 2836–2841.

[5] I.A. Bakhtin, The contraction mapping principle in quasi-metric spaces, *Funct. Anal. Unianowsk Gos. Ped. Inst.* **30** (1989), 26–37.

[6] S. Banach, Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales, *Fundam. Math.* **3** (1922), 133–181.

[7] T.G. Bhaskar and V. Lakshmikantham, Fixed point theory in partially ordered metric spaces and applications, *Nonlinear Anal., Theory Methods Appl.* **65** (2006), 1379–1393.

[8] M. Boriceanu, Fixed point theory for multivalued generalized contraction on a set with two b-metrics, *Studia Univ Babes, Bolya: Math.* **LIV** (3) (1989), 1–14.

[9] M. Boriceanu, M. Bota and A. Petru, Multivalued fractals in b-metrics, *Central European J. Math.* **8** (2010), 367–377.

[10] A. Branciari, A fixed point theorem for mappings satisfying a general contractive condition of integral type, *International Journal of Mathematical and Mathematical Sciences* **29** (2002), 531–536.

[11] D. Butnariu, Fixed points for fuzzy mappings, *Fuzzy Sets and Systems* **7** (1982), 191–207.

[12] S. Czerwik, Contraction mapping in b-metrics, *Acta Mathematica et Informatica Universitatis Ostraviensis* **1** (1993), 5–11.

[13] S. Czerwik, Nonlinear set-valued contraction mappings in b-metrics, *Atti Sem. Mat. Univ. Modena* **46** (1998), 263–276.

[14] S. Czerwik, K. Dlutek and S.L. Singh, Round-off stability of iteration procedures for operator in b-metrics, *J. Natur. Phys. Sci.* **11** (1997), 87–94.

[15] V.D. Estruch and A. Vidal, A note on fixed fuzzy point for fuzzy mappings, *Rend Ist. Mat. Univ. Trieste* **32** (2001), 39–45.

[16] Y. Feng and S. Liu, Fixed point theorem for multi-valued mappings and multi-valued Caristi type mapping, *Journal of Mathematical Analysis and Application* **317** (2006), 103–112.

[17] S. Heilpern, Fuzzy mappings and fixed point theorem, *Journal for Mathematical Analysis and Application* **83** (1981), 566–569.

[18] A. Ishak, M. Gülhan and D. Hacer, Multivalued F-contractions on complete metric spaces, *Journal of Nonlinear and Convex Anlysis* **16** (2015), 659–666.

[19] E. Karapinar, M.A. Ksutbi, H. Piri and D. O'Regan, Fixed points of conditionally F-contractions in complete metric-like spaces, *Journal of Nonlinear and Convex Anlysis* **16** (2015), 659–666.

[20] D. Klim and D. Wardowski, Fixed points of dynamic processes of set-valued F-contractions and application to functional equations, *Fixed Point Theory and Applications* **22** (2015), 9 p.

[21] S.B. Nadler, Jr., Multivalued contraction mappings, *Pacific Journal of Mathematicals* **30** (1969), 475–488.

[22] A. Padcharoen, D. Gopal, P. Chaipunya and P. Kumam, Fixed point and periodic point results for α-type F-contractions in modular metric spaces, *Fixed Point Theory and Applications* **39** (2016), 12 p.

[23] S. Phiangsungnoen and P. Kumam, Fuzzy fixed point theorem for multivalued fuzzy contractions in b-metric spaces, *J. Nonlinear Sci. Appl.* **8** (2015), 55–63.

[24] D. Turkoglu and B.E. Rhoades, A fixed fuzzy point for fuzzy mapping in complete metrics spaces, *Fixed Point Theory Appl.* **94** (2012), 6 p.

[25] D. Wardowski, Fixed point theory of a new type of contractive mappings in complete metric spaces, *Fixed Point Theory Appl.* **94** (2012), 6 p.

[26] D. Wardowski and N.V. Dung, Fixed points of F-weak contractions on complete metric spaces, *Demonstr. Math.* **1** (2014), 146–155.

[27] L.A. Zadeh, Fuzzy sets, *Information and Control* **8** (1965), 338–353.