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Research Article

On Neutrosophic Implicative Filters of BL-Algebra

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Abstract. We put forward the ideas of the neutrosophic implicative and n-fold implicative filters of BL-algebras. Additionally, we demonstrate that every implicative filter, including the n-fold implicative filter, is a neutrosophic filter. Moreover, we obtain an extension property for the neutrosophic implication. We then look at some comparable circumstances for neutrosophic implicative filters.

Keywords. BL-algebra, Filter, Neutrosophic filter, Neutrosophic implicative filter, Neutrosophic n-fold implicative filter

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1. Introduction

Ward and Dilworth [11] developed the concept of residuated lattices as a generalisation of the form of a ring's set of ideals. BL-algebras are the most well-known example of residuated lattices in logic. Hájek [2] created Basic Logic algebra (BL-algebra), a type of logical algebra, to offer an algebraic demonstration of completeness of '*Basic Logic*'. Xu and Qin [12] first proffered the conception of filter and implication filter in lattice implication algebras.

Filter theory is a crucial component of the study of innumerable logical algebras (Park and Ahn [7], and Zhang *et al*. [14]). They play a key role in the case made for the completeness of certain logical algebras. Researchers from several academic fields have looked into the conception of filters. Neutrosophy is acknowledged as a scientific study, investigates the origin, nature, and scope of neutralities (Salama and Alagamy [8], and Smarandache [9]). Fuzzy [13], intuitionistic fuzzy sets and logic are generalised as neutrosophic sets and neutrosophic logic (Atanassov [1]). Recently, the authors examined some of the features of the neutrosophic filter, neutrosophic fantastic filter of BL-algebras (Ibrahim and Gunaseeli [4,5]).

In Section 2, the basic notions and outcomes are recalled. In Section 3, we explore the concept of neutrosophic implicative filter. In Section 4, we exhibit the conception of neutrosophic n-fold implicative filter.

2. Preliminaries

In this part, few of the definitions and findings from the literature are referred to progress the major conclusions.

Definition 2.1 ([2, 3])**.** A BL-algebra $(\mathcal{G},\vee,\wedge,\circ,\rightarrow,0,1)$ of type $(2,2,2,2,0,0)$ such that the subsequent requirements are persuaded for all $g_1,h_1,i_1\in\mathcal{G}$,

(i) $(\mathcal{G},\vee,\wedge,0,1)$ is a bounded lattice,

(ii) $(\mathcal{G},\circ,1)$ is a commutative monoid,

(iii) '$\circ$' and '$\rightarrow$' form an adjoint pair, that is, $i_1\le g_1\rightarrow h_1$ if and only if $g_1\circ i_1\le h_1$, for all $g_1,h_1,i_1\in\mathcal{G}$,

(iv) $g_1\wedge h_1=g_1\circ(g_1\rightarrow h_1)$,

(v) $(g_1\rightarrow h_1)\vee(h_1\rightarrow g_1)=1$.

Proposition 2.2 ([6, 10])**.** *The succeeding requirements are persuaded in a BL-algebra $\mathcal{G}$ for all $g_1,h_1,i_1\in\mathcal{G}$,*

(i) $h_1\rightarrow(g_1\rightarrow i_1)=g_1\rightarrow(h_1\rightarrow i_1)=(g_1\circ h_1)\rightarrow i_1$,

(ii) $1\rightarrow g_1=g_1$,

(iii) $g_1\le h_1$ *if and only if* $g_1\rightarrow h_1=1$,

(iv) $g_1\vee h_1=((g_1\rightarrow h_1)\rightarrow h_1)\wedge((h_1\rightarrow g_1)\rightarrow g_1)$,

(v) $g_1\le h_1$ *implies* $h_1\rightarrow i_1\le g_1\rightarrow i_1$,

(vi) $g_1\le h_1$ *implies* $i_1\rightarrow g_1\le i_1\rightarrow h_1$,

(vii) $g_1\rightarrow h_1\le(i_1\rightarrow g_1)\rightarrow(i_1\rightarrow h_1)$,

(viii) $g_1\rightarrow h_1\le(h_1\rightarrow i_1)\rightarrow(g_1\rightarrow i_1)$,

(ix) $g_1\le(g_1\rightarrow h_1)\rightarrow h_1$,

(x) $g_1\circ(g_1\rightarrow h_1)=g_1\wedge h_1$,

(xi) $g_1\circ h_1\le g_1\wedge h_1$,

(xii) $g_1\rightarrow h_1\le(g_1\circ i_1)\rightarrow(h_1\circ i_1)$,

(xiii) $g_1\circ(h_1\rightarrow i_1)\le h_1\rightarrow(g_1\circ i_1)$,

(xiv) $(g_1\rightarrow h_1)\circ(h_1\rightarrow i_1)\le g_1\rightarrow i_1$,

(xv) $(g_1\circ g_1^*)=0$.

Definition 2.3 ([9]). A neutrosophic subset C of the universe U is a triple (T_C, I_C, F_C) where $T_C : U \to [0,1]$, $I_C : U \to [0,1]$ and $F_C : U \to [0,1]$ represents truth membership, indeterminacy and false membership functions respectively where $0 \le T_C(g_1) + I_C(g_1) + F_C(g_1) \le 3$, for all $g_1 \in U$.

Definition 2.4 ([5]). A neutrosophic set C of an algebra $\mathcal{G}$ is called a neutrosophic filter, if it persuades the following:

(i) $T_C(g_1) \le T_C(1)$, $I_C(g_1) \ge I_C(1)$ and $F_C(g_1) \ge F_C(1)$,

(ii) $\min\{T_C(g_1 \to h_1), T_C(g_1)\} \le T_C(h_1)$,
$\min\{I_C(g_1 \to h_1), I_C(g_1)\} \ge I_C(h_1)$, and
$\min\{F_C(g_1 \to h_1), F_C(g_1)\} \ge F_C(h_1)\}$, for all $g_1, h_1 \in \mathcal{G}$.

Proposition 2.5 ([5]). *Let C be a neutrosophic filter of $\mathcal{G}$ if and only if*

(i) *If $g_1 \le h_1$ then $T_C(g_1) \le T_C(h_1)$, $I_C(g_1) \ge I_C(h_1)$ and $F_C(g_1) \ge F_C(h_1)$,*

(ii) *$T_C(g_1 \circ h_1) \ge \min\{T_C(g_1), T_C(h_1)\}$, $I_C(g_1 \circ h_1) \le \min\{I_C(g_1), I_C(h_1)\}$ and $F_C(g_1 \circ h_1) \le \min\{F_C(g_1), F_C(h_1)\}$, for all $g_1, h_1 \in \mathcal{G}$.*

Proposition 2.6 ([4,5]). *Let C be a neutrosophic filter of $\mathcal{G}$, for all $g_1, h_1, i_1 \in \mathcal{G}$ then the following hold:*

(i) *$T_C(g_1 \to h_1) = T_C(1)$, then $T_C(g_1) \le T_C(h_1)$,*
$I_C(g_1 \to h_1) = I_C(1)$, then $I_C(g_1) \ge I_C(h_1)$,
$F_C(g_1 \to h_1) = F_C(1)$, then $F_C(g_1) \ge F_C(h_1)$,

(ii) *$T_C(g_1 \wedge h_1) = \min\{T_C(g_1), T_C(h_1)\}$,*
$I_C(g_1 \wedge h_1) = \min\{I_C(g_1), I_C(h_1)\}$,
$F_C(g_1 \wedge h_1) = \min\{F_C(g_1), F_C(h_1)\}$,

(iii) *$T_C(g_1 \circ h_1) = \min\{T_C(g_1), T_C(h_1)\}$,*
$I_C(g_1 \circ h_1) = \min\{I_C(g_1), I_C(h_1)\}$,
$F_C(g_1 \circ h_1) = \min\{F_C(g_1), F_C(h_1)\}$,

(iv) *$T_C(0) = \min\{T_C(g_1), T_C(g_1^*)\}$,*
$I_C(0) = \min\{I_C(g_1), I_C(g_1^*)\}$,
$F_C(0) = \min\{F_C(g_1), F_C(g_1^*)\}$.

3. Neutrosophic Implicative Filter

Here we put forward the conception of a neutrosophic implicative filter and confer its features with illustrations.

Definition 3.1. Let C be a neutrosophic filter of a BL-algebra $\mathcal{G}$. C is called a neutrosophic implicative filter if it persuades the following:

(i) $T_C(g_1) \le T_C(1)$, $I_C(g_1) \ge I_C(1)$ and $F_C(g_1) \ge F_C(1)$,

(ii) $\min\{T_C(g_1 \to (h_1 \to i_1)), T_C(g_1 \to h_1)\} \le T_C(g_1 \to i_1)$,
$\min\{_C(g_1 \to (h_1 \to i_1)), I_C(g_1 \to h_1)\} \ge I_C(g_1 \to i_1)$,
$\min\{F_C(g_1 \to (h_1 \to i_1)), F_C(g_1 \to h_1)\} \ge F_C(g_1 \to i_1)$, for all $g_1, h_1, i_1 \in \mathcal{G}$.

Example 3.2. Let $C = \{0, g_1, h_1, i_1, 1\}$. The bi-fold operations are specified by Tables 1 and 2.

Table 1. '$\circ$' operation

$\circ$	0	g_1	h_1	i_1	1
0	1	g_1	0	0	0
g_1	g_1	1	g_1	g_1	g_1
h_1	0	g_1	1	h_1	h_1
i_1	0	g_1	h_1	1	i_1
1	0	g_1	h_1	i_1	1

Table 2. '$\rightarrow$' operation

$\rightarrow$	0	g_1	h_1	i_1	1
0	1	1	1	1	1
g_1	g_1	1	1	1	1
h_1	0	g_1	1	1	1
i_1	0	g_1	h_1	1	1
1	0	g_1	h_1	i_1	1

Consider
$C = \{(0,[0.5,0.4,0.4]),(g_1,[0.5,0.4,0.4]),(h_1,[0.5,0.4,0.4]),(i_1,[0.5,0.4,0.4]),(1,[0.6,0.3,0.3])\}$.
It is evident that C assures Definition 3.1. Hence, C is a neutrosophic implicative filter of $\mathcal{G}$.

Proposition 3.3. *Every neutrosophic implicative filter of $\mathcal{G}$ is a neutrosophic filter. But, the converse is not true.*

Proof. Let C be a neutrosophic implicative filter of $\mathcal{G}$.

To prove: C is a neutrosophic filter of $\mathcal{G}$.
Taking $g_1 = 1$ in Definition 3.1, we get

$$\min\{T_C(1 \rightarrow (h_1 \rightarrow i_1)), T_C(1 \rightarrow h_1)\} \leq T_C(1 \rightarrow i_1), \quad \text{for all } g_1, h_1, i_1 \in \mathcal{G},$$

which implies

$$T_C(i_1) \geq \min\{T_C(h_1 \rightarrow i_1), T_C(h_1)\}.$$

Similarly,

$$I_C(i_1) \leq \min\{I_C(h_1 \rightarrow i_1), I_C(h_1)\}, F_C(i_1) \leq \min\{F_C(h_1 \rightarrow i_1), F_C(h_1)\}.$$

Thus, from Definition 2.4, C is a neutrosophic filter of $\mathcal{G}$. □

The converse part may not be true. This can be proved by an illustration.

Example 3.4. Let $C = \{0, g_1, h_1, 1\}$. The bi-fold operations are specified by Tables 3 and 4.

Table 3. '$\circ$' operation

$\circ$	0	g_1	h_1	1
0	0	0	0	0
g_1	0	0	g_1	h_1
h_1	0	g_1	h_1	h_1
1	0	g_1	h_1	1

Table 4. '$\rightarrow$' operation

$\rightarrow$	0	g_1	h_1	1
0	1	1	1	1
g_1	g_1	1	1	1
h_1	0	g_1	1	1
1	0	g_1	h_1	1

Consider $C = \{(0,[0.9,0.2,0.1]),(g_1,[0.5,0.3,0]),(h_1,[0.5,0.3,0]),(1,[0.9,0.2,0.1])\}$.
Here, C is not a neutrosophic implicative filter.
Since, $T_C(h_1 \rightarrow 1) = T_C(h_1) = 0.5 \not\geq 0.9 = T_C(0)$.

Proposition 3.5. *Let C be a neutrosophic filter of a BL-algebra $\mathcal{G}$. The following are equivalent for all $g_1,h_1,i_1 \in \mathcal{G}$.*

(i) *C is a neutrosophic implicative filter.*

(ii) $T_C(g_1 \to h_1) \geq T_C(g_1 \to (g_1 \to h_1))$,
$I_C(g_1 \to h_1) \leq I_C(g_1 \to (g_1 \to h_1))$,
$F_C(g_1 \to h_1) \leq F_C(g_1 \to (g_1 \to h_1))$,

(iii) $T_C(g_1 \to h_1) = T_C(g_1 \to (g_1 \to h_1))$,
$I_C(g_1 \to h_1) = I_C(g_1 \to (g_1 \to h_1))$,
$F_C(g_1 \to h_1) = F_C(g_1 \to (g_1 \to h_1))$.

Proof. (i)⇒(ii): Assume that C is a neutrosophic implicative filter of $\mathcal{G}$.
Put $i_1 = h_1$, $h_1 = g_1$ in Definition 3.1, we get

$$\begin{aligned} T_C(g_1 \to h_1) &\geq \min\{T_C(g_1 \to (g_1 \to h_1)), T_C(g_1 \to g_1)\} \\ &\geq \min\{T_C(g_1 \to (g_1 \to h_1)), T_C(1)\} \\ &= T_C(g_1 \to (g_1 \to h_1)). \end{aligned}$$

Therefore,

$$T_C(g_1 \to h_1) \geq T_C(g_1 \to (g_1 \to h_1)).$$

Similarly, we can prove for I_C, F_C.
Hence (ii) holds.

(ii)⇒(iii): Let $T_C(g_1 \to h_1) \geq T_C(g_1 \to (g_1 \to h_1))$.
Since $g_1 \to h_1 \leq g_1 \to (g_1 \to h_1)$ and from Proposition 2.6, we have

$$T_C(g_1 \to h_1) \leq T_C(g_1 \to (g_1 \to h_1)), \quad \text{for all } g_1,h_1 \in \mathcal{G}$$

and from (ii) we get

$$T_C(g_1 \to h_1) = T_C(g_1 \to (g_1 \to h_1)).$$

Similarly, we can prove for I_C, F_C. Hence (iii) holds.

(iii)⇒(i): Let $T_C(g_1 \to h_1) = T_C(g_1 \to (g_1 \to h_1))$.
If C is a neutrosophic filter of $\mathcal{G}$, then from Proposition 2.6,

$$\min\{T_C(g_1 \to (h_1 \to i_1)), T_C(g_1 \to h_1)\} \leq T_C(g_1 \to i_1), \quad \text{for all } g_1,h_1,i_1 \in \mathcal{G}.$$

Similarly, we can prove for I_C, F_C.
Hence, C is a neutrosophic implicative filter of $\mathcal{G}$. □

Proposition 3.6. *Let C and D be two neutrosophic filters of $\mathcal{G}$. Let $C \subseteq D$, $T_C(1) = T_D(1)$, $I_C(1) = I_D(1)$, $F_C(1) = F_D(1)$. If C is a neutrosophic implicative filter, then so is D.*

Proof. Let C and D be two neutrosophic filters of $\mathcal{G}$.
From Proposition 3.5, we only prove that

$$T_C(g_1 \to h_1) \geq T_C(g_1 \to (g_1 \to h_1)),$$
$$I_C(g_1 \to h_1) \leq I_C(g_1 \to (g_1 \to h_1)),$$
$$F_C(g_1 \to h_1) \leq F_C(g_1 \to (h_1 \to i_1)).$$

Let $x_1 = g_1 \to (g_1 \to h_1)$.
Then, $g_1 \to (g_1 \to (x_1 \to h_1)) = x_1 \to (g_1 \to (g_1 \to h_1)) = 1$.
Suppose, C is a neutrosophic implicative filter of $\mathcal{G}$, then from (iii) of Proposition 3.5 and since $C \subseteq D$, $T_C(1) = T_D(1)$,

$$\begin{aligned} T_D(x_1 \to (g_1 \to h_1)) &= T_D(g_1 \to (x_1 \to h_1)) \\ &\geq T_C(g_1 \to (x_1 \to h_1)) \\ &= T_C(g_1 \to (g_1 \to (x_1 \to h_1)) \\ &= T_C(x_1 \to (g_1 \to (g_1 \to h_1)) \\ &= T_C(1) \\ &= T_D(1). \end{aligned}$$

Thus,

$$T_D(x_1 \to (g_1 \to h_1)) \geq T_D(1).$$

This together with (i) of Definition 3.1,

$$T_D(x_1 \to (g_1 \to h_1)) \leq T_D(1)$$

imply that

$$T_D(x_1 \to (g_1 \to h_1)) = T_D(1).$$

Since, D is a neutrosophic filter then by Definition 2.4, we have

$$\begin{aligned} T_D(g_1 \to h_1) &\geq \min\{T_D(x_1 \to (g_1 \to h_1)), T_D(x_1)\} \\ &= \min\{T_D(1), T_D(x_1)\} \\ &= T_D(x_1) \\ &= T_D(g_1 \to (g_1 \to h_1)). \end{aligned}$$

Hence,

$$T_D(g_1 \to h_1) \geq T_D(g_1 \to (g_1 \to h_1)).$$

Similarly, we can prove for I_D, F_D.
Therefore, from (ii) of Proposition 3.5, D is a neutrosophic implicative filter. □

4. Neutrosophic n-fold Implicative Filter

Here, we put forward the conception of the neutrosophic n-fold implicative filter and confer its features with illustrations.

For any element g_1 and h_1 of a BL-algebra $\mathcal{G}$ and a positive integer n, let $g_1^n \to h_1$ signify $g_1 \to (g_1 \to \dots (g_1 \to h_1))$ where g_1 happens n-times and $g_1^0 \to h_1 = h_1$.

Definition 4.1. Let C be a neutrosophic filter of a BL-algebra $\mathcal{G}$. C is called a neutrosophic n-fold implicative filter if it persuades,

(i) $T_C(1) \geq T_C(g_1)$, $I_C(1) \leq I_C(g_1)$ and $F_C(1) \leq F_C(g_1)$,
(ii) $T_C(g_1^n \to i_1) \geq \min\{T_C(g_1^n \to (h_1 \to i_1)), T_C(g_1^n \to h_1)\}$,
$I_C(g_1^n \to i_1) \leq \min\{I_C(g_1^n \to (h_1 \to i_1)), I_C(g_1^n \to h_1)\}$,
$F_C(g_1^n \to i_1) \leq \min\{F_C(g_1^n \to (h_1 \to i_1)), F_C(g_1^n \to h_1)\}$, for all $g_1, h_1, i_1 \in \mathcal{G}$.

Note. The neutrosophic 1-fold implicative filter is the same as neutrosophic implicative filter.

Example 4.2. Let $C = \{0, g_1, h_1, i_1, j_1, 1\}$. The bi-fold operations are specified by Tables 5 and 6. Consider $C = \{(0,[0.6,0.4,0.4]),(g_1,[0.6,0.4,0.4]),(h_1,[0.8,0.3,0.3]),(i_1,[0.8,0.3,0.3]),(j_1,[0.6,0.4,0.4]),(1,[0.8,0.3,0.3])\}$.

Table 5. '$\circ$' operation

$\circ$	0	g_1	h_1	i_1	j_1	1
1	1	1	1	1	1	1
g_1	i_1	1	h_1	i_1	h_1	1
h_1	j_1	g_1	1	h_1	g_1	1
i_1	g_1	g_1	1	1	g_1	1
j_1	h_1	1	1	h_1	1	1
1	0	g_1	h_1	i_1	j_1	1

Table 6. '$\to$' operation

$\to$	0	g_1	h_1	i_1	j_1	1
0	1	1	1	1	1	1
g_1	1	1	h_1	i_1	h_1	i_1
h_1	1	g_1	1	h_1	g_1	j_1
i_1	1	g_1	1	1	g_1	g_1
j_1	1	1	1	h_1	1	h_1
1	1	g_1	h_1	i_1	j_1	0

It is evident that C assures Definition 3.1. Hence, C is a neutrosophic n-fold implicative filter of $\mathcal{G}$.

Proposition 4.3. *Every neutrosophic n-fold implicative filter of a BL-algebra $\mathcal{G}$ is a neutrosophic filter but the adverse is not true.*

Proof. Let C be a neutrosophic n-fold implicative filter of $\mathcal{G}$.
Taking $g_1 = 1$ in (ii) of Definition 4.1 and from (ii) of Proposition 2.2, we get

$$T_C(i_1) \geq \min\{T_C(h_1 \to i_1), T_C(h_1)\},$$

$$I_C(i_1) \leq \min\{I_C(h_1 \to i_1), I_C(h_1)\},$$

$$F_C(i_1) \leq \min\{F_C(h_1 \to i_1), F_C(h_1)\}, \quad \text{for all } h_1, i_1 \in \mathcal{G}.$$

Thus, (ii) of Definition 2.4 holds.
Hence, C is a neutrosophic filter of $\mathcal{G}$. □

The adverse of the proposition may not be true. It can be verified by an illustration.

Example 4.4. Let $D = \{0, g_1, h_1, i, j_1, 1\}$. The bi-fold operations are specified by Tables 5 and 6. Consider $D = \{(0,[0.6,0.4,0.4]),(g_1,[0.6,0.4,0.4]),(h_1,[0.6,0.4,0.4]),(i_1,[0.6,0.4,0.4]),(j_1,[0.6,0.4,0.4]),(1,[0.8,0.3,0.3])\}$.
Here, D is not a neutrosophic n-fold implicative filter of $\mathcal{G}$.
Since, $T_D(j_1 \to i_1) = T_D(h_1) = 0.6 \not\geq 0.8 = T_D(1)$.

Proposition 4.5. *Let C be a neutrosophic filter of a BL-algebra $\mathcal{G}$. Then the succeeding requirements are equivalent.*

(i) *C is a neutrosophic n-fold implicative filter of $\mathcal{G}$.*

(ii) $T_C(g_1^n \to h_1) \geq T_C(g_1^{n+1} \to h_1), I_C(g_1^n \to h_1) \leq I_C(g_1^{n+1} \to h_1)$
$F_C(g_1^n \to h_1) \leq F_C(g_1^{n+1} \to h_1)$, *for all* $g_1, h_1 \in \mathcal{G}$.

(iii) $T_C((g_1^n \to h_1) \to (g_1^n \to i_1)) \geq T_C(g_1^n \to (h_1 \to i_1))$,
$I_C((g_1^n \to h_1) \to (g_1^n \to i_1)) \leq I_C(g_1^n \to (h_1 \to i_1))$,
$F_C((g_1^n \to h_1) \to (g_1^n \to i_1)) \leq F_C(g_1^n \to (h_1 \to i_1))$, *for all* $g_1, h_1, i_1 \in \mathcal{G}$.

Proof. (i)⇒(ii): Let C be a neutrosophic n-fold implicative filter of $\mathcal{G}$.
Putting $i_1 = h_1$, $h_1 = g_1$ in (ii) of Definition 4.1,

$$\begin{aligned}T_C(g_1^n \to h_1) &\geq \min\{T_C(g_1^n \to (g_1 \to h_1)), T_C(g_1^n \to g_1)\}\\ &= \min\{T_C(g_1^{n+1} \to h_1), T_C(1)\}\\ &= T_C(g_1^{n+1} \to h_1).\end{aligned}$$

Hence,

$$T_C(g_1^n \to h_1) \geq T_C(g_1^{n+1} \to h_1), \quad \text{for all } g_1, h_1 \in \mathcal{G}.$$

Similarly, we can prove for I_C, F_C.

(ii)⇒(iii): Let (ii) holds.
Since,

$$g_1^n \to (h_1 \to i_1) \leq g_1^n \to (g_1^n \to h_1) \to (g_1^n \to i_1)),$$

we have

$$T_C(g_1^n \to ((g_1^n \to h_1) \to (g_1^n \to i_1))) \geq T_C(g_1^n \to (h_1 \to i_1)) \quad \text{(from Definition 3.1)}.$$

Since,

$$\begin{aligned}g_1^{n+1} \to ((g_1^{n-1} \to ((g_1^n \to h_1) \to i_1)) &= g_1^n \to ((g_1^n \to ((g_1^n \to h_1) \to i_1))\\ &= g_1^n \to ((g_1^n \to h_1) \to (g_1^n \to i_1))\end{aligned}$$

and using (ii), we have

$$\begin{aligned}T_C(g_1^{n+1} \to ((g_1^{n-2} \to ((g_1^n \to h_1) \to i_1))) &= T_C(g_1^n \to ((g_1^{n-1} \to ((g_1^n \to h_1) \to i_1))))\\ &\geq T_C(g_1^{n+1} \to ((g_1^{n-1} \to ((g_1^n \to h_1) \to i_1)))\\ &= T_C(g_1^n \to ((g_1^n \to h_1) \to (g_1^n \to i_1)))\\ &\geq T_C(g_1^n \to (h_1 \to i_1)).\end{aligned}$$

Repeating the process, we conclude that

$$\begin{aligned}T_C((g_1^n \to h_1) \to (g_1^n \to i_1)) &= T_C(g_1^n \to ((g_1^n \to h_1) \to i_1))\\ &\geq T_C(g_1^n \to (h_1 \to i_1)).\end{aligned}$$

Similarly, we can prove for I_C, F_C.
Therefore, (iii) holds.

(iii)⇒(i): Let (iii) holds.
By (iii) and (ii) of Definition 3.1,

$$\begin{aligned}T_C(g_1^n \to i_1) &\geq \min\{T_C((g_1^n \to h_1) \to (g_1^n \to i_1)), T_C(g_1^n \to h_1)\}\\ &\leq \min\{T_C(g_1^n \to (h_1 \to i_1)), T_C(g_1^n \to h_1)\}, \quad \text{for all } g_1, h_1, i_1 \in \mathcal{G}.\end{aligned}$$

Similarly, we can prove for I_C, F_C. Thus, C is a neutrosophic n-fold implicative filter.

5. Conclusion

In BL-algebras, we have put forth the conception of a neutrosophic implication in filters. We have also demonstrated the neutrosophic nature of every implicative and n-fold implicative filter. Moreover, other analogous circumstances for neutrosophic implicative filters are conferred. Further, research on the structure of BL-algebras and the above study will give us a wide range of applications in medical, industrial, and other fields.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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