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Research Article

Product Binary *L*-Cordial Labeling of Various Degree Splitting Graphs

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Abstract. This study examines the probability of the degree splitting operation results from various graphs being product binary L-cordial graphs. The graphs under investigation include the path graph (P_n), comb graph ($P_n \odot k_1$), double comb graph ($P_n \odot 2k_1$), cycle graph (C_n), shell graph (S_n) and ladder graph (L_n). The application of product binary L-cordial labeling demonstrates that these analyzed graphs exhibit characteristics of product binary L-cordial graphs.

Keywords. Product binary L-cordial labeling, Product binary L-cordial graph, Degree splitting graph

Mathematics Subject Classification (2020). 05C78, 05C76

1. Introduction

A graph labeling involves assigning distinct labels to the edges and vertices of a graph. For standardized terminology and notations in graph theory, the work of Clark and Holton [5] is consulted. Gallian [6] provides a comprehensive survey with extensive bibliographic references on various graph labeling problems. To conduct our analysis, we begin with a simple, finite, connected, and undirected graph denoted as $G = (V(G), E(G))$. The subsequent section provides a concise overview of definitions and relevant information essential for the current investigation.

2. Preliminaries

L-cordial labeling was introduced by Bapat [3] and is described in Definition 2.1. He also shown that the triangular snakes, star, path and cycle are L-cordial graphs (Bapat [1–3]).

Then, he extended his research to include $FL(C_n)$, C_n, bull (C_n), shell graph S_n, S_n^+ and S_n^{+t}, demonstrating that these graphs are likewise L-cordial.

Definition 2.1 ([3])**.** A graph $G(V,E)$ possesses an L-cordial labeling if there exists a bijection function r from $E(G)$ to $\{1,2,3,\dots,|E|\}$. Each vertex u is labeled 0 if the highest label on its incident edges is even and 1 otherwise. The terms $v_r(1)$ and $v_r(0)$ denote the total count of vertices u's labeled as 1 and 0, respectively. The labeling function r is considered an L-cordial labeling of the graph if the absolute difference between $v_r(1)$ and $v_r(0)$ is less than or equal to 1. It's important to note that isolated vertices are not included in this labeling process.

Steffi and Subhashini [9] have introduced binary L-cordial labeling and product binary L-cordial labeling, detailed in Definition 2.2 and Definition 2.3, respectively. They have explored the product binary L-cordial labeling of graphs such as path, comb graph $P_n \odot k_1$, dumbell graph, flag graph and crown graph $C_n \odot k_1$.

Definition 2.2 ([9])**.** A graph $G(V,E)$ featuring a binary operation exhibits a binary L-cordial labeling when there exists a bijective function r from $E(G)$ to $\{1,2,3,\dots,|E|\}$. In this scenario, each vertex u is assigned the label 1 or 0 based on a binary operation '$*$', where uv_i, uv_j possess the smallest and greatest r values, respectively. Label 1 is assigned if $r(uv_i) * r(uv_j)$ is odd, and 0 is assigned otherwise. The quantities $v_r(1)$ and $v_r(0)$ denote the total number of vertices labeled as 1 and 0, respectively. The labeling function r is deemed a binary L-cordial labeling of the graph if the absolute difference between $v_r(1)$ and $v_r(0)$ is less than or equal to 1. This labeling scheme is represented by the acronym B_{LCL} and graph which hold the said labeling viz., binary L-cordial graph is represented by the acronym B_{LCG}.

Definition 2.3 ([9])**.** A graph $G(V,E)$ featuring a binary operation exhibits a product binary L-cordial labeling when there exists a bijective function r from $E(G)$ to $\{1,2,3,\dots,|E|\}$. In this scenario, each vertex u is assigned the label 1 or 0 based on a multiplication binary operation '$\cdot$', where uv_i, uv_j possess the smallest and greatest r values, respectively. Label 1 is assigned if $r(uv_i)\cdot r(uv_j)$ is odd, and 0 is assigned otherwise. The quantities $v_r(1)$ and $v_r(0)$ denote the total number of vertices labeled as 1 and 0, respectively. The labeling function r is deemed a product binary L-cordial labeling of the graph if the absolute difference between $v_r(1)$ and $v_r(0)$ is less than or equal to 1. This labeling scheme is represented by the acronym PB_{LCL} and graph which hold the said labeling viz., product binary L-cordial graph is represented by the acronym PB_{LCG}.

Now, the descriptions of graphs used in the present investigation are provided below.

Definition 2.4 ([4])**.** Degree splitting graph of G is denoted by $DS(G)$, where $G = (V(G),E(G))$ is a graph with $V = C_1 \cup C_2 \cup C_3 \cup \dots C_p \cup M$. In this representation, each C_k is a set of vertices having same degree (at least two vertices) and $M = V - \cup_{k=1}^{p} C_k$. Degree splitting graph of G is constructed by adding vertices $u_1, u_2, u_3, \dots, u_p$ to the graph G and joining u_k to each vertex of C_k, for $1 \le k \le p$.

Definition 2.5. A shell graph S_n is graph made by taking $(n-3)$ concurrent chords in C_n.

Definition 2.6 ([7])**.** Cartesian product of graphs G_p and G_q is denoted by $G_p \times G_q$. This graph possesses a vertex set $V(G_p)\times V(G_q) = \{(w,c)/w \in V(G_p)$ and $c \in V(G_q)\}$, where (w,c) is connected to (w',c') if and only if either $w = w'$ and $cc' \in E(G_q)$ or $c = c'$ and $ww' \in E(G_p)$.

Definition 2.7. Ladder graph L_n is defined as $L_n = P_n \times K_2$.

Definition 2.8 ([8])**.** Corona product of graphs viz., G_p and G_q is denoted by $G_p \odot G_q$. The $G_p \odot G_q$ is constructed by taking one copy of G_p and $|V(G_p)|$ copies of G_q, and joining each vertex of the jth copies of G_q to jth vertex of G_p, for $1,2,3,\dots,|V(G)|$.

By using corona product of graphs as defined in Definition 2.8 three more graphs are generated which are defined below.

Definition 2.9. Crown graph is defined as $C_n \odot K_1$ in which C_n is the cycle of n vertices.

Definition 2.10. Comb graph is defined as $P_n \odot K_1$ in which P_n is the cycle of n vertices.

Definition 2.11. Double comb graph is defined as $C_n \odot 2K_1$ in which P_n is the cycle of n vertices.

3. Results and Discussions

Theorem 3.1. *$DS(P_n; n \geq 2)$ is a PB_{LCG}.*

Proof. Let $V(DS(P_n)) = \{u, v, v_k : 1 \leq k \leq n\}$ and $E(DS(P_n)) = \{v_k v_{k+1} : 1 \leq k \leq n-1\} \cup \{uv_{k+1} : 1 \leq k \leq n-2\} \cup \{vv_1, vv_n\}$, where $|V(DS(P_n)| = n+2$ and $|E(DS(P_n))| = 2n-1$.
Labeling function $r : E(DS(P_n)) \to \{1,2,3,\dots,|E(DS(P_n))|\}$ is defined as follows:

Case 1: n is odd:

Subcase 1: For $n = 3$,

$$r(p) = \begin{cases} 2k-1, & \text{for } p = v_k v_{k+1},\ 1 \leq k \leq 2; \\ 5, & \text{for } p = vv_1; \\ 4, & \text{for } p = vv_n; \\ 2, & \text{for } p = uv_2. \end{cases}$$

Subcase 2: For $n > 3$,

$$r(p) = \begin{cases} 2k-1, & \text{for } p = v_k v_{k+1},\ 1 \leq k \leq n-1; \\ 2k, & \text{for } p = uv_{k+1},\ 1 \leq k \leq \frac{n-1}{2}; \\ n+2k+1, & \text{for } p = uv_{\frac{n+2k+1}{2}},\ 1 \leq k \leq \frac{n-3}{2}; \\ 2n-1, & \text{for } p = vv_1; \\ n+1, & \text{for } p = vv_n. \end{cases}$$

Consequently, $v_r(1) = \frac{n+1}{2}$ and $v_r(0) = \frac{n+3}{2}$.

Case 2: n is even:

Subcase 1: For $n = 4$,

$$r(p) = \begin{cases} 2k-1, & \text{for } p = v_k v_{k+1}, 1 \le k \le 3; \\ 2, & \text{for } p = uv_2; \\ 4, & \text{for } p = uv_3; \\ 7, & \text{for } p = vv_1; \\ 6, & \text{for } p = vv_4. \end{cases}$$

Subcase 2: For $n > 4$,

$$r(p) = \begin{cases} 2k-1, & \text{for } p = v_k v_{k+1}, 1 \le k \le n-1; \\ 2k, & \text{for } p = uv_{k+1}, 1 \le k \le \frac{n}{2}; \\ n+2k+2, & \text{for } p = uu_{\frac{n+2k+2}{2}}, 1 \le k \le \frac{n-4}{2}; \\ 2n-1, & \text{for } p = vv_1; \\ n+2, & \text{for } p = vv_n. \end{cases}$$

Consequently, $v_r(1) = \frac{n+2}{2} = v_r(0)$.

The observation of $|v_r(1) - v_r(0)| \le 1$ in all the cases in Theorem 3.1 highlights that the $DS(P_n; n \ge 2)$ is in fact a PB_{LCG}. □

Example 3.1. $DS(P_6)$ is a PB_{LCG} which is shown in Figure 1.

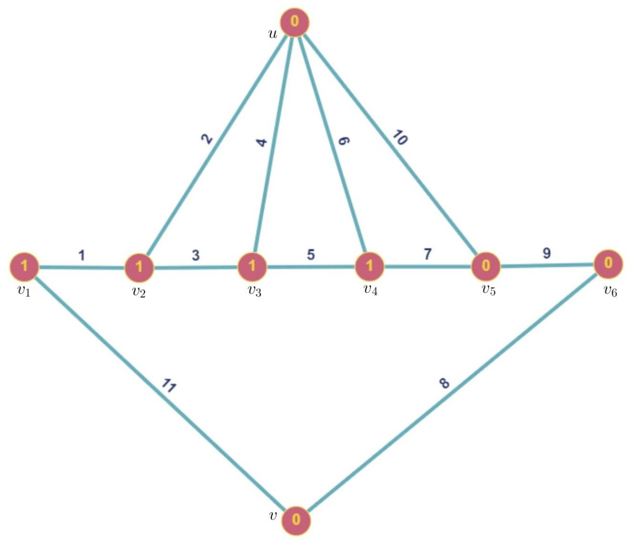

Figure 1. $DS(P_6)$ with its PB_{LCL}

Theorem 3.2. *$DS(P_n \odot k_1; n \ge 2)$ is a PB_{LCG}.*

Proof. Let $V(DS(P_n \odot k_1)) = \{u, v, q, u_k, w_k : 1 \le k \le n\}$ and $E(DS(P_n \odot k_1)) = \{w_k w_{k+1} : 1 \le k \le n-1\} \cup \{u_k w_k : 1 \le k \le n\} \cup \{uu_k : 1 \le k \le n\} \cup \{qw_1, qw_n\} \cup \{vw_{k+1} : 1 \le k \le n-2\}$, where $|V(DS(P_n \odot k_1)| = 2n+3$ and $|E(DS(P_n \odot k_1))| = 4n-1$.

Labeling function $r : E(DS(P_n \odot k_1)) \to \{1,2,3,\dots,|E(DS(P_n \odot k_1))|\}$ is defined as follows:

$$r(p) = \begin{cases} 2k-1, & \text{for } p = w_k w_{k+1},\ 1 \le k \le n-1; \\ 2k, & \text{for } p = u_k w_k,\ 1 \le k \le n; \\ 2n+2k, & \text{for } p = uu_k,\ 1 \le k \le n-1; \\ 4n-1, & \text{for } p = uu_n; \\ 4n-5, & \text{for } p = qw_1; \\ 4n-3, & \text{for } p = qw_n; \\ 2n+2k-3, & \text{for } p = vw_{k+1},\ 1 \le k \le n-2. \end{cases}$$

Consequently, $v_r(1) = n+2$ and $v_r(0) = n+1$.

The observation of $|v_r(1) - v_r(0)| \le 1$ in all the cases in Theorem 3.2 highlights that the $DS(P_n \odot k_1; n \ge 2)$ is in fact a PB_{LCG}.

Example 3.2. $DS(P_5 \odot k_1)$ is a PB_{LCG} which is shown in Figure 2.

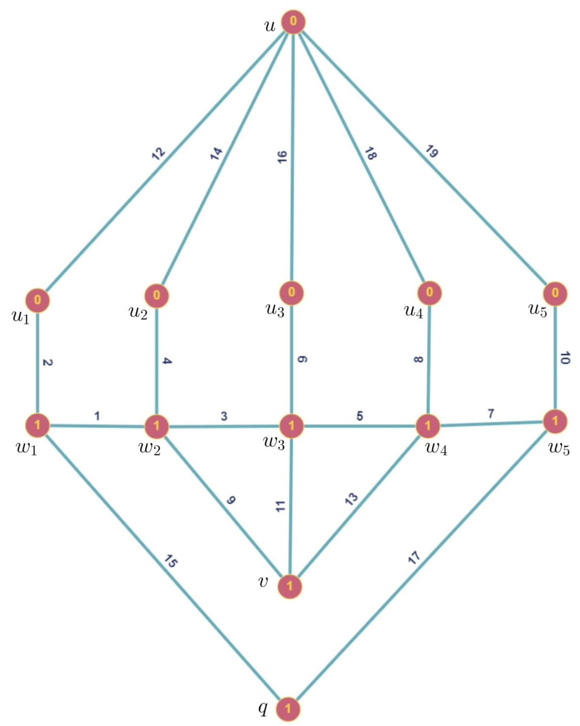

Figure 2. The graph $DS(P_5 \odot k_1)$ with its PB_{LCL}

Theorem 3.3. *The graph $DS(P_n \odot 2k_1; n \ge 2)$ is a PB_{LCG}.*

Proof. Let $V(DS(P_n \odot 2k_1)) = \{u, v, q, u_k, v_k, w_k : 1 \le k \le n\}$ and $E(DS(P_n \odot 2k_1)) = \{w_k w_{k+1} : 1 \le k \le n-1\} \cup \{u_k w_k : 1 \le k \le n\} \cup \{v_k w_k : 1 \le k \le n\} \cup \{uu_k : 1 \le k \le n\} \cup \{uv_k : 1 \le k \le n\} \cup \{qw_1, qw_n\} \cup \{vw_{k+1} : 1 \le k \le n-2\}$, where $|V(DS(P_n \odot 2k_1)| = 3n+3$ and $|E(DS(P_n \odot 2k_1))| = 6n-1$.

Labeling function $r : E(DS(P_n \odot 2k_1)) \to \{1,2,3,\dots,|E(DS(P_n \odot 2k_1))|\}$ is defined as follows:

Case 1: n is odd:

$$r(p) = \begin{cases} 2k-1, & \text{for } p = w_k v_k, 1 \le k \le n; \\ 2n+k, & \text{for } p = v v_k, 1 \le k \le n; \\ 4n+2k-1, & \text{for } p = u_k w_k, 1 \le k \le n; \\ 2k, & \text{for } p = u w_k, 2 \le k \le n-1; \\ 3n+k, & \text{for } p = w_k w_{k+1}, 1 \le k \le n-2; \\ 4n+2k-2, & \text{for } p = v u_k, 2 \le k \le n; \\ 4n, & \text{for } p = w_{n-1} w_n; \\ 4n-1, & \text{for } p = v u_1; \\ 2, & \text{for } p = q w_1; \\ 2n, & \text{for } p = q w_n. \end{cases}$$

Consequently, $v_r(0) = 3\lfloor \frac{n}{2} \rfloor + 3 = v_r(1)$.

Case 2: n is even:

Subcase 1: For $n = 2$ (Figure 3),

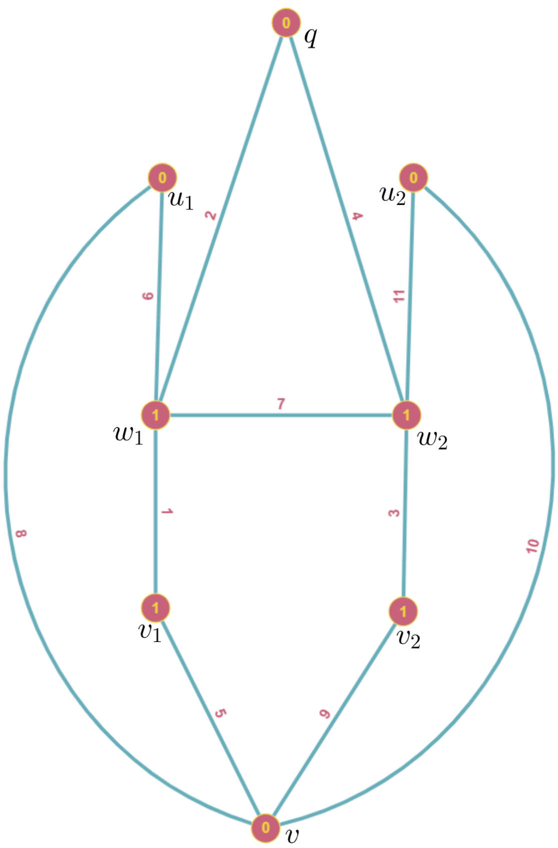

Figure 3. The graph $DS(L_2)$ with its PB_{LCL}

Subcase 2: For $n > 2$,

$$r(p) = \begin{cases} 2k-1, & \text{for } p = w_k v_k, 1 \le k \le n; \\ 2n+k, & \text{for } p = v v_k, 1 \le k \le n-1; \\ 3n+k, & \text{for } p = w_k w_{k+1}, 1 \le k \le n-1; \\ 4n+2k-1, & \text{for } p = u_k w_k, 1 \le k \le n-1; \\ 3n, & \text{for } p = u_n w_n; \\ 6n-2, & \text{for } p = v v_n; \\ 4n+2k-2, & \text{for } p = v u_k, 1 \le k \le n-1; \\ 6n-1, & \text{for } p = v u_n; \\ 2k, & \text{for } p = u w_k, 2 \le k \le n-1; \\ 2, & \text{for } p = q w_1; \\ 2n, & \text{for } p = q w_n. \end{cases}$$

Consequently, $v_r(1) = \frac{3n}{2} + 1$ and $v_r(0) = \frac{3n}{2} + 2$.

The observation of $|v_r(1) - v_r(0)| \le 1$ in all the cases in Theorem 3.3 highlights that the $DS(P_n \odot 2k_1; n \ge 2)$ is in fact a PB_{LCG}.

Example 3.3. $DS(P_5 \odot 2k_1)$is a PB_{LCG} which is shown in Figure 4.

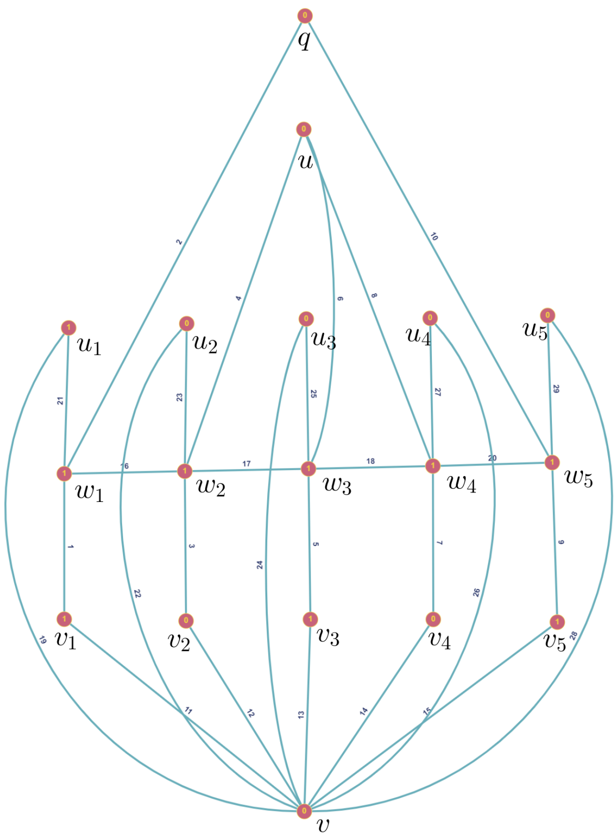

Figure 4. The graph $DS(P_5 \odot 2k_1)$ with its PB_{LCL}

Theorem 3.4. *The graph $DS(C_n)$ is a PB_{LCG}.*

Proof. Let $V(DS(C_n)) = \{v, u_k : 1 \le k \le n\}$ and $E(DS(C_n)) = \{u_k u_{k+1} : 1 \le k \le n-1\} \cup \{u_n u_1\} \cup \{v u_k : 1 \le k \le n\}$, where $|V(DS(C_n)| = n+1$ and $|E(DS(C_n))| = 2n$.
Labeling function $r : E(DS(C_n)) \to \{1,2,3,\dots,|E(DS(C_n))|\}$ is defined as follows:

Case 1: n is odd:

$$r(p) = \begin{cases} 2k-1, & \text{for } p = u_k u_{k+1},\ 1 \le k \le \frac{n-1}{2}; \\ n-2k+3, & \text{for } p = u_{\frac{n-1}{2}+k} u_{\frac{n+1}{2}+k},\ 1 \le k \le \frac{n-1}{2}; \\ 2, & \text{for } p = u_n u_1; \\ n+2k-2, & \text{for } p = v u_k,\ 1 \le k \le \frac{n+1}{2}; \\ n+2k+1, & \text{for } p = v u_{\frac{n+1}{2}+k},\ 1 \le k \le \frac{n-1}{2}. \end{cases}$$

Consequently, $v_r(1) = \lceil \frac{n}{2} \rceil = v_r(0)$.

Case 2: n is even:

$$r(p) = \begin{cases} 2k-1, & \text{for } p = u_k u_{k+1},\ 1 \le k \le \frac{n}{2}; \\ 2k, & \text{for } p = u_{\frac{n}{2}+k} u_{\frac{n}{2}+k+1},\ 1 \le k \le \frac{n-2}{2}; \\ n, & \text{for } p = u_n u_1; \\ n+2k-1, & \text{for } p = v u_k,\ 1 \le k \le \frac{n}{2}; \\ n+2k, & \text{for } p = v u_{\frac{n}{2}+k},\ 1 \le k \le \frac{n}{2}. \end{cases}$$

Consequently, $v_r(1) = \frac{n}{2}$ and $v_r(0) = \frac{n}{2} + 1$. □

The observation of $|v_r(1) - v_r(0)| \le 1$ in all the cases in Theorem 3.4 highlights that the $DS(C_n)$ is in fact a PB_{LCG}.

Example 3.4. $DS(C_7)$ is a PB_{LCG} which is shown in Figure 5.

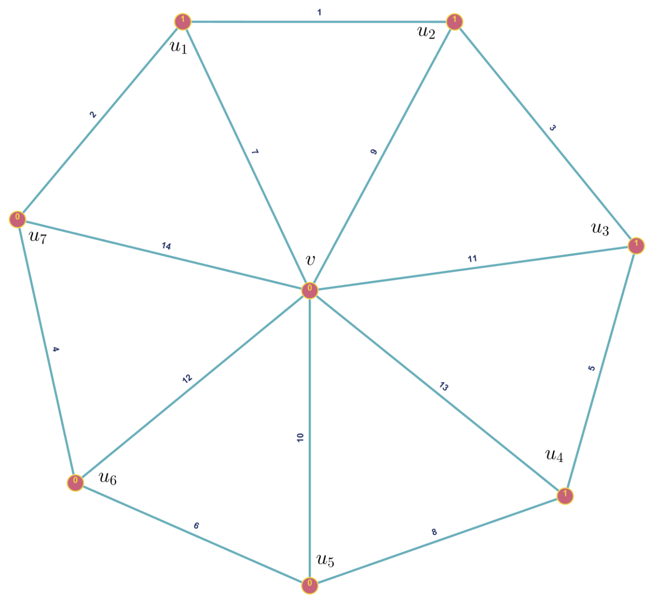

Figure 5. The graph $DS(C_7)$ with its PB_{LCL}

Theorem 3.5. *The graph $DS(S_n)$ is a PB_{LCG}.*

Proof. Let $V(DS(S_n)) = \{u', u'', u_k : 1 \le k \le n\}$ and $E(DS(S_n)) = \{u_k u_{k+1} : 1 \le k \le n-1\} \cup \{u_1 u_k : 3 \le k \le n-1\} \cup \{u_n u_1, u' u_n, u' u_2\} \cup \{u'' u_k : 3 \le k \le n-1\}$, where $|V(DS(S_n)| = n+2$ and $|E(DS(S_n))| = 3n-4$.
Labeling function $r : E(DS(S_n)) \to \{1,2,3,\dots,|E(DS(S_n))|\}$ is defined as follows:

Case 1: n is odd:

$$r(p) = \begin{cases} 2k-1, & \text{for } p = u_k u_{k+1},\ 1 \le k \le \frac{n+1}{2}; \\ 2k, & \text{for } p = u_{\frac{n+1}{2}+k} u_{\frac{n+3}{2}+k},\ 1 \le k \le \frac{n-3}{2}; \\ n-1, & \text{for } p = u_n u_1; \\ n+2k, & \text{for } p = u_1 u_{k+2},\ 1 \le k \le \frac{n-3}{2}; \\ n+2k-1, & \text{for } p = u_1 u_{\frac{n+1}{2}+k},\ 1 \le k \le \frac{n-3}{2}; \\ 2n-2, & \text{for } p = u' u_n; \\ 2n-1, & \text{for } p = u' u_2; \\ 2n+2k-1, & \text{for } p = u'' u_{k+2},\ 1 \le k \le \frac{n-3}{2}; \\ 2n+2k-2, & \text{for } p = u'' u_{\frac{n+1}{2}+k},\ 1 \le k \le \frac{n-3}{2}. \end{cases}$$

Consequently, $v_r(1) = \frac{n+1}{2}$ and $v_r(0) = \frac{n+3}{2}$.

Case 2: n is even:

$$r(p) = \begin{cases} 2k-1, & \text{for } p = u_k u_{k+1},\ 1 \le k \le \frac{n}{2}; \\ n-2k+2, & \text{for } p = u_{\frac{n}{2}+k} u_{\frac{n+2}{2}+k},\ 1 \le k \le \frac{n-2}{2}; \\ 2, & \text{for } p = u_n u_1; \\ n+2k-1, & \text{for } p = u_1 u_{k+2},\ 1 \le k \le \frac{n-2}{2}; \\ n+2k, & \text{for } p = u_1 u_{\frac{n+2}{2}+k},\ 1 \le k \le \frac{n-4}{2}; \\ 2n-2, & \text{for } p = u' u_n; \\ 3n-4, & \text{for } p = u' u_2; \\ 2n+2k-3, & \text{for } p = u'' u_{k+2},\ 1 \le k \le \frac{n-2}{2}; \\ 2n+2k-2, & \text{for } p = u'' u_{\frac{n+2}{2}+k},\ 1 \le k \le \frac{n-4}{2}. \end{cases}$$

Consequently, $v_r(1) = \frac{n+2}{2} = v_r(0)$.

The observation of $|v_r(1) - v_r(0)| \le 1$ in all the cases in Theorem 3.5 highlights that the $DS(S_n)$ is in fact a PB_{LCG}.

Example 3.5. $DS(S_8)$ is a PB_{LCG} which is shown in Figure 6.

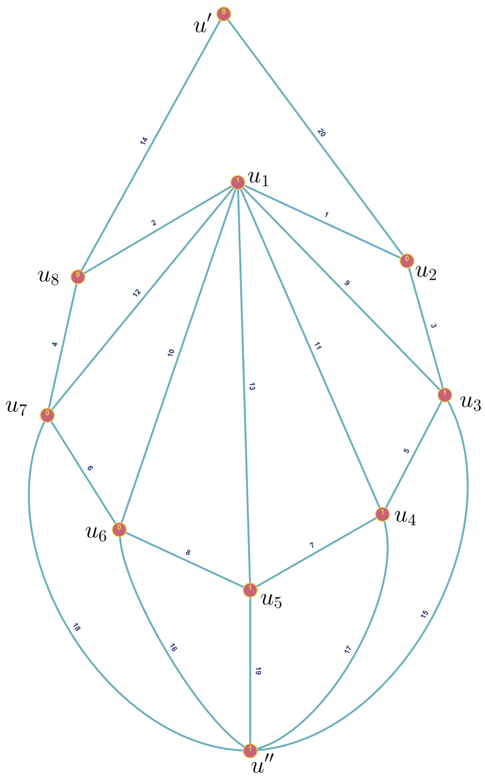

Figure 6. The graph $DS(S_8)$ with its PB_{LCL}

Theorem 3.6. *The graph $DS(L_n)$ is a PB_{LCG}.*

Proof. Let $V(DS(L_n)) = \{u', w', u_k, w_k : 1 \le k \le n\}$ and $E(DS(L_n)) = \{u_k u_{k+1}, w_k w_{k+1} : 1 \le k \le n-1\} \cup \{u'w_k, u'u_k : 2 \le k \le n-1\} \cup \{w'u_1, w'u_n, w'w_1, w'w_n\}$, where $|V(DS(L_n)| = 2n+2$ and $|E(DS(L_n))| = 5n-2$.
Labeling function $r : E(DS(L_n)) \to \{1, 2, 3, \dots, |E(DS(L_n))|\}$ is defined as follows:

Case 1: n is odd:

Subcase 1: For $n = 3$ (Figure 7),

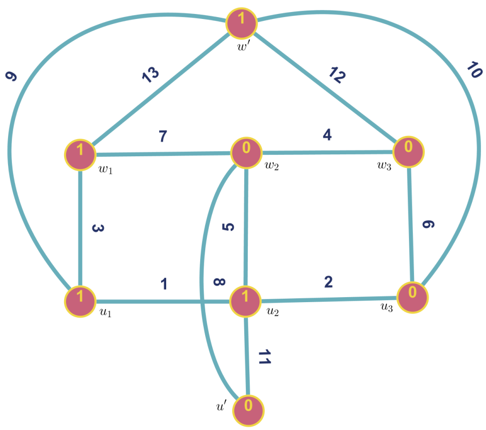

Figure 7. The graph $DS(L_3)$ with its PB_{LCL}

Subcase 2: For $n = 5$,

$$r(p) = \begin{cases} n-2k, & \text{for } p = u_k u_{k+1},\ 1 \le k \le \frac{n-1}{2};\\ 2k, & \text{for } p = u_{\frac{n-1}{2}+k} u_{\frac{n+1}{2}+k},\ 1 \le k \le \frac{n-1}{2};\\ 2n+2k-1, & \text{for } p = w_k w_{k+1},\ 1 \le k \le \frac{n-1}{2};\\ n+2k-1, & \text{for } p = w_{\frac{n-1}{2}+k} w_{\frac{n+1}{2}+k},\ 1 \le k \le \frac{n-1}{2};\\ n+2k-2, & \text{for } p = u_k w_k,\ 1 \le k \le \frac{n+1}{2};\\ 2n+2k-2, & \text{for } p = u_{\frac{n+1}{2}+k} w_{\frac{n+1}{2}+k},\ 1 \le k \le \frac{n-1}{2};\\ 3n+2k-2, & \text{for } p = u' u_{k+1},\ 1 \le k \le \frac{n-1}{2};\\ 3n+2k-3, & \text{for } p = u' u_{\frac{n+1}{2}+k},\ 1 \le k \le \frac{n-3}{2};\\ 5n-4, & \text{for } p = u' w_{\frac{n-1}{2}};\\ 4n+2k-6, & \text{for } p = u' w_{\frac{n-1}{2}+k},\ 1 \le k \le \frac{n-1}{2};\\ 5n-6, & \text{for } p = w' u_1;\\ 5n-2, & \text{for } p = w' w_1;\\ 5n-5, & \text{for } p = w' u_n;\\ 5n-3, & \text{for } p = w' w_n. \end{cases}$$

Subcase 3: For $n \ge 7$, In addition to edge labeling conditions required for $n = 5$ this case requires one more additional edge labeling condition which is given below.

$$r(p) = 4n + 2k - 3, \quad \text{for } p = u' w_{k+1}, \quad 1 \le k \le \frac{n-5}{2}.$$

Consequently, $v_r(1) = n + 1 = v_r(0)$.

Case 2: For n is even:

Subcase 1: For $n = 4$ (Figure 8),

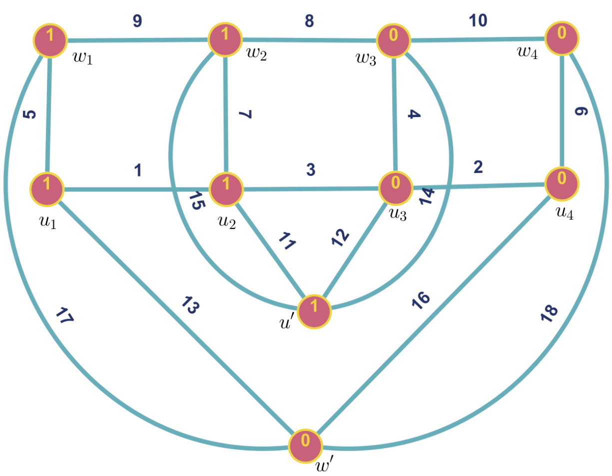

Figure 8. The graph $DS(L_4)$ with its PB_{LCL}

Subcase 2: For $n \geq 6$,

$$r(p)=\begin{cases} 2k-1, & \text{for } p=u_ku_{k+1},\ 1\leq k\leq \frac{n}{2};\\ 2k, & \text{for } p=u_{\frac{n}{2}+k}u_{\frac{n+2}{2}+k},\ 1\leq k\leq \frac{n-2}{2};\\ 2n+2k-1, & \text{for } p=w_kw_{k+1},\ 1\leq k\leq \frac{n-2}{2};\\ 2n+2k-2, & \text{for } p=w_{\frac{n-2}{2}+k}w_{\frac{n}{2}+k},\ 1\leq k\leq \frac{n}{2};\\ n+2k-1, & \text{for } p=u_kw_k,\ 1\leq k\leq \frac{n}{2};\\ n+2k-2, & \text{for } p=u_{\frac{n}{2}+k}w_{\frac{n}{2}+k},\ 1\leq k\leq \frac{n}{2};\\ 3n+2k-3, & \text{for } p=u'u_{k+1},\ 1\leq k\leq \frac{n-2}{2};\\ 3n+2k-2, & \text{for } p=u'u_{\frac{n}{2}+k},\ 1\leq k\leq \frac{n-2}{2};\\ 5n-5, & \text{for } p=u'w_{\frac{n}{2}};\\ 4n+2k-5, & \text{for } p=u'w_{k+1},\ 1\leq k\leq \frac{n-4}{2};\\ 4n+2k-4, & \text{for } p=u'w_{\frac{n}{2}+k},\ 1\leq k\leq \frac{n-2}{2};\\ 5n-7, & \text{for } p=w'u_1;\\ 5n-3, & \text{for } p=w'w_1;\\ 5n-4, & \text{for } p=w'u_n;\\ 5n-2, & \text{for } p=w'w_n. \end{cases}$$

Consequently, $v_r(1)=n+1=v_r(0)$.

The observation of $|v_r(1)-v_r(0)|\leq 1$ in all the cases in Theorem 3.6 highlights that the $DS(L_n)$ is in fact a PB_{LCG}.

Example 3.6. $DS(L_7)$ is a PB_{LCG} which is shown in Figure 9.

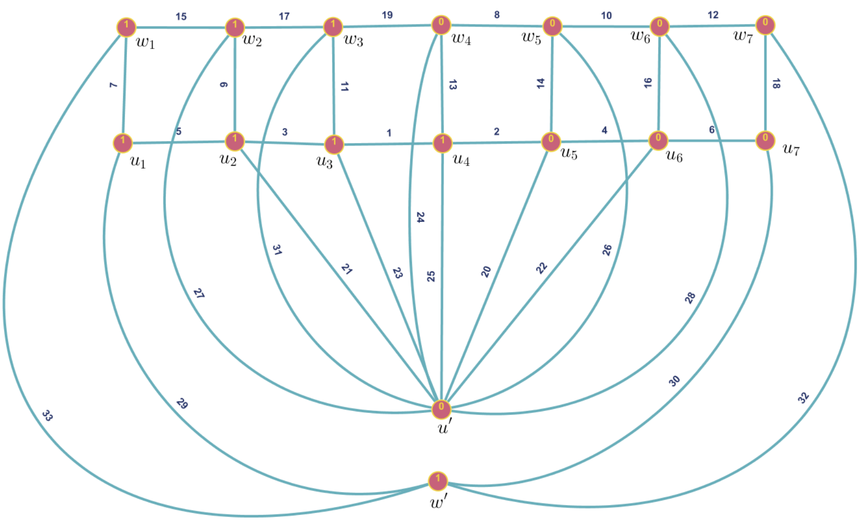

Figure 9. The graph $DS(L_7)$ with its PB_{LCL}

4. Conclusions

The potential that the degree splitting graphs of path graph (P_n), comb graph ($P_n \odot k_1$), double comb graph ($P_n \odot 2k_1$), cycle graph (C_n), shell graph (S_n) and ladder graph (L_n) are product binary L-cordial graph has been investigated in this study. It has been determined that the examined graphs are, in fact, product binary L-cordial graphs by using product binary L-cordial labeling. Future study can also be conducted in a similar manner on other operations viz., shadow, mirror etc. and other types of graphs.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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