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The Solutions of the Periodic Rational Recursive Systems

Research Article

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Abstract. In this study, we obtain the solutions of some periodic rational difference equation systems. Then we examine the period of solutions of these systems.

Keywords. Periodic discrete system; Non-linear discrete system

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1. Introduction

Rational recursive systems are also called rational difference equation systems. These systems have been studied in many branches of mathematics as well as other sciences for the years. There are many applications in several branches of science [1, 2, 3, 4, 7, 9]. These types seem very simple and some of their properties can also be observed and conjectured by computers simulations, however, it is extremely difficult to prove completely the properties observed and conjectured by computers simulation, for example, see [5, 6, 10]. In this study, we consider the following systems by using the results in [8]

$$x_{n+1}=\frac{Ay_{n-1}}{y_n(y_{n-2}+z_{n-2})}+\frac{B}{y_{n-1}+z_{n-1}},\quad y_{n+1}=\frac{B}{y_{n-1}+z_{n-1}},\tag{1.1}$$

$$z_{n+1}=\frac{C}{x_{n-1}-y_{n-1}}-\frac{B}{y_{n-1}+z_{n-1}},\quad (0\le n),\tag{1.2}$$

with initial values $x_{-1},x_0,y_{-2},y_{-1},y_0,z_{-2},z_{-1},z_0$, $A,B,C\in\mathbb{R}^+$, $(x_{-1}\neq y_{-1},\ x_0\neq y_0)$ and

$$x_{n+1}=\frac{Ay_{n-1}}{y_n(y_{n-3}+z_{n-3})}+\frac{B}{y_{n-2}+z_{n-2}},\quad y_{n+1}=\frac{B}{y_{n-2}+z_{n-2}},\tag{1.3}$$

$$z_{n+1}=\frac{C}{x_{n-2}-y_{n-2}}-\frac{B}{y_{n-2}+z_{n-2}},\quad (0\le n), \tag{1.4}$$

with initial values $x_{-2},x_{-1},x_0,y_{-3},y_{-2},y_{-1},y_0,z_{-3},z_{-2},z_{-1},z_0,A,B,C\in\mathbb{R}^+$, $(x_{-2}\ne y_{-2},x_{-1}\ne y_{-1},x_0\ne y_0)$.

Firstly, we give basic preliminary definitions and a theorem. Let I_1,I_2 and I_3 be some intervals of real numbers and let $F_1:I_2\times I_3\to I_1,F_2:I_2\times I_3\to I_2$ and $F_3:I_1\times I_2\times I_3\to I_3$ be three continuously differentiable functions. For every initial condition $(x_s,y_s,z_s)\in I_1\times I_2\times I_3$, it is obvious that the system of difference equations (1.3)

$$x_{n+1}=F_1(y_n,z_n),\quad y_{n+1}=F_2(y_n,z_n),\quad z_{n+1}=F_3(x_n,y_n,z_n) \tag{1.5}$$

has a unique solution $\{x_n,\ y_n,\ z_n\}$.

- A solution $\{x_n,y_n,z_n\}$ of the system of difference equations (1.3) is periodic if there exist a positive integer p such that

  $$x_{n+p}=x_n,\quad y_{n+p}=y_n,\quad z_{n+p}=z_n,$$

  the smallest such positive integer p is called the prime period of the solution of difference equation system (1.3).
- A point $(\bar{x},\bar{y},\bar{z})\in I_1\times I_2\times I_3$ is called an equilibrium point of system (1.3), if

  $$\bar{x}=F_1(\bar{y},\bar{z}),\quad \bar{y}=F_2(\bar{y},\bar{z}),\quad \bar{z}=F_3(\bar{x},\bar{y},\bar{z}).$$

- The equilibrium point $(\bar{x},\bar{y},\bar{z})$ of the difference equation system (1.3) is called stable (or locally stable), if for every $\varepsilon>0$, there exist $\delta>0$ such that for all $(x_s,y_s,z_s)\in I_1\times I_2\times I_3$ with

  $$\|(x_s,y_s,z_s)-(\bar{x},\bar{y},\bar{z})\|<\delta \quad \text{implies} \quad \|(x_n,y_n,z_n)-(\bar{x},\bar{y},\bar{z})\|<\varepsilon$$

  for all $n\ge 0$. Otherwise equilibrium point is called unstable.
- The equilibrium point $(\bar{x},\bar{y},\bar{z})$ of the difference equation system (1.3) is called asymptotically stable (or locally asymptotically stable), if it is stable and there exist $\gamma>0$ such that for all $(x_s,y_s,z_s)\in I_1\times I_2\times I_3$ with

  $$\|(x_s,y_s,z_s)-(\bar{x},\bar{y},\bar{z})\|<\gamma \quad \text{implies} \quad \lim_{n\to\infty}\|(x_n,y_n,z_n)-(\bar{x},\bar{y},\bar{z})\|=0.$$

- The equilibrium point $(\bar{x},\bar{y},\bar{z})$ of the difference equation system (1.3) is called global asymptotically stable, if it is stable and for every $(x_s,y_s,z_s)\in I_1\times I_2\times I_3$ we have

  $$\lim_{n\to\infty}\|(x_n,y_n,z_n)-(\bar{x},\bar{y},\bar{z})\|=0.$$

- Let $I_1\times I_2\times I_3$ be an interval of real numbers. For all initial values $x_{-2},x_{-1},x_0\in I_1$, $y_{-3},y_{-2},y_{-1},y_0\in I_2$ and $z_{-3},z_{-2},z_{-1},z_0\in I_3$, $(x_{-2}\ne y_{-2},x_{-1}\ne y_{-1},x_0\ne y_0)$, if we have

  $$\lim_{n\to\infty}\|(x_n,y_n,z_n)\|=(\bar{x},\bar{y},\bar{z}),$$

  then the equilibrium point $(\bar{x},\bar{y},\bar{z})$ of the system of difference equations (1.3) is global attractor [7, 9].

2. Main results

In this section all results have been obtained by using [8]. The following theorems show us the period of solutions of the systems (1.1)-(1.2) and (1.3)-(1.4).

Theorem 2.1. *Suppose that* $\{x_n, y_n, z_n\}$ *are the solutions of the difference equation system* (1.1)-(1.2)*. All solutions of the system* (1.1)-(1.2) *are periodic with period* 6 *if and only if* $A = C$.

Proof. From the system (1.1)-(1.2), it is obtained the following equalities

$$x_{n+1} = \frac{Ay_{n-1}}{y_n(y_{n-2}+z_{n-2})} + \frac{B}{y_{n-1}+z_{n-1}},$$
$$y_{n+1} = \frac{B}{y_{n-1}+z_{n-1}},$$
$$z_{n+1} = \frac{C}{x_{n-1}-y_{n-1}} - \frac{B}{y_{n-1}+z_{n-1}},$$
$$x_{n+2} = \frac{Ay_n}{B} + \frac{B}{y_n+z_n},$$
$$y_{n+2} = \frac{B}{y_n+z_n},$$
$$z_{n+2} = \frac{C}{x_n-y_n} + \frac{B}{y_n+z_n},$$
$$x_{n+3} = \frac{A}{y_{n-1}+z_{n-1}} + \frac{B(x_{n-1}-y_{n-1})}{C},$$
$$y_{n+3} = \frac{B(x_{n-1}-y_{n-1})}{C},$$
$$z_{n+3} = \frac{Cy_n(y_{n-2}+z_{n-2})}{Ay_{n-1}} - \frac{B(x_{n-1}-y_{n-1})}{C},$$
$$x_{n+4} = \frac{A}{y_n+z_n} + \frac{B(x_n-y_n)}{C},$$
$$y_{n+4} = \frac{B(x_n-y_n)}{C},$$
$$z_{n+4} = \frac{BC}{Ay_n} - \frac{B(x_n-y_n)}{C},$$
$$x_{n+5} = \frac{A(x_{n-1}-y_{n-1})}{C} + \frac{BAy_{n-1}}{Cy_n(y_{n-2}+z_{n-2})},$$
$$y_{n+5} = \frac{BAy_{n-1}}{Cy_n(y_{n-2}+z_{n-2})},$$
$$z_{n+5} = \frac{C(y_{n-1}+z_{n-1})}{A} - \frac{BAy_{n-1}}{Cy_n(y_{n-2}+z_{n-2})},$$
$$x_{n+6} = \frac{A}{C}(x_n-y_n) + \frac{A}{C}y_n = \frac{A}{C}x_n,$$
$$y_{n+6} = \frac{A}{C}y_n, \; z_{n+6} = \frac{C}{A}(y_n+z_n) - \frac{A}{C}y_n.$$

From $A = C$, all solutions of the system (1.1)-(1.2) are periodic with 6 period. Thus we have

$$x_{n+6} = x_n, \quad y_{n+6} = y_n, \quad z_{n+6} = z_n. \qquad \square$$

Theorem 2.2. *Suppose that* $\{x_n, y_n, z_n\}$ *are the solutions of the difference equation system* (1.3)-(1.4)*. All solutions of the system* (1.3)-(1.4) *are periodic with period* 8 *if and only if* $A = C$.

Proof. From the system (1.3)-(1.4), it is obtained the following equalities

$$\begin{aligned}
x_{n+1} &= \frac{A y_{n-1}}{y_n(y_{n-3}+z_{n-3})} + \frac{B}{y_{n-2}+z_{n-2}}, \\
y_{n+1} &= \frac{B}{y_{n-2}+z_{n-2}}, \\
z_{n+1} &= \frac{C}{x_{n-2}-y_{n-2}} - \frac{B}{y_{n-2}+z_{n-2}}, \\
x_{n+2} &= \frac{A y_n}{B} + \frac{B}{y_{n-1}+z_{n-1}}, \\
y_{n+2} &= \frac{B}{y_{n-1}+z_{n-1}}, \\
z_{n+2} &= \frac{C}{x_{n-1}-y_{n-1}} - \frac{B}{y_{n-1}+z_{n-1}}, \\
x_{n+3} &= \frac{A}{y_{n-2}+z_{n-2}} + \frac{B}{y_n+z_n}, \\
y_{n+3} &= \frac{B}{y_n+z_n}, \\
z_{n+3} &= \frac{C}{x_n-y_n} - \frac{B}{y_n+z_n}, \\
x_{n+4} &= \frac{A}{y_{n-1}+z_{n-1}} + \frac{B(x_{n-2}-y_{n-2})}{C}, \\
y_{n+4} &= \frac{B(x_{n-2}-y_{n-2})}{C}, \\
z_{n+4} &= \frac{C y_n(y_{n-3}+z_{n-3})}{A y_{n-1}} - \frac{B(x_{n-2}-y_{n-2})}{C}, \\
x_{n+5} &= \frac{A}{y_n+z_n} + \frac{B(x_{n-1}-y_{n-1})}{C}, \\
y_{n+5} &= \frac{B(x_{n-1}-y_{n-1})}{C}, \\
z_{n+5} &= \frac{CB}{A y_n} - \frac{B(x_{n-1}-y_{n-1})}{C}, \\
x_{n+6} &= \frac{A(x_{n-2}-y_{n-2})}{C} + \frac{B(x_n-y_n)}{C}, \\
y_{n+6} &= \frac{B(x_n-y_n)}{C},
\end{aligned}$$

$$
\begin{aligned}
z_{n+6} &= \frac{C(y_{n-2}+z_{n-2})}{A} - \frac{B(x_n-y_n)}{C},\\
x_{n+7} &= \frac{A(x_{n-1}-y_{n-1})}{C} + \frac{ABy_{n-1}}{Cy_n(y_{n-3}+z_{n-3})},\\
y_{n+7} &= \frac{ABy_{n-1}}{Cy_n(y_{n-3}+z_{n-3})},\\
z_{n+7} &= \frac{C(y_{n-1}+z_{n-1})}{A} - \frac{ABy_{n-1}}{Cy_n(y_{n-3}+z_{n-3})},\\
x_{n+8} &= \frac{A}{C}(x_n-y_n) + \frac{A}{C}y_n = \frac{A}{C}x_n,\\
y_{n+8} &= \frac{A}{C}y_n,\ z_{n+8} = \frac{C}{A}(y_n+z_n) - \frac{A}{C}y_n.
\end{aligned}
$$

From $A=C$, all solutions of the system (1.3)-(1.4) are periodic with 8 period. Thus we have

$$x_{n+8}=x_n, \quad y_{n+8}=y_n, \quad z_{n+8}=z_n. \qquad \square$$

Theorem 2.3. *Suppose that* $\{x_n, y_n, z_n\}$ *are the solutions of the difference equation system* (1.1)-(1.2) *with initial values* $x_{-1}=a$, $x_0=b$, $y_{-2}=c$, $y_{-1}=d$, $y_0=e$, $z_{-2}=f$, $z_{-1}=g$, $z_0=h$, A,B,C $(x_{-1}\neq y_{-1},\ x_0\neq y_0)\in\mathbb{R}^+$. *For* $n\geq 0$, *all solutions of the system* (1.1)-(1.2) *are periodic with period* 6

$$
\begin{aligned}
x_{6n+1} &= \frac{Ad}{e(c+f)} + \frac{B}{d+g},\\
y_{6n+1} &= \frac{B}{d+g},\\
z_{6n+1} &= \frac{A}{a-d} - \frac{B}{d+g},\\
x_{6n+2} &= \frac{Ae}{B} + \frac{B}{e+h},\\
y_{6n+2} &= \frac{B}{e+h},\\
z_{6n+2} &= \frac{A}{b-e} + \frac{B}{e+h},\\
x_{6n+3} &= \frac{A}{d+g} + \frac{B(a-d)}{A},\\
y_{6n+3} &= \frac{B(a-d)}{A},\\
z_{6n+3} &= \frac{Ae(c+f)}{Ad} - \frac{B(a-d)}{A},\\
x_{6n+4} &= \frac{A}{e+h} + \frac{B(b-e)}{A},\\
y_{6n+4} &= \frac{B(b-e)}{A},\\
z_{8n+4} &= \frac{B}{e} - \frac{B(b-e)}{A},
\end{aligned}
$$

$$x_{6n+5} = (b-e) + \frac{Bd}{e(c+f)},$$
$$y_{6n+5} = \frac{Bd}{e(c+f)},$$
$$z_{6n+5} = (d+g) - \frac{Bd}{e(c+f)},$$
$$x_{6n+6} = b,$$
$$y_{6n+6} = e,$$
$$z_{6n+6} = h.$$

if and only if $A = C$.

Proof. Let us use the principle of mathematical induction on n. For $n = 0$, it is easy to see. Assume that it is true for all positive integers n. From the system (1.1)-(1.2), it is obtained the following equalities

$$x_{6n+7} = \frac{Ay_{6n+5}}{y_{6n+6}(y_{6n+4}+z_{6n+4})} + \frac{B}{y_{6n+5}+z_{6n+5}} = \frac{Ad}{e(c+f)} + \frac{B}{d+g},$$
$$y_{6n+7} = \frac{B}{y_{6n+5}+z_{6n+5}} = \frac{B}{d+g},$$
$$z_{6n+7} = \frac{C}{x_{6n+5}-y_{6n+5}} - \frac{B}{y_{6n+5}+z_{6n+5}} = \frac{A}{a-d} - \frac{B}{d+g},$$
$$x_{6n+8} = \frac{Ay_{6n+6}}{B} + \frac{B}{y_{6n+6}+z_{6n+6}} = \frac{Ae}{B} + \frac{B}{e+h},$$
$$y_{6n+8} = \frac{B}{y_{6n+6}+z_{6n+6}} = \frac{B}{e+h},$$
$$z_{6n+8} = \frac{C}{x_{6n+6}-y_{6n+6}} + \frac{B}{y_{6n+6}+z_{6n+6}} = \frac{A}{b-e} + \frac{B}{e+h},$$
$$x_{6n+9} = \frac{A}{y_{6n+5}+z_{6n+5}} + \frac{B(x_{6n+5}-y_{6n+5})}{C} = \frac{A}{d+g} + \frac{B(a-d)}{A},$$
$$y_{6n+9} = \frac{B(x_{6n+5}-y_{6n+5})}{C} = \frac{B(a-d)}{A},$$
$$z_{6n+9} = \frac{Cy_{6n+6}(y_{6n+4}+z_{6n+4})}{Ay_{n-1}} - \frac{B(x_{6n+5}-y_{6n+5})}{C} = \frac{Ae(c+f)}{Ad} - \frac{B(a-d)}{A},$$
$$x_{6n+10} = \frac{A}{y_{6n+6}+z_{6n+6}} + \frac{B(x_{6n+6}-y_{6n+6})}{C} = \frac{A}{e+h} + \frac{B(b-e)}{A},$$
$$y_{6n+10} = \frac{B(x_{6n+6}-y_{6n+6})}{C} = \frac{B(b-e)}{A},$$
$$z_{6n+10} = \frac{BC}{Ay_{6n+6}} - \frac{B(x_{6n+6}-y_{6n+6})}{C} = \frac{B}{e} - \frac{B(b-e)}{A},$$
$$x_{6n+11} = \frac{A(x_{6n+5}-y_{6n+5})}{C} + \frac{BAy_{6n+5}}{Cy_{6n+6}(y_{6n+4}+z_{6n+4})} = (b-e) + \frac{Bd}{e(c+f)},$$

$$y_{6n+11} = \frac{BAy_{6n+5}}{Cy_{6n+6}(y_{6n+4}+z_{6n+4})} = \frac{Bd}{e(c+f)},$$
$$z_{6n+11} = \frac{C(y_{6n+5}+z_{6n+5})}{A} - \frac{BAy_{6n+5}}{Cy_{6n+6}(y_{6n+4}+z_{6n+4})} = (d+g) - \frac{Bd}{e(c+f)},$$
$$x_{6n+12} = \frac{A}{C}x_{6n+6},$$
$$y_{6n+12} = \frac{A}{C}y_{6n+6},$$
$$z_{6n+12} = \frac{C}{A}(y_{6n+6}+z_{6n+6}) - \frac{A}{C}y_{6n+6}.$$

From $A = C$, all solutions of the system (1.1)-(1.2) are periodic with 6 period. Thus we have

$$x_{6n+12} = b, \quad y_{6n+12} = e, \quad z_{6n+12} = h.$$

Therefore we have the required formulates on n. □

Theorem 2.4. *Suppose that* $\{x_n, y_n, z_n\}$ *are the solutions of the difference equation system* (1.3)-(1.4) *with initial values* $x_{-2} = a$, $x_{-1} = b$, $x_0 = c$, $y_{-3} = d$, $y_{-2} = e$, $y_{-1} = f$, $y_0 = g$, $z_{-3} = h$, $z_{-2} = p$, $z_{-1} = r$, $z_0 = s$, A, B, C $(x_{-2} \neq y_{-2}, x_{-1} \neq y_{-1},\ x_0 \neq y_0) \in \mathbb{R}^+$. *For* $n \geq 0$, *all solutions of the system* (1.3)-(1.4) *are periodic with period* 8

$$x_{8n+1} = \frac{Af}{g(d+h)} + \frac{B}{e+p},$$
$$y_{8n+1} = \frac{B}{e+p},$$
$$z_{8n+1} = \frac{A}{a-e} - \frac{B}{e+p},$$
$$x_{8n+2} = \frac{Ag}{B} + \frac{B}{f+r},$$
$$y_{8n+2} = \frac{B}{f+r},$$
$$z_{8n+2} = \frac{A}{b-f} - \frac{B}{f+r},$$
$$x_{8n+3} = \frac{A}{e+p} + \frac{B}{g+s},$$
$$y_{8n+3} = \frac{B}{g+s},$$
$$z_{8n+3} = \frac{A}{c-g} - \frac{B}{g+s},$$
$$x_{8n+4} = \frac{A}{f+r} + \frac{B(a-e)}{A},$$
$$y_{8n+4} = \frac{B(a-e)}{A},$$

$$z_{8n+4}=\frac{g(d+h)}{f}-\frac{B(a-e)}{A},$$
$$x_{8n+5}=\frac{A}{g+s}+\frac{B(b-f)}{A},$$
$$y_{8n+5}=\frac{B(b-f)}{A},$$
$$z_{8n+5}=\frac{B}{g}-\frac{B(b-f)}{A},$$
$$x_{8n+6}=(a-e)+\frac{B(c-g)}{A},$$
$$y_{8n+6}=\frac{B(c-g)}{A},$$
$$z_{8n+6}=(e+p)-\frac{B(c-g)}{A},$$
$$x_{8n+7}=(b-f)+\frac{Bf}{g(d+h)},$$
$$y_{8n+7}=\frac{Bf}{g(d+h)},$$
$$z_{8n+7}=(f+r)-\frac{Bf}{g(d+h)},$$
$$x_{8n+8}=c,$$
$$y_{8n+8}=g,$$
$$z_{8n+8}=s.$$

if and only if $A=C$.

Proof. Let us use the principle of mathematical induction on n. For $n=0$, it is easy to see. Assume that it is true for all positive integers n. From the system (1.3)-(1.4), it is obtained the following equalities

$$x_{8n+9}=\frac{Ay_{8n+7}}{y_{8n+8}(y_{8n+5}+z_{8n+5})}+\frac{B}{y_{8n+6}+z_{8n+6}}=\frac{Af}{g(d+h)}+\frac{B}{e+p},$$
$$y_{8n+9}=\frac{B}{y_{8n+6}+z_{8n+6}}=\frac{B}{e+p},$$
$$z_{8n+9}=\frac{C}{x_{8n+6}-y_{8n+6}}-\frac{B}{y_{8n+6}+z_{8n+6}}=\frac{A}{a-e}-\frac{B}{e+p},$$
$$x_{8n+10}=\frac{Ay_{8n+8}}{B}+\frac{B}{y_{8n+7}+z_{8n+7}}=\frac{Ag}{B}+\frac{B}{f+r},$$
$$y_{8n+10}=\frac{B}{y_{8n+7}+z_{8n+7}}=\frac{B}{f+r},$$
$$z_{8n+10}=\frac{C}{x_{8n+7}-y_{8n+7}}-\frac{B}{y_{8n+7}+z_{8n+7}}=\frac{A}{b-f}-\frac{B}{f+r},$$

$$x_{8n+11} = \frac{A}{y_{8n+6}+z_{8n+6}} + \frac{B}{y_{8n+8}+z_{8n+8}} = \frac{A}{e+p} + \frac{B}{g+s},$$
$$y_{8n+11} = \frac{B}{y_{8n+8}+z_{8n+8}} = \frac{B}{g+s},$$
$$z_{8n+11} = \frac{C}{x_{8n+8}-y_{8n+8}} - \frac{B}{y_{8n+8}+z_{8n+8}} = \frac{A}{c-g} - \frac{B}{g+s},$$
$$x_{8n+12} = \frac{A}{y_{8n+7}+z_{8n+7}} + \frac{B(x_{8n+6}-y_{8n+6})}{C} = \frac{A}{f+r} + \frac{B(a-e)}{A},$$
$$y_{8n+12} = \frac{B(x_{8n+6}-y_{8n+6})}{C} = \frac{B(a-e)}{A},$$
$$z_{8n+12} = \frac{Cy_{8n+8}(y_{8n+5}+z_{8n+5})}{Ay_{8n+7}} - \frac{B(x_{8n+6}-y_{8n+6})}{C} = \frac{g(d+h)}{f} - \frac{B(a-e)}{A},$$
$$x_{8n+13} = \frac{A}{y_{8n+8}+z_{8n+8}} + \frac{B(x_{8n+7}-y_{8n+7})}{C} = \frac{A}{g+s} + \frac{B(b-f)}{A},$$
$$y_{8n+13} = \frac{B(x_{8n+7}-y_{8n+7})}{C} = \frac{B(b-f)}{A},$$
$$z_{8n+13} = \frac{CB}{Ay_{8n+8}} - \frac{B(x_{8n+7}-y_{8n+7})}{C} = \frac{B}{g} - \frac{B(b-f)}{A},$$
$$x_{8n+14} = \frac{A(x_{8n+6}-y_{8n+6})}{C} + \frac{B(x_{8n+8}-y_{8n+8})}{C} = (a-e) + \frac{B(c-g)}{A},$$
$$y_{8n+14} = \frac{B(x_{8n+8}-y_{8n+8})}{C} = \frac{B(c-g)}{A},$$
$$z_{8n+14} = \frac{C(y_{8n+6}+z_{8n+6})}{A} - \frac{B(x_{8n+8}-y_{8n+8})}{C} = (e+p) - \frac{B(c-g)}{A},$$
$$x_{8n+15} = \frac{A(x_{8n+7}-y_{8n+7})}{C} + \frac{ABy_{8n+7}}{Cy_{8n+8}(y_{8n+5}+z_{8n+5})} = (b-f) + \frac{Bf}{g(d+h)},$$
$$y_{8n+15} = \frac{ABy_{8n+7}}{Cy_{8n+8}(y_{8n+5}+z_{8n+5})} = \frac{Bf}{g(d+h)},$$
$$z_{8n+15} = \frac{C(y_{8n+7}+z_{8n+7})}{A} - \frac{ABy_{8n+7}}{Cy_{8n+8}(y_{8n+5}+z_{8n+5})} = (f+r) - \frac{Bf}{g(d+h)},$$
$$x_{8n+16} = \frac{A}{C}c,$$
$$y_{8n+16} = \frac{A}{C}g,$$
$$z_{8n+16} = \frac{C}{A}(g+s) - \frac{A}{C}g.$$

From $A = C$, all solutions of the system (1.3)-(1.4) are periodic with 8 period. Thus we have

$$x_{8n+16} = c, \quad y_{8n+16} = g, \quad z_{8n+16} = s.$$

Therefore we have the required formulates on n. □

3. Conclusion

The difference systems given in (1.1)-(1.2) and (1.3)-(1.4) can be generalized to high order difference system. Thus the solutions and period of this new system can be examinated again.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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