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Research Article

Common Fixed Point Theorem for Three Self-maps in G_{JS}-Metric Space

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Abstract. In this article, we prove a common fixed point theorem for three self-maps using the contractive modulus function in a recently emerged generalized metric space known as G_{JS}-metric space and verified its uniqueness. We illustrated the main theorem with an example.

Keywords. G_{JS}-metric space, G_{JS}-continuous mapping, Common fixed point, Compatible maps, Associated sequence, Contractive modulus

Mathematics Subject Classification (2020). 54H25, 47H10

1. Introduction

In the field of non-linear analysis, fixed point theory plays an eminent role. Due to the invention of the Banach contraction principle by Banach [1], many new results emerged, and it initiated the generalization of many metric spaces. S-metric space was initiated by Sedghi *et al*. [10] has been generalized into S^{JS}-metric space by Beg *et al*. [2] through JS-metric space which in turn is proposed by Mohamed and Samet [7]. In similar footsteps, G-metric space initiated by Mustafa *et al*. [8] has been generalized into G_{JS}-metric space through JS-metric by Srilatha and Kiran [12]. Moreover, common fixed point results on compatible self-maps were given by Jungck [4] and the concept of compatible mappings was also introduced by him with the aim of generalizing the notion of weak commutativity. In 2017, Vishnu and Dolhare [13] proved the common fixed point theorem for three self-maps in a generalized metric space, and these results were extended to four, five and six self-maps in various metric spaces by Kumar *et al*. [6], Rauf *et al*. [9] and Goud and Rangamma [3], respectively. The main aim of this paper is to generalize the common fixed point result for three self-maps given by Singh [11] using the contractive modulus function in G_{JS}-metric space, which is a recently emerged metric space. It is a well-known fact that if $U : E \to E$ is an identity map on any metric space, then $U(\chi) = \chi$, for every $\chi \in E$, which implies that 'χ' is a fixed point of U, whereas if V is any self-map on E such that $U(\chi) = V(\chi) = t$, for some $t \in E$, then 'χ' is called the common fixed point of U and V. In this article, we extend this concept to three self-maps and try to find the common fixed point of three self-maps in G_{JS}-metric space and verify its uniqueness.

2. Preliminaries

In preliminaries, we give some basic definitions which are required for our main result.

Definition 2.1 ([12])**.** Assume that E is a non-void set and $G_{JS} : E^3 \to [0,\infty]$ is a mapping satisfying the following conditions:

($G_{JS}1$): $G_{JS}(\chi,\psi,\xi) = 0$ if and only if $\chi = \psi = \xi$,

($G_{JS}2$): $0 < G_{JS}(\chi,\chi,\psi)$ for all $\chi,\psi \in E$ with $\chi \neq \psi$,

($G_{JS}3$): $G_{JS}(\chi,\chi,\psi) < G_{JS}(\chi,\psi,\xi)$, for all $\chi,\psi,\xi \in E$ with $\psi \neq \xi$,

($G_{JS}4$): $G_{JS}(\chi,\psi,\xi) = G_{JS}(\sigma(\chi,\psi,\xi))$, for all $\chi,\psi,\xi \in E$ where $\sigma(\chi,\psi,\xi)$ is a permutation of the set $\{\chi,\psi,\xi\}$, and

($G_{JS}5$): there is a constant $c > 0$ such that for $(\chi,\psi,\xi) \in E^3$ and $\langle\chi_n\rangle \in \mathcal{G}(G_{JS},E,\chi)$,

$$G_{JS}(\chi,\psi,\xi) \leq c \limsup_{n\to\infty} G_{JS}(\chi_n,\psi,\xi),$$

where $\mathcal{G}(G_{JS},E,\chi) = \{\langle\chi_n\rangle \subset E : \lim_{n\to\infty} G_{JS}(\chi_n,\chi,\chi) = 0\}$.

Then, the mapping G_{JS} is called a G_{JS}-metric on E and the pair (E,G_{JS}) is called a G_{JS}-metric space.

Definition 2.2 ([5])**.** Two self-maps U,V of G_{JS}-metric space (E,G_{JS}) are said to be compatible if for all $\chi \in E$, $\lim_{n\to\infty} G_{JS}(UV\chi_n,UV\chi_n,VU\chi_n) = 0$, where $\langle\chi_n\rangle$ is a sequence in E such that $\lim_{n\to\infty} U\chi_n = \lim_{n\to\infty} V\chi_n = t$, for some $t \in E$.

Definition 2.3. A function $\phi : [0,\infty) \to [0,\infty)$ is called a contractive modulus if $\phi(0) = 0$ and $\phi(t) < t$, for $t > 0$.

Definition 2.4. A self-map $U : E \to E$ is said to be G_{JS}-continuous at a point χ_0 in E if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that $G_{JS}(U\chi,U\chi,\chi_0) < \varepsilon$ whenever $G_{JS}(\chi,\chi,\chi_0) < \delta$, for every $\chi \in E$.

Definition 2.5. Let E be a non-empty set and U,V and W be three self-maps on E such that $U(E) \cup V(E) \subseteq W(E)$. Then a sequence $\langle\chi_n\rangle$ is called an associated sequence of $\chi_0 \in E$ relative to three self-maps U,V and W if $U\chi_{2n} = W\chi_{2n+1}, V\chi_{2n+1} = W\chi_{2n+2}$ for $n \geq 0$.

Definition 2.6. A point $\chi \in E$ is said to be common fixed point of three self-maps U,V and W on E if $U\chi = V\chi = W\chi = \chi$.

3. Main Result

Theorem 3.1. *In a G_{JS}-metric space, (E, G_{JS}), E be a non-void set and U, V and W be three self-maps of E which are commutative, fulfilling the following conditions:*

(i) $U(E) \cup V(E) \subseteq W(E)$.

(ii) W *is* G_{JS}*-continuous.*

(iii) *Either* (V, W) *or* (U, W) *is a compatible pair.*

(iv) *For* $\chi_0 \in E$, *we can find an associate sequence* $\langle \chi_n \rangle$ *relative to* U, V *and* W *such that the sequence* $U\chi_0, V\chi_1, U\chi_2, V\chi_3, \dots, U\chi_{2n}, V\chi_{2n+1}$ *converges to some point* $\xi \in E$.

(v)
$$\begin{aligned} G_{JS}(U\chi, U\chi, V\psi) \le \max\{&\phi(G_{JS}(W\chi, W\chi, W\psi) + G_{JS}(U\chi, U\chi, W\chi)), \\ &\phi(G_{JS}(U\chi, U\chi, W\chi) + G_{JS}(V\psi, V\psi, W\psi)), \\ &\phi(G_{JS}(W\chi, W\chi, W\psi) + G_{JS}(V\psi, V\psi, W\psi))\}, \end{aligned} \tag{3.1}$$

where ϕ *is a contractive modulus.*

Then U, V *and* W *will have* ξ *as the unique common fixed point.*

Proof. Let us consider the case when (U, W) is a compatible pair and ϕ be a contractive modulus. Due to the fact that each of the sequence $U\chi_{2n}$ and $V\chi_{2n+1}$ converges to $\xi \in E$ and $U\chi_{2n} = W\chi_{2n+1}$ and $V\chi_{2n+1} = W\chi_{2n+2}$ for $n \ge 0$. As $n \to \infty$, we can have

$$U\chi_{2n}, V\chi_{2n+1}, W\chi_{2n+1}, W\chi_{2n+2} \text{ and hence } W\chi_{2n} \to \xi \text{ as } n \to \infty. \tag{3.2}$$

Since W is continuous as $n \to \infty$, we can have

$$WU\chi_{2n} \to W\xi,\ W^2\chi_{2n} \to W\xi. \tag{3.3}$$

Also, since U, W are compatible, we have

$$\lim_{n\to\infty} G_{JS}(WU\chi_{2n}, WU\chi_{2n}, UW\chi_{2n}) = 0. \tag{3.4}$$

Since $U\chi_{2n}, W\chi_{2n} \to \xi$ as $n \to \infty$, by (3.2), using (3.3) and (3.4), we get

$$UW\chi_{2n} \to W\xi \text{ as } n \to \infty.$$

Similarly, if (V, W) is a compatible pair and W is continuous, we get

$$W^2\chi_{2n+1} \to W\xi,\ WV\chi_{2n+1} \to W\xi \text{ and } VW\chi_{2n+1} \to W\xi \text{ as } n \to \infty.$$

Now, using (3.1), we know that

$$\begin{aligned} &G_{JS}(UW\chi_{2n}, UW\chi_{2n}, V\chi_{2n+1}) \\ &\quad \le \max\{\phi(G_{JS}(W^2\chi_{2n}, W^2\chi_{2n}, W\chi_{2n+1}) + G_{JS}(UW\chi_{2n}, UW\chi_{2n}, W^2\chi_{2n})), \\ &\qquad \phi(G_{JS}(UW\chi_{2n}, UW\chi_{2n}, W^2\chi_{2n}) + G_{JS}(V\chi_{2n+1}, V\chi_{2n+1}, W\chi_{2n+1})), \\ &\qquad \phi(G_{JS}(W^2\chi_{2n}, W^2\chi_{2n}, W\chi_{2n+1}) + G_{JS}(V\chi_{2n+1}, V\chi_{2n+1}, W\chi_{2n+1}))\}. \end{aligned}$$

Letting $n \to \infty$, we get

$$\begin{aligned} G_{JS}(W\xi, W\xi, \xi) \le \max\{&\phi(G_{JS}(W\xi, W\xi, \xi) + G_{JS}(W\xi, W\xi, W\xi)), \\ &\phi(G_{JS}(W\xi, W\xi, W\xi) + G_{JS}(\xi, \xi, \xi)), \\ &\phi(G_{JS}(W\xi, W\xi, \xi) + G_{JS}(\xi, \xi, \xi))\}. \end{aligned}$$

Using the fact that $\phi(0) = 0$, we get

$$G_{JS}(W\xi, W\xi, \xi) \le \max\{\phi(G_{JS}(W\xi, W\xi, \xi)), 0, \phi(G_{JS}(W\xi, W\xi, \xi))\} = \phi(G_{JS}(W\xi, W\xi, \xi)).$$

Thus, we get

$$G_{JS}(W\xi,W\xi,\xi)\le\phi(G_{JS}(W\xi,W\xi,\xi)).$$

This leads to a contradiction if $W\xi\neq\xi$ as $\phi(t)<t$, for $t>0$. Hence

$$W\xi=\xi. \tag{3.5}$$

Again,

$$\begin{aligned}G_{JS}(U\xi,U\xi,V\chi_{2n+1})\le\max\{&\phi(G_{JS}(W\xi,W\xi,W\chi_{2n+1})+G_{JS}(U\xi,U\xi,W\xi)),\\&\phi(G_{JS}(U\xi,U\xi,W\xi)+G_{JS}(V\chi_{2n+1},V\chi_{2n+1},W\chi_{2n+1})),\\&\phi(G_{JS}(W\xi,W\xi,W\chi_{2n+1})+G_{JS}(V\chi_{2n+1},V\chi_{2n+1},W\chi_{2n+1}))\}.\end{aligned}$$

Letting $n\to\infty$, we get

$$\begin{aligned}G_{JS}(U\xi,U\xi,\xi)&\le\max\{\phi(G_{JS}(\xi,\xi,\xi)+G_{JS}(W\xi,W\xi,\xi)),\phi(G_{JS}(W\xi,W\xi,\xi)+0),\\&\qquad\phi(G_{JS}(\xi,\xi,\xi)+0)\}\\&\le\max\{\phi(G_{JS}(U\xi,U\xi,\xi)),\phi(G_{JS}(U\xi,U\xi,\xi)),0\}.\end{aligned}$$

Thus

$$G_{JS}(U\xi,U\xi,\xi)\le\phi(G_{JS}(U\xi,U\xi,\xi)).$$

This leads to a contradiction if $U\xi\neq\xi$ as $\phi(t)<t$, for $t>0$. Hence

$$U\xi=\xi. \tag{3.6}$$

Similarly, we can see that

$$\begin{aligned}G_{JS}(U\chi_{2n},U\chi_{2n},V\xi)\le\max\{&\phi(G_{JS}(W\chi_{2n},W\chi_{2n},W\xi)+G_{JS}(U\chi_{2n},U\chi_{2n},W\chi_{2n})),\\&\phi(G_{JS}(U\chi_{2n},U\chi_{2n},W\chi_{2n})+G_{JS}(V\xi,V\xi,W\xi)),\\&\phi(G_{JS}(W\chi_{2n},W\chi_{2n},W\xi)+G_{JS}(V\xi,V\xi,W\xi))\}.\end{aligned}$$

As $n\to\infty$ and using equation (3.5), we get

$$\begin{aligned}G_{JS}(\xi,\xi,V\xi)&\le\max\{\phi(G_{JS}(\xi,\xi,\xi)+G_{JS}(\xi,\xi,\xi)),\phi(G_{JS}(\xi,\xi,\xi)+G_{JS}(V\xi,V\xi,\xi)),\\&\qquad\phi(G_{JS}(\xi,\xi,\xi)+G_{JS}(V\xi,V\xi,\xi))\}\\&\le\max\{0,\phi(G_{JS}(V\xi,V\xi,\xi)),\phi(G_{JS}(V\xi,V\xi,\xi))\}.\end{aligned}$$

Thus $G_{JS}(\xi,\xi,V\xi)\le\phi(G_{JS}(V\xi,V\xi,\xi))$, which leads to a contradiction if $V\xi\neq\xi$ as $\phi(t)<t$, for $t>0$. Hence

$$V\xi=\xi. \tag{3.7}$$

Thus, from (3.5), (3.6) and (3.7), we get

$$U\xi=V\xi=W\xi=\xi.$$

Hence U,V and W has ξ as a common fixed point.
Now to prove its uniqueness, let us take $\xi'\neq\xi$ as some other common fixed point of U,V and W. Then $U\xi=V\xi=W\xi=\xi$ and $U\xi'=V\xi'=W\xi'=\xi'$,

$$\begin{aligned}G_{JS}(\xi,\xi,\xi')=G_{JS}(U\xi,U\xi,V\xi')&\le\max\{\phi(G_{JS}(W\xi,W\xi,W\xi')+G_{JS}(U\xi,U\xi,W\xi)),\\&\qquad\phi(G_{JS}(U\xi,U\xi,W\xi)+G_{JS}(V\xi',V\xi',W\xi')),\\&\qquad\phi(G_{JS}(W\xi,W\xi,W\xi')+G_{JS}(V\xi',V\xi',W\xi'))\}\\&\le\max\{\phi(G_{JS}(\xi,\xi,\xi')),0,\phi(G_{JS}(\xi,\xi,\xi'))\}.\end{aligned}$$

Thus $G_{JS}(\xi,\xi,\xi') \le \phi(G_{JS}(\xi,\xi,\xi'))$, which will be a contradiction if $\xi \ne \xi'$.
Hence $\xi = \xi'$.
Therefore, the common fixed point of U,V and W is unique. □

Example 3.2. Let $G_{JS} : E^3 \to [0,\infty]$ be a G_{JS}-metric on $E = [0,1]$ defined by,

$$G_{JS}(\xi,\psi,\chi) = |\xi - \psi| + |\psi - \chi| + |\chi - \xi|, \quad \text{for } \chi,\psi,\xi \in E.$$

Define the self-maps U,V and W of E by, $U(\chi) = \frac{\chi^3}{32}$, $V(\chi) = \frac{\chi}{2}$ and $W(\chi) = \frac{(\chi^2+\chi)}{2}$, where $U(E) = \left[0,\frac{1}{32}\right]$, $V(E) = \left[0,\frac{1}{2}\right]$ and $W(E) = [0,1]$.
Thus $U(E) \cup V(E) \subseteq W(E)$, and we can observe that W is G_{JS}-continuous on E.
If we establish a sequence $\langle \chi_n \rangle$ in a way that $\chi_n \to 0$ as $n \to \infty$, then

$$\lim_{n\to\infty} W\chi_n = \lim_{n\to\infty} U\chi_n = 0.$$

Moreover, $\lim_{n\to\infty} G_{JS}(WU\chi_n, WU\chi_n, UW\chi_n) = 0$, showing that (U,W) is a compatible pair.
Let $\phi(t) = \sqrt{t}$ be a contractive modulus.
Now we will check condition (3.1) for all possible values of χ,ψ.

Case 1: Let $\chi = \psi = 0$.
In this case,

$$U(\chi) = V(\chi) = W(\chi) = 0,$$
$$U(\psi) = V(\psi) = W(\psi) = 0,$$

so that (3.1) is obvious.

Case 2: If $\chi = 0$, $\psi \ne 0$.
Then

$$G_{JS}(U\chi,U\chi,V\psi) = G_{JS}\left(0,0,\frac{\psi}{2}\right) = \psi, \tag{3.8}$$

$$G_{JS}(W\chi,W\chi,W\psi) = G_{JS}\left(0,0,\frac{\psi^2+\psi}{2}\right) = \psi^2 + \psi,$$

$$G_{JS}(U\chi,U\chi,W\chi) = 0,$$

$$G_{JS}(V\psi,V\psi,W\psi) = G_{JS}\left(\frac{\psi}{2},\frac{\psi}{2},\frac{\psi^2+\psi}{2}\right) = \psi^2,$$

$$\begin{aligned}\max\{\phi(\psi^2+\psi),\phi(\psi^2),\phi(2\psi^2+\psi)\} &= \max\{\phi(G_{JS}(W\chi,W\chi,W\psi)+G_{JS}(U\chi,U\chi,W\chi)),\\ &\qquad \phi(G_{JS}(U\chi,U\chi,W\chi)+G_{JS}(V\psi,V\psi,W\psi)),\\ &\qquad \phi(G_{JS}(W\chi,W\chi,W\psi)+G_{JS}(V\psi,V\psi,W\psi))\}\\ &= \phi(2\psi^2+\psi)\\ &= \sqrt{2\psi^2+\psi}.\end{aligned} \tag{3.9}$$

Thus, from (3.8) and (3.9), we can say that condition (3.1) is satisfied in this case.

Case 3: If $\chi \ne 0$, $\psi = 0$.
Then we can easily verify that the result remains same as seen in *Case* 2.

Case 4: If $\chi \neq 0$, $\psi \neq 0$,

$$G_{JS}(U\chi, U\chi, V\psi) = G_{JS}\left(\frac{\chi^3}{32}, \frac{\chi^3}{32}, \frac{\chi}{2}\right) = \left|\frac{16\psi - \chi^3}{16}\right|, \tag{3.10}$$

$$\begin{aligned} G_{JS}(W\chi, W\chi, W\psi) &= G_{JS}\left(\frac{\chi^2+\chi}{2}, \frac{\chi^2+\chi}{2}, \frac{\psi^2+\psi}{2}\right) \\ &= |\chi^2 - \psi^2 + \chi - \psi| G_{JS}(U\chi, U\chi, W\chi) \\ &= G_{JS}\left(\frac{\chi^3}{32}, \frac{\chi^3}{32}, \frac{\chi^2+\chi}{2}\right) \\ &= \frac{|\chi^3 - 32\chi^2 - 32\chi|}{16} \end{aligned}$$

$$G_{JS}(V\psi, V\psi, W\psi) = G_{JS}\left(\frac{\psi}{2}, \frac{\psi}{2}, \frac{\psi^2+\psi}{2}\right) = \psi^2,$$

$$\begin{aligned} &\max\{\phi(G_{JS}(W\chi, W\chi, W\psi) + G_{JS}(U\chi, U\chi, W\chi)), \phi(G_{JS}(U\chi, U\chi, W\chi) + G_{JS}(V\psi, V\psi, W\psi)), \\ &\quad \phi(G_{JS}(W\chi, W\chi, W\psi) + G_{JS}(V\psi, V\psi, W\psi))\} \\ &\quad = \max\left\{\phi\left(|\chi^2 - \psi^2 + \chi - \psi| + \frac{|\chi^3 - 32\chi^2 - 32\chi|}{16}\right), \phi\left(\frac{|\chi^3 - 32\chi^2 - 32\chi|}{16} + \psi^2\right),\right. \\ &\qquad \left.\phi(|\chi^2 - \psi^2 + \chi - \psi| + \psi^2)\right\}. \end{aligned} \tag{3.11}$$

Thus, from (3.10) and (3.11), we can say that condition (3.1) is satisfied in this case.

Hence in all the cases, condition (3.1) is satisfied for all $\chi, \psi \in E$.

Let $\chi_0 = 0 \in E$ so that $U\chi_0 = 0$ and there exists $\chi_1 \in E$ such that $U\chi_0 = W\chi_1 \Rightarrow 0 = \chi_1$.

Hence $\chi_1 = 0$.

Now $\chi_2 \in E$ with $V\chi_1 = W\chi_2$ which implies $0 = \chi_2$.

Thus $\chi_2 = 0$.

Proceeding in the same manner we can construct an associated sequence of χ_0, that is $U\chi_0, V\chi_1, U\chi_2, V\chi_3, \dots$ converging to a point $0 \in E$.

Hence '0' is the common fixed point of U, V and W.

Corolllary 3.3. *If U, V and W are three self-maps of (E, G_{JS}) which are commutative among themselves satisfying the conditions from* (i) *to* (iv) *of Theorem* 3.1. *Further,*

$$\begin{aligned} G_{JS}(U\chi, U\psi, V\psi) \leq \max\{&\phi(G_{JS}(V\psi, V\psi, W\psi))[G_{JS}(W\chi, W\chi, W\psi) + G_{JS}(U\chi, U\chi, W\chi)], \\ &\phi(G_{JS}(W\chi, W\chi, W\psi))[G_{JS}(U\chi, U\chi, W\chi) + G_{JS}(V\psi, V\psi, W\psi)], \\ &\phi(G_{JS}(U\chi, U\chi, W\chi))[G_{JS}(W\chi, W\chi, W\psi) + G_{JS}(V\psi, V\psi, W\psi)]\}, \end{aligned} \tag{3.12}$$

where ϕ is a contractive modulus.

Then U, V and W has one and only one common fixed point as ξ.

Proof. Using (3.12), we know that

$G_{JS}(UW\chi_{2n}, UW\chi_{2n}, V\chi_{2n+1})$

$\leq \max\{\phi(G_{JS}(V\chi_{2n+1}, V\chi_{2n+1}, W\chi_{2n+1}))[G_{JS}(W^2\chi_{2n}, W^2\chi_{2n}, W\chi_{2n+1}) + G_{JS}(UW\chi_{2n}, UW\chi_{2n}, W^2\chi_{2n})],$

$$\phi(G_{JS}(W^2\chi_{2n},W^2\chi_{2n},W\chi_{2n+1}))[G_{JS}(UW\chi_{2n},UW\chi_{2n},W^2\chi_{2n})+G_{JS}(V\chi_{2n+1},V\chi_{2n+1},W\chi_{2n+1})],$$
$$\phi(G_{JS}(UW\chi_{2n},UW\chi_{2n},W^2\chi_{2n}))[G_{JS}(W^2\chi_{2n},W^2\chi_{2n},W\chi_{2n+1})+G_{JS}(V\chi_{2n+1},V\chi_{2n+1},W\chi_{2n+1})]\}.$$

Letting $n\to\infty$, we get

$$\begin{aligned}G_{JS}(W\xi,W\xi,\xi) &\le \max\{\phi(G_{JS}(\xi,\xi,\xi))[G_{JS}(W\xi,W\xi,\xi)+G_{JS}(W\xi,W\xi,W\xi)],\\ &\qquad \phi(G_{JS}(W\xi,W\xi,\xi))[G_{JS}(W\xi,W\xi,W\xi)+G_{JS}(\xi,\xi,\xi)],\\ &\qquad \phi(G_{JS}(W\xi,W\xi,W\xi))[G_{JS}(W\xi,W\xi,\xi)+G_{JS}(\xi,\xi,\xi)]\}\\ &\le \max\{\phi(0)[G_{JS}(W\xi,W\xi,\xi)+0],\phi(G_{JS}(W\xi,W\xi,\xi))[0+0],\\ &\qquad \phi(0)[G_{JS}(W\xi,W\xi,\xi)+0]\}\\ &\le \max\{0,0,0\}.\end{aligned}$$

Thus, $G_{JS}(W\xi,W\xi,\xi)=0$, this implies

$$W\xi=\xi. \tag{3.13}$$

Following the same procedure, we can easily show that $G_{JS}(U\xi,U\xi,\xi)=0$ and

$$G_{JS}(\xi,\xi,V\xi)=0 \text{ which implies, } U\xi=\xi \text{ and } V\xi=\xi, \text{ respectively.} \tag{3.14}$$

Thus, from (3.13) and (3.14), we can see that

$$U\xi=V\xi=W\xi=\xi.$$

Clearly, ξ is the common fixed point of U,V and W.

If ξ and $\xi'$are two common fixed points of U,V and W, by following the same procedure as above, we can prove that $G_{JS}(\xi,\xi,\xi')=0$.

This implies $\xi=\xi'$.

Thus, the common fixed points of three self-maps U,V and W on G_{JS}-metric space is unique. □

Remark 3.1. The result of Corollary 3.3 remains same even if we replace '+' sign with '−' sign in inequality (3.12).

4. Conclusion

We proved a theorem on G_{JS}-metric space using the contractive modulus, in which we have shown that a common fixed point for three self-maps can be found if they satisfy certain conditions. An example is given to support the theorem in which all possible cases were discussed.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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