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Research Article

An Application of Differential Transform Method to Solve an Epidemic Model — Ebola Virus Disease Outbreaks

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Abstract. An epidemic is defined as an unusually large, short-term disease outbreak. Various factors influence a disease's spread from person to person. These include the infectious agent itself, its mode of transmission, infectious period and its susceptibility and resistance. In this work, we consider a system of non-linear differential equations which constructed as a mathematical model of disease due to Ebola Virus Disease. This model is divided into five compartments as SIRDP (*Susceptible-Infected-Recovered-Deceased-Pathogens*). Further, we solve this model by one of the novel techniques the *Differential Transform Method* (DTM). Moreover, the simulation of solution derived by DTM is compared with VIM.

Keywords. Ebola Virus Disease (EVD), Differential transform method, Variational iteration method

Mathematics Subject Classification (2020). 65L06, 92C60, 92D25, 92D30

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1. Introduction

In 1976, near the Ebola river in the Democratic Republic of Congo, the Ebola virus was initially detected (Ndanguza *et al.* [25], and Rachah and Torres [26]). Since then, the virus has infected humans on a sporadic basis, causing outbreaks in a number of African countries. The origin of the Ebola virus is unknown to scientists. They believe that EVD is animal-borne, with bats or nonhuman primates as the most likely source based on similar viruses. Virus carrying in animals such as apes, monkeys, duikers and humans can spread the virus to other animal and humans. Only four of the viruses have caused sickness in humans (Ebola, Sudan, Ta Forest, Bundibugyo). Individuals get diagnosed with the disease by close interaction with animal carcasses, biological fluids and cells. The infection then replicates to more persons by direct contact with the body fluids of an EVD patient or a person who has died from the disease. When a person communicates with infectious bodily excretions or objects that are contaminated, it could occur. The virus then penetrates via cracks in the skin or mucous membranes in the eyes, nose and mouth. Physical intercourse with those who are sick with EVD or has rehabilitated with it can alert individuals to the virus. After healing from the sickness, the disease can be observed in various bodily fluids. Furthermore, the Ebola virus is not recognized to still be spread through food. Moreover, in certain parts of the world, the Ebola virus can be transmitted by touching and eating wild animal meat or hunting Ebola-infected wild creatures. There seems to be no evidence that mosquitoes or other insects may spread the Ebola virus. Signs can develop somewhere between 12 and 20 days after viral transmission, with just an average of 8 to 10 days. As the individual goes worse, the illness generally develops from 'dry' symptoms (fever, aches and pains, and weariness) to 'wet' symptoms (diarrhea and vomiting) (Agusto *et al.* [2], Area *et al.* [4], Berge *et al.* [7], Bibby *et al.* [9], Chowell *et al.* [10], Fisman *et al.* [11], Ivorra *et al.* [16], Juga *et al.* [17], and Leroy *et al.* [22, 23], ¹).

The keynotes of the transmission of the disease helps us to frame SIR (*Susceptible-Infected-Recovered*) model, which usually shows the flow design linking the compartments. The origin of such model works start began by Ross [27] in 1910, Ross and Hudson [28] in 1917, Kermack and McKendic [19] in 1927, and Kendall [18] in 1956. Furthermore, SIRD-SIRV-MSIR-SEIR-SEIS-MSEIR-MSEIRS etc. epidemiological model has been developed by many with ordinary differential equation either in deterministic or in stochastic (random) (Legrand *et al.* [20], and Lekone *et al.* [21]).

In this paper, SIRDP (*Susceptible-Infected-Recovered-Deceased-Pathogens*) model is proposed by *Differential Transform Method* (DTM) for analytical solution and *Variation Iteration Method* (VIM) for numerical solution with their initial and parametric values. The linear and nonlinear differential equation can be solved by DTM. The calculations are carried out by using DTM and VIM to show the efficiency of the proposed model. The finding discloses that the DTM can achieve more suitable results in predicting the solution of such problems (Akinboro *et al.* [3], Ayaz [5], and Hatami *et al.* [13]).

¹UNO - African Region, *Ebola Disease*, external situation report, (2022), <https://www.afro.who.int/health-topics/ebola-virus-disease>

2. Mathematical Formulation

The Ebola virus in the habitat is classified into five compartments: susceptible population $f(t)$, infected individuals $l(t)$, recovered $q(t)$, infected deceased $w(t)$, and the Ebola virus pathogen in the environment is classified as $g(t)$. Various equations have been used to explain the model and its behavior, including [24]

$$\begin{aligned}
 \frac{df(t)}{dt} &= \alpha_1 - [\beta_1 l(t) + \beta_2 w(t) + \gamma g(t)] f(t) - \zeta f(t) ; \\
 \frac{dl(t)}{dt} &= \alpha_2 + [\beta_1 l(t) + \beta_2 w(t) + \gamma g(t)] f(t) - (\zeta + \theta + \epsilon) l(t) ; \\
 \frac{dq(t)}{dt} &= \epsilon l(t) - \zeta q(t) ; \\
 \frac{dw(t)}{dt} &= (\zeta + \theta) l(t) - \omega w(t) ; \\
 \frac{dg(t)}{dt} &= \nu + \chi l(t) + \rho w(t) - \xi g(t),
 \end{aligned} \tag{2.1}$$

where the susceptible cases are increasing by migrant and new births at a constant rate α_1 , the susceptible infectious one valued as β_1 , the diseased individual β_2 and the polluted environment γ might acquire infection. Natural death was computed as ζ , and infectious human death was calculated as θ . The ϵ is seen as a recovered person. The infected population may expand as a result of the birth of an infected infant or the migration of an infected person, as demonstrated by a constant rate α_2 . The rate of human entombing after death is ω . The constant rate at which the Ebola virus disease is adulterate the environment through all possible ways such as wildlife, fruit bats and so on is ν . Furthermore, χ , ξ and ρ are the rates at which infectious and deceased persons gather the environment.

Table 1. Ebola virus pathobiology initial values ¹

Initial value	Values
$f(0)$	3470
$l(0)$	3317
$q(0)$	2287
$w(0)$	1171
$g(0)$	0

Table 2. Parametric values for Ebola Virus Disease (EVD) (Nazir *et al.* [24])

Variable	Values	Variable	Values
α_1	10.000	γ	0.01
α_2	3.000	θ	0.05
β_1	0.006	ρ	0.04
β_2	0.012	ζ	0.50
ν	0.000	ϵ	0.06
ω	0.800	χ	0.04
ξ	0.030		

3. Differential Transform Method and its Properties

In this section, we discussed about the basic definitions and operation properties of differential transformation method. This method consists of a given system of differential equations and related initial conditions. These are transformed into a system of recurrence equations that finally leads to a system of algebraic equations whose solutions are the coefficients of a power series solution.

The differential transformation of a function is defined as follows:

$$Y(p) = \frac{1}{p!} \left[\frac{d^p y(t)}{dt^p} \right]_{t=t_0}. \quad (3.1)$$

In equation (3.1), $y(t)$ is the original function and $Y(p)$ is the transformed function, which is called T -function. Differential inverse transform for $Y(p)$ is defined as:

$$y(t) = \sum_{p=0}^{\infty} Y(p)(t-t_0)^p. \quad (3.2)$$

From equations (3.1) and (3.2), we obtain

$$y(t) = \sum_{p=0}^{\infty} \frac{(t-t_0)^p}{p!} \left[\frac{d^p y(t)}{dt^p} \right]_{t=t_0}. \quad (3.3)$$

Equation (3.3) implies that the concept of differential transform is derived from the Taylor series expansion and relative derivatives are calculated by an iterative way which is described by the transformed equations of the original functions. The function $y(t)$ is expressed as finite series then (3.2) can be written

$$y(t) \approx \sum_{p=0}^N Y(p)(t-t_0)^p \quad (3.4)$$

with N is a convergence of natural frequency. For the properties and its applications one can refer [5, 13, 30]

4. Variational Iteration Method

In 1997, He [15] introduced another semi-analytical method namely *Variational Iteration Method* (VIM) (also see He [14]) to solve large class of non-linear differential equations effectively and it is observed that this method helps to get fast convergence. Following He [15], there are numerous works among those [1, 3, 8, 12, 29] have been considered and applied the VIM to solve the system of linear or non-linear ODEs and obtained solutions.

Now, consider the non-linear equation:

$$L\beta(\tau) + N\beta(\tau) = \alpha(\tau),$$

where L is a linear operator, N is non-linear operator and $\alpha(\tau)$ is analytic function. Due to VIM, we take the correction functional as

$$\beta^{(n+1)}(\tau) = \beta^{(n)}(\tau) + \int_0^\tau \lambda(t)(L\beta(t) + N\beta(t) - \alpha(t))dt,$$

where λ is known as a general Lagrange multiplier and evaluated by variational theory, $\beta^{(0)}(\tau)$ is an initial approximation with possible unknowns and $\tilde{\beta}^{(n)}(\tau)$ is a restricted variation, that is $\delta \tilde{\beta}^{(n)}(\tau) = 0$ (He [15]).

5. DTM Proposal

Let $\mathcal{F}(t)$, $\mathcal{L}(t)$, $\mathcal{Q}(t)$, $\mathcal{W}(t)$ and $\mathcal{G}(t)$ represent the differential transform of $f(t)$, $l(t)$, $q(t)$, $w(t)$ and $g(t)$, respectively. With the properties of differential transform method, the recurrence relations to each equation in Section 2 can be written as:

$$\begin{aligned} \mathcal{F}(p+1) &= \frac{1}{(p+1)} \left[\alpha_1 \delta(p) - \sum_{r=0}^p \mathcal{F}(r) (\beta_1 \mathcal{L}(p-r) + \beta_2 \mathcal{W}(p-r) + \gamma \mathcal{G}(p-r)) - \zeta \mathcal{F}(p) \right]; \\ \mathcal{L}(p+1) &= \frac{1}{(p+1)} \left[\alpha_2 \delta(p) + \sum_{r=0}^p \mathcal{F}(r) (\beta_1 \mathcal{L}(p-r) + \beta_2 \mathcal{W}(p-r) + \gamma \mathcal{G}(p-r)) - (\zeta + \theta + \epsilon) \mathcal{L}(p) \right]; \\ \mathcal{Q}(p+1) &= \frac{1}{(p+1)} [\epsilon \mathcal{L}(p) - \zeta \mathcal{Q}(p)]; \\ \mathcal{W}(p+1) &= \frac{1}{(p+1)} [(\zeta + \theta) \mathcal{L}(p) - \omega \mathcal{W}(p)]; \\ \mathcal{G}(p+1) &= \frac{1}{(p+1)} [\nu \delta(p) + \chi \mathcal{L}(p) + \rho \mathcal{W}(p) - \xi \mathcal{G}(p)]. \end{aligned} \tag{5.1}$$

Additionally, applying the initial values and parameter values from Table 1 and Table 2 to form the series of $f(t)$, $l(t)$, $q(t)$, $w(t)$ and $g(t)$ upto certain orders by inverse differential transform method we get:

$$\begin{aligned} f(t) &= 3470 - 116075.3800t + 714521.3145t^2 + 24337984.34t^3 + \dots; \\ l(t) &= 3317 + 115800.0100t - 778859.1625t^2 - 24060529.42t^3 + \dots; \\ q(t) &= 2287 - 944.4800000t + 3710.120300t^2 - 16195.53663t^3 + \dots; \\ w(t) &= 1171 + 887.5500000t + 31489.98275t^2 - 151188.1752t^3 + \dots; \\ g(t) &= 179.5200000t + 2331.058400t^2 - 9988.232980t^3 + \dots. \end{aligned} \tag{5.2}$$

6. VIM Proposal

Now we apply the variational iteration method to solve the equation (2.1) and we write the correction functional as:

$$\begin{aligned} f_{n+1}(t) &= f_n(t) + \int_0^t \lambda_1(\tau) \left[\frac{df_n(\tau)}{d\tau} - \alpha_1 + (\beta_1 l_n(\tau) + \beta_2 w_n(\tau) + \gamma g_n(\tau)) f_n(\tau) + \zeta f_n(\tau) \right] d\tau, \\ l_{n+1}(t) &= l_n(t) + \int_0^t \lambda_2(\tau) \left[\frac{dl_n(\tau)}{d\tau} - \alpha_2 - (\beta_1 l_n(\tau) + \beta_2 w_n(\tau) + \gamma g_n(\tau)) f_n(\tau) + (\zeta + \theta + \epsilon) l_n(\tau) \right] d\tau, \\ q_{n+1}(t) &= q_n(t) + \int_0^t \lambda_3(\tau) \left[\frac{dq_n(\tau)}{d\tau} - \epsilon l_n(\tau) + \zeta q_n(\tau) \right] d\tau, \\ w_{n+1}(t) &= w_n(t) + \int_0^t \lambda_4(\tau) \left[\frac{dw_n(\tau)}{d\tau} - (\zeta + \theta) l_n(\tau) + \omega w_n(\tau) \right] d\tau, \\ g_{n+1}(t) &= g_n(t) + \int_0^t \lambda_5(\tau) \left[\frac{dg_n(\tau)}{d\tau} - \nu - \chi l_n(\tau) - \rho w_n(\tau) + \xi g_n(\tau) \right] d\tau. \end{aligned} \tag{6.1}$$

Here $\lambda_i, i = 1, 2, 3, 4, 5$ are general Lagrange multipliers. By making the above equations stationary with respect to $f_n(t), l_n(t), q_n(t), w_n(t),$ and $g_n(t)$ and by definition $\delta f_n(\tau) = \delta l_n(\tau) = \delta q_n(\tau) = \delta w_n(\tau) = \delta g_n(\tau) = 0,$ provides

$$\begin{aligned} \delta f_{n+1}(t) &= \delta f_n(t) + \delta \int_0^t \lambda_1(\tau) \left[\frac{df_n(\tau)}{d\tau} - \alpha_1 + (\beta_1 l_n(\tau) + \beta_2 w_n(\tau) + \gamma g_n(\tau))f_n(\tau) + \zeta f_n(\tau) \right] d\tau ; \\ \delta l_{n+1}(t) &= \delta l_n(t) + \delta \int_0^t \lambda_2(\tau) \left[\frac{dl_n(\tau)}{d\tau} - \alpha_2 - (\beta_1 l_n(\tau) + \beta_2 w_n(\tau) + \gamma g_n(\tau))f_n(\tau) + (\zeta + \theta + \epsilon)l_n(\tau) \right] d\tau ; \\ \delta q_{n+1}(t) &= \delta q_n(t) + \delta \int_0^t \lambda_3(\tau) \left[\frac{dq_n(\tau)}{d\tau} - \epsilon l_n(\tau) + \zeta q_n(\tau) \right] d\tau ; \\ \delta w_{n+1}(t) &= \delta w_n(t) + \delta \int_0^t \lambda_4(\tau) \left[\frac{dw_n(\tau)}{d\tau} - (\zeta + \theta) l_n(\tau) + \omega w_n(\tau) \right] d\tau ; \\ \delta g_{n+1}(t) &= \delta g_n(t) + \delta \int_0^t \lambda_5(\tau) \left[\frac{dg_n(\tau)}{d\tau} - \nu - \chi l_n(\tau) - \rho w_n(\tau) + \xi g_n(\tau) \right] d\tau. \end{aligned} \tag{6.2}$$

Hence, the Lagrange multiplier can be easily identified as $\lambda_i(\tau) = -1,$ where $i = 1, 2, 3, 4, 5.$ As a result, the iterations are obtained as follows:

$$\begin{aligned} f_{n+1}(t) &= f_n(t) - \int_0^t \left[\frac{df_n(\tau)}{d\tau} - \alpha_1 + (\beta_1 l_n(\tau) + \beta_2 w_n(\tau) + \gamma g_n(\tau))f_n(\tau) + \zeta f_n(\tau) \right] d\tau ; \\ l_{n+1}(t) &= l_n(t) - \int_0^t \left[\frac{dl_n(\tau)}{d\tau} - \alpha_2 - (\beta_1 l_n(\tau) + \beta_2 w_n(\tau) + \gamma g_n(\tau))f_n(\tau) + (\zeta + \theta + \epsilon)l_n(\tau) \right] d\tau ; \\ q_{n+1}(t) &= q_n(t) - \int_0^t \left[\frac{dq_n(\tau)}{d\tau} - \epsilon l_n(\tau) + \zeta q_n(\tau) \right] d\tau ; \\ w_{n+1}(t) &= w_n(t) - \int_0^t \left[\frac{dw_n(\tau)}{d\tau} - (\zeta + \theta) l_n(\tau) + \omega w_n(\tau) \right] d\tau ; \\ g_{n+1}(t) &= g_n(t) - \int_0^t \left[\frac{dg_n(\tau)}{d\tau} - \nu - \chi l_n(\tau) - \rho w_n(\tau) + \xi g_n(\tau) \right] d\tau. \end{aligned} \tag{6.3}$$

Applying the initial approximations and parametric values, which in turn yields the successive approximations for the above equations as:

$$\begin{aligned} f(t) &= 3470.0 - 116075.3800t + 714521.3142t^2 + 27364610.65t^3 + \dots ; \\ l(t) &= 3317.0 + 115800.0100t - 778859.1622t^2 - 27364610.65t^3 + \dots ; \\ q(t) &= 2287.0 - 1073.240000t + 294.9365000t^2 + 580.6435717t^3 + \dots ; \\ w(t) &= 1171.0 + 887.5500000t + 31489.98275t^2 - 151188.1751t^3 + \dots ; \\ g(t) &= 179.5200000t + 2331.058400t^2 - 9988.232977t^3 + \dots. \end{aligned} \tag{6.4}$$

7. Result And Discussion

As this work is proceeded by two methods, one by getting analytical solutions and another by obtaining numerical solutions in the form of series expansion by differential transform method and variation iteration method respectively. Comparison of both the methods (5.2) and (6.4) gives the closest form of solution. Graphical representation (Figures 1–5) shows the behaviour of the considered subclasses.

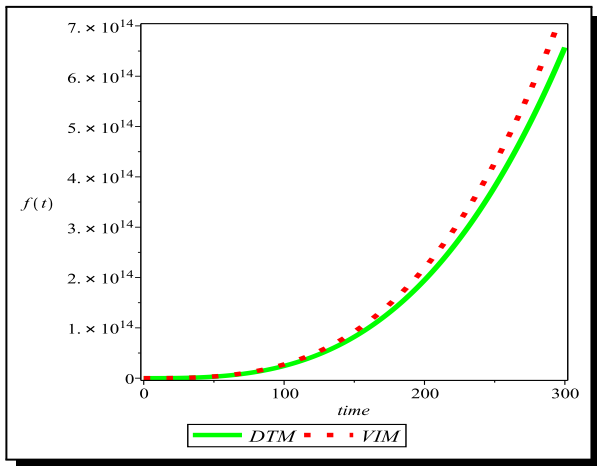


Figure 1. Graphical representation for venerable population $f(t)$ at time t by DTM and VIM

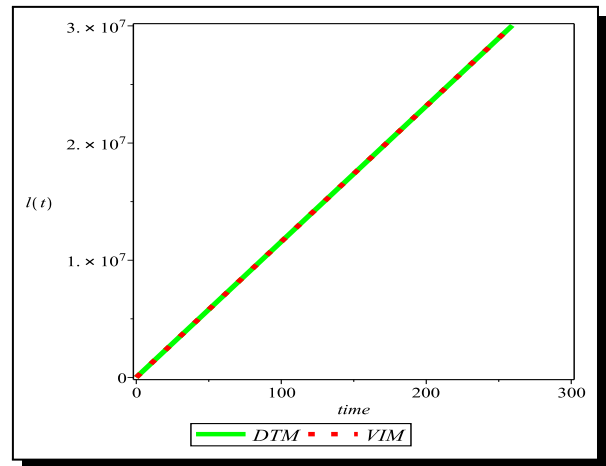


Figure 2. Graphical representation for infected individuals $l(t)$ at time t by DTM and VIM

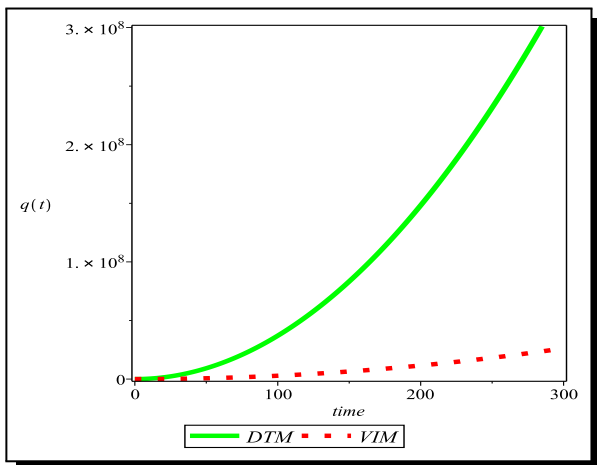


Figure 3. Graphical representation for recovered population $q(t)$ at time t by DTM and VIM

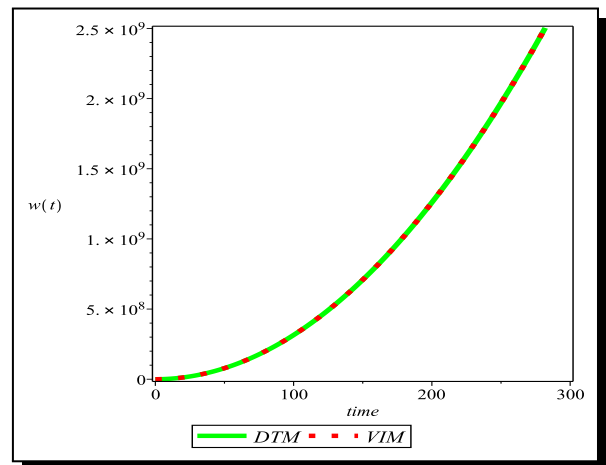


Figure 4. Graphical representation for recovered population $w(t)$ at time t by DTM and VIM

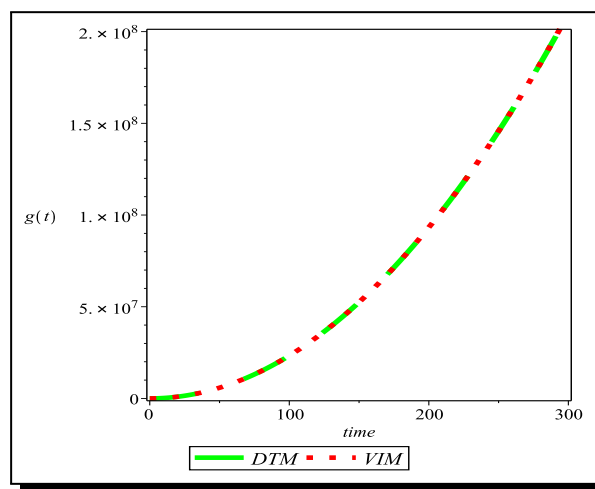


Figure 5. Graphical representation for spread disease $g(t)$ at time t by DTM and VIM

8. Conclusion

The EVD (*Ebola Virus Disease*) model is considered for evaluation. Real-time situation has been discussed and the spreading of virus is considered as the main stage, the disease transmission stages and the rate of change that occurs while transmitting from one stage to another with more infectious which causes the virus more efficient leading to death. Finally, the burial stage takes important part in controlling the disease transmission. All these aspects are given in the form of differential equations which are solved by using differential transform method to acquire the analytic solution and graphical representation shows the situation and controlling in spread of disease.

This Ebola virus has come across more than 15 outbreaks, either to end this epidemic or to bring under control there are two ways:

- (i) Instantaneous isolation of the infected individuals, and
- (ii) Careful burial of deceased bodies.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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