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Research Article

# Impacts of Fisheries Harvesting on Fish-Bird Model for the Salton Sea Region

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**Abstract.** This study investigates the impact of fisheries harvesting on a Fish-bird model, focusing specifically on the presence of illnesses in fish and the consequential effects on bird populations. Tilapia fish, serving as the primary dietary source for pelican birds in the Salton Sea region, face the risk of overexploitation due to increased fishing activities. The study reveals that heightened harvesting rates can lead to the eventual extinction of bird species over time, highlighting the delicate balance between fishing sustainability and ecological preservation. The equilibrium and stability of the system are analyzed, with numerical simulations illustrating various dynamical behaviors such as chaotic attractors and quasi-periodic oscillations. The harvesting model considers the extraction of both susceptible and infected fish, demonstrating that an increase in the harvesting rate of susceptible fish poses a significant threat to bird populations. Conversely, while maintaining a constant harvesting rate of infected fish, the bird population remains viable even for varying levels of infected harvesting rates. Ultimately, the study underscores the importance of sustainable fishing practices, cautioning against the overexploitation of susceptible fish, which could lead to the extinction of birds in the Salton Sea region.

**Keywords.** Fish-bird model, Harvesting, Local stability analysis, Chaotic attractor, Quasi periodic ascillation

**Mathematics Subject Classification (2020).** 34D20, 92B05, 92D25, 34C23, 37Gxx

## 1. Introduction

Ecology and epidemiology were traditionally studied as distinct fields for a considerable period of time. The classical Lotka-Volterra model, developed by Lotka in 1925 [12] and and Volterra in 1926 [20], is an example of an early ecological model. The Lotka-Volterra model has been a useful tool for researchers studying interaction models between two or more species. Its application has led to numerous studies in the field of ecology, investigating the dynamics of species interactions and their impacts on ecosystem health. However, Kermack and McKendrick [10] were the first to use mathematical modeling to analyze the spread of diseases by using SIR epidemic model (Capasso and Serio [4]).

Over the past thirty years, there has been a rising interest in the examination of infectious diseases in the context of prey-predator interaction models. This has led to an increased focus on the study of eco-epidemiological systems, where the spread of disease is analyzed alongside the interactions between species in an ecosystem. Understanding these complex interactions is important for identifying effective strategies for disease control and maintaining the health of ecosystems.

The Predator-Prey model with infection in prey studied by Chattophyay and Arino [6], and Adak and Bairagi [1].

In eco-epidemiological research, the focus is on the spread of diseases among populations that interact with one another. The study of dynamics is crucial from both an ecological and mathematical perspective. Numerous researchers have explored various interaction models to understand the spread of diseases became an important issue from both Banerjee *et al*. [2] and Upadhyay and Roy [17]. Pelicans are at risk in Salton sea – an eco-epidemiological model studied by Chattopadhyay *et al*. [5, 7].

Additionally, there have been numerous studies investigating the transmission of disease in the fish populations of Salton Sea (Greenhalgh *et al*. [8], and Upadhyay *et al*. [18, 19]). In the summer the weather reaches 128 degree Fahrenheit and the water evaporates very quickly, leaving the salt behind. The salinity of water progressively rises as a result of the presence of dissolved salts, leading to a reduction in oxygen levels. This phenomenon occurs because the binding of salt in water is more challenging compared to freshwater. The Salton Sea serves as a primary destination for several migrating bird species, including pelicans. However, a significant number of water birds, mostly pelicans, as well as fish, have perished in substantial quantities. The precise aetiology of this phenomenon remains uncertain; nevertheless, mounting evidence suggests a growing association with the proliferation of hazardous algal blooms. Numerous aquatic organisms have perished as a result of the depletion of oxygen levels. Chattopadhyay and Bairagi [5] and Upadhyay *et al*. [18] have undertaken research on mathematical models that elucidate the dynamics of the interaction between fish, namely tilapia, and birds, specifically pelicans.

Furthermore, Upadhayay *et al*. [19] conducted a study on an ecosystem in a state of crisis. They examined an eco-epidemic model that included seasonal disturbances. The study demonstrates that the model exhibits stable focus, a limit cycle, and chaotic dynamics. The research discovered that variations in the contact rate, environmental carrying capacity ($k$), and predator population death rate contribute to the eco-epidemiological system exhibiting chaotic dynamics within certain ranges of amplitude and frequency of contact rate fluctuations.

In 2014, Upadhayay and Roy [17] studied an eco-epidemic model, which considered the prey species divided into two compartments: susceptible fish and infected fish population. The model also included the predator tilapia birds.

The odor emitted by dying fishes is unpleasant, and migratory birds visiting the sea during the winter months are particularly impacted by it.

The pedator-prey model did eases with harvesting studied by many researchers, e.g., Barman and Ghosh [3], Mahata *et al*. [13], and Panja *et al*. [14] but on Salton sea harvesting as fishing did not studied by anyone.

In this article harvesting can be included as a factor that affects the size and structure of the fish population. For example, the model can simulate different levels of fishing pressure and examine the impact on the fish population's dynamics and the spread of diseases or parasites. Harvesting can also be used as a control measure to manage the spread of diseases or parasites. For instance, if a disease outbreak occurs, harvesting can be temporarily increased to reduce the density of infected fish and limit the spread of the disease. The model assumes that only fish and birds interact as predator and prey in a closed system. However, in reality, there are many other factors that can influence the dynamics of predator-prey interactions. One of these factors is harvesting, which refers to the act of catching fish for human consumption. Harvesting can have a significant impact on the dynamics between the fish and bird populations, as it can alter the predator-prey relationship and affect the sustainability of the ecosystem. The model formulation is presented in Section 2. Section 3 is devoted to the analysis of the positivity of the solution, equilibrium, and stability. A new model, referred to as model (4.1), is developed in Section 4, where harvesting is applied to both susceptible and infected fish. The results of numerical simulations are discussed in Section 5. Finally, the discussion section is presented in Section 6.

## 2. Model Formulation

The model consists fish population as prey and pelican bird as a predator. $N(t)$ and $P(t)$ are the fishes and pelican density at time $t$. Some assumptions for model formulation as follows:

I. If disease not present, the fishes grows logistically with intrinsic growth rate $r$ and carrying capacity $k$ such that

$$\frac{dN}{dt} = rN\left(1-\frac{N}{k}\right). \tag{2.1}$$

II. If disease present in the system then total fishes divided into two compartment, namely, susceptible $S(t)$ and infected prey $I(t)$, that is at time $t$ net population of fishes as:

$$N(t) = S(t) + I(t). \tag{2.2}$$

III. Reproduction of fishes only due to susceptible fishes. In logistic growth birth rate positive as population can not be negative. The infected prey removed with positive death rate $d_1$ or not be able for reproducing. However, infected population contributes with $S$ to population growth towards the carrying capacity.

IV. Diseases transmitted by the law of mass action means disease spread with linear function $\phi(t) = \lambda SI$, where $\lambda$ represent rate of disease transmission. Infected fishes only due to disease spread in prey fishes not genetically inherited. The infected fishes does not recover or become immune,

$$\frac{dS}{dt} = rS\left(1-\frac{S+I}{k}\right) - \lambda SI. \tag{2.3}$$

V. Predator (Pelican) bird are not smart enough to predict which fish is infected and which is not, so pelican birds consumes fishes according to modified Holling type-II. In the absence of fishes(prey) pelican(predator) population decay exponentially.

VI. We add one more term in fish growth as harvesting due to fishing, for fishing we take linear harvesting. Hence, the rate of change of susceptible fish population can be written as

$$\frac{dS}{dt} = rS\left(1-\frac{S+I}{k}\right) - \lambda .SI - \frac{pSB}{(mS+I+c)} - hS. \tag{2.4}$$

Based on the assumptions outlined above, the model can be mathematically formulated as a system of nonlinear differential equations, which can be expressed as follows:

$$\frac{dS}{dt} = rS\left(1-\frac{S+I}{k}\right) - \lambda SI - \frac{pSB}{(mS+I+c)} - hS, \tag{2.5a}$$

$$\frac{dI}{dt} = \lambda SI - \frac{m_1 IB}{(mS+I+c)} - dI, \tag{2.5b}$$

$$\frac{dB}{dt} = \frac{p_1 SB}{(mS+I+c)} + \frac{m_2 IB}{(mS+I+c)} - d_1 B \tag{2.5c}$$

with $s(0) > 0$, $I(0) = I_0 > 0$ and $B(0) > 0$. Also, all others parameters have been considered positive for biological feasible reason.

**Table 1.** Parameters of model (2.5) and model (4.1)

| Symbol | Description | Unit |
|---|---|---|
| $S$ | Susceptible fish | Number per unit area (tones) |
| $I$ | Infected fish | Number per unit area (tones) |
| $B$ | Pelican bird (Predator) | Number per unit area (tones) |
| $r$ | Intrinsic growth rate of fish population | Per day |
| $k$ | Carrying capacity | Number per unit area (tones) |
| $\lambda$ | Disease transmission rate | Per day |
| $h_2$ | Harvesting rate | Per day |
| $d$ | Death rate of infected fish | Per day |
| $d_1$ | Death rate of birds | Per day |
| $m$ | Predator preference rate between $S$ and $I$ | Per day |
| $p$, $m_1$ | Predation rate | Per day |
| $m_2$ | Conversion rate of infected fish to pelican's bird | Per day |
| $p_1$ | Conversion rate of susceptible fish to pelican's bird | Per day |

## 3. Theoretical Studies

The existence and boundedness of the solution of model (2.5) are same as the paper (Upadhayay and Roy [18], and Kumar and Sinha [11]) so this part omitted.

**Theorem 3.1.** *Solution of the given model* (2.5) *with initial value* $S(0) \geq 0$, $I(0) \geq 0$ *and* $B(0) \geq 0$ *is positive.*

*Proof.* Since the function $Sf_1$, $If_2$, and $Bf_3$ are taken from right side of model (2.5) are continuous function and locially Lipschitzian on $R^3_+$, implies that the solution $(S(t), I(t), B(t))$ exist and unique on $[0,\epsilon]$, where $0 < \epsilon < \infty$ (see Hale [9], and Shaikh and Das [16]). For positivity integrating model (2.5), with respect to initial condition we get solution as follows;

$$S(t) = S(0)e^{\int_0^t f_1(S(s),I(s),B(s))ds} \geq 0,$$

$$I(t) = I(0)e^{\int_0^t f_2(S(s),I(s),B(s))ds} \geq 0,$$

$$B(t) = B(0)e^{\int_0^t f_3(S(s),I(s),B(s))ds} \geq 0,$$

where $S(0) = S_0 \geq 0$, $I(0) = I_0 \geq 0$ and $B(0) = B_0 \geq 0$. Hence the theorem proved. □

### 3.1 Equilibrium and Their Stability Analysis

To find for the system we need to solve $\frac{dS}{dt} = 0$, $\frac{dI}{dt} = 0$, and $\frac{dB}{dt} = 0$, simultaneously. We have the following results obtained as follows:

(I) The trivial equilibrium always exist.

(II) The axial equilibrium $E_1(S^*,0,0) = \left(k\left(1-\frac{h}{k}\right),0,0\right)$ is biological feasible if $h < k$.

(III) The disease-free equilibrium point $E(S^*,0,B^*)$, where $S^* = \frac{cd_1}{(p_1-d_1m)}$ and $B^* = \frac{\left\{rS^*\left(1-\frac{S^*}{k}\right)-h\right\}(mS^*+c)}{p}$ is feasible if $p_1 > d_1m$ and $rS^*\left(1-\frac{S^*}{k}\right) > h$.

(IV) A non-zero equilibrium solution of the model is called as coexistence of equilibrium, $E(S^*,I^*,B^*)$, where $S^* > 0$, $I^* > 0$, $B^* > 0$, where $B^* = \frac{(mS+I+c)(\lambda S-d)}{m_1}$, $I = \frac{d_1mS+dd_1-p_1S}{m_2-d_1}$ and $S^* = \frac{m_1(d_1-m_2)(h-r)+m_1(r+\lambda)(I+d)+Bd_1(d_1-m_2)}{(d_1-m_2)(m_1r-p\lambda k)-m_1mk(r+\lambda)\}(d_1-m_2)m_1}$.

#### 3.1.1 Feasible Region for Coexistence

We consider the set of parameters $r = 2.4$, $K = 80$, $m = 3.1$, $m_1 = 3.1$, $d = 5$, $m_2 = 3.25$, $\lambda = 0.8$, $p = 1.2$, $p_1 = 4.45$, $d = 2.98$, $d_1 = 1.3$. For parameter $h$ the feasibility condition for interior equilibrium $(S^*I^*B^*)$ is $0 \leq h \leq 0.34$ (see Figure 1), where $S^* = -7.320575769h + 6.274477838$, $I^* = 1.791125645h + 1.713554153$, $B^* = 39.48890341h^2 - 63.18183191h + 17.21436086$ for the above set of parameters.

### 3.2 Local Stability Analysis

For local stability of equilibrium point first we have to find Jacobian matrix about the interior equilibrium. If all the eigenvalue of the jacobian matrix are negative real parts then that equilibrium point is locally asymptotically stable (Perko [15]).

$$J = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \tag{3.1}$$

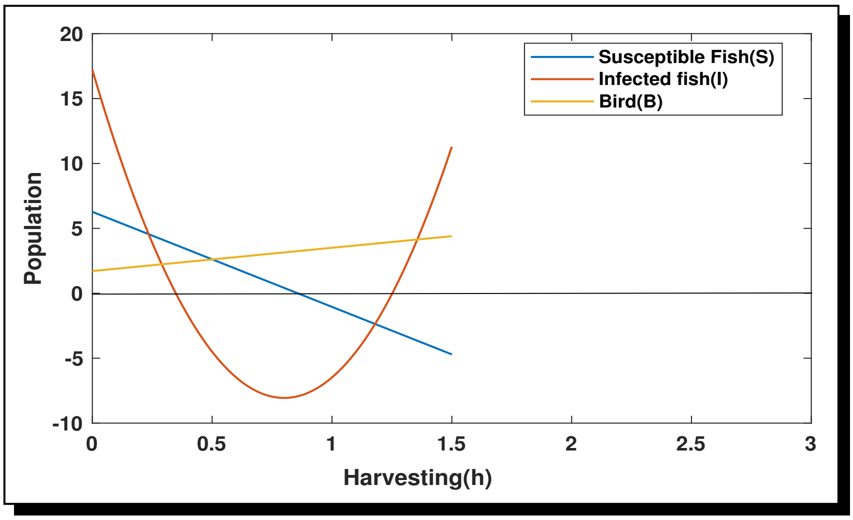


**Figure 1.** Feasible region $h$ for the co-existence of all the three population densities. Taking into account all the parameters mentioned in equation (5.1)

where $a_{11} = r(1-\frac{(2S+I)}{k}) - \lambda I - \frac{pB(I+c)}{(mS+I+c)^2} - h$, $a_{12} = \frac{-rS}{K} - \lambda S + \frac{pSB}{(mS+I+c)^2}$, $a_{13} = \frac{pS}{mS+I+c}$, $a_{21} = \lambda I + \frac{m_1 mIB}{(mS+I+c)^2}$, $a_{22} = \lambda S - \frac{m_1 B(mS+I+c)}{(mS+I+c)^2} - d$, $a_{23} = \frac{-m_1 I}{mS+I+c}$, $a_{32} = \frac{-p_1 SB}{(mS+I+c)^2} + \frac{m_2 B(mS+c)}{(mS+I+c)^2}$, and $a_{33} = \frac{p_1 S}{(mS+I+c)} + \frac{m_2 I}{(mS+I+c)} - d_1$.

The characteristic equation as follows:

$$\lambda^3 - A_1\lambda^2 + A_2\lambda - A_3 = 0. \tag{3.2}$$

So interior equilibrium is asymptotically stable if $A_1 > 0$, $A_2 > 0$ and $A_1A_2 > A_3$, this is the Routh-Hurwitz criterion so proof omitted.

(i) *Stability of trivial Equilibrium:* The eigenvalues of trivial equilibrium are $r-h$, $-d$, $d_1$. So there is an unstable manifold along $S$-direction and a stable manifold along $I$ and $B$-direction.

(ii) *Stability of Axial equilibrium Point:*

$$J_{(S^*,0,0)} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} h-r & (\frac{h}{r}-1)(-r-\lambda k) & a_{13} \\ 0 & \frac{m_1 p}{mk(1-h/r)+c} - d & 0 \\ 0 & 0 & \frac{p_1 k(r-h)}{mkr(r-h)+cr} - d_1 \end{bmatrix}. \tag{3.3}$$

Here eigenvalues of the matrix are diagonal elements of the matrix because $J_{E_1}$ as a diagonal matrix. If all the eigenvalues are negative then axial equilibrium is asymptotically stable.

(iii) *Disease Free Equilibrium:*

$$J_{(S^*,0,B^*)} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \tag{3.4}$$

where $a_{11} = r(1-\frac{(2S)}{k}) - \frac{pBc)}{(mS+c)^2} - h$, $a_{12} = \frac{-rS}{K} - \lambda S + \frac{pSB}{(mS+I+c)^2}$, $a_{13} = \frac{pS}{mS+I+c}$, $a_{21} = \lambda I + \frac{m_1 mIB}{(mS+I+c)^2}$, $a_{22} = \lambda S - \frac{m_1 B(mS+I+c)}{(mS+I+c)^2} - d$, $a_{23} = 0$, $a_{32} = \frac{-p_1 SB}{mS+c)^2} + \frac{m_2 B(ms+c)}{mS+c)^2}$, and $a_{33} = \frac{p_1 S}{(mS+c)} - d_1$. Stability of diseases free equilibrium calculated as similar to the interior equilibrium.

# 4. Fish-Bird Model With Harvesting Term in Both Infected Fish and Susceptible Fish Population

In this model, we considered a harvesting term used in both susceptible and infected prey populations. All the assumptions and parameters are taken to be the same as in the model(2.5).

$$\frac{dS}{dt} = rS\left(1-\frac{S+I}{k}\right)-\lambda SI-\frac{pSB}{(mS+I+c)}-hS, \tag{4.1a}$$

$$\frac{dI}{dt} = \lambda SI-\frac{m_1 IB}{(mS+I+c)}-dI-h_2 I, \tag{4.1b}$$

$$\frac{dB}{dt} = \frac{p_1 SB}{(mS+I+c)}+\frac{m_2 IB}{(mS+I+c)}-d_1 B \tag{4.1c}$$

with $S(0)>0$, $I(0)=I_0>0$ and $B(0)>0$. Also, all others parameters have been considered positive for biological feasible reason, where $h$ and $h_1$ are the used for harvesting in susceptible and infected fishes both.

# 5. Numerical Simulation

In this section, we discuss the numerical simulation in which we studied the dynamical behavior of the models numerically. The dynamical behavior of the solution of the models was studied through phase portraits and time series plots, as discussed in the subsection below. For the numerical simulation we use the Runga-Kutta method and the MATLAB 2018a software. we considered a fixed set of parameters, which are listed below:

$$r=2.4,\ k=80,\ m=3.1,\ m_1=3.1,\ d=5,\ m_2=3.25,\ \lambda=0.8,\ p=1.2,\ d_1=2.98, \tag{5.1}$$
$$d_2=1.3,\ h=0.1,\ p_1=4.45.$$

For the numerical simulation we use the Runga-Kutta method and the MATLAB 2018a software used.

## 5.1 Numerical Simulation for Model (2.5)

In this subsection discuss the impact of susceptible harvesting of the model (2.5). We plot phase portraits and time series for the different harvesting parameter.

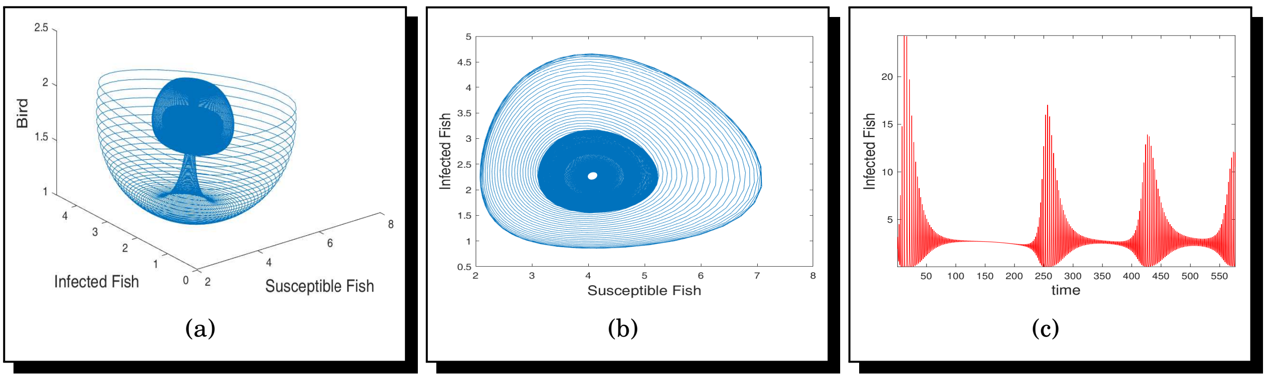


**Figure 2.** (a) and (b) indicates the quasi periodic attractor, and (c) indicates the quasi periodic oscillation for $h=0.010$ and $d_2=1.31$ for model (2.5) and taking the initial condition (4.5,1.5,4.4). Taking into account all the parameters mentioned in equation (5.1)

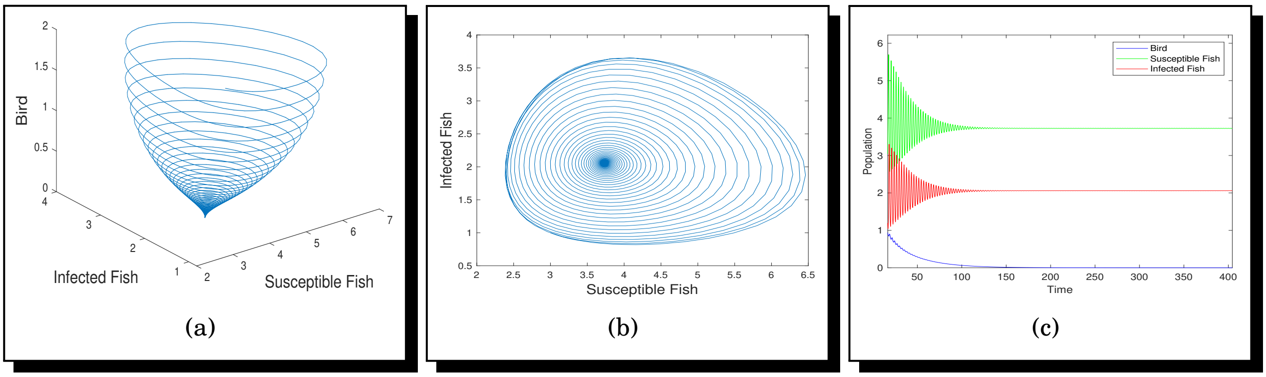


**Figure 3.** (a), (b) indicates the stable focus, and (c) indicates the time series for $h = 0.3$. Taking all the other parameters are same as equation (5.1)

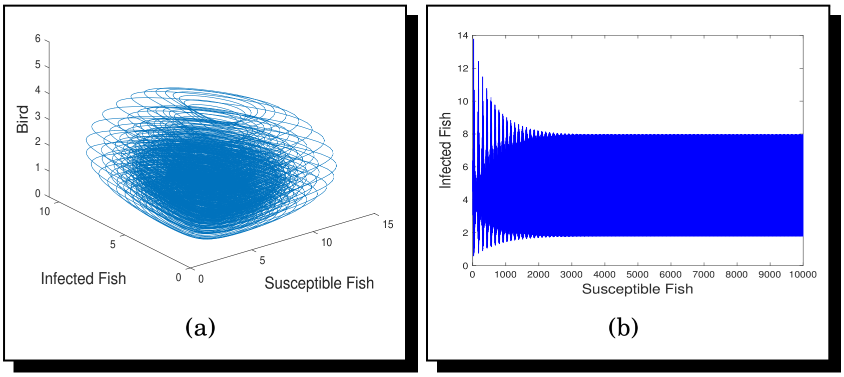


**Figure 4.** (a) Phase portrait and (b) time series plot for $d_1 = 1.2854$ and $h = 0.01$. Taking all the others parameters are same as equation (5.1)

Figure 4(a) displays the attractor, which represents the long-term behavior of the system, and Figure 4(b) shows the corresponding time series, which illustrates the dynamics of the system over time, converging to the attractor.

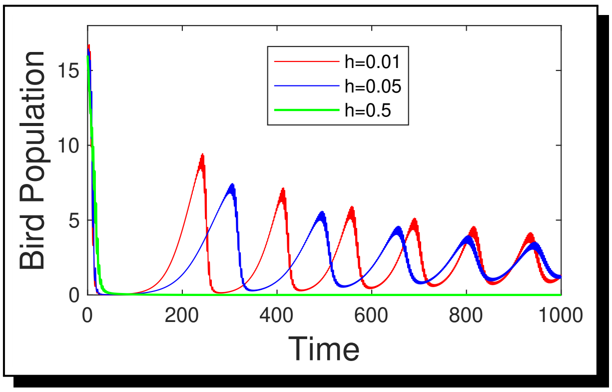


**Figure 5.** This diagram shows the variation of bird population with time for the various harvesting value and all others parameters are same as fixed set of parameters for model (2.5). As the harvesting rate ($h$) increases, the bird population is more likely to go to extinction

### 5.2 Numerical Simulation of Model (4.1)

For the numerical simulation of model (4.1), we used the same fixed parameters as in equation (5.1), but with different values of harvesting, $h$ and $h_1$.

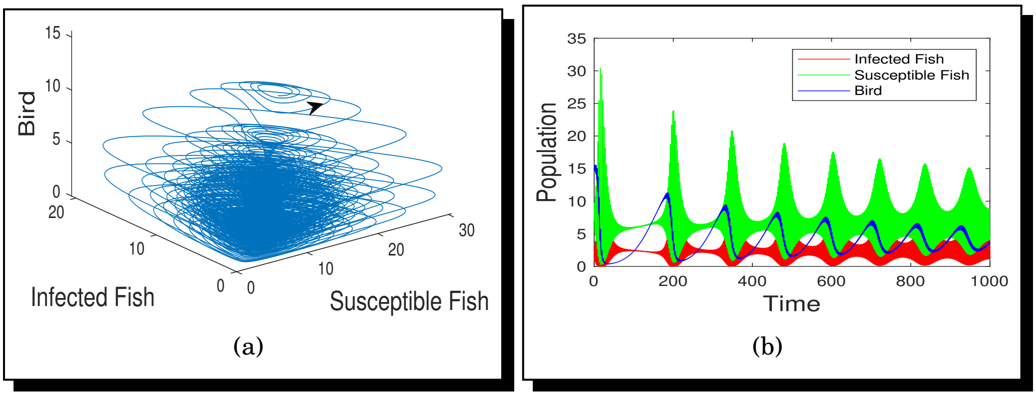


**Figure 6.** This shows chaotic attractor and time series about unstable interior equilibrium plot for the fixed parameter and $h_1 = 1.5$ and $h = 0.1$ and initial condition$(7, 1, 18)$

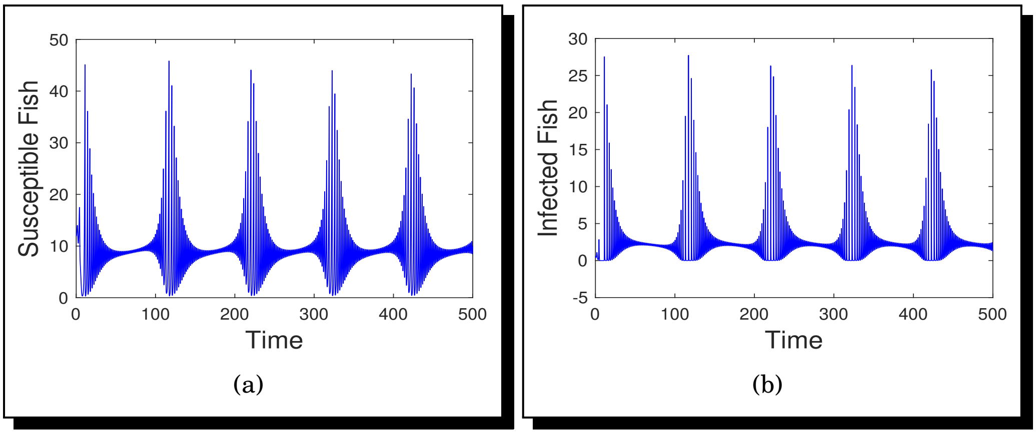


**Figure 7.** This shows quasi periodic oscillation time series about unstable interior equilibrium plot for the fixed parameter and $h_1 = 1.5$ and $h = 0.1$ for model (4.1)

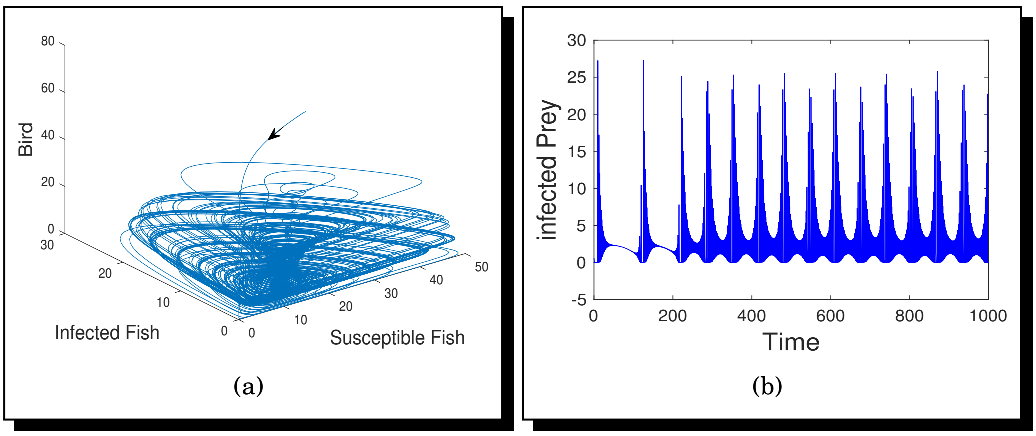


**Figure 8.** This shows chaotic attractor and time series about unstable interior equilibrium plot for the fixed parameter and $h_1 = 5.5$ and $h = 0.1$ for model (4.1), taking initial condition $(15, 1, 79)$

The term "quasi-periodic" is used to describe a certain kind of behaviour that is shown by some dynamic system. It is situated within the spectrum, which includes periodic phenomena, characterised by regular patterns, and chaotic phenomena, characterised by high levels of unpredictability. Quasi-periodic systems exhibit a discernible level of regularity in their patterns, although they lack the exact repetition characteristic of periodic systems.

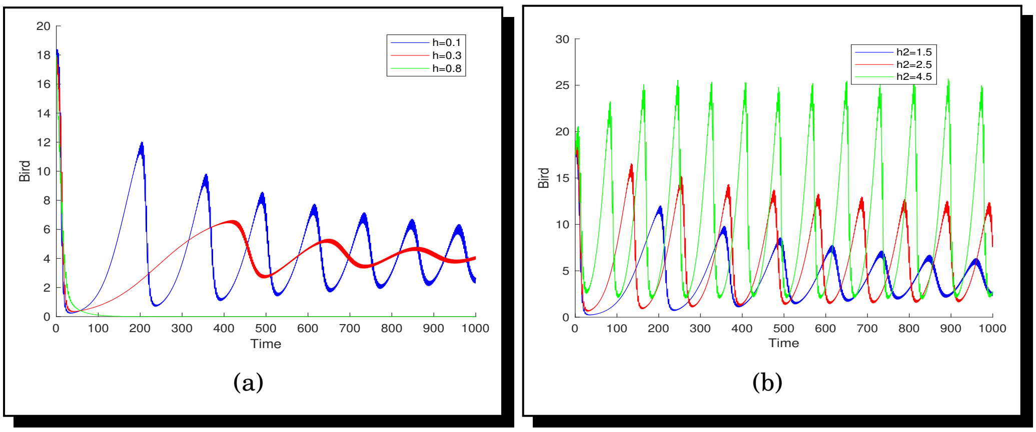


**Figure 9.** In (a) and (b) are the time series variation of bird population with time for the different value harvesting $h$ and $h_1$, all other parameters are same as fixed and (a) for $h_1 = 1.5$; (b) for $h = 0.1$

Figure 9(a) illustrates that when we fix the infected fish harvesting rate $h_1 = 1.5$ and increase the susceptible harvesting rate $h$, the bird population tends towards extinction. This suggests that the harvesting of susceptible fish has a more significant impact on the bird population. In contrast, Figure 9(b) shows that the harvesting of infected prey has a relatively minimal impact on the bird population. As $h_1$ increases, we observe a transition from damped oscillations to periodic oscillations, as seen in the case where $h = 4.5$, which exhibits periodic oscillations.

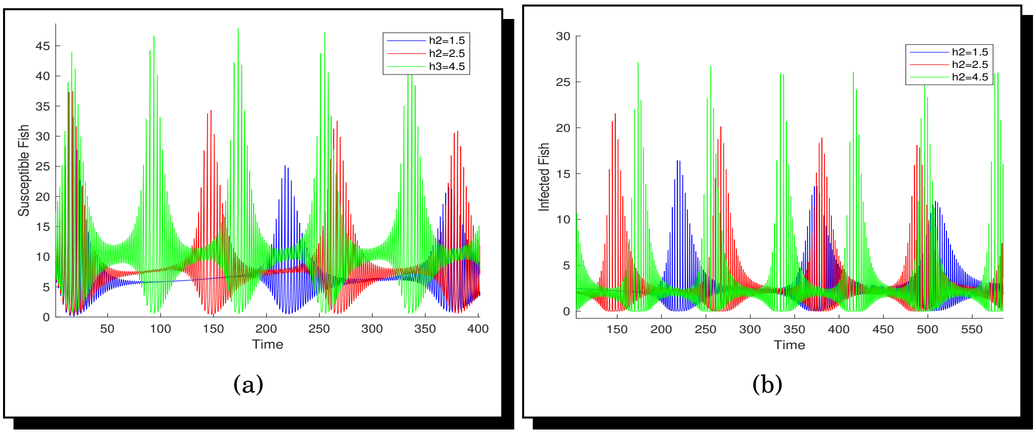


**Figure 10.** This figure shows the time series of infected and susceptible fish with respect to time for the different value of $h_1$ for the model (4.1) and all others are same as fixed parameters

Figures 10(a) and (b) present time series plots for the susceptible fish and infected fish populations, respectively, as a function of time. These plots reveal that the harvesting rate of infected fish, $h_1$ is the primary factor responsible for the type of oscillations observed in the system.

## 6. Discussion

This study investigates the effects of fisheries harvesting on two Fish-bird model (2.5) and (4.1), specifically focusing on the presence of illnesses in fish. Tilapia fish serve as the primary dietary source for pelican bird in the Salton Sea region. If there is an increase in fishing, it may have a detrimental impact on bird populations, as they may have difficulties owing to the insufficient availability of fish as a food source. Figure 5 illustrates the consequences of harvesting on bird populations. It is evident from the figure that an increase in the harvesting rate leads to the eventual extinction of the bird species over a certain period of time. Fishing is feasible under the constraints of a restricted fishing pace. The equilibrium and stability of the system have been studied, and numerical simulations have been conducted to study the phase portrait and time series (5). The harvesting model (4.1) considers the extraction of both susceptible and infected fish. Through this study, it is seen that an increase in the harvesting rate of susceptible fish leads to the extinction of birds, assuming a constant harvesting rate of infected fish (Figure 9). It is important to note that the collection of susceptible fish should be significantly reduced in comparison to infected fish in order to ensure the survival of pelican bird populations. Furthermore, it is evident that while the susceptible harvesting rate remains constant, the bird population will not go extinct for varying values of the infected harvesting rate, as seen in Figure 10. In the present work also describe the dynamical behaviour such as chaotic attractor, quasi periodic attractor, quasi periodic oscillation and dumping oscillator are studied through numerical simulation see Figures 2, 3, 6, 7 and 8. This study concludes that the fishing of infected fish can be increase, however, the fishing of susceptible fish may lead to the extinction of birds in Salton sea.

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### Competing Interests

The authors declare that they have no competing interests.

### Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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