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Research Article

A New Fixed Point Theorem for Generalized (α,ψ)-Contraction Mapping of Quadratic Type With Applications

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Abstract. In this paper, we acquaint generalized (α,ψ)-contraction mappings of quadratic type and establish common fixed point results in the setting of rectangular quasi b-metric spaces. Our results extend and generalize related fixed point results of Karapinar and Lakzian $(\alpha-\psi)$-Contractive mappings on generalized quasi metric spaces, *Journal of Function Spaces* **2014** (2014), 914398, 7 pages), Alharbi *et al*. (α-Contractive mappings on rectangular b-metric spaces and an application to integral equations, *Journal of Mathematical Analysis* **9**(3) (2018), 47 – 60), and Khuangsatung *et al*. (The rectangular quasi-metric space and common fixed point theorem for ψ-contraction and ψ-Kannan mappings, *Thai Journal of Mathematics* (Special Issue (2020): Annual Meeting in Mathematics 2019), (2020), 89 – 101). We applied our result to facilitate the existence of a solution to an integral equation.

Keywords. 47H10, 54H25

Mathematics Subject Classification (2020). Fixed point, (α,ψ)-contraction, b-metric space

1. Introduction

Fixed point theory is an significant tool in study of nonlinear analysis. It is considered to be the key association between pure and applied mathematics. It is also widely applied in different fields of study such as Economics, Chemistry, Physics and almost all Engineering areas. The contraction mapping principle, introduced by Banach [6] has wide scope of applications in fixed point theory. The Banach contraction principle has been extended and generalized in distinct way by different researchers (see [5–22]). In 2014, Lin *et al.* [19] introduced the concept of rectangular quasi metric space and proved fixed point theorem for the Meir-Keeler contractive mappings. Also, Karapinar and Lakzian [15] acquaint (α,ψ)-contractive mapping in rectangular quasi metric space and proved fixed point theorems for the maps introduced. In 2015, George *et al.* [12] declared the notion of rectangular b-metric space as a generalization of b-metric space and rectangular metric space.

Recently, Alharbi *et al.* [3] defined (α)-contractive mapping and proved fixed point theorems in rectangular b-metric space. Afterward, several research papers were published on the existence of fixed point results for single valued and multi valued mappings in the setting of rectangular quasi metric spaces. Very recently, Khuangsatung *et al.* [16] introduced the notion (ψ) contraction mappings in complete rectangular quasi metric spaces and proved the existence and uniqueness of fixed points.

2. Preliminaries

We present some definitions which will be useful in the sequel.

Definition 2.1 ([9]). Let (X,d) be a b-metric space with coefficient $s \geq 1$ be a given real number. A function $d: X \times X \to R^+$ is a b-metric space if and only if for all $x,y,z \in X$, the following conditions are satisfied:

(I) $d(x,y)=0$ if and only if $x=y$;

(II) $d(x,y)=d(y,x)$;

(III) $d(x,z) \leq s[d(x,y)+d(y,z)]$.

The pair (X,d) is called a b-metric space.

Definition 2.2 ([8]). Let X be a non empty set and $d: X \times X \to R^+$ be a function satisfying the following conditions:

(I) $d(x,y)=0$ if and only if $x=y$;

(II) $d(x,y)=d(y,x)$;

(III) $d(x,y) \leq d(x,u)+d(u,v)+d(v,y)$. for all $x,y \in X$ and all distinct point $u,v \in X/\{x,y\}$.

Then d is called rectangular metric on X, and the pair (X,d) is called rectangular metric space.

Definition 2.3 ([12]). Let X be a non empty set, $s \geq 1$ be a given real number and $d: X \times X \to R^+$ be a function satisfying the following conditions:

(I) $d(x,y)=0$ if and only if $x=y$;

(II) $d(x,y)=d(y,x)$;

(III) $d(x,y) \leq s[d(x,u)+d(u,v)+d(v,y)]$, for all $x,y \in X$ and all distinct point $u,v \in X/\{x,y\}$.

Then d is called rectangular b-metric on X, and the pair (X,d) is called rectangular b-metric space.

Inspired and motivated by the works of Karapinar and Lakzian [15], Alharbi *et al*. [3] and Khuangsatung *et al*. [16], the main purpose of this paper is to introduce (α,ψ) contraction mapping of quadratic type and establish fixed point results in the setting of rectangular quasi b-metric spaces. Further, an application of our result is furnished.

3. Main Result

We introduce the following:

Definition 3.1. Let (X,d) be a rectangular quasi b-metric space and $T:X\to X$ be a given mapping. We say that T is a generalized (α,ψ)-contraction mapping of quadratic type if there exist two functions $\alpha:X\times X\to R^+$ and $\psi\in\Psi$ such that

$$\alpha(x,y)d(Tx,Ty)\le\psi(M(x,y)),\quad \text{for all } x,y\in X, \tag{3.1}$$

where $M(x,y)=\max\left\{d^2(x,y),d(x,Tx)\cdot d(y,Ty),d(x,Tx)\cdot d(x,Ty)\right\}$.

Now, we state and prove the following fixed point theorem.

Theorem 3.2. *Let (X,d) be a complete rectangular quasi b-metric space and $T:X\to X$ be generalized (α,ψ)-contraction of quadratic type mapping. Suppose that*

(I) *T is an α admissible mapping;*

(II) *there exists $x_0\in X$ such that $\alpha(x_0,Tx_0)\ge1$, $\alpha(Tx_0,x_0)\ge1$, $\alpha(x_0,T^2x_0)\ge1$ and $\alpha(T^2x_0,x_0)\ge1$;*

(III) *T is continuous.*

Then T has a fixed point.

Proof. By (ii) above, there exists $x_0\in X$ such that $\alpha(x_0,Tx_0)\ge1$ and $\alpha(Tx_0,x_0)\ge1$. Now, we produce sequence $\{x_n\}$ in X by $x_{n+1}=Tx_n=T^{n+1}x_0$, for all $n\ge0$. Suppose that $x_{n_0}=x_{n_0+1}$ for some $n_0\ge0$. Since $Tx_{n_0}=x_{n_0+1}$, the point $u=x_{n_0}$ forms a fixed point of T. That completes the proof. We assume that $x_n\ne x_{n+1}$ for all $n\ge0$.

Since T is a α-admissible, we have $\alpha(x_0,x_1)=\alpha(x_0,Tx_0)\ge1\implies\alpha(Tx_0,Tx_1)=\alpha(x_1,x_2)\ge1$.

Applying the expression above, we obtain that

$$\alpha(x_n,x_{n+1})\ge1,\quad \text{for all } n=0,1,2,\dots \tag{3.2}$$

and

$$\alpha(x_1,x_0)=\alpha(Tx_0,x_0)\ge1 \text{ implies } \alpha(Tx_1,Tx_0)=\alpha(x_2,x_1)\ge1.$$

We obtain

$$\alpha(x_{n+1},x_n)\ge1,\quad \text{for all } n=0,1,2,\dots. \tag{3.3}$$

Likewise, we derive that

$$\alpha(x_0,x_2)=\alpha(x_0,T^2x_0)\ge1 \text{ implies } \alpha(Tx_0,Tx_2)=\alpha(x_1,x_3)\ge1.$$

Recursively, we get

$$\alpha(x_n,x_{n+2}) \geq 1m \quad \text{for all } n=0,1,2,\dots. \tag{3.4}$$

Similarly, we can easily derive that

$$\alpha(x_{n+2},x_n) \geq 1, \quad \text{for all } n=0,1,2. \tag{3.5}$$

Step 1: We show that

$$\lim_{n\to\infty} d(x_n,x_{n+1}) = 0 = \lim_{n\to\infty} d(x_{n+1},x_n)$$

and

$$\lim_{n\to\infty} d(x_n,x_{n+2}) = 0 = \lim_{n\to\infty} d(x_{n+2},x_n).$$

from (3.1), we have

$$\begin{aligned} d(x_n,x_{n+1}) &= d(Tx_{n-1},x_n) \\ &\leq \alpha(x_{n_1},x_n)d(Tx_{n_1},Tx_n) \\ &\leq \psi\left(M(x_{n_1},x_n)\right), \quad \text{for all } n\geq 1, \end{aligned} \tag{3.6}$$

where

$$M(x,y)=\max\{d^2(x,y),d(x,Tx)\cdot d(y,Ty),d(x,Tx)\cdot d(x,Ty)\},$$

$$\begin{aligned} M(x_{n-1},x_n) &= \max\{d^2(x_{n-1},x_n),d(x_{n-1},Tx_{n-1})\cdot d(x_n,Tx_n),d(x_{n-1},Tx_{n-1})\cdot d(x_{n-1},Tx_{n-1})\} \\ &= \max\{d^2(x_{n-1},x_n),d(x_{n-1},x_n)\cdot d(x_n,x_{n+1}),d(x_{n-1},x_n)\cdot d(x_{n-1},x_n)\} \\ &= \max\{d^2(x_{n-1},x_n),d(x_{n-1},x_n)\cdot d(x_n,x_{n+1}),d(x_{n-1},x_n)\cdot d(x_{n-1},x_n) \\ &= \max\{d^2(x_{n-1},x_n),x_n),d(x_{n-1},x_n)\cdot d(x_n,x_{n+1})\} \\ &= \max\{d^2(x_{n-1},x_n),d^2(x_n,x_{n+1})\}. \end{aligned}$$

If $M(x_{n-1},x_n)=d^2(x_n,x_{n+1})$, then from (3.6), we get

$$\begin{aligned} d^2(x_n,x_{n+1}) &\leq \psi(d^2(x_n,x_{n+1})) \\ &\leq s\ \psi(d^2(x_n,x_{n+1})) \\ &< d^2(x_n,x_{n+1}), \end{aligned}$$

which is a contradiction. Hence, $M(x_{n-1},x_n)=d^2(x_{n-1},x_n)$.

We let $e_n=d^2(x_n,x_{n+1}), l_n=d^2(x_{n+1},x_n), e_n^*=d^2(x_n,x_{n+2})$ and $l_n^*=d^2(x_{n+2},x_n)$, for all $n\geq 0$. By using (3.6), we get

$$\begin{aligned} e_n &= d^2(x_n,x_{n+1}) \\ &= d^2(Tx_{n-1},Tx_n) \\ &\leq \psi(d^2(x_{n-1},x_n)) \\ &= \psi(d^2(Tx_{n-2},Tx_{n-1})) \\ &\leq \psi^2(d^2(x_{n-2},x_{n-1})) \\ &\ \ \vdots \\ &\leq \psi^n(d^2(x_0,x_1)) = \psi^n(e_0) \to 0 \text{ as } n\to\infty. \end{aligned} \tag{3.7}$$

Also,

$$\begin{aligned} l_n &= d^2(x_{n+1},x_n) \\ &= d^2(Tx_n,Tx_{n-1}) \\ &\le \alpha(x_n,x_{n-1})d^2(Tx_n,Tx_{n-1}) \\ &\le \psi(M(x_n,x_{n-1})), \quad \text{for all } n\ge 1, \end{aligned} \tag{3.8}$$

where

$$M(x_n,x_{n-1}) = \max\{d^2(x_n,x_{n-1}),d(x_n,Tx_n)\cdot d(x_{n-1},Tx_{n-1}),d(x_n,Tx_n)\cdot d(x_n,Tx_{n-1})\}.$$

We deal with three different cases as follows:

Case (i): If $M(x_n,x_{n-1}) = d^2(x_{n-1},x_n)$ then using (3.8), we get

$$\begin{aligned} d^2(x_{n+1},x_n) &\le \psi(d^2(x_{n-1},x_n)) \\ &\le \psi^n(d^2(x_0,x_1)) \\ &= \psi^n(e_0) \to 0 \text{ as } n\to\infty. \end{aligned}$$

Case(ii): If $M(x_n,x_{n-1}) = d(x_n,Tx_n)\cdot d(x_{n-1},Tx_{n-1})$, then using (3.8), we get

$$\begin{aligned} d^2(x_{n+1},x_n) &\le \psi(d^2(x_n,x_{n+1})) \\ &\le \psi^n(d^2(x_0,x_1)) \\ &= \psi^n(e_0) \to 0 \text{ as } n\to\infty. \end{aligned}$$

Case(iii) If $M(x_n,x_{n-1}) = d(x_n,Tx_n)\cdot d(x_n,Tx_{n-1})$ then using (3.8) we get

$$\begin{aligned} l_n = d^2(x_{n+1},x_n) = d^2(Tx_n,Tx_{n-1}) &\le \psi(d(x_n,x_{n-1}) \\ &= \psi(d^2(Tx_{n-1},Tx_n-2) \\ &\le \psi^2(d^2(x_{n-1},x_{n-2})) \\ &\vdots \\ &\le \psi^n(d^2(x_1,x_0)) \\ &= \psi^n(l_0) \to 0 \text{ as } n\to\infty. \end{aligned}$$

From Case (i)-Case (iii), we get

$$l_n = d^2(x_{n+1},x_n) \to 0 \text{ as } n\to\infty. \tag{3.9}$$

From (3.7) and (3.9), we deduce that

$$\lim_{n\to\infty} d^2(x_n,x_{n+1}) = 0 = \lim_{n\to\infty} d^2(x_{n+1},x_n). \tag{3.10}$$

Now, we show that

$$\lim_{n\to\infty} d^2(x_n,x_{n+2}) = 0 = \lim_{n\to\infty} d^2(x_{n+2},x_n).$$

Also,

$$e_n^* = d^2(x_n,x_{n+2})$$

$$
\begin{aligned}
&= d^2(Tx_{n-1},Tx_{n+1}) \\
&\le \alpha(x_{n-1},x_{n+1})\cdot d^2(Tx_{n-1},Tx_{n+1}) \\
&\le \psi(M(x_{n-1},x_{n+1})), \quad \text{for all } n\ge 1,
\end{aligned}
\tag{3.11}
$$

where

$$
\begin{aligned}
M(x_{n-1},x_{n+1}) = \max\{&d^2(x_{n-1},x_{n+1}), d(x_{n-1},Tx_{n-1})\cdot d(x_{n+1},Tx_{n+1}), \\
&d(x_{n-1},Tx_{n-1})\cdot d(x_{n+1},Tx_{n+1})\} \\
&= \max\{d^2(x_{n-1},x_{n+1}), d(x_{n-1},x_n)\cdot d(x_{n+1},x_{n+2}), d(x_{n-1},x_n)\cdot d(x_{n+1},x_{n+2})\} \\
&= \max\{d^2(x_{n-1},x_{n+1}), d(x_{n-1},x_n)\cdot d(x_{n+1},x_{n+2})\}.
\end{aligned}
$$

Again, we deal with three different cases as follows:

Case (i): If $M(x_n,x_{n-1}) = d^2(x_{n-1},x_{n+1})$, then using (3.11), we get

$$
\begin{aligned}
d^2(x_n,x_{n+2}) &\le \psi(d^2(x_{n-1},x_{n+1})) \\
&\le \psi^{n-1}(d^2(x_0,x_2)) \\
&= \psi^{n-1}(e_0^*) \to 0 \text{ as } n\to\infty.
\end{aligned}
$$

Case (ii): If $M(x_n,x_{n-1}) = d(x_{n-1},x_n)\cdot d(x_{n+1},x_{n+2})$ then using (3.11), we get

$$
\begin{aligned}
d^2(x_n,x_{n+2}) &\le \psi(d(x_{n-1},x_n)\cdot d(x_{n+1},x_{n+2})) \\
&\le \psi^{n-1}(d(x_0,x_1)) \\
&= \psi^{n-1}(e_0) \to 0 \text{ as } n\to\infty.
\end{aligned}
$$

Case (iii): If $M(x_{n-1},x_{n+1}) = d(x_{n-1},x_n)\cdot d(x_{n+1},x_{n+2})$ then by (3.11), we get

$$
\begin{aligned}
d^2(x_n,x_{n+2}) &\le \psi(d^2(x_{n+1},x_{n+2})) \\
&\le \psi^{n+1}(d(x_0,x_1)) \\
&= \psi^{n+1}(e_0) \to 0 \text{ as } n\to\infty
\end{aligned}
$$

From Case (i)-Case (iii), we get

$$
e_n^* = d^2(x_n,x_{n+2}) \to 0 \text{ as } n\to\infty. \tag{3.12}
$$

$$
\begin{aligned}
d^2(x_{n+2},x_n) &= d^2(Tx_{n+1},Tx_{n-1}) \\
&\le \alpha(x_{n+1},x_{n-1}) d^2(Tx_{n+1},Tx_{n-1}) \\
&\le \psi(M(x_{n+1},x_{n-1})), \quad \text{for all } n\ge 1,
\end{aligned}
\tag{3.13}
$$

where

$$
\begin{aligned}
M(x_{n+1},x_{n-1}) = \max\{&d^2(x_{n+1},x_{n-1}), d(x_{n+1},Tx_{n+1})\cdot d(x_{n-1},Tx_{n-1}), \\
&d(x_{n+1},Tx_{n+1})\cdot d(x_{n+1},Tx_{n-1})\} \\
&= \max\{d^2(x_{n+1},x_{n-1}), d(x_{n+1},x_{n+2})\cdot d(x_{n-1},x_n), d(x_{n+1},x_{n+2})\cdot d(x_{n+1},x_n)\} \\
&= \max\{d^2(x_{n+1},x_{n-1}), d(x_{n+1},x_{n+2})\cdot d(x_{n-1},x_n), d(x_{n+1},x_{n+2})\cdot d(x_{n+1},x_n)\}.
\end{aligned}
$$

Again, we deal with three different cases as follows:

Case (i): If $M(x_{n+1},x_{n-1}) = d^2(x_{n+1},x_{n-1})$ then using (3.13), we get

$$\begin{aligned} d^2(x_{n+2},x_n) &\le \psi(d^2(x_{n+1},x_{n-1})) \\ &\le \psi^{n+1}(d(x_2,x_0)) \\ &= \psi^{n+1}(l_0^*) \to 0 \text{ as } n \to \infty. \end{aligned}$$

Case (ii): If $M(x_{n+1},x_{n-1}) = d(x_{n+1},x_{n+2}) \cdot d(x_{n-1},x_n)$ then using (3.13), we get

$$\begin{aligned} d^2(x_{n+2},x_n) &\le \psi(d^2(x_{n+1},x_{n+2})) \\ &\le \psi^{n+1}(d^2(x_0,x_1)) \\ &= \psi^{n+1}(e_0) \to 0 \text{ as } n \to \infty. \end{aligned}$$

Case (iii): If $M(x_{n+1},x_{n-1}) = d(x_{n+1},x_{n+2}) \cdot d(x_{n+1},x_n)$ then using (3.13), we get

$$\begin{aligned} d^2(x_{n+2},x_n) &\le \psi(d^2(x_{n-1},x_n)) \\ &\le \psi^{n-1}((d^2(x_0,x_1)) \\ &= \psi^{n-1}(e_0) \to 0 \text{ as } n \to \infty. \end{aligned} \tag{3.14}$$

From (3.12) and (3.14), we deduce that

$$\lim_{n\to\infty} d^2(x_n,x_{n+2}) = 0 = \lim_{n\to\infty} d^2(x_{n+2},x_n).$$

Step 2: We shall prove that $\{x_n\}$ is a rectangular quasi b-Cauchy sequence, that is,

$$\lim_{n\to\infty} d^2(x_n,x_{n+p}) = 0 = \lim_{n\to\infty} d^2(x_{n+p},x_n), \quad \text{for all } p \in N.$$

Case (i): Suppose that for some $n,m \in N$ with $m > n$ and $x_n = x_m$, using (3.10),

$$\begin{aligned} d^2(x_n,x_{n+1}) &= d^2(x_n,Tx_n) \\ &= d^2(x_m,Tx_m) \\ &= d^2(x_m,x_{m+1}) \\ &\le \psi^{m-n}(d^2(x_n,x_{n+1})) \\ &\le s\psi(d^2(x_n,x_{n+1})) \\ &< d^2(x_n,x_{n+1}), \end{aligned}$$

which is a contradiction.

Case (ii): Suppose that for some $n,m \in N$ with $m > n$, and $x_n = x_m$, using (3.10),

$$\begin{aligned} d^2(x_{m+1},x_m) &= d^2(Tx_m,x_m) \\ &= d^2(Tx_n,x_n) \\ &= d^2(x_{n+1},x_n) \\ &\le \psi^{n-m}(d^2(x_{m+1},x_m)) \\ &\le s\psi(d^2(x_{m+1},x_m)) \\ &< d^2(x_{m+1},x_m), \end{aligned}$$

which is a contradiction.

Therefore, from Case (i) and Case (ii), $x_n \neq x_m$ for $n \neq m$. The case $p = 1$ and $p = 2$ is proved.

Now we take $p \geq 3$; arbitrary. We discriminate four different cases as follows:

Case (i): Let $p = 2m$, where $m \geq 2$. By rectangular inequality, we get

$$\begin{aligned}
d^2(x_n, x_{n+2m}) &\leq s[d^2(x_n, x_{n+2}) + sd^2(x_{n+2}, x_{n+3}) + sd^2(x_{n+3}, x_{n+2m})] \\
&\leq sd^2(x_n, x_{n+2}) + sd^2(x_{n+2}, x_{n+3}) + sd^2(x_{n+3}, x_{n+2m}) \\
&\quad + s^2[x_{n+3}, x_{n+4}) + d(x_{n+4}, x_{n+5}) + d(x_{n+5}, x_{n+2m}) \\
&= sd(x_n, x_{n+2}) + sd(x_{n+2}, x_{n+3}) + s^2d^2(x_{n+3}, x_{n+4}) + s^2d^2(x_{n+4}, x_{n+5}) \\
&\quad + s^2d^2(x_{n+5}, x_{n+2m}) \\
&\;\;\vdots \\
&\leq sd^2(x_n, x_{n+2}) + s^3d^2(x_{n+2}, x_{n+3}) + s^4d^2(x_{n+3}, x_{n+4}) \\
&\quad + s^5d^2(x_{n+4}, x_{n+5}) + s^{2m}d^2(x_{n+2m-1}, x_{n+2m}) \\
&= sd^2(x_n, x_{n+2}) + \sum_{k=n+2}^{n+2m-1} s^{k-n+1}d^2(x_k, x_{k+1}) \\
&\leq sd^2(x_n, x_{n+2}) + \sum_{k=n+2}^{n+2m-1} s^k\psi^k(e_0) \\
&= sd^2(x_n, x_{n+2}) + \sum_{k=n+2}^{\infty} s^k\psi^k(e_0).
\end{aligned}$$

From (3.14),

$$\lim_{n\to\infty} d^2(x_n, x_{n+2}) = 0$$

and

$$\sum_{k=n+2}^{\infty} s^k\psi^k(e_0) \to 0 \text{ as } n \to \infty.$$

Therefore,

$$\lim_{n,m\to\infty} d^2(x_n, x_{n+2}) = 0.$$

Case (ii): Let $p = 2m+1$, where $m \geq 1$. By rectangular inequality, we get

$$\begin{aligned}
d^2(x_n, x_{n+2m+1}) &\leq s[d^2(x_n, x_{n+1}) + sd^2(x_{n+1}, x_{n+2}) + sd^2(x_{n+2}, x_{n+2m+1})] \\
&\leq sd^2(x_n, x_{n+1}) + sd^2(x_{n+1}, x_{n+2}) + sd^2(x_{n+1}, x_{n+2}) + s^2d^2(x_{n+2}, x_{n+3}) \\
&\quad + s^2d^2(x_{n+3}, x_{n+4}) + s^2d^2(x_{n+4}, x_{n+2m+1})] \\
&\;\;\vdots \\
&\leq sd^2(x_n, x_{n+1}) + s^2d^2(x_{n+1}, x_{n+2}) \\
&\quad + s^3d^2(x_{n+2}, x_{n+3}) + s^4d^2(x_{n+3}, x_{n+4}) \ldots + s^{2m+1}d^2(x_{n+2m}, x_{n+2m+1}) \\
&= \sum_{k=n}^{n+2m} s^{k-n+1}d^2(x_k, x_{k+1}) \\
&= \sum_{k=n}^{n+2m} s^{k-n+1}\psi^k(e_0)
\end{aligned}$$

$$\le \sum_{k=n}^{n+2m} s^k \psi^k(e_0)$$
$$= \sum_{k=n}^{\infty} s^k \psi^k(e_0) \to 0 \text{ as } n \to \infty.$$

Thus, we obtain

$$d^2(x_n, x_{n+2m+1}) = 0.$$

Case (iii): Let $p = 2m$, where $m \ge 2$. By rectangular inequality, we get

$$\begin{aligned}
d^2(x_{n+2m}, x_n) &\le s[d^2(x_{n+2m}, x_{n+2m-2}) + sd^2(x_{n+2m-2}, x_{n+2m-3}) + sd^2(x_{n+2m-3}, x_n)] \\
&\le sd^2(x_{n+2m}, x_{n+2m-2}) + sd^2(x_{n+2m-2}, x_{n+2m-3}) + s^2d^2(x_{n+2m-3}, x_{n+2m-4}) \\
&\quad + s^2d^2(x_{n+2m-4}, x_{n+2m-5}) + s^2d^2(x_{n+2m-5}, x_n) \\
&\vdots \\
&\le sd^2(x_{n+2m}, x_{n+2m-2}) + s^{n+2m-2}d^2(x_{n+2m-2}, x_{n+2m-3}) \\
&\quad + s^{n+2m-3}d^2(x_{n+2m-3}, x_{n+2m-4}) + s^{n+2m-4}d^2(x_{n+2m-4}, x_{n+2m-5}) \dots \\
&\quad + s^{n-1}d^2(x_{n-1}, x_n) \\
&= sd^2(x_{n+2m}, x_{n+2m-2}) + \sum_{k=n-1}^{n+2m-1} s^k d^2(x_k, x_{k+1}) \\
&\le sd^2(x_{n+2m}, x_{n+2m-2}) + \sum_{k=n-1}^{n+2m-1} s^k \psi^k(l_0^*) \\
&\le sd^2(x_{n+2m}, x_{n+2m-2}) + \sum_{k=n-1}^{\infty} s^k \psi^k(l_0^*).
\end{aligned}$$

Since

$$\lim_{n,m\to\infty} d^2(x_{n+2m}, x_{n+2m-2}) = 0 \text{ and } \sum_{k=n-1}^{\infty} s^k \psi^k(l_0^*) \to 0 \text{ as } n \to \infty,$$

we have

$$\lim_{n,m\to\infty} d^2(x_{n+2m}, x_n) = 0.$$

Case (iv): Let $p = 2m+1$, where $m \ge 1$. By the rectangular inequality, we get

$$\begin{aligned}
d^2(x_{n+2m+1}, x_n) &\le s[d^2(x_{n+2m+1}, x_{n+2m}) + sd^2(x_{n+2m}, x_{n+2m-1}) + sd^2(x_{n+2m-1}, x_n)] \\
&\le sd^2(x_{n+2m+1}, x_{n+2m}) + sd^2(x_{n+2m}, x_{n+2m-1}) + s^2d^2(x_{n+2m-1}, x_{n+2m-2}) \\
&\quad + s^2d^2(x_{n+2m-2}, x_{n+2m-3}) + s^2d^2(x_{n+2m-3}, x_n) \\
&= sd^2(x_{n+2m}, x_{n+2m-2}) + sd^2(x_{n+2m}, x_{n+2m-1}) \\
&\quad + s^2d^2(x_{n+2m-1}, x_{n+2m-2}) + s^2d^2(x_{n+2m-2}, x_{n+2m-3}) + s^2d^2(x_{n+2m-3}, x_n) \\
&\vdots \\
&\le s^{n+2m-1}d^2(x_{n+2m+1}, x_{n+2m}) + s^{n+2m}d^2(x_{n+2m}, x_{n+2m-1}) \\
&\quad + s^{n+2m-1}d^2(x_{n+2m-1}, x_{n+2m-2}) + s^{n+2m-2}d^2(x_{n+2m-2}, x_{n+2m-3}) \dots
\end{aligned}$$

$$\begin{aligned}
&+s^{n+1}d^2(x_{n+1},x_n)\\
&=\sum_{k=n+1}^{n+2m} s^k d^2(x_{k+1},x_k)\\
&=\sum_{k=n+1}^{n+2m} s^{k-n+1}\psi^k(l_0)\\
&\le\sum_{k=n+1}^{n+2m} s^k\psi^k(l_0)\\
&\le\sum_{k=n+1}^{\infty} s^k\psi^k(l_0)\to 0 \text{ as } n\to\infty.
\end{aligned}$$

Thus, we obtain

$$d^2(x_{n+2m+1},x_n)=0.$$

Finally, from *Case* (i)–*Case* (iv), we get

$$\lim_{n\to\infty} d^2(x_n,x_{n+p})=0=\lim_{n\to\infty} d^2(x_{n+p},x_n), \quad \text{for all } p\ge 3.$$

Thus, $\{x_n\}$ is a rectangular quasi b-sequence in (X,d).

Since X is a complete rectangular quasi b-metric space, there exists $u\in X$ such that

$$\lim_{n\to\infty} x_n=u, \text{ i.e., } \lim_{n\to\infty} d^2(x_n,u)=0=\lim_{n\to\infty} d^2(u,x_n). \tag{3.15}$$

Now, we show that u is a fixed point of T.

Since T is a continuous, from (3.15), we have $u=\lim_{n\to\infty} x_{n+1}=\lim_{n\to\infty} Tx_n=T(\lim_{n\to\infty} x_n)=Tu$, which gives $Tu=u$. Thus u is fixed point of T. □

Now, we provide the succeeding fixed point theorem by withdraw the continuity supposition of T from Theorem 3.2.

Theorem 3.3. *Let (X,d) be a complete rectangular quasi b-metric space and $T:X\to X$ be generalized (α,ψ)-contraction mapping. Suppose that*

(I) *T is an α admissible mapping;*

(II) *there exists $x_0\in X$ such that $\alpha(x_0,Tx_0)\ge 1$, $\alpha(Tx_0,x_0)\ge 1$, $\alpha(x_0,T^2x_0)\ge 1$ and $\alpha(T^2(x_0,x_0))\ge 1$;*

(III) *if $\{x_n\}$ is a sequence in X such that $\alpha(x_n,x_{n+1})$ for all $n\ge 0$ and $x_n\to x\in X$ as $n\to\infty$, then $\alpha(x_n,x)\ge 1$, for all $n\ge 0$. Then, T has a fixed point.*

Proof. Succeeding the proof of Theorems 3.2, we know that the sequence $\{x_n\}$ defined by $x_{n+1}=Tx_n$ for all $n\ge 0$ is a rectangular quasi b-convergence to a point $u\in X$. It is sufficient to show that T acknowledge a fixed point. By rectangular inequality property of ψ, and (iii), we have

$$\begin{aligned}
d^2(u,Tu)&\le sd^2(u,x_n)+sd^2(x_n,x_{n+1})+sd^2(x_{n+1},Tu)\\
&=sd^2(u,x_n)+sd^2(x_n,x_{n+1})+sd^2(Tx_n,Tu)\\
&\le sd^2(u,x_n)+sd^2(x_n,x_{n+1})+s\alpha(x_n,u)d^2(Tx_n,Tu)
\end{aligned}$$

$$\leq sd^2(u,x_n)+sd^2(x_n,x_{n+1})+s\psi M(x_n,u), \tag{3.16}$$

where

$$\begin{aligned} M(x_n,u) &= \max\{d^2(x_n,u), d(x_n,x_{n+1})\cdot d(y,Ty), d(x,Tx)\cdot d(x,Ty)\} \\ &= \max\{d^2(x_n,u), d(x_n,Tx_n)\cdot d(y,Ty), d(x,Tx)\cdot d(x,Ty)\} \\ &= \max\{d^2(x_n,u), d(x_n,Tx_n)\cdot d(u,Tu), d(x_n,Tx_n)\cdot (x_n,Tu)\}. \end{aligned}$$

We deal with three different cases as follows:

Case (i): If $M(x_n,u)=d^2(x_n,u)$, then using (3.16), we get

$$\begin{aligned} d^2(u,Tu) &\leq sd^2(u,x_n)+sd^2(x_n,x_{n+1})+s\psi(d^2(u,x_n)) \\ &< sd^2(u,x_n)+sd^2(x_n,x_{n+1})+(d^2(u,x_n)). \end{aligned}$$

Letting $n\to\infty$ in the above inequality, from (3.10) and (3.15), we get $d^2(Tu,u)\leq 0$.

Case (ii): If $M(x_n,u)=d(x_n,x_{n+1})\cdot d(u,Tu)$, then by (3.16), we get

$$\begin{aligned} d^2(u,Tu) &\leq sd^2(u,x_n)+sd^2(x_n,x_{n+1})+s^2\psi(d^2(x_n,x_{n+1})) \\ &< sd^2(u,x_n)+sd^2(x_n,x_{n+1})+(d^2(x_n,x_{n+1})). \end{aligned}$$

Letting $n\to\infty$ in the above inequality, from (3.10) and (3.15), we get $d^2(u,Tu)\leq 0$.

Case (iii): If $M(x_n,u)=d(x_n,Tx_n)\cdot d(x_n,Tu)$, then by (3.16), we get

$$\begin{aligned} d^2(u,Tu) &\leq sd^2(u,x_n)+sd^2(x_n,x_{n+1})+s\psi(d^2(x_n,Tu)) \\ &< sd^2(u,x_n)+sd^2(x_n,x_{n+1})+(d^2(x_n,Tu)). \end{aligned}$$

Letting $n\to\infty$ in the above inequality, from (3.10) and (3.15), we get $0\leq 0$, which is trivial.

Clearly, $d^2(u,Tu)\geq 0$, from Case (i)-Case (iii), we obtain

$$d^2(u,Tu)=0. \tag{3.17}$$

Also,

$$\begin{aligned} d^2(Tu,u) &\leq sd^2(Tu,x_n)+sd^2(x_n,x_{n+1})+sd^2(x_{n+1},u) \\ &= sd^2(Tu,Tx_{n-1})+sd^2(x_n,x_{n+1})+sd^2(x_{n+1},u) \\ &\leq s\alpha(u,x_{n-1})d^2(Tu,Tx_{n-1})+sd^2(x_n,x_{n+1})+sd^2(x_{n+1},u) \\ &\leq s\psi(M(u,x_{n-1}))+sd^2(x_n,x_{n+1})+sd^2(x_{n+1},u), \end{aligned} \tag{3.18}$$

where

$$\begin{aligned} M(u,x_{n-1}) &= \max\{d^2(u,x_{n-1}), d(u,Tu)\cdot d(x_{n-1},Tx_{n-1}), d(u,Tu)\cdot d(u,Tx_{n-1})\} \\ &= \max\{d^2(u,x_{n-1}), d(u,Tu)\cdot d(x_{n-1},x_n), d(u,Tu)\cdot d(u,x_n)\}. \end{aligned}$$

Again, we deal with three different cases as follows:

Case (i): If $M(u,x_{n-1})=d^2(u,x_{n-1})$, then using (3.18), we get

$$\begin{aligned} d^2(Tu,u) &\leq s\psi(d^2(u,x_{n-1}))+sd^2(x_n,x_{n+1})+sd^2(x_{n+1},u) \\ &< d^2(u,x_{n-1}))+sd^2(x_n,x_{n+1})+sd^2(x_{n+1},u). \end{aligned}$$

Letting $n\to\infty$ in the above inequality, using from (3.10) and (3.15), we get $d^2(u,Tu)\le 0$.

Case (ii): If $M(u,x_{n-1})=d(u,Tu)\cdot d(x_{n-1},x_n)$, then by (3.18), we get

$$\begin{aligned}d^2(Tu,u)&\le s\psi(d(u,Tu)d(x_{n-1},x_n))+sd(u,Tu)\cdot d(x_{n-1},x_n)+sd^2(x_n,u)\\&< d(u,Tu)\cdot d(x_{n-1},x_n))+sd(x_{n-1},x_n)+sd^2(x_n,u).\end{aligned}$$

Letting $n\to\infty$ in the above inequality, using (3.10) and (3.15), we get

$$d^2(u,Tu)\le 0.$$

Case (iii): If $M(u,x_{n-1})=d(u,Tu)\cdot d(u,x_n)$, then (3.18), we get

$$\begin{aligned}d^2(Tu,u)&\le s\psi(d(u,Tu)d(u,x_n))+sd^2(x_n,x_{n+1})+sd^2(x_{n+1},u)\\&< d(u,Tu)\cdot d(u,x_n))+sd(u,x_n)+sd^2(x_{n+1},u).\end{aligned}$$

Letting $n\to\infty$ in the above inequality, using (3.10), (3.15) and (3.17), we get $0\le 0$ which is trivial. Clearly, $d^2(Tu,u)\ge 0$, from *Case* (i)–*Case* (iii), we obtain

$$d^2(Tu,u)=0. \tag{3.19}$$

From (3.17) and (3.19), it follows that $d^2(u,Tu)=0=d^2(Tu,u)$. So that, $Tu=u$. Thus u has a fixed point of T. □

To confirm the uniqueness of fixed point of T, we will consider the following condition.

Property U. For all $x,y\in Fix(T)$, we have $\alpha(x,y)\ge 1$ and $\alpha(y,x)\ge 1$, where Fix T denotes the set of all fixed points of T.

Theorem 3.4. *Adding property U to the hypothesis of Theorem* 3.2 (*resp., Theorem* 3.3) *one obtain uniqueness of the fixed point of T.*

Proof. From the proofs of Theorem 3.2 and Theorem 3.3, Fix $(T)\ne\phi$. Suppose that u and v are two distinct fixed points of T. By property U, $\alpha(Tu,Tv)=\alpha(u,v)\ge 1$ and $\alpha(Tv,Tu)=\alpha(v,u)\ge 1$.

Thus, by α-admissibility of T and the above relation, we obtain

$$d^2(u,v)\le\alpha(u,v)d^2(u,v)=\alpha(Tu,Tv)\cdot d^2(u,v)\le\psi(M(u,v)),$$

where

$$\begin{aligned}M(u,v)&=\max\{d^2(u,v),d(u,Tu)\cdot d(v,Tv),d(u,Tu)\cdot d(u,Tv)\}\\&=\max\{d^2(u,v),d(u,u)\cdot d(v,v),d(u,u)\cdot d(u,v)\}\\&=\max\{d^2(u,v)\}.\end{aligned}$$

Since $s\psi(t)<t$, for all $t>0$, and the inequality above, we get

$$d^2(u,v)\le\psi(d^2(u,v))\le s\psi(d^2(u,v))<d^2(u,v), \tag{3.20}$$

which is a contradiction. Similarly,

$$d^2(v,u)\le\alpha(v,u)d^2(v,u)=\alpha(Tv,Tu)\cdot d^2(v,u)\le\psi(M(v,u)),$$

where

$$\begin{aligned}M(v,u) &= \max\{d^2(v,u), d(v,Tv)\cdot d(u,Tu), d(u,Tv)\cdot d(u,Tu)\}\\ &= \max\{d^2(v,u), \cdot d(v,v)\cdot d(u,u), d(u,u)\cdot d(v,u)\}\\ &= \max\{d^2((v,u))\}.\end{aligned}$$

Since $s\psi(t) < t$, for all $t > 0$, and the inequality above, we get

$$d^2(v,u) \le \psi(d^2(v,u)) \le s\psi(d^2(v,u)) < d^2(v,u), \tag{3.21}$$

which is a contradiction. From (3.20) and (3.21), we get that $d^2(u,v) = d^2(v,u) = 0$. Therefore, $u = v$.

Thus T has a unique fixed point. □

Corollary 3.5. *Let (X,d) be a complete rectangular quasi b-metric space and $T: X \to X$ be an (α,ψ)-contraction quadratic type mapping, that is,*

$$\alpha(x,y)d^2(Tx,Ty) \le \Psi(d^2(x,y)), \quad \textit{for all } x,y \in X.$$

Then T has a fixed point.

Remark 3.6. By taking $s = 1$ in Corollary 3.5, we get similar results of Karapinar [13] and Lakzian [18] in quadratic version.

Proof. The results follows by taking $M(x,y) = d^2(x,y)$, for all $x,y \in X$ in the proof of Theorem 3.2 (or Theorem 3.3). □

Remark 3.7. By taking $s = 1$ in Corollary 3.5, we get the works by Khuangsatung *et al.* [16] in quadratic form as follows:

Corollary 3.8. *Let (X,d) be a complete rectangular quasi b-metric space and $T: X \to X$ be a continuous quadratic type mapping if there exists function $\psi \in \Psi$ such that*

$$d^2(Tx,Ty) \le \Psi(d^2(x,y)), \quad \textit{for all } x,y \in X.$$

Then T has a unique fixed point.

Proof. The results follows by taking $\alpha(x,y) = 1$ and $M(x,y) = d^2(x,y)$, for all $x,y \in X$ in the proof of Theorem 3.2. □

Corollary 3.9. *Let (X,d) be a complete rectangular quasi b-metric space and $T: X \to X$ be a continuous quadratic type mapping. Suppose that there exists $k \in [0,1)$ such that*

$$d^2(Tx,Ty) \le k(d^2(x,y)), \quad \textit{for all } x,y \in X.$$

Then T has a unique fixed point.

Proof. The results follows by taking $\psi(t) = kt$, where $k \in [0,1)$ and $t \ge 0$ in Corollary 3.5. □

4. Application to Integral Equation

In this section, we provide an existence theorem for a solution of the following integral equation,

$$x(t) = \int_0^1 K(t,r,x(r))\,dr, \tag{4.1}$$

where $K : [0,1]\times[0,1]\times R \to R$ is continuous functions.

Throughout this section, let $X = (C[0,1],R)$ be the set of real continuous functions defined on $[0,1]$. Take the rectangular quasi b-metric $d : X \times X \to [0,\infty)$ given by

$$d(x,y) = \begin{cases} \|(x-y)\|^4{}_\infty + \|x\|_\infty^2, & \text{if } x \neq y,\\ 0, & \text{if } x = y,\end{cases}$$

where

$$\|u\|_\infty = \max_{r\in[0,1]} |u(s)|, \quad \text{for all } u \in X.$$

It is know that (X,d) is a complete rectangular quasi b-metric space $s = \frac{3}{2}$. Now we prove the following result.

Theorem 4.1. *Suppose the following hypotheses hold*

(I) *there exists $k \in (0,1)$ and $g : X \times X \to [0,\infty)$ such that for all $x,y \in X$ with $x(t) \le y(t)$ for all $t \in [0,1]$ and for every $r \in [0,1]$, we have*

$$0 \le |K(t,r,x(r)) - K(t,r,y(r))| \le g(t,r)|x-r|,$$

and

$$\sup_{t\in[0,1]} \int_0^1 g(t,r)\,dr = k.$$

(II) *K is a non-decreasing in its third variable;*

(III) *there exists $x_0 \in X$ such that for all $t \in [0,1]$, we have*

$$x_0(t) \le \int_0^1 K(t,r,x_0(r))dr$$

and

$$x_0(t) \le \int_0^1 K(t,r,\int_0^1 K(t,r,x_0(r)dr)\,dr.$$

Then (4.1) *has a solution in X.*

Proof. For all $x \in X$ and $t \in [0,1]$, acquaint the mapping $T : X \to X$ by $Tx(t) = \int_0^1 K(t,r,x(r))dr$, and $\alpha : X \times X \to [0,\infty)$ by

$$\alpha(x,y) = \begin{cases} 1, & \text{if } x \le y,\\ 0, & \text{otherwise.}\end{cases}$$

Take $\psi(t) = kt$, so $\psi(t) < \frac{t}{s}$ (since $s = \frac{3}{2}$). We give $x,y \in X$, $x \le y$ if and only if $x(t) \le y(t)$, for all $t \in [0,1]$. Let $x,y \in X$ such that $\alpha(x,y) \ge 1$, so $x \le y$, hence $x(t) \le y(t)$ for all $t \in [0,1]$. Thus, by condition (i),

$$|Tx(t) - Ty(t)|^2 \le \int_0^1 |K(t,r,x_r - K(t,r,y_r|^2 dr$$

$$
\begin{aligned}
&\le \int_0^1 g(t,r)|x(r)-y(r)|^2 dr \\
&= \int_0^1 g(t,r)\sqrt{(x(r)-y(r))^4} dr \\
&\le k^4 \sqrt{\|x-y\|_\infty^4}.
\end{aligned}
$$

Again,

$$
\begin{aligned}
|Tx(t)|^2 &\le \int_0^1 |K(t,r,x_r|^2 dr \\
&\le \int_0^1 g(t,r)|x(r)|^2 dr \\
&\le k^2 \|x\|_\infty^2.
\end{aligned}
$$

We deduce that for all $x,y \in X$ with $x \le y$,

$$
\begin{aligned}
d^2(Tx-Ty) &= \|Tx(t)-Ty(t)\|_\infty^4 + \|x\|_\infty^2 \\
&\le k^4 \|x-y\|_\infty^4 + k^2 \|x\|_\infty^2 \\
&\le k^2 d^2(x,y) \\
&= \psi(d^2(x,y)) \\
&= \psi(M(x,y)).
\end{aligned}
$$

Since K is non decreasing in its third variable, so for all $x,y \in X$ with $x \le y$, we get $T^2x(t) \le T^2y(t)$ for all $t \in [0,1]$, that is if $\alpha(x,y) \ge 1$, we obtain $\alpha(Tx,Ty) \ge 1$. Furthermore the condition (iii) yields that there exists $x_0 \in X$ such that $\alpha(x_0,Tx_0) \ge 1$, $\alpha(Tx_0,x_0) \ge 1$ and $\alpha(x_0,T^2x_0) \ge 1$ and $\alpha(T^2x_0,x_0) \ge 1$. Therefore, all conditions of Theorem 3.3 are confirmed with $s = \frac{3}{2}$ and hence T has a fixed point, which is a solution of (4.1) in X. □

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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