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Research Article

A Novel Approach to Plithogenic Neutrosophic Hypersoft Rough Set and Its Application to Decision Making Problem With Reference to Bakery Industry

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Abstract. The neutrosophic set, plithogenic hypersoft set, and rough set are distinct mathematical models to deal with different types of uncertainties in data and information. In this paper, we advance the study of plithogenic neutrosophic hypersoft rough set by combining plithogenic neutrosophic hypersoft set with rough set. Using plithogenic neutrosophic hypersoft rough set and the correspondence between objects, we develop an algorithm to solve a decision making problem occurring in Bakery industry. Finally a real life example is solved using the algorithm developed.

Keywords. Neutrosophic hypersoft sets, Plithogenic neutrosophic hypersoft sets, Plithogenic neutrosophic hypersoft rough sets, Decision making algorithm

Mathematics Subject Classification (2020). 60L90, 03E72, 08A72, 94D05, 20F10, 90B50

1. Introduction

To deal with real-world issues involving data in uncertainty, a powerful mathematical instrument is always required. For critical situations, researchers have developed several mathematical tools to make it easier. The idea of fuzzy set was first suggested by Zadeh [23] in 1965. In this concept, each element gives a membership degree in the form of a single crisp value in the range $[0,1]$. A generalization of this notion was put forward by Smarandache [18] in 1998, who presented the idea of a neutrosophic set, which is membership (T), indeterminacy (I), and non-membership (F) degrees were separately quantified, i.e., $T,I,F \in [0;1]$, and the sum $T+I+F$ need not be less than or equal to 3.

Incomplete information be evaluated successfully using the rough set theory, which was introduced by Pawlak [14] in 1982 as a method for dealing with ambiguity and imperfection. Furthermore, a variety of new rough set models, including rough fuzzy sets, fuzzy rough sets, generalised fuzzy rough sets have been developed by numerous researchers (Dubois and Prade [7], and Wu *et al*. [20]) based on various characteristics, including the universe, relations, objects and operators. The fundamental characteristics of rough neutrosophic set were established by Broumi *et al*. [4,5].

In order to deal with a different type of uncertainty, Molodtsov [12] proposed the soft set in 1999. This can be a broad mathematical tool. A parameterized family of subsets of a discourse universe was characterized by Molodtsov [12] is soft set. Aggregation operations on soft sets were presented in 2002 by Maji *et al*. [11] (also see Maji [9]). Maji *et al*. [10] have proposed fuzzy soft set theory. Al-Quran *et al*. [3] discussed a novel approach to neutrosophic soft rough set under uncertainty, Ozturk *et al*. [13] proposed neutrosophic soft sets, Das *et al*. [6] combined the neutrosophic soft set with rough set theory and Yolcu *et al*. [22] proved some results on neutrosophic soft rough sets.

Smarandache [17] introduced the concept of hypersoft set as a generalisation of soft set in 2018. Hypersoft set, hypersoft point, neutrosophic hypersoft set (Abbas *et al*. [1], Saeed *et al*. [15], and Saqlain *et al*. [16]) are some interesting notions studied by researchers. Additionally, Smarandache [17] introduced the concept of plithogenic hypersoft set with crisp, fuzzy, intuitionistic fuzzy, neutrosophic, and plithogenic sets. In 2020, Ahmad *et al*. [2] presented a study on the effectiveness of the plithogenic hypersoft set. Subha and Selvakumar [19] introduced a new approach to neutrosophic hypersoft rough set in 2023. Multicriteria decision making method using the correlation coefficient under cosine similarity measure was discussed by Kholood [8], Ye [21], and Zhao and Zhang [24].

The rest of this article is organized as follows. In Section 2, we briefly review some basic notions, including the definitions of soft set, hypersoft set, plithogenic set, and plithogenic hypersoft set (PHSSs) along with an illustrative example. Section 3 deals with plithogenic neutrosophic hypersoft rough set ($\mathsf{Pn_{hss}R}$) and its properties. In Section 4, we discuss an algorithm to handle decision making problem in Bakery industries based on *plithogenic neutrosophic hypersoft rough set* (PHSSs) and the significant steps of the proposed decision making approach along with an example. Section 5 provides the conclusion of this paper.

2. Preliminaries

In this section, we present some basic definitions including neutrosophic set, soft set, rough set, neutrosophic hypersoft set, and plithogenic neutrosophic hypersoft set.

Definition 2.1 ([19])**.** Let $\Im$ be the universe of discourse. A neutrosophic set $\mathbb{N}$ on $\Im$ is defined as $\mathbb{N} = \{\langle x, \mu_{\mathbb{N}}(x), \eta_{\mathbb{N}}(x), \nu_{\mathbb{N}}(x)\rangle : x \in \Im\}$, where $0 \le \mu_{\mathbb{N}}(x) + \eta_{\mathbb{N}}(x) + \nu_{\mathbb{N}}(x) \le 3$ and $\mu, \eta, \nu : \Im \to [0,1]$.

Definition 2.2 ([19])**.** Let $\Im$ be the universe, $P(\Im)$ the power set of $\Im$ and $\Delta_1,\Delta_2,\dots,\Delta_n$ a pairwise disjoint collection of subsets of the parameter set Δ. Let δ_j be a non empty subset of Δ_j for each $i=1,2,3,\dots,n$. A hypersoft set can be identified by the pair $(F,\delta_1\times\delta_2\times\dots\times\delta_n)$, where $F:\delta_1\times\delta_2\times\dots\times\delta_n\to P(\Im)$.

Definition 2.3 ([19])**.** Let $\Im$ be the universe, K be a subset of $\Im$. Then, the lower, upper and boundary approximations of K are defined as

$$\mathfrak{R}_-(K)=\bigcup_{x\in\Im}\{\mathfrak{R}(x):\mathfrak{R}(x)\subseteq K\},$$

$$\mathfrak{R}^-(K)=\bigcup_{x\in\Im}\{\mathfrak{R}(x):\mathfrak{R}(x)\cap K\neq\phi\}$$

and

$$B\mathfrak{R}(K)=\mathfrak{R}^-(K)\setminus\mathfrak{R}_-(K),$$

respectively.

Here $\mathfrak{R}$ is an indiscernibility relation $\mathfrak{R}\subseteq\Im\times\Im$ which indicates our knowledge or information about elements of $\Im$. The set K is said to be defined if $\mathfrak{R}^-(K)=\mathfrak{R}_-(K)$. If $\mathfrak{R}^-(K)\neq\mathfrak{R}_-(K)$ i.e., $B\mathfrak{R}(K)\neq\phi$, the set K is called a rough set with respect to $\mathfrak{R}$.

Definition 2.4 ([19])**.** Let $\Im$, $P(\Im)$ be as above. Consider $\{\Delta_1,\Delta_2,\dots,\Delta_n\}$ for $n\geq 1$, n well-defined attributes, whose corresponding attributive values are the set $\{\delta_1,\delta_2,\dots,\delta_n\}$ respectively with $\delta_i\cap\delta_j=\phi$, for $i\neq j$ and $i,j\in\{1,2,3,\dots,n\}$ and their relation $\{\delta_1\times\delta_2\times\dots\times\delta_n\}=\prod_{i=1}^n\delta_j$. Then, the pair $\left(\mathbb{N},\prod_{i=1}^n\delta_i\right)$ is said to be *Neutrosophic Hypersoft Set* (NHSS) over $\Im$, where $\mathbb{N}:\prod_{i=1}^n\delta_j\to P(\Im)$ and $\mathbb{N}\left(\prod_{i=1}^n\delta_j\right)=\left\{\left\langle x,\mu\left(\mathbb{N}\left(\prod_{i=1}^n\delta_j\right)\right),\eta\left(\mathbb{N}\left(\prod_{i=1}^n\delta_j\right)\right),\nu\left(\mathbb{N}\left(\prod_{i=1}^n\delta_j\right)\right)\right\rangle,x\in\Im\right\}$, μ is the membership value of truthfulness, η is the membership value of indeterminacy and ν is the membership value of falsity such that $\mu,\eta,\nu:\Im\to[0,1]$ and $0\leq\mu\left(\mathbb{N}\left(\prod_{i=1}^n\delta_j\right)\right)+\eta\left(\mathbb{N}\left(\prod_{i=1}^n\delta_j\right)\right)+\nu\left(\mathbb{N}\left(\prod_{i=1}^n\delta_j\right)\right)\leq 3$.

Definition 2.5 ([17])**.** Let $\Im$ be the universe and $\prod_{i=1}^n\delta_j$ is the set of all attribute values. Each attribute value γ possesses a corresponding appurtenance degree $d_N(x,\gamma)$ of the member $x\in\Im$, in accordance with some given condition or criteria. The attribute value degree of appurtenance is a plithogenic neutrosophic hypersoft set ($\mathsf{Pn}_{\mathsf{hss}}$) that is defined by $d_N:\Im\times\prod_{i=1}^n\delta_j\to P([0,1]^3)$, $\forall\ x\in\Im$, such that $d_N(x,\gamma)\in[0,1]^3$, and $P([0,1]^3)$ is the power set of $[0,1]^3$.

3. Plithogenic Neutrosophic Hypersoft Rough Sets ($\mathsf{Pn}_{\mathsf{hss}}\mathsf{R}$)

Some introduction about the content of this section.

Definition 3.1. Let $\Im$ and $\prod_{j=1}^n\delta_j$ be defined in Section 2. A $\mathsf{Pn}_{\mathsf{hss}}$ can be identified by the pair $\left(\mathbb{N},\Im\times\prod_{j=1}^n\delta_j\right)$. Then $\left(\mathbb{N},\left(\Im\times\prod_{j=1}^n\delta_j\right)\right)$ is called plithogenic neutrosophic hypersoft approximation space. The <u>*lower*</u> and $\overline{\textit{upper}}$ plithogenic neutrosophic hypersoft approximation spaces of $K\subseteq\mathsf{Pn}_{\mathsf{hss}}\left(\Im,\left(\Im\times\prod_{j=1}^n\delta_j\right)\right)$ with respect to $\left(\mathbb{N},\left(\Im\times\prod_{j=1}^n\delta_j\right)\right)$ are denoted by $\underline{\mathsf{Pn}_{\mathsf{hss}}}(K)$ and $\overline{\mathsf{Pn}_{\mathsf{hss}}}(K)$ respectively, defined by

$$\underline{\mathsf{Pn}_{\mathsf{hss}}}(K)=\Big\{\Big(\Big(\Im\times\prod_{j=1}^{n}\delta_j\Big)\Big\langle x,\frac{\gamma}{\underline{\mu}_{(\Im\times\prod_{j=1}^{n}\delta_j)}(x,\gamma),\underline{\eta}_{(\Im\times\prod_{j=1}^{n}\delta_j)}(x,\gamma),\underline{\nu}_{(\Im\times\prod_{j=1}^{n}\delta_j)}(x,\gamma)}\Big\rangle\Big),$$
$$\forall\Big(\Im\times\prod_{j=1}^{n}\delta_j\Big)\in\Big(\Im\times\prod_{j=1}^{n}\Delta_j\Big),\ \forall\,\gamma\in\prod_{j=1}^{n}\delta_j \text{ and } x\in\Im\Big\},$$

$$\overline{\mathsf{Pn}_{\mathsf{hss}}}(K)=\Big\{\Big(\Big(\Im\times\prod_{j=1}^{n}\delta_j\Big)\Big\langle x,\frac{\gamma}{\overline{\mu}_{(\Im\times\prod_{j=1}^{n}\delta_j)}(x,\gamma),\overline{\eta}_{(\Im\times\prod_{j=1}^{n}\delta_j)}(x,\gamma),\overline{\nu}_{(\Im\times\prod_{j=1}^{n}\delta_j)}(x,\gamma)}\Big\rangle\Big),$$
$$\forall\Big(\Im\times\prod_{j=1}^{n}\delta_j\Big)\in\Big(\Im\times\prod_{j=1}^{n}\Delta_j\Big),\ \forall\,\gamma\in\prod_{j=1}^{n}\delta_j \text{ and } x\in\Im\Big\},$$

where

$$\underline{\mu}_{(\Im\times\prod_{j=1}^{n}\delta_j)}(x,\gamma)=\Big\{\bigwedge\mu_{(\Im\times\prod_{j=1}^{n}\delta_j)}(x,\gamma):\mu_{(\Im\times\prod_{j=1}^{n}\delta_j)}(x,\gamma)\in K\cap\Big(\mathbb{N}_j,\Big(\Im\times\prod_{j=1}^{n}\delta_j\Big)\Big);$$
$$\Big(\mathbb{N}_j,\Big(\Im\times\prod_{j=1}^{n}\delta_j\Big)\Big)\subseteq K,\ \forall\Big(\Im\times\prod_{j=1}^{n}\delta_j\Big)\in\Big(\Im\times\prod_{j=1}^{n}\Delta_j\Big),\ \forall\,\gamma\in\prod_{j=1}^{n}\delta_j \text{ and } x\in\Im\Big\},$$

$$\underline{\eta}_{(\Im\times\prod_{j=1}^{n}\delta_j)}(x,\gamma)=\Big\{\bigwedge\eta_{(\Im\times\prod_{j=1}^{n}\delta_j)}(x,\gamma):\eta_{(\Im\times\prod_{j=1}^{n}\delta_j)}(x,\gamma)\in K\cap\Big(\mathbb{N}_j,\Big(\Im\times\prod_{j=1}^{n}\delta_j\Big)\Big);$$
$$\Big(\mathbb{N}_j,\Big(\Im\times\prod_{j=1}^{n}\delta_j\Big)\Big)\subseteq K,\ \forall\Big(\Im\times\prod_{j=1}^{n}\delta_j\Big)\in\Big(\Im\times\prod_{j=1}^{n}\Delta_j\Big),\ \forall\,\gamma\in\prod_{j=1}^{n}\delta_j \text{ and } x\in\Im\Big\},$$

$$\underline{\nu}_{(\Im\times\prod_{j=1}^{n}\delta_j)}(x,\gamma)=\Big\{\bigvee\nu_{(\Im\times\prod_{j=1}^{n}\delta_j)}(x,\gamma):\nu_{(\Im\times\prod_{j=1}^{n}\delta_j)}(x,\gamma)\in K\cap\Big(\mathbb{N}_j,\Big(\Im\times\prod_{j=1}^{n}\delta_j\Big)\Big);$$
$$\Big(\mathbb{N}_j,\Big(\Im\times\prod_{j=1}^{n}\delta_j\Big)\Big)\subseteq K,\ \forall\Big(\Im\times\prod_{j=1}^{n}\delta_j\Big)\in\Big(\Im\times\prod_{j=1}^{n}\Delta_j\Big),\ \forall\,\gamma\in\prod_{j=1}^{n}\delta_j \text{ and } x\in\Im\Big\},$$

$$\overline{\mu}_{(\Im\times\prod_{j=1}^{n}\delta_j)}(x,\gamma)=\Big\{\bigvee\mu_{(\Im\times\prod_{j=1}^{n}\delta_j)}(x,\gamma):\mu_{(\Im\times\prod_{j=1}^{n}\delta_j)}(x,\gamma)\in K\cap\Big(\mathbb{N}_j,\Big(\Im\times\prod_{j=1}^{n}\delta_j\Big)\Big);$$
$$\Big(\mathbb{N}_j,\Big(\Im\times\prod_{j=1}^{n}\delta_j\Big)\Big)\subseteq K,\ \forall\Big(\Im\times\prod_{j=1}^{n}\delta_j\Big)\in\Big(\Im\times\prod_{j=1}^{n}\Delta_j\Big),\ \forall\,\gamma\in\prod_{j=1}^{n}\delta_j \text{ and } x\in\Im\Big\},$$

$$\overline{\eta}_{(\Im\times\prod_{j=1}^{n}\delta_j)}(x,\gamma)=\Big\{\bigvee\eta_{(\Im\times\prod_{j=1}^{n}\delta_j)}(x,\gamma):\eta_{(\Im\times\prod_{j=1}^{n}\delta_j)}(x,\gamma)\in K\cap\Big(\mathbb{N}_j,\Big(\Im\times\prod_{j=1}^{n}\delta_j\Big)\Big);$$
$$\Big(\mathbb{N}_j,\Big(\Im\times\prod_{j=1}^{n}\delta_j\Big)\Big)\subseteq K,\ \forall\Big(\Im\times\prod_{j=1}^{n}\delta_j\Big)\in\Big(\Im\times\prod_{j=1}^{n}\Delta_j\Big),\ \forall\,\gamma\in\prod_{j=1}^{n}\delta_j \text{ and } x\in\Im\Big\},$$

$$\overline{\nu}_{(\Im\times\prod_{j=1}^{n}\delta_j)}(x,\gamma)=\Big\{\bigwedge\nu_{(\Im\times\prod_{j=1}^{n}\delta_j)}(x,\gamma):\nu_{(\Im\times\prod_{j=1}^{n}\delta_j)}(x,\gamma)\in K\cap\Big(\mathbb{N}_j,\Big(\Im\times\prod_{j=1}^{n}\delta_j\Big)\Big);$$
$$\Big(\mathbb{N}_j,\Big(\Im\times\prod_{j=1}^{n}\delta_j\Big)\Big)\subseteq K,\ \forall\Big(\Im\times\prod_{j=1}^{n}\delta_j\Big)\in\Big(\Im\times\prod_{j=1}^{n}\Delta_j\Big),\ \forall\,\gamma\in\prod_{j=1}^{n}\delta_j \text{ and } x\in\Im\Big\},$$

where $\bigwedge$ and $\bigvee$ mean 'min' and 'max' operators, respectively. It is easy to see that $\underline{\mathsf{Pn}_{\mathsf{hss}}}(K)$ and $\overline{\mathsf{Pn}_{\mathsf{hss}}}(K)$ are two $\mathsf{Pn}_{\mathsf{hss}}$s on $\big(\mathbb{N},\Im\times\prod_{j=1}^{n}\delta_j\big)$. If $\underline{\mathsf{Pn}_{\mathsf{hss}}}(K)=\overline{\mathsf{Pn}_{\mathsf{hss}}}(K)$, then K is said to be plithogenic neutrosophic hypersoft definable set, otherwise it is called a $\mathsf{Pn}_{\mathsf{hss}}\mathsf{R}$.

Definition 3.2. Let $\left(\mathbb{N}_1,\Im\times\prod_{j=1}^{n}\delta_j\right)$ and $\left(\mathbb{N}_2,\Im\times\prod_{j=1}^{n}\delta_j\right)$ be two neutrosophic hypersoft sets over the universe set $\Im$. Then, defined some operations in them as follows:

(i) $$\left(\mathbb{N}_1,\Im\times\prod_{j=1}^{n}\delta_j\right)\cup\left(\mathbb{N}_2,\Im\times\prod_{j=1}^{n}\delta_j\right)$$
$$=\Big\{\Im\times\prod_{j=1}^{n}\delta_j,\big\langle x,\max\{\mu_{\mathbb{N}_1,\Im\times\prod_{j=1}^{n}\delta_j}(x),\mu_{\mathbb{N}_2,\Im\times\prod_{j=1}^{n}\delta_j}(x)\},\max\{\eta_{\mathbb{N}_1,\Im\times\prod_{j=1}^{n}\delta_j}(x),\eta_{\mathbb{N}_2,\Im\times\prod_{j=1}^{n}\delta_j}(x)\},$$
$$\min\{\nu_{\mathbb{N}_1,\Im\times\prod_{j=1}^{n}\delta_j}(x),\nu_{\mathbb{N}_2,\Im\times\prod_{j=1}^{n}\delta_j}(x)\}\big\rangle:x\in\Im:\Im\times\prod_{j=1}^{n}\delta_j\in\Im\times\prod_{j=1}^{n}\Delta_j\Big\},$$

(ii) $$\left(\mathbb{N}_1,\Im\times\prod_{j=1}^{n}\delta_j\right)\cap\left(\mathbb{N}_2,\Im\times\prod_{j=1}^{n}\delta_j\right)$$
$$=\Big\{\Im\times\prod_{j=1}^{n}\delta_j,\big\langle x,\min\{\mu_{\mathbb{N}_1,\Im\times\prod_{j=1}^{n}\delta_j}(x),\mu_{\mathbb{N}_2,\Im\times\prod_{j=1}^{n}\delta_j}(x)\},\min\{\eta_{\mathbb{N}_1,\Im\times\prod_{j=1}^{n}\delta_j}(x),\eta_{\mathbb{N}_2,\Im\times\prod_{j=1}^{n}\delta_j}(x)\},$$
$$\max\{\nu_{\mathbb{N}_1,\Im\times\prod_{j=1}^{n}\delta_j}(x),\nu_{\mathbb{N}_2,\Im\times\prod_{j=1}^{n}\delta_j}(x)\}\big\rangle:x\in\Im:\Im\times\prod_{j=1}^{n}\delta_j\in\Im\times\prod_{j=1}^{n}\Delta_j\Big\},$$

(iii) $$\left(\mathbb{N}_1,\Im\times\prod_{j=1}^{n}\delta_j\right)\oplus\left(\mathbb{N}_2,\Im\times\prod_{j=1}^{n}\delta_j\right)$$
$$=\Big\{\Im\times\prod_{j=1}^{n}\delta_j,\big\langle x,\{\mu_{\mathbb{N}_1,\prod_{i=1}^{n}\delta_j}(x)+\mu_{\mathbb{N}_2,\prod_{i=1}^{n}\delta_j}(x)\}-\mu_{\mathbb{N}_1,\prod_{i=1}^{n}\delta_j}(x)\cdot\mu_{\mathbb{N}_2,\prod_{i=1}^{n}\delta_j}(x)\},$$
$$\{\eta_{\mathbb{N}_1,\prod_{i=1}^{n}\delta_j}(x)\cdot\eta_{\mathbb{N}_2,\prod_{i=1}^{n}\delta_j}(x)\},\{\nu_{\mathbb{N}_1,\prod_{i=1}^{n}\delta_j}(x)\cdot\nu_{\mathbb{N}_2,\prod_{i=1}^{n}\delta_j}(x)\}\big\rangle:x\in\Im:\Im\times\prod_{j=1}^{n}\delta_j\in\Im\times\prod_{j=1}^{n}\Delta_j\Big\},$$

(iv) $$\left(\mathbb{N}_1,\Im\times\prod_{j=1}^{n}\delta_j\right)\otimes\left(\mathbb{N}_2,\Im\times\prod_{j=1}^{n}\delta_j\right)$$
$$=\Big\{\Im\times\prod_{j=1}^{n}\delta_j,\big\langle x,\{\mu_{\mathbb{N}_1,\prod_{i=1}^{n}\delta_j}(x)\cdot\mu_{\mathbb{N}_2,\prod_{i=1}^{n}\delta_j}(x)\},\{\eta_{\mathbb{N}_1,\prod_{i=1}^{n}\delta_j}(x)+\eta_{\mathbb{N}_2,\prod_{i=1}^{n}\delta_j}(x)\}$$
$$-\eta_{\mathbb{N}_1,\prod_{i=1}^{n}\delta_j}(x)\cdot\eta_{\mathbb{N}_2,\prod_{i=1}^{n}\delta_j}(x)\},\Big\{\nu_{\mathbb{N}_1,\prod_{i=1}^{n}\delta_j}(x)+\nu_{\mathbb{N}_2,\prod_{i=1}^{n}\delta_j}(x)\Big\}$$
$$-\nu_{\mathbb{N}_1,\prod_{i=1}^{n}\delta_j}(x)\cdot\nu_{\mathbb{N}_2,\prod_{i=1}^{n}\delta_j}(x)\}\big\rangle:x\in\Im:\Im\times\prod_{j=1}^{n}\delta_j\in\Im\times\prod_{j=1}^{n}\Delta_j\Big\}.$$

Definition 3.3. Let $\Omega(x_j)$, $j=\{1,2,\dots,n\}$ be $\mathsf{Pn}_{\mathsf{hss}}$ over $\Im$ and Ω^* be the ideal $\mathsf{Pn}_{\mathsf{hss}}$, then the $\mathcal{CSM}$ is defined as

$$COS_{Pn_{hss}}(\Omega(x_j),\Omega^*)$$
$$=\left(\Im\times\left(\prod_{j=1}^{n}\delta_j,\prod_{j=1}^{n}\delta_j^*\right)\right)\left\langle x,\frac{\gamma}{\frac{\sum_{i=1}^{n}\left(\underline{\mu}_{(\Omega_i)}(x,\gamma)\underline{\mu}_{(\Omega_i^*)}(x,\gamma)+\underline{\eta}_{(\Omega_i)}(x,\gamma)\underline{\eta}_{(\Omega_i^*)}(x,\gamma)+\underline{\nu}_{(\Omega_i)}(x,\gamma)\underline{\nu}_{(\Omega_i^*)}(x,\gamma)\right)}{\sum_{i=1}^{n}\left(\sqrt{(\underline{\mu}_{(\Omega_i)}(x,\gamma))^2+(\underline{\eta}_{(\Omega_i)}(x,\gamma))^2+(\underline{\nu}_{(\Omega_i)}(x,\gamma))^2}\times\sqrt{(\underline{\mu}_{(\Omega_i^*)}(x,\gamma))^2+(\underline{\eta}_{(\Omega_i^*)}(x,\gamma))^2+(\underline{\nu}_{(\Omega_i^*)}(x,\gamma))^2}\right)}}\right\rangle,$$

$\forall$ $(\Omega(x_j)\in\left(\Im\times\prod_{j=1}^{n}\delta_j\right)$, $\Omega^*\in\left(\Im\times\prod_{j=1}^{n}\delta_j^*)\right)\in\left(\Im\times\prod_{j=1}^{n}\Delta_j\right)$, $\forall\ \gamma\in\left(\prod_{j=1}^{n}\delta_j,\prod_{j=1}^{n}\delta_j^*\right)$ and $x\in\Im$.

The concept of ideal point has been used to identify the best alternative in the decision set. Although the ideal alternate does not exist in real world, it does provide a useful theoretical construct against which we may evaluate alternatives. Hence, we can compute the $\mathcal{CSM}$ between each $\mathsf{Pn_{hss}}$ element and the ideal $\mathsf{Pn_{hss}}$. The bigger the value of the $\mathcal{CSM}$ ($COS_{Pn_{hss}}(\Omega(x_j),\Omega^*)$) is, the better the alternative is close to the ideal alternative

$$\Omega^* = \Big\{\Big(\Big(\Im \times \prod_{j=1}^{n} \delta_j^*\Big)\Big\langle x, \frac{\gamma}{\underline{\mu}_{(\Omega_i^*)}(x,\gamma), \underline{\eta}_{(\Omega_i^*)}(x,\gamma), \underline{\nu}_{(\Omega_i^*)}(x,\gamma)}\Big\rangle\Big),$$

$$\forall \Big(\Im \times \prod_{j=1}^{n} \delta_j^*\Big) \in \Big(\Im \times \prod_{j=1}^{n} \Delta_j\Big),\ \forall\ \gamma \in \prod_{j=1}^{n} \delta_j^* \text{ and } x \in \Im\ \forall\ i = 1,2,3\dots,n\Big\}$$

$$=\{(1,0,0),(1,0,0),(1,0,0),\dots,(1,0,0)\}.$$

By computing $\mathcal{CSM}$ value, the ranking of alternatives can be determined, and can obtain the optimal alternative.

4. An Application of Plithogenic Neutrosophic Hypersoft Rough Sets to Decision-Making Problem with Reference to Bakery Industry

In this section, we give algorithm based on the proposed cosign similarity measure. Then we apply it for the selection of best bakery problem. Now we will go over how to put the proposed DM for $\mathsf{Pn_{hss}R}$ into practice. Here we present our DM approaches for the selection of best bakery based on the $\mathsf{Pn_{hss}R}$. Let $\Im = \{x_1, x_2, \dots, x_n\}$ represent different sites separate set of geographical areas. Consider $\{\Delta_1, \Delta_2, \dots, \Delta_n\}$ for $n \geq 1$, n well-defined attributes, whose corresponding attributive values are the set $\{\delta_1, \delta_2, \dots, \delta_n\}$ respectively with $\delta_i \cap \delta_j = \phi$, for $i \neq j$ and $i, j \in \{1,2,3,\dots,n\}$ and their relation $\prod_{i=1}^{n} \delta_j$. Now we will go over how to put the proposed cosine similarity metrics for neutrosophic hypersoft rough sets into practice.

In the following, a procedure for selecting the best one and choosing the most desirable alternatives are given

Algorithm

Step 1: By Definition 3.1, we calculate the plithogenic neutrosophic hypersoft rough set $\underline{\mathsf{Pn_{hss}}}(K)$, $\overline{\mathsf{Pn_{hss}}}(K)$ of $\big(\mathbb{N}, (\Im \times \prod_{j=1}^{n} \delta_j)\big)$.

Step 2: By Definition 3.2(iii), we get $(\underline{\mathsf{Pn_{hss}}}(K) \oplus \overline{\mathsf{Pn_{hss}}}(K))$.

Step 3: By Definition 3.3, we compute the cosine similarity measure between each $\mathsf{Pn_{hss}}$ element

$$COS_{Pn_{hss}}(\Omega(x_j),\Omega^*)$$

$$= \Big(\Im \times \Big(\prod_{j=1}^{n} \delta_j, \prod_{j=1}^{n} \delta_j^*\Big)\Big)\Big\langle x, \frac{\gamma}{\frac{\sum_{i=1}^{n}\left(\underline{\mu}_{(\Omega_i)}(x,\gamma)\underline{\mu}_{(\Omega_i^*)}(x,\gamma)+\underline{\eta}_{(\Omega_i)}(x,\gamma)\underline{\eta}_{(\Omega_i^*)}(x,\gamma)+\underline{\nu}_{(\Omega_i)}(x,\gamma)\underline{\nu}_{(\Omega_i^*)}(x,\gamma)\right)}{\sum_{i=1}^{n}\left(\sqrt{(\underline{\mu}_{(\Omega_i)}(x,\gamma))^2+(\underline{\eta}_{(\Omega_i)}(x,\gamma))^2+(\underline{\nu}_{(\Omega_i)}(x,\gamma))^2} \times \sqrt{(\underline{\mu}_{(\Omega_i^*)}(x,\gamma))^2+(\underline{\eta}_{(\Omega_i^*)}(x,\gamma))^2+(\underline{\nu}_{(\Omega_i^*)}(x,\gamma))^2}\right)}}\Big\rangle$$

$$\forall\ (\Omega(x_j) \in \Big(\Im \times \prod_{j=1}^{n} \delta_j\Big),\ \Omega^* \in \Big(\Im \times \prod_{j=1}^{n} \delta_j^*)\Big) \in \Big(\Im \times \prod_{j=1}^{n} \Delta_j\Big),\ \forall\ \gamma \in \Big(\prod_{j=1}^{n} \delta_j, \prod_{j=1}^{n} \delta_j^*\Big) \text{ and } x \in \Im,$$

in ($\underline{\mathsf{Pn}_{\mathsf{hss}}}(K) \oplus \overline{\mathsf{Pn}_{\mathsf{hss}}}(K)$) and the ideal $\mathsf{Pn}_{\mathsf{hss}}$ element

$$\Omega^* = \left\{\left(\left(\Im \times \prod_{j=1}^{n} \delta_j^*\right)\left\langle x, \frac{\gamma}{\underline{\mu}_{(\Omega_i^*)}(x,\gamma), \underline{\eta}_{(\Omega_i^*)}(x,\gamma), \underline{\nu}_{(\Omega_i^*)}(x,\gamma)}\right\rangle\right),\right.$$
$$\left. \forall \left(\Im \times \prod_{j=1}^{n} \delta_j^*\right) \in \left(\Im \times \prod_{j=1}^{n} \Delta_j\right),\ \forall\ \gamma \in \prod_{j=1}^{n} \delta_j^* \text{ and } x \in \Im,\ \forall\ i = 1,2,3,\dots,n\right\}$$
$$= \{(1,0,0),(1,0,0),(1,0,0),\dots,(1,0,0)\}.$$

Step 4: For each combinations of attributes the decision is to select $\Omega(x_j)$ if

$$COS_{Pn_{hss}}(\Omega(x_j),\Omega^*) = \left(\Im \times \prod_{j=1}^{n} \delta_j\right), \max_{j=1,2,\dots,n} COS_{Pn_{hss}}(\Omega(x_j),\Omega^*),$$

when j has more than one value for each combinations of attributes, then each $\Omega(x_j)$ will be the decision. In this case, the each combinations of attributes may select separate decision and will be chosen as the most frequently selected one will be the optimal solution.

Step 5: Suppose all the combinations of attributes are different decisions, so, we find *Avg of* $COS_{Pn_{hss}}(\Omega,\Omega^*)$ to calculate optimal solution.

Using this algorithm, we investigate a numerical example to illustrate the application of $\mathsf{Pn}_{\mathsf{hss}}\mathsf{R}$ over the universe.

Example 4.1. Let $\Im$ be the set of the Bakery in Chidambaram that is $\Im = \{x_1, x_2, x_3, x_4\}$, where $\Im$ stands for APPLE, BLACK FOREST, CASINO and SHANMUGAVILLA'S, respectively.

Consider an algorithm and select the most desirable alternatives Figure 1 shows the procedure to choose the best alternatives as

Product's Price (Δ_1): Setting the right prices for bakery products is indeed crucial for the success and profitability of a bakery business. Cheap (δ_{11}): A 5% profit margin is considered low. Budget-friendly (δ_{12}): An average profit margin for a bakery business is around 10%. Costly (δ_{13}): A 20% profit margin is considered expensive.

Quality (Δ_2): It is evident that maintaining safety and quality in a bakery production environment is crucial to avoid issues such as mould, pathogens, food borne illnesses, and product recalls. Customer loyalty (δ_{21}): Satisfied and loyal customers are more likely to repeatedly choose your brand and try new products you introduce. They also tend to spend more than new customers. Improved safety (δ_{22}): Maintaining high standards of food safety and sanitation is essential in the bakery business. Customer referrals (δ_{23}): Satisfied and loyal customers often recommend their favourite brands to others, resulting in customer referrals and word-of-mouth marketing. Reduced liability risk (δ_{24}): Bakery businesses, like any other, face the risk of liability for third-party injuries or damages caused by their products. It is crucial to address potential liability risks associated with design defects, production or manufacturing flaws, and marketing defects.

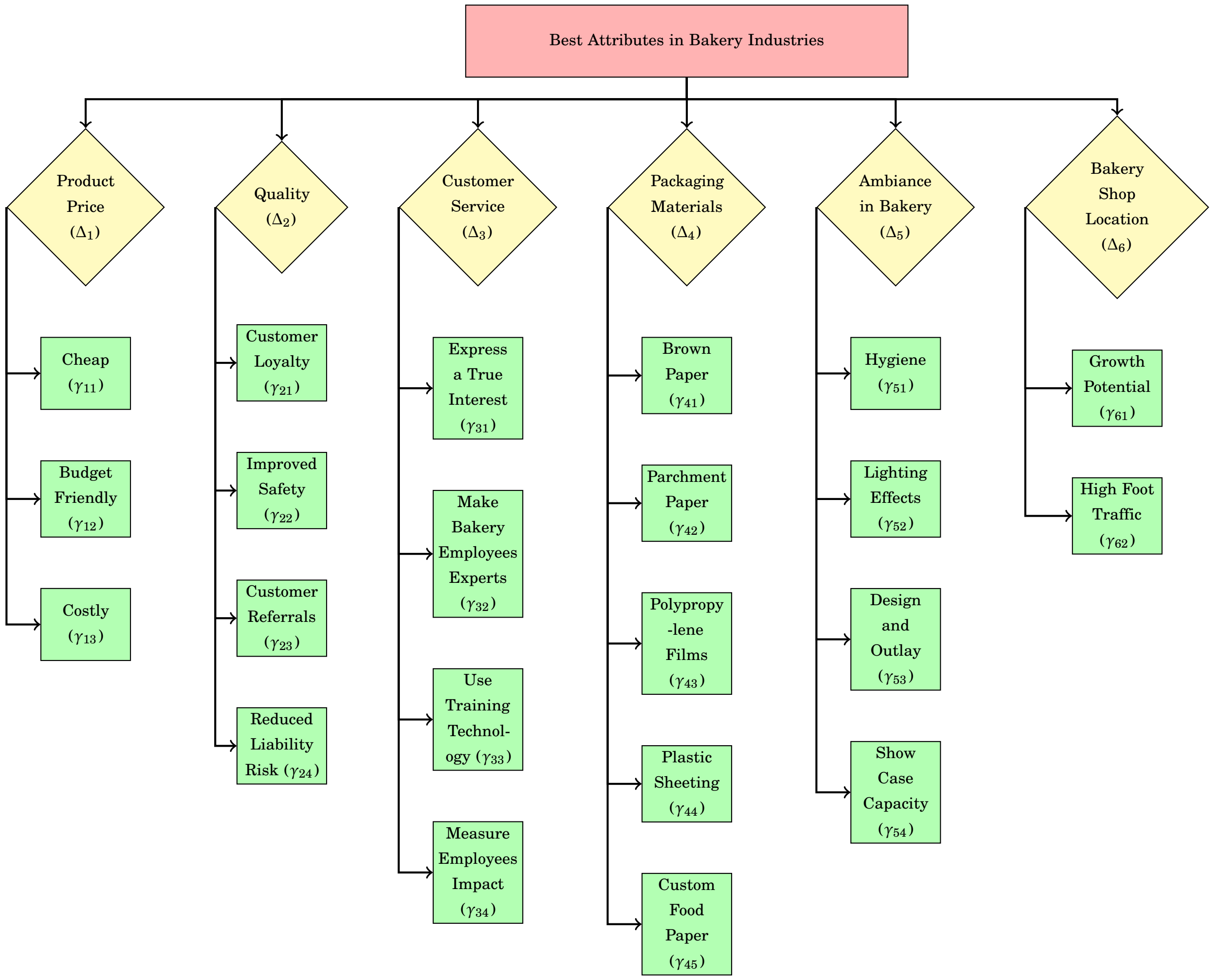

Figure 1. Best attributes in bakery industries

Customer Service (Δ_3): Customer service is a vital aspect of a business, and there are several strategies and approaches that can be used to provide excellent support to customers. Express a true interest (δ_{31}): Offering personalized greetings, Using names, Practice assertive hospitality, Ask questions, Cosset, Anticipate, Remember preferences, Pay attention to details, Display a sense of urgency, Solicit feedback, Offer personal and Follow up on service. Make bakery employees experts (δ_{32}):Ensuring product knowledge, Minimizing and Spotting manufacturing errors. Use training technology (δ_{33}): Customer service training,Interpersonal communication, Product knowledge, Conflict resolution and Crisis management. Measure employees' impact (δ_{34}): Graphic rating scales, 360-degree and Check-lists.

Packaging Materials (Δ_4): It is true that bakery product packaging serves a dual purpose of attracting customers with its visual appeal and ensuring the freshness and integrity of the products. Brown Paper (δ_{41}): Brown paper is a cost-effective packaging material commonly used in bakeries. Parchment Paper (δ_{42}): Parchment paper is widely used for baking and storing bakery products. Polypropylene Films (δ_{43}):Polypropylene films are suitable for bakery goods that require an oxygen barrier. Plastic Sheeting (δ_{44}): Clear plastic sheeting allows customers

Table 1. We consider single element on $\mathsf{Pn}_{\mathsf{hss}}$

Combinations of attributes ($\mathsf{Pn}_{\mathsf{hss}}$)	Bakerys	**Product Price** (Δ_1) Cheep (δ_{11})	Budget Friently (δ_{12})	Costly (δ_{13})	**Quality** (Δ_2) Customer Loyalty (δ_{21})	Improved Safety (δ_{22})	Customer Referrals (δ_{23})	Reduced Liability (δ_{24})	**Customer service** (Δ_3) Express a True Interest (δ_{31})	Make Bakery Employees Experts (δ_{32})	Use Training Technology (δ_{33})	Measure Employees Impact (δ_{34})
$(\gamma_{12},\gamma_{24},\gamma_{33},\gamma_{44},\gamma_{54},\gamma_{61})$	x_1	-	$\langle .2,.5,.8\rangle$	-	-	-	-	$\langle .6,.5,.9\rangle$	-	-	$\langle .3,.6,.8\rangle$	-
	x_2	-	$\langle .3,.8,.2\rangle$	-	-	-	-	$\langle .5,.4,.8\rangle$	-	-	$\langle .3,.6,.1\rangle$	-
	x_3	-	$\langle .1,.3,.5\rangle$	-	-	-	-	$\langle .3,.6,.4\rangle$	-	-	$\langle .8,.5,.8\rangle$	-
	x_4	-	$\langle .4,.8,.2\rangle$	-	-	-	-	$\langle .9,.5,.1\rangle$	-	-	$\langle .5,.4,.8\rangle$	-

Continue...	**Packing Materials** (Δ_4) Brown paper (δ_{41})	Parchment Paper(δ_{42})	Polypropylene Films (δ_{43})	Plastic Sheeting (δ_{44})	Custom Food Paper (δ_{45})	**Ambiance in bakery** (Δ_5) Hygiene (δ_{51})	Lighting Effects (δ_{52})	Design and Outlay (δ_{53})	Show Case Capacity (δ_{54})	**Shop Location** (Δ_6) Growth Potential (δ_{61})	High Food Traffic (δ_{62})
	-	-	-	$\langle .2,.8,.2\rangle$	-	-	-	-	$\langle .6,.8,.3\rangle$	$\langle .9,.3,.1\rangle$	-
	-	-	-	$\langle .4,.3,.9\rangle$	-	-	-	-	$\langle .2,.7,.3\rangle$	$\langle .7,.9,.2\rangle$	-
	-	-	-	$\langle .3,.7,.9\rangle$	-	-	-	-	$\langle .2,.8,.2\rangle$	$\langle .6,.1,.7\rangle$	-
	-	-	-	$\langle .3,.8,.1\rangle$	-	-	-	-	$\langle .9,.4,.2\rangle$	$\langle .6,.3,.1\rangle$	-

to have a clear view of the bakery products inside the packaging. Custom Food Paper (δ_{45}): Custom food paper offers several benefits for bakery product packaging. It is an affordable option and can be composted, making it environmentally friendly.

Ambiance in a Bakery(Δ_5) is style and appearance must be help to attract and retain customers, Hygiene (δ_{51}): Maintaining proper hygiene is crucial for a bakery to ensure food safety and prevent the growth of harmful bacteria. Here are some important steps to keep your bakery clean: Clean work, Proper waste management, Personal hygiene, Regular equipment cleaning and Proper food storage. Lighting effects (δ_{52}): Appropriate lighting can enhance the presentation of your bakery products and create a welcoming ambiance. Here are some tips for effective. Highlight bakery displays, Balance lighting intensity. Design and layout (δ_{53}): An effective bakery layout and floor plan maximize efficiency, accessibility, and customer experience. Consider the following points when designing your bakery's layout. Accessibility, Balancing creativity. Showcase capacity (δ_{54}): Confectionery showcases and pastry cabinets play a vital role in displaying and preserving baked goods. Here are some benefits of well-designed show. Aesthetic appeal, Protection and freshness, Energy efficiency.

Bakery Shop Location (Δ_6), Growth potential (δ_{61}): It is crucial to assess the growth potential of the location where you plan to set up your bakery. Analyse the long-term prospects of the location and ensure it aligns with your bakery's goals. If the chosen location lacks flexibility and does not support your long-term aspirations, it might be advisable to seek a better alternative. High foot traffic (δ_{62}) Foot traffic refers to the number of customers entering a store, mall, or specific location. Store owners, especially in retail establishments like department stores, closely monitor foot traffic numbers. Higher foot traffic generally correlates with increased sales and revenue.

In best bakery selection practice, a customers can see different bakeries and get different experience. To increase the arrival of customers, we examine each and every comments of customers and taken into account then choosing the attributes. Based on this, the set of customers are described as $\mathsf{Pn_{hss}R}$. In this example, a bakeries $\left(\mathbb{N},(\Im\times\prod_{j=1}^{n}\delta_j)\right)$ are illustrated by a $\mathsf{Pn_{hss}R}$ in the universe U which are obtained from customers as follows.

Step 1: By Definition 3.1, we calculate the plithogenic neutrosophic hypersoft rough set $\underline{\mathsf{Pn_{hss}}}(K)$, $\overline{\mathsf{Pn_{hss}}}(K)$ of $\left(\mathbb{N},(\Im\times\prod_{j=1}^{n}\delta_j)\right)$ as follows:

We consider following $\mathsf{Pn_{hss}}$ like us Tablet1,

Suppose that $\delta_1=\{\gamma_{11},\gamma_{12}\}$, $\delta_2=\{\gamma_{21},\gamma_{22},\gamma_{24}\}$, $\delta_3=\{\gamma_{32},\gamma_{33}\}$, $\delta_4=\{\gamma_{41},\gamma_{42},\gamma_{44}\}$, $\delta_5=\{\gamma_{51},\gamma_{54}\}$, $\delta_6=\{\gamma_{61}\}$ that is $\prod_{j=1}^{n}\delta_j\subseteq\prod_{j=1}^{n}\Delta_j$, $j=1,2,3$. Let the $\mathsf{Pn_{hss}}$,

$$\left(\mathbb{N}_1,\prod_{j=1}^{3}\delta_i\right)=\Bigg\{\Bigg((\gamma_{11},\gamma_{21},\gamma_{32},\gamma_{41},\gamma_{51},\gamma_{61}),$$
$$\Bigg\{x_2,\Bigg(\frac{\gamma_{11}}{\langle .3,.2,.6\rangle},\frac{\gamma_{21}}{\langle .2,.4,.8\rangle},\frac{\gamma_{32}}{\langle .3,.5,.6\rangle},\frac{\gamma_{41}}{\langle .3,.6,.7\rangle},\frac{\gamma_{51}}{\langle .1,.6,.3\rangle},\frac{\gamma_{61}}{\langle .6,.3,.7\rangle}\Bigg),$$
$$x_4,\Bigg(\frac{\gamma_{11}}{\langle .4,.6,.2\rangle},\frac{\gamma_{21}}{\langle .8,.4,.1\rangle},\frac{\gamma_{32}}{\langle .7,.9,.3\rangle},\frac{\gamma_{41}}{\langle .6,.4,.8\rangle},\frac{\gamma_{51}}{\langle .9,.1,.3\rangle},\frac{\gamma_{61}}{\langle .4,.6,.8\rangle}\Bigg)\Bigg\}\Bigg),$$

$$\Big((\gamma_{12},\gamma_{22},\gamma_{33},\gamma_{42},\gamma_{54},\gamma_{61}),$$
$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .7,.4,.3\rangle},\frac{\gamma_{22}}{\langle .7,.3,.1\rangle},\frac{\gamma_{33}}{\langle .3,.6,.6\rangle},\frac{\gamma_{42}}{\langle .6,.6,.3\rangle},\frac{\gamma_{54}}{\langle .7,.4,.8\rangle},\frac{\gamma_{61}}{\langle .3,.8,.4\rangle}\Big),$$
$$x_2,\Big(\frac{\gamma_{12}}{\langle .6,.7,.8\rangle},\frac{\gamma_{22}}{\langle .1,.4,.6\rangle},\frac{\gamma_{33}}{\langle .8,.5,.2\rangle},\frac{\gamma_{42}}{\langle .6,.3,.1\rangle},\frac{\gamma_{54}}{\langle .3,.5,.8\rangle},\frac{\gamma_{61}}{\langle .2,.9,.4\rangle}\Big),$$
$$x_3,\Big(\frac{\gamma_{12}}{\langle .6,.7,.9\rangle},\frac{\gamma_{22}}{\langle .3,.6,.8\rangle},\frac{\gamma_{33}}{\langle .1,.8,.3\rangle},\frac{\gamma_{42}}{\langle .6,.4,.9\rangle},\frac{\gamma_{54}}{\langle .3,.8,.7\rangle},\frac{\gamma_{61}}{\langle .9,.4,.2\rangle}\Big),$$
$$x_4,\Big(\frac{\gamma_{12}}{\langle .5,.2,.7\rangle},\frac{\gamma_{22}}{\langle .5,.8,.9\rangle},\frac{\gamma_{33}}{\langle .6,.1,.6\rangle},\frac{\gamma_{42}}{\langle .6,.2,.7\rangle},\frac{\gamma_{54}}{\langle .9,.3,.1\rangle},\frac{\gamma_{61}}{\langle .4,.3,.9\rangle}\Big)\Big\}\Big),$$
$$\Big((\gamma_{11},\gamma_{22},\gamma_{32},\gamma_{42},\gamma_{51},\gamma_{61}),$$
$$\Big\{x_2,\Big(\frac{\gamma_{11}}{\langle .3,.6,.8\rangle},\frac{\gamma_{22}}{\langle .4,.6,.1\rangle},\frac{\gamma_{32}}{\langle .7,.6,.2\rangle},\frac{\gamma_{42}}{\langle .9,.5,.1\rangle},\frac{\gamma_{51}}{\langle .6,.4,.8\rangle},\frac{\gamma_{61}}{\langle .3,.5,.9\rangle}\Big)\Big\}\Big),$$
$$\Big((\gamma_{11},\gamma_{21},\gamma_{33},\gamma_{42},\gamma_{51},\gamma_{61}),$$
$$\Big\{x_2,\Big(\frac{\gamma_{11}}{\langle .4,.6,.2\rangle},\frac{\gamma_{21}}{\langle .8,.2,.7\rangle},\frac{\gamma_{33}}{\langle .2,.6,.8\rangle},\frac{\gamma_{42}}{\langle .9,.4,.7\rangle},\frac{\gamma_{51}}{\langle .8,.6,.4\rangle},\frac{\gamma_{61}}{\langle .6,.2,.8\rangle}\Big),$$
$$x_4,\Big(\frac{\gamma_{11}}{\langle .8,.3,.6\rangle},\frac{\gamma_{21}}{\langle .7,.2,.6\rangle},\frac{\gamma_{33}}{\langle .7,.8,.1\rangle},\frac{\gamma_{42}}{\langle .4,.7,.8\rangle},\frac{\gamma_{51}}{\langle .1,.3,.8\rangle},\frac{\gamma_{61}}{\langle .7,.3,.7\rangle}\Big)\Big\}\Big),$$
$$\Big((\gamma_{12},\gamma_{22},\gamma_{32},\gamma_{41},\gamma_{54},\gamma_{61}),$$
$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .2,.7,.9\rangle},\frac{\gamma_{22}}{\langle .3,.6,.8\rangle},\frac{\gamma_{32}}{\langle .6,.8,.2\rangle},\frac{\gamma_{41}}{\langle .7,.3,.7\rangle},\frac{\gamma_{54}}{\langle .7,.3,.6\rangle},\frac{\gamma_{61}}{\langle .9,.6,.1\rangle}\Big),$$
$$x_3,\Big(\frac{\gamma_{12}}{\langle .4,.8,.8\rangle},\frac{\gamma_{22}}{\langle .1,.5,.7\rangle},\frac{\gamma_{32}}{\langle .7,.4,.7\rangle},\frac{\gamma_{41}}{\langle .4,.8,.2\rangle},\frac{\gamma_{54}}{\langle .6,.9,.3\rangle},\frac{\gamma_{61}}{\langle .6,.8,.1\rangle}\Big)\Big\}\Big),$$
$$\Big((\gamma_{11},\gamma_{22},\gamma_{32},\gamma_{42},\gamma_{51},\gamma_{61}),$$
$$\Big\{x_2,(\frac{\gamma_{11}}{\langle .4,.8,.1\rangle},\frac{\gamma_{22}}{\langle .8,.3,.2\rangle},\frac{\gamma_{32}}{\langle .5,.7,.4\rangle},\frac{\gamma_{42}}{\langle .3,.4,.7\rangle},\frac{\gamma_{51}}{\langle .2,.6,.9\rangle},\frac{\gamma_{61}}{\langle .1,.5,.7\rangle}),$$
$$x_4,\Big(\frac{\gamma_{11}}{\langle .3,.5,.7\rangle},\frac{\gamma_{22}}{\langle .2,.8,.9\rangle},\frac{\gamma_{32}}{\langle .1,.5,.8\rangle},\frac{\gamma_{42}}{\langle .3,.5,.4\rangle},\frac{\gamma_{51}}{\langle .1,.7,.9\rangle},\frac{\gamma_{61}}{\langle .1,.8,.5\rangle}\Big)\Big\}\Big),$$
$$\Big((\gamma_{12},\gamma_{22},\gamma_{33},\gamma_{44},\gamma_{51},\gamma_{61}),$$
$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .2,.7,.9\rangle},\frac{\gamma_{22}}{\langle .1,.3,.4\rangle},\frac{\gamma_{33}}{\langle .4,.9,.2\rangle},\frac{\gamma_{44}}{\langle .1,.4,.7\rangle},\frac{\gamma_{51}}{\langle .9,.2,.4\rangle},\frac{\gamma_{61}}{\langle .9,.1,.3\rangle}\Big),$$
$$x_2,\Big(\frac{\gamma_{12}}{\langle .6,.7,.2\rangle},\frac{\gamma_{22}}{\langle .9,.5,.1\rangle},\frac{\gamma_{33}}{\langle .7,.4,.3\rangle},\frac{\gamma_{44}}{\langle .8,.5,.2\rangle},\frac{\gamma_{51}}{\langle .6,.6,.2\rangle},\frac{\gamma_{61}}{\langle .8,.5,.1\rangle}\Big),$$
$$x_3,\Big(\frac{\gamma_{12}}{\langle .2,.4,.7\rangle},\frac{\gamma_{22}}{\langle .8,.3,.2\rangle},\frac{\gamma_{33}}{\langle .6,.1,.5\rangle},\frac{\gamma_{44}}{\langle .6,.8,.4\rangle},\frac{\gamma_{51}}{\langle .7,.2,.6\rangle},\frac{\gamma_{61}}{\langle .5,.4,.8\rangle}\Big),$$
$$x_4,\Big(\frac{\gamma_{12}}{\langle .5,.1,.4\rangle},\frac{\gamma_{22}}{\langle .8,.4,.3\rangle},\frac{\gamma_{33}}{\langle .8,.4,.1\rangle},\frac{\gamma_{44}}{\langle .7,.3,.8\rangle},\frac{\gamma_{51}}{\langle .2,.8,.3\rangle},\frac{\gamma_{61}}{\langle .7,.6,.3\rangle}\Big)\Big\}\Big),$$
$$\Big((\gamma_{12},\gamma_{24},\gamma_{33},\gamma_{44},\gamma_{51},\gamma_{61}),$$
$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .2,.8,.3\rangle},\frac{\gamma_{24}}{\langle .4,.6,.8\rangle},\frac{\gamma_{33}}{\langle .3,.6,.8\rangle},\frac{\gamma_{44}}{\langle .3,.8,.1\rangle},\frac{\gamma_{51}}{\langle .8,.4,.8\rangle},\frac{\gamma_{61}}{\langle .3,.9,.3\rangle}\Big),$$

$$x_2,\Big(\frac{\gamma_{12}}{\langle .5,.2,.1\rangle},\frac{\gamma_{24}}{\langle .6,.2,.8\rangle},\frac{\gamma_{33}}{\langle .5,.3,.8\rangle},\frac{\gamma_{44}}{\langle .1,.4,.9\rangle},\frac{\gamma_{51}}{\langle .3,.2,.7\rangle},\frac{\gamma_{61}}{\langle .1,.5,.8\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{12}}{\langle .4,.7,.1\rangle},\frac{\gamma_{24}}{\langle .6,.8,.2\rangle},\frac{\gamma_{33}}{\langle .4,.6,.7\rangle},\frac{\gamma_{44}}{\langle .4,.4,.6\rangle},\frac{\gamma_{51}}{\langle .8,.5,.1\rangle},\frac{\gamma_{61}}{\langle .4,.6,.8\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .4,.3,.2\rangle},\frac{\gamma_{22}}{\langle .7,.5,.3\rangle},\frac{\gamma_{33}}{\langle .9,.4,.3\rangle},\frac{\gamma_{44}}{\langle .7,.5,.1\rangle},\frac{\gamma_{51}}{\langle .6,.3,.8\rangle},\frac{\gamma_{61}}{\langle .4,.2,.9\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{22},\gamma_{33},\gamma_{44},\gamma_{54},\gamma_{61}),$$

$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .3,.7,.8\rangle},\frac{\gamma_{22}}{\langle .1,.5,.7\rangle},\frac{\gamma_{33}}{\langle .4,.6,.7\rangle},\frac{\gamma_{44}}{\langle .1,.6,.7\rangle},\frac{\gamma_{54}}{\langle .3,.6,.9\rangle},\frac{\gamma_{61}}{\langle .6,.8,.9\rangle}\Big),$$

$$x_2,\Big(\frac{\gamma_{12}}{\langle .6,.2,.6\rangle},\frac{\gamma_{22}}{\langle .7,.2,.8\rangle},\frac{\gamma_{33}}{\langle .4,.2,.7\rangle},\frac{\gamma_{44}}{\langle .9,.3,.7\rangle},\frac{\gamma_{54}}{\langle .7,.9,.3\rangle},\frac{\gamma_{61}}{\langle .2,.8,.3\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{12}}{\langle .4,.6,.2\rangle},\frac{\gamma_{22}}{\langle .8,.6,.3\rangle},\frac{\gamma_{33}}{\langle .9,.1,.5\rangle},\frac{\gamma_{44}}{\langle .6,.3,.6\rangle},\frac{\gamma_{54}}{\langle .8,.3,.6\rangle},\frac{\gamma_{61}}{\langle .7,.9,.1\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .1,.6,.3\rangle},\frac{\gamma_{22}}{\langle .8,.6,.1\rangle},\frac{\gamma_{33}}{\langle .4,.3,.8\rangle},\frac{\gamma_{44}}{\langle .1,.4,.9\rangle},\frac{\gamma_{54}}{\langle .3,.5,.2\rangle},\frac{\gamma_{61}}{\langle .4,.1,.9\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{24},\gamma_{33},\gamma_{44},\gamma_{54},\gamma_{61}),$$

$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .2,.5,.8\rangle},\frac{\gamma_{24}}{\langle .6,.5,.9\rangle},\frac{\gamma_{33}}{\langle .3,.6,.8\rangle},\frac{\gamma_{44}}{\langle .2,.8,.2\rangle},\frac{\gamma_{54}}{\langle .6,.8,.3\rangle},\frac{\gamma_{61}}{\langle .9,.3,.1\rangle}\Big),$$

$$x_2,\Big(\frac{\gamma_{12}}{\langle .3,.8,.2\rangle},\frac{\gamma_{22}}{\langle .5,.4,.8\rangle},\frac{\gamma_{33}}{\langle .3,.6,.1\rangle},\frac{\gamma_{44}}{\langle .4,.3,.9\rangle},\frac{\gamma_{54}}{\langle .2,.7,.3\rangle},\frac{\gamma_{61}}{\langle .7,.9,.2\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{12}}{\langle .1,.3,.5\rangle},\frac{\gamma_{24}}{\langle .3,.6,.4\rangle},\frac{\gamma_{33}}{\langle .8,.5,.8\rangle},\frac{\gamma_{44}}{\langle .3,.7,.9\rangle},\frac{\gamma_{54}}{\langle .2,.8,.2\rangle},\frac{\gamma_{61}}{\langle .6,.1,.7\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .4,.8,.2\rangle},\frac{\gamma_{22}}{\langle .9,.5,.1\rangle},\frac{\gamma_{33}}{\langle .5,.4,.8\rangle},\frac{\gamma_{44}}{\langle .3,.8,.1\rangle},\frac{\gamma_{54}}{\langle .9,.4,.2\rangle},\frac{\gamma_{61}}{\langle .6,.3,.1\rangle}\Big)\Big\}\Big)\Big\}.$$

Suppose that $\alpha_1=\{\gamma_{12},\gamma_{13}\}$, $\alpha_2=\{\gamma_{22},\gamma_{24}\}$, $\alpha_3=\{\gamma_{31},\gamma_{32},\gamma_{33}\}$, $\alpha_4=\{\gamma_{42},\gamma_{43},\gamma_{44}\}$, $\alpha_5=\{\gamma_{51},\gamma_{52},\gamma_{54}\}$, $\alpha_6=\{\gamma_{61},\gamma_{62}\}$ that is $\prod_{j=1}^{n}\alpha_j\subseteq\prod_{j=1}^{n}\Delta_j$, $j=1,2,3$. Let the $\mathsf{Pn}_{\mathsf{hss}}$,

$$\Big(\mathbb{N}_1,\prod_{j=1}^{3}\alpha_i\Big)=\Big\{\Big((\gamma_{12},\gamma_{22},\gamma_{31},\gamma_{42},\gamma_{51},\gamma_{61}),$$

$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .3,.1,.5\rangle},\frac{\gamma_{22}}{\langle .8,.3,.6\rangle},\frac{\gamma_{31}}{\langle .1,.7,.8\rangle},\frac{\gamma_{42}}{\langle .2,.6,.8\rangle},\frac{\gamma_{51}}{\langle .1,.6,.8\rangle},\frac{\gamma_{61}}{\langle .3,.7,.2\rangle}\Big),$$

$$x_2,\Big(\frac{\gamma_{12}}{\langle .7,.3,.7\rangle},\frac{\gamma_{22}}{\langle .3,.7,.9\rangle},\frac{\gamma_{31}}{\langle .2,.3,.6\rangle},\frac{\gamma_{42}}{\langle .7,.4,.8\rangle},\frac{\gamma_{51}}{\langle .3,.5,.8\rangle},\frac{\gamma_{61}}{\langle .4,.6,.7\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{12}}{\langle .2,.4,.8\rangle},\frac{\gamma_{22}}{\langle .3,.5,.6\rangle},\frac{\gamma_{31}}{\langle .2,.1,.9\rangle},\frac{\gamma_{42}}{\langle .7,.5,.3\rangle},\frac{\gamma_{51}}{\langle .8,.2,.7\rangle},\frac{\gamma_{61}}{\langle .3,.7,.8\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .1,.6,.8\rangle},\frac{\gamma_{22}}{\langle .3,.4,.7\rangle},\frac{\gamma_{31}}{\langle .8,.2,.6\rangle},\frac{\gamma_{42}}{\langle .6,.3,.4\rangle},\frac{\gamma_{51}}{\langle .6,.2,.8\rangle},\frac{\gamma_{61}}{\langle .1,.7,.8\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{13},\gamma_{24},\gamma_{32},\gamma_{43},\gamma_{52},\gamma_{62}),$$

$$\Big\{x_1,\Big(\frac{\gamma_{13}}{\langle .4,.3,.8\rangle},\frac{\gamma_{24}}{\langle .2,.5,.8\rangle},\frac{\gamma_{32}}{\langle .2,.7,.6\rangle},\frac{\gamma_{43}}{\langle .1,.9,.3\rangle},\frac{\gamma_{52}}{\langle .4,.8,.1\rangle},\frac{\gamma_{62}}{\langle .8,.2,.4\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{13}}{\langle .1,.7,.9\rangle},\frac{\gamma_{24}}{\langle .2,.4,.1\rangle},\frac{\gamma_{32}}{\langle .6,.3,.7\rangle},\frac{\gamma_{43}}{\langle .2,.5,.9\rangle},\frac{\gamma_{52}}{\langle .8,.3,.8\rangle},\frac{\gamma_{62}}{\langle .3,.6,.8\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{13},\gamma_{22},\gamma_{33},\gamma_{44},\gamma_{54},\gamma_{61}),$$

$$\Big\{x_3,\Big(\frac{\gamma_{13}}{\langle .2,.5,.7\rangle},\frac{\gamma_{22}}{\langle .7,.3,.1\rangle},\frac{\gamma_{33}}{\langle .8,.3,.2\rangle},\frac{\gamma_{44}}{\langle .8,.3,.8\rangle},\frac{\gamma_{54}}{\langle .1,.3,.6\rangle},\frac{\gamma_{61}}{\langle .5,.2,.8\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{24},\gamma_{32},\gamma_{44},\gamma_{52},\gamma_{62}),$$

$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .3,.2,.7\rangle},\frac{\gamma_{24}}{\langle .3,.3,.9\rangle},\frac{\gamma_{32}}{\langle .1,.7,.3\rangle},\frac{\gamma_{44}}{\langle .8,.3,.5\rangle},\frac{\gamma_{52}}{\langle .6,.4,.6\rangle},\frac{\gamma_{62}}{\langle .8,.4,.7\rangle}\Big),$$

$$x_2,\Big(\frac{\gamma_{12}}{\langle .3,.7,.8\rangle},\frac{\gamma_{24}}{\langle .6,.9,.2\rangle},\frac{\gamma_{32}}{\langle .4,.8,.8\rangle},\frac{\gamma_{44}}{\langle .4,.8,.9\rangle},\frac{\gamma_{52}}{\langle .4,.5,.7\rangle},\frac{\gamma_{62}}{\langle .8,.3,.1\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{12}}{\langle .7,.6,.7\rangle},\frac{\gamma_{24}}{\langle .8,.4,.2\rangle},\frac{\gamma_{32}}{\langle .8,.3,.8\rangle},\frac{\gamma_{44}}{\langle .1,.6,.3\rangle},\frac{\gamma_{52}}{\langle .7,.5,.8\rangle},\frac{\gamma_{62}}{\langle .3,.7,.1\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .4,.2,.1\rangle},\frac{\gamma_{24}}{\langle .6,.7,.3\rangle},\frac{\gamma_{32}}{\langle .3,.8,.2\rangle},\frac{\gamma_{44}}{\langle .8,.3,.6\rangle},\frac{\gamma_{52}}{\langle .6,.2,.9\rangle},\frac{\gamma_{62}}{\langle .6,.3,.9\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{13},\gamma_{22},\gamma_{32},\gamma_{42},\gamma_{51},\gamma_{62}),$$

$$\Big\{x_1,\Big(\frac{\gamma_{13}}{\langle .2,.1,.4\rangle},\frac{\gamma_{22}}{\langle .4,.6,.8\rangle},\frac{\gamma_{32}}{\langle .1,.6,.8\rangle},\frac{\gamma_{42}}{\langle .7,.2,.7\rangle},\frac{\gamma_{51}}{\langle .5,.3,.1\rangle},\frac{\gamma_{62}}{\langle .6,.2,.8\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{13}}{\langle .1,.4,.7\rangle},\frac{\gamma_{22}}{\langle .4,.7,.9\rangle},\frac{\gamma_{32}}{\langle .1,.3,.8\rangle},\frac{\gamma_{42}}{\langle .4,.2,.5\rangle},\frac{\gamma_{51}}{\langle .7,.5,.6\rangle},\frac{\gamma_{62}}{\langle .8,.2,.4\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{24},\gamma_{32},\gamma_{43},\gamma_{51},\gamma_{61}),$$

$$\Big\{x_2,(\frac{\gamma_{12}}{\langle .1,.4,.6\rangle},\frac{\gamma_{24}}{\langle .6,.3,.8\rangle},\frac{\gamma_{32}}{\langle .1,.2,.8\rangle},\frac{\gamma_{43}}{\langle .8,.3,.1\rangle},\frac{\gamma_{51}}{\langle .3,.8,.9\rangle},\frac{\gamma_{61}}{\langle .1,.7,.8\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{12}}{\langle .1,.4,.6\rangle},\frac{\gamma_{24}}{\langle .4,.8,.2\rangle},\frac{\gamma_{32}}{\langle .4,.8,.4\rangle},\frac{\gamma_{43}}{\langle .1,.4,.7\rangle},\frac{\gamma_{51}}{\langle .5,.1,.4\rangle},\frac{\gamma_{61}}{\langle .5,.2,.6\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .4,.2,.1\rangle},\frac{\gamma_{24}}{\langle .6,.1,.2\rangle},\frac{\gamma_{32}}{\langle .5,.8,.8\rangle},\frac{\gamma_{43}}{\langle .7,.4,.2\rangle},\frac{\gamma_{51}}{\langle .1,.5,.7\rangle},\frac{\gamma_{61}}{\langle .8,.3,.5\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{22},\gamma_{33},\gamma_{44},\gamma_{51},\gamma_{61}),$$

$$\Big\{x_1,(\frac{\gamma_{12}}{\langle .5,.8,.2\rangle},\frac{\gamma_{22}}{\langle .5,.7,.9\rangle},\frac{\gamma_{33}}{\langle .1,.2,.4\rangle},\frac{\gamma_{44}}{\langle .6,.3,.8\rangle},\frac{\gamma_{51}}{\langle .2,.5,.9\rangle},\frac{\gamma_{61}}{\langle .2,.3,.4\rangle}\Big),$$

$$x_2,\Big(\frac{\gamma_{12}}{\langle .4,.3,.1\rangle},\frac{\gamma_{22}}{\langle .4,.9,.3\rangle},\frac{\gamma_{33}}{\langle .5,.2,.7\rangle},\frac{\gamma_{44}}{\langle .1,.8,.3\rangle},\frac{\gamma_{51}}{\langle .,6.4,.3\rangle},\frac{\gamma_{61}}{\langle .8,.6,.2\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{12}}{\langle .6,.1,.7\rangle},\frac{\gamma_{22}}{\langle .3,.8,.2\rangle},\frac{\gamma_{33}}{\langle .7,.3,.6\rangle},\frac{\gamma_{44}}{\langle .7,.4,.2\rangle},\frac{\gamma_{51}}{\langle .8,.3,.4\rangle},\frac{\gamma_{61}}{\langle .1,.4,.9\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .6,.3,.8\rangle},\frac{\gamma_{22}}{\langle .1,.6,.3\rangle},\frac{\gamma_{33}}{\langle .7,.5,.4\rangle},\frac{\gamma_{44}}{\langle .3,.5,.2\rangle},\frac{\gamma_{51}}{\langle .8,.6,.5\rangle},\frac{\gamma_{61}}{\langle .8,.3,.7\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{24},\gamma_{33},\gamma_{44},\gamma_{51},\gamma_{61}),$$

$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .1,.2,.6\rangle},\frac{\gamma_{24}}{\langle .7,.4,.9\rangle},\frac{\gamma_{33}}{\langle .2,.4,.6\rangle},\frac{\gamma_{44}}{\langle .9,.6,.4\rangle},\frac{\gamma_{51}}{\langle .1,.2,.5\rangle},\frac{\gamma_{61}}{\langle .7,.7,.3\rangle}\Big),$$

$$x_2,\Big(\frac{\gamma_{12}}{\langle .4,.3,.2\rangle},\frac{\gamma_{24}}{\langle .7,.1,.3\rangle},\frac{\gamma_{33}}{\langle .5,.4,.3\rangle},\frac{\gamma_{44}}{\langle .9,.5,.2\rangle},\frac{\gamma_{51}}{\langle .6,.4,.8\rangle},\frac{\gamma_{61}}{\langle .1,.4,.3\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{12}}{\langle .1,.2,.3\rangle},\frac{\gamma_{24}}{\langle .5,.6,.6\rangle},\frac{\gamma_{33}}{\langle .7,.8,.8\rangle},\frac{\gamma_{44}}{\langle .9,.2,.5\rangle},\frac{\gamma_{51}}{\langle .6,.1,.4\rangle},\frac{\gamma_{61}}{\langle .6,.1,.4\rangle}\Big),$$

$$x_4,\left(\frac{\gamma_{12}}{\langle .6,.4,.3\rangle},\frac{\gamma_{24}}{\langle .6,.2,.7\rangle},\frac{\gamma_{33}}{\langle .3,.7,.2\rangle},\frac{\gamma_{44}}{\langle .8,.6,.1\rangle},\frac{\gamma_{51}}{\langle .8,.3,.5\rangle},\frac{\gamma_{61}}{\langle .7,.2,.1\rangle}\right)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{22},\gamma_{33},\gamma_{44},\gamma_{54},\gamma_{61}),$$

$$\Big\{x_1,(\frac{\gamma_{12}}{\langle .4,.8,.3\rangle},\frac{\gamma_{22}}{\langle .6,.1,.7\rangle},\frac{\gamma_{33}}{\langle .3,.5,.8\rangle},\frac{\gamma_{44}}{\langle .4,.5,.6\rangle},\frac{\gamma_{54}}{\langle .9,.3,.4\rangle},\frac{\gamma_{61}}{\langle .5,.7,.3\rangle}\right),$$

$$x_2,\left(\frac{\gamma_{12}}{\langle .3,.5,.1\rangle},\frac{\gamma_{22}}{\langle .7,.4,.3\rangle},\frac{\gamma_{33}}{\langle .8,.4,.3\rangle},\frac{\gamma_{44}}{\langle .6,.3,.1\rangle},\frac{\gamma_{54}}{\langle .3,.7,.9\rangle},\frac{\gamma_{61}}{\langle .3,.8,.5\rangle}\right),$$

$$x_3,\left(\frac{\gamma_{12}}{\langle .1,.6,.8\rangle},\frac{\gamma_{22}}{\langle .3,.4,.8\rangle},\frac{\gamma_{33}}{\langle .9,.2,.7\rangle},\frac{\gamma_{44}}{\langle .3,.8,.3\rangle},\frac{\gamma_{54}}{\langle .1,.3,.8\rangle},\frac{\gamma_{61}}{\langle .2,.3,.9\rangle}\right),$$

$$x_4,\left(\frac{\gamma_{12}}{\langle .2,.1,.4\rangle},\frac{\gamma_{22}}{\langle .8,.5,.4\rangle},\frac{\gamma_{33}}{\langle .2,.5,.8\rangle},\frac{\gamma_{44}}{\langle .9,.4,.3\rangle},\frac{\gamma_{54}}{\langle .8,.4,.1\rangle},\frac{\gamma_{61}}{\langle .6,.3,.9\rangle}\right)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{24},\gamma_{33},\gamma_{44},\gamma_{54},\gamma_{61}),$$

$$\Big\{x_1,\left(\frac{\gamma_{12}}{\langle .8,.2,.6\rangle},\frac{\gamma_{24}}{\langle .5,.3,.8\rangle},\frac{\gamma_{33}}{\langle .1,.3,.6\rangle},\frac{\gamma_{44}}{\langle .8,.4,.9\rangle},\frac{\gamma_{54}}{\langle .3,.5,.7\rangle},\frac{\gamma_{61}}{\langle .1,.6,.7\rangle}\right),$$

$$x_2,\left(\frac{\gamma_{12}}{\langle .3,.2,.8\rangle},\frac{\gamma_{24}}{\langle .6,.5,.9\rangle},\frac{\gamma_{33}}{\langle .4,.2,.5\rangle},\frac{\gamma_{44}}{\langle .7,.2,.9\rangle},\frac{\gamma_{54}}{\langle .5,.4,.1\rangle},\frac{\gamma_{61}}{\langle .9,.5,.8\rangle}\right),$$

$$x_3,\left(\frac{\gamma_{12}}{\langle .8,.2,.6\rangle},\frac{\gamma_{24}}{\langle .7,.4,.1\rangle},\frac{\gamma_{33}}{\langle .8,.3,.8\rangle},\frac{\gamma_{44}}{\langle .4,.1,.8\rangle},\frac{\gamma_{54}}{\langle .6,.7,.9\rangle},\frac{\gamma_{61}}{\langle .3,.4,.7\rangle}\right),$$

$$x_4,\left(\frac{\gamma_{12}}{\langle .3,.8,.4\rangle},\frac{\gamma_{24}}{\langle .9,.1,.9\rangle},\frac{\gamma_{33}}{\langle .3,.6,.8\rangle},\frac{\gamma_{44}}{\langle .3,.4,.9\rangle},\frac{\gamma_{54}}{\langle .3,.8,.2\rangle},\frac{\gamma_{61}}{\langle .1,.8,.8\rangle}\right)\Big\}\Big)\Big\}.$$

Suppose that $\beta_1=\{\gamma_{12},\gamma_{13}\}$, $\beta_2=\{\gamma_{22},\gamma_{23},\gamma_{24}\}$, $\beta_3=\{\gamma_{32},\gamma_{33},\gamma_{34}\}$, $\beta_4=\{\gamma_{42},\gamma_{44},\gamma_{45}\}$, $\beta_5=\{\gamma_{51},\gamma_{53},\gamma_{54}\}$, $\beta_6=\{\gamma_{61},\gamma_{62}\}$ that is $\prod_{j=1}^{n}\beta_j\subseteq\prod_{j=1}^{n}\Delta_j$, $j=1,2,3$. Let the $\mathsf{Pn}_{\mathsf{hss}}$,

$$\Big(\mathbb{N}_1,\prod_{j=1}^{3}\beta_i\Big)=\Big\{\Big((\gamma_{12},\gamma_{22},\gamma_{32},\gamma_{42},\gamma_{51},\gamma_{61}),$$

$$\Big\{x_1,\left(\frac{\gamma_{12}}{\langle .4,.1,.8\rangle},\frac{\gamma_{22}}{\langle .2,.4,.7\rangle},\frac{\gamma_{32}}{\langle .1,.2,.8\rangle},\frac{\gamma_{42}}{\langle .4,.6,.8\rangle},\frac{\gamma_{51}}{\langle .1,.2,.5\rangle},\frac{\gamma_{61}}{\langle .7,.3,.8\rangle}\right),$$

$$x_2,\left(\frac{\gamma_{12}}{\langle .3,.1,.8\rangle},\frac{\gamma_{22}}{\langle .1,.2,.6\rangle},\frac{\gamma_{32}}{\langle .8,.1,.8\rangle},\frac{\gamma_{42}}{\langle .3,.6,.8\rangle},\frac{\gamma_{51}}{\langle .3,.4,.6\rangle},\frac{\gamma_{61}}{\langle .1,.6,.9\rangle}\right)$$

$$x_3,\left(\frac{\gamma_{12}}{\langle .1,.5,.8\rangle},\frac{\gamma_{22}}{\langle .2,.7,.4\rangle},\frac{\gamma_{32}}{\langle .8,.1,.8\rangle},\frac{\gamma_{42}}{\langle .3,.6,.8\rangle},\frac{\gamma_{51}}{\langle .1,.3,.7\rangle},\frac{\gamma_{61}}{\langle .3,.7,.2\rangle}\right)$$

$$x_4,\left(\frac{\gamma_{12}}{\langle .4,.9,.1\rangle},\frac{\gamma_{22}}{\langle .3,.5,.8\rangle},\frac{\gamma_{32}}{\langle .1,.2,.8\rangle},\frac{\gamma_{42}}{\langle .2,.8,.1\rangle},\frac{\gamma_{51}}{\langle .8,.3,.8\rangle},\frac{\gamma_{61}}{\langle .2,.7,.9\rangle}\right)\Big\}\Big),$$

$$\Big((\gamma_{13},\gamma_{24},\gamma_{34},\gamma_{45},\gamma_{54},\gamma_{62}),$$

$$\Big\{x_2,\left(\frac{\gamma_{13}}{\langle .1,.8,.5\rangle},\frac{\gamma_{24}}{\langle .4,.7,.1\rangle},\frac{\gamma_{34}}{\langle .8,.3,.8\rangle},\frac{\gamma_{45}}{\langle .1,.7,.3\rangle},\frac{\gamma_{54}}{\langle .6,.2,.6\rangle},\frac{\gamma_{62}}{\langle .1,.8,.3\rangle}\right),$$

$$x_3,\left(\frac{\gamma_{13}}{\langle .2,.6,.1\rangle},\frac{\gamma_{24}}{\langle .6,.2,.6\rangle},\frac{\gamma_{34}}{\langle .1,.9,.2\rangle},\frac{\gamma_{45}}{\langle .4,.7,.2\rangle},\frac{\gamma_{54}}{\langle .6,.1,.5\rangle},\frac{\gamma_{62}}{\langle .2,.7,.1\rangle}\right),$$

$$x_4,\left(\frac{\gamma_{13}}{\langle .1,.3,.6\rangle},\frac{\gamma_{24}}{\langle .9,.4,.2\rangle},\frac{\gamma_{34}}{\langle .7,.3,.8\rangle},\frac{\gamma_{45}}{\langle .5,.3,.1\rangle},\frac{\gamma_{54}}{\langle .6,.3,.6\rangle},\frac{\gamma_{62}}{\langle .2,.5,.9\rangle}\right)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{23},\gamma_{33},\gamma_{44},\gamma_{53},\gamma_{61}),$$

$$\Big\{x_3,\Big(\frac{\gamma_{12}}{\langle .1,.7,.9\rangle},\frac{\gamma_{23}}{\langle .2,.4,.9\rangle},\frac{\gamma_{33}}{\langle .1,.4,.5\rangle},\frac{\gamma_{44}}{\langle .2,.8,.1\rangle},\frac{\gamma_{53}}{\langle .6,.4,.9\rangle},\frac{\gamma_{61}}{\langle .1,.4,.9\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{13},\gamma_{24},\gamma_{33},\gamma_{44},\gamma_{53},\gamma_{62}),$$

$$\Big\{x_1,\Big(\frac{\gamma_{13}}{\langle .2,.6,.1\rangle},\frac{\gamma_{24}}{\langle .6,.3,.4\rangle},\frac{\gamma_{33}}{\langle .6,.1,.9\rangle},\frac{\gamma_{44}}{\langle .2,.4,.2\rangle},\frac{\gamma_{53}}{\langle .6,.3,.1\rangle},\frac{\gamma_{62}}{\langle .5,.3,.9\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{13}}{\langle .3,.1,.8\rangle},\frac{\gamma_{24}}{\langle .3,.2,.9\rangle},\frac{\gamma_{33}}{\langle .5,.4,.1\rangle},\frac{\gamma_{44}}{\langle .4,.9,.2\rangle},\frac{\gamma_{53}}{\langle .8,.3,.9\rangle},\frac{\gamma_{62}}{\langle .1,.4,.3\rangle}\Big)$$

$$x_4,\Big(\frac{\gamma_{13}}{\langle .2,.4,.9\rangle},\frac{\gamma_{24}}{\langle .8,.4,.2\rangle},\frac{\gamma_{33}}{\langle .6,.3,.1\rangle},\frac{\gamma_{44}}{\langle .2,.4,.6\rangle},\frac{\gamma_{53}}{\langle .8,.3,.9\rangle},\frac{\gamma_{62}}{\langle .4,.3,.9\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{24},\gamma_{32},\gamma_{45},\gamma_{51},\gamma_{62}),$$

$$\Big\{x_1,(\frac{\gamma_{12}}{\langle .2,.1,.9\rangle},\frac{\gamma_{24}}{\langle .4,.5,.6\rangle},\frac{\gamma_{32}}{\langle .4,.1,.9\rangle},\frac{\gamma_{45}}{\langle .4,.3,.9\rangle},\frac{\gamma_{51}}{\langle .2,.1,.8\rangle},\frac{\gamma_{62}}{\langle .3,.8,.2\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .2,.5,.5\rangle},\frac{\gamma_{24}}{\langle .6,.9,.7\rangle},\frac{\gamma_{32}}{\langle .2,.4,.1\rangle},\frac{\gamma_{45}}{\langle .7,.3,.9\rangle},\frac{\gamma_{51}}{\langle .1,.4,.7\rangle},\frac{\gamma_{62}}{\langle .3,.1,.8\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{13},\gamma_{22},\gamma_{34},\gamma_{42},\gamma_{54},\gamma_{61}),$$

$$\Big\{x_2,\Big(\frac{\gamma_{13}}{\langle .3,.6,.1\rangle},\frac{\gamma_{22}}{\langle .3,.7,.4\rangle},\frac{\gamma_{34}}{\langle .6,.3,.8\rangle},\frac{\gamma_{42}}{\langle .3,.1,.8\rangle},\frac{\gamma_{54}}{\langle .4,.6,.9\rangle},\frac{\gamma_{61}}{\langle .3,.6,.4\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{13}}{\langle .4,.2,.7\rangle},\frac{\gamma_{22}}{\langle .4,.2,.6\rangle},\frac{\gamma_{34}}{\langle .8,.1,.6\rangle},\frac{\gamma_{42}}{\langle .4,.1,.7\rangle},\frac{\gamma_{54}}{\langle .2,.3,.7\rangle},\frac{\gamma_{61}}{\langle .3,.1,.7\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{22},\gamma_{33},\gamma_{44},\gamma_{51},\gamma_{61}),$$

$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .4,.9,.1\rangle},\frac{\gamma_{22}}{\langle .5,.8,.3\rangle},\frac{\gamma_{33}}{\langle .9,.2,.1\rangle},\frac{\gamma_{44}}{\langle .5,.3,.9\rangle},\frac{\gamma_{51}}{\langle .1,.4,.9\rangle},\frac{\gamma_{61}}{\langle .1,.5,.9\rangle}\Big),$$

$$x_2,\Big(\frac{\gamma_{12}}{\langle .3,.9,.1\rangle},\frac{\gamma_{22}}{\langle .8,.3,.7\rangle},\frac{\gamma_{33}}{\langle .6,.3,.4\rangle},\frac{\gamma_{44}}{\langle .1,.6,.2\rangle},\frac{\gamma_{51}}{\langle .6,.8,.4\rangle},\frac{\gamma_{61}}{\langle .9,.7,.2\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{12}}{\langle .4,.7,.1\rangle},\frac{\gamma_{22}}{\langle .5,.8,.3\rangle},\frac{\gamma_{33}}{\langle .6,.9,.2\rangle},\frac{\gamma_{44}}{\langle .5,.7,.3\rangle},\frac{\gamma_{51}}{\langle .1,.3,.9\rangle},\frac{\gamma_{61}}{\langle .3,.8,.6\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .3,.9,.3\rangle},\frac{\gamma_{22}}{\langle .9,.2,.7\rangle},\frac{\gamma_{33}}{\langle .3,.9,.3\rangle},\frac{\gamma_{44}}{\langle .1,.6,.8\rangle},\frac{\gamma_{51}}{\langle .1,.8,.2\rangle},\frac{\gamma_{61}}{\langle .8,.3,.6\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{24},\gamma_{33},\gamma_{44},\gamma_{51},\gamma_{61}),$$

$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .3,.1,.5\rangle},\frac{\gamma_{24}}{\langle .7,.3,.6\rangle},\frac{\gamma_{33}}{\langle .9,.3,.1\rangle},\frac{\gamma_{44}}{\langle .3,.6,.3\rangle},\frac{\gamma_{51}}{\langle .9,.5,.7\rangle},\frac{\gamma_{61}}{\langle .3,.7,.9\rangle}\Big),$$

$$x_2,\Big(\frac{\gamma_{12}}{\langle .3,.7,.1\rangle},\frac{\gamma_{24}}{\langle .5,.8,.3\rangle},\frac{\gamma_{33}}{\langle .4,.1,.9\rangle},\frac{\gamma_{44}}{\langle .7,.2,.9\rangle},\frac{\gamma_{51}}{\langle .3,.7,.9\rangle},\frac{\gamma_{61}}{\langle .1,.8,.2\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{12}}{\langle .4,.1,.8\rangle},\frac{\gamma_{24}}{\langle .4,.3,.9\rangle},\frac{\gamma_{33}}{\langle .3,.8,.2\rangle},\frac{\gamma_{44}}{\langle .5,.3,.8\rangle},\frac{\gamma_{51}}{\langle .2,.8,.5\rangle},\frac{\gamma_{61}}{\langle .3,.8,.1\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .1,.4,.7\rangle},\frac{\gamma_{24}}{\langle .5,.2,.2\rangle},\frac{\gamma_{33}}{\langle .2,.8,.5\rangle},\frac{\gamma_{44}}{\langle .1,.9,.4\rangle},\frac{\gamma_{51}}{\langle .2,.2,.6\rangle},\frac{\gamma_{61}}{\langle .5,.7,.8\rangle}\Big)\Big\}\Big)$$

$$\Big((\gamma_{12},\gamma_{22},\gamma_{33},\gamma_{44},\gamma_{54},\gamma_{61}),$$

$$\Big\{x_1,(\frac{\gamma_{12}}{\langle .6,.1,.9\rangle},\frac{\gamma_{22}}{\langle .3,.5,.2\rangle},\frac{\gamma_{33}}{\langle .9,.5,.9\rangle},\frac{\gamma_{44}}{\langle .8,.2,.5\rangle},\frac{\gamma_{54}}{\langle .6,.1,.5\rangle},\frac{\gamma_{61}}{\langle .3,.6,.9\rangle}\Big),$$

$$x_2,\left(\frac{\gamma_{12}}{\langle .3,.2,.9\rangle},\frac{\gamma_{22}}{\langle .1,.6,.9\rangle},\frac{\gamma_{33}}{\langle .3,.5,.1\rangle},\frac{\gamma_{44}}{\langle .4,.8,.2\rangle},\frac{\gamma_{54}}{\langle .6,.1,.8\rangle},\frac{\gamma_{61}}{\langle .4,.2,.4\rangle}\right),$$
$$x_3,\left(\frac{\gamma_{12}}{\langle .3,.2,.1\rangle},\frac{\gamma_{22}}{\langle .3,.7,.1\rangle},\frac{\gamma_{33}}{\langle .8,.4,.9\rangle},\frac{\gamma_{44}}{\langle .4,.3,.9\rangle},\frac{\gamma_{54}}{\langle .8,.3,.1\rangle},\frac{\gamma_{61}}{\langle .6,.4,.2\rangle}\right),$$
$$x_4,\left(\frac{\gamma_{12}}{\langle .1,.3,.7\rangle},\frac{\gamma_{22}}{\langle .1,.9,.3\rangle},\frac{\gamma_{33}}{\langle .5,.2,.8\rangle},\frac{\gamma_{44}}{\langle .9,.5,.9\rangle},\frac{\gamma_{54}}{\langle .3,.6,.1\rangle},\frac{\gamma_{61}}{\langle .9,.2,.1\rangle}\right)\Big\}\Big)$$
$$\Big((\gamma_{12},\gamma_{24},\gamma_{33},\gamma_{44},\gamma_{54},\gamma_{61}),$$
$$\Big\{x_1,(\frac{\gamma_{12}}{\langle .4,.6,.1\rangle},\frac{\gamma_{24}}{\langle .4,.3,.9\rangle},\frac{\gamma_{33}}{\langle .1,.5,.9\rangle},\frac{\gamma_{44}}{\langle .3,.7,.1\rangle},\frac{\gamma_{54}}{\langle .8,.3,.3\rangle},\frac{\gamma_{61}}{\langle .6,.3,.1\rangle}\Big),$$
$$x_2,\left(\frac{\gamma_{12}}{\langle .3,.4,.1\rangle},\frac{\gamma_{24}}{\langle .6,.3,.3\rangle},\frac{\gamma_{33}}{\langle .8,.3,.5\rangle},\frac{\gamma_{44}}{\langle .9,.1,.4\rangle},\frac{\gamma_{54}}{\langle .4,.4,.6\rangle},\frac{\gamma_{61}}{\langle .9,.4,.8\rangle}\right),$$
$$x_3,\left(\frac{\gamma_{12}}{\langle .5,.3,.7\rangle},\frac{\gamma_{24}}{\langle .5,.9,.3\rangle},\frac{\gamma_{33}}{\langle .9,.4,.1\rangle},\frac{\gamma_{44}}{\langle .5,.3,.5\rangle},\frac{\gamma_{54}}{\langle .9,.4,.8\rangle},\frac{\gamma_{61}}{\langle .1,.2,.6\rangle}\right),$$
$$x_4,\left(\frac{\gamma_{12}}{\langle .4,.4,.2\rangle},\frac{\gamma_{24}}{\langle .6,.3,.2\rangle},\frac{\gamma_{33}}{\langle .6,.2,.5\rangle},\frac{\gamma_{44}}{\langle .6,.6,.4\rangle},\frac{\gamma_{54}}{\langle .8,.2,.6\rangle},\frac{\gamma_{61}}{\langle .5,.4,.1\rangle}\right)\Big\}\Big)\Big\}.$$

Suppose that $\zeta_1=\{\gamma_{11},\gamma_{12},\gamma_{13}\}$, $\zeta_2=\{\gamma_{21},\gamma_{22},\gamma_{23},\gamma_{24}\}$, $\zeta_3=\{\gamma_{31},\gamma_{32},\gamma_{34}\}$, $\zeta_4=\{\gamma_{42},\gamma_{43},\gamma_{44},\gamma_{45}\}$, $\zeta_5=\{\gamma_{51},\gamma_{53},\gamma_{54}\}$, $\zeta_6=\{\gamma_{61},\gamma_{62}\}$ that is $\prod_{j=1}^{n}\zeta_j\subseteq\prod_{j=1}^{n}\Delta_j$, $j=1,2,3$. Let the $\mathsf{Pn}_{\mathsf{hss}}$,

$$\Big(\mathbb{N}_1,\prod_{j=1}^{3}\zeta_i\Big)=\Big\{\Big((\gamma_{11},\gamma_{21},\gamma_{31},\gamma_{42},\gamma_{51},\gamma_{61}),$$
$$\Big\{x_2,(\frac{\gamma_{11}}{\langle .3,.2,.7\rangle},\frac{\gamma_{21}}{\langle .1,.6,.8\rangle},\frac{\gamma_{31}}{\langle .3,.2,.9\rangle},\frac{\gamma_{42}}{\langle .1,.2,.7\rangle},\frac{\gamma_{51}}{\langle .3,.6,.7\rangle},\frac{\gamma_{61}}{\langle .3,.4,.7\rangle}\Big),$$
$$x_4,\left(\frac{\gamma_{11}}{\langle .1,.5,.7\rangle},\frac{\gamma_{21}}{\langle .5,.8,.3\rangle},\frac{\gamma_{31}}{\langle .7,.5,.2\rangle},\frac{\gamma_{42}}{\langle .4,.3,.7\rangle},\frac{\gamma_{51}}{\langle .1,.4,.7\rangle},\frac{\gamma_{61}}{\langle .4,.7,.1\rangle}\right)\Big\}\Big),$$
$$\Big((\gamma_{12},\gamma_{22},\gamma_{32},\gamma_{42},\gamma_{53},\gamma_{62}),$$
$$\Big\{x_2,(\frac{\gamma_{12}}{\langle .4,.1,.8\rangle},\frac{\gamma_{22}}{\langle .3,.7,.9\rangle},\frac{\gamma_{32}}{\langle .2,.7,.9\rangle},\frac{\gamma_{42}}{\langle .7,.4,.8\rangle},\frac{\gamma_{53}}{\langle .3,.8,.9\rangle},\frac{\gamma_{62}}{\langle .3,.7,.9\rangle}\Big),$$
$$x_3,\left(\frac{\gamma_{12}}{\langle .2,.6,.9\rangle},\frac{\gamma_{22}}{\langle .4,.1,.7\rangle},\frac{\gamma_{32}}{\langle .8,.4,.7\rangle},\frac{\gamma_{42}}{\langle .1,.7,.3\rangle},\frac{\gamma_{53}}{\langle .8,.4,.8\rangle},\frac{\gamma_{62}}{\langle .2,.7,.9\rangle}\right),$$
$$x_4,\left(\frac{\gamma_{12}}{\langle .5,.3,.6\rangle},\frac{\gamma_{22}}{\langle .7,.3,.1\rangle},\frac{\gamma_{32}}{\langle .8,.7,.3\rangle},\frac{\gamma_{42}}{\langle .6,.4,.8\rangle},\frac{\gamma_{53}}{\langle .1,.4,.7\rangle},\frac{\gamma_{62}}{\langle .3,.7,.3\rangle}\right)\Big\}\Big),$$
$$\Big((\gamma_{13},\gamma_{23},\gamma_{34},\gamma_{43},\gamma_{53},\gamma_{61}),$$
$$\Big\{x_3,(\frac{\gamma_{13}}{\langle .4,.2,.1\rangle},\frac{\gamma_{23}}{\langle .8,.1,.3\rangle},\frac{\gamma_{34}}{\langle .7,.5,.3\rangle},\frac{\gamma_{43}}{\langle .1,.8,.3\rangle},\frac{\gamma_{53}}{\langle .5,.7,.8\rangle},\frac{\gamma_{61}}{\langle .1,.8,.4\rangle}\Big)\Big\}\Big),$$
$$\Big((\gamma_{13},\gamma_{24},\gamma_{34},\gamma_{44},\gamma_{54},\gamma_{61}),$$
$$\Big\{x_3,(\frac{\gamma_{13}}{\langle .2,.6,.9\rangle},\frac{\gamma_{24}}{\langle .1,.6,.8\rangle},\frac{\gamma_{34}}{\langle .3,.7,.9\rangle},\frac{\gamma_{44}}{\langle .4,.8,.1\rangle},\frac{\gamma_{54}}{\langle .6,.8,.9\rangle},\frac{\gamma_{61}}{\langle .3,.4,.5\rangle}\Big),$$
$$x_4,\left(\frac{\gamma_{13}}{\langle .1,.3,.7\rangle},\frac{\gamma_{24}}{\langle .8,.5,.1\rangle},\frac{\gamma_{34}}{\langle .3,.6,.8\rangle},\frac{\gamma_{44}}{\langle .8,.5,.3\rangle},\frac{\gamma_{54}}{\langle .2,.6,.7\rangle},\frac{\gamma_{61}}{\langle .1,.3,.6\rangle}\right)\Big\}\Big),$$
$$\Big((\gamma_{11},\gamma_{24},\gamma_{31},\gamma_{45},\gamma_{53},\gamma_{62}),$$

$$\Big\{x_1,\Big(\frac{\gamma_{11}}{\langle .4,.6,.8\rangle},\frac{\gamma_{24}}{\langle .1,.8,.4\rangle},\frac{\gamma_{31}}{\langle .2,.7,.8\rangle},\frac{\gamma_{45}}{\langle .4,.6,.7\rangle},\frac{\gamma_{53}}{\langle .4,.5,.2\rangle},\frac{\gamma_{62}}{\langle .6,.3,.8\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{11}}{\langle .1,.3,.7\rangle},\frac{\gamma_{24}}{\langle .3,.6,.4\rangle},\frac{\gamma_{31}}{\langle .8,.5,.1\rangle},\frac{\gamma_{45}}{\langle .6,.4,.2\rangle},\frac{\gamma_{53}}{\langle .6,.3,.7\rangle},\frac{\gamma_{62}}{\langle .1,.7,.4\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{22},\gamma_{34},\gamma_{45},\gamma_{54},\gamma_{61}),$$

$$\Big\{x_1,(\frac{\gamma_{12}}{\langle .7,.4,.8\rangle},\frac{\gamma_{22}}{\langle .2,.8,.4\rangle},\frac{\gamma_{34}}{\langle .6,.1,.3\rangle},\frac{\gamma_{45}}{\langle .6,.4,.2\rangle},\frac{\gamma_{54}}{\langle .6,.4,.8\rangle},\frac{\gamma_{61}}{\langle .3,.3,.2\rangle}\Big),$$

$$x_2,\Big(\frac{\gamma_{12}}{\langle .1,.6,.8\rangle},\frac{\gamma_{22}}{\langle .3,.4,.5\rangle},\frac{\gamma_{34}}{\langle .7,.4,.2\rangle},\frac{\gamma_{45}}{\langle .6,.4,.8\rangle},\frac{\gamma_{54}}{\langle .1,.3,.7\rangle},\frac{\gamma_{61}}{\langle .1,.6,.4\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{12}}{\langle .4,.2,.1\rangle},\frac{\gamma_{22}}{\langle .5,.3,.8\rangle},\frac{\gamma_{34}}{\langle .2,.3,.6\rangle},\frac{\gamma_{45}}{\langle .4,.5,.6\rangle},\frac{\gamma_{54}}{\langle .1,.3,.6\rangle},\frac{\gamma_{61}}{\langle .3,.7,.6\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .4,.3,.9\rangle},\frac{\gamma_{22}}{\langle .1,.2,.4\rangle},\frac{\gamma_{34}}{\langle .4,.4,.7\rangle},\frac{\gamma_{45}}{\langle .3,.4,.7\rangle},\frac{\gamma_{54}}{\langle .2,.5,.8\rangle},\frac{\gamma_{61}}{\langle .1,.7,.8\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{22},\gamma_{33},\gamma_{44},\gamma_{51},\gamma_{61}),$$

$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .2,.6,.4\rangle},\frac{\gamma_{22}}{\langle .8,.3,.1\rangle},\frac{\gamma_{33}}{\langle .7,.4,.1\rangle},\frac{\gamma_{44}}{\langle .1,.7,.8\rangle},\frac{\gamma_{51}}{\langle .1,.3,.6\rangle},\frac{\gamma_{61}}{\langle .8,.4,.3\rangle}\Big),$$

$$x_2,\Big(\frac{\gamma_{12}}{\langle .3,.1,.8\rangle},\frac{\gamma_{22}}{\langle .3,.1,.8\rangle},\frac{\gamma_{33}}{\langle .3,.3,.9\rangle},\frac{\gamma_{44}}{\langle .1,.8,.3\rangle},\frac{\gamma_{51}}{\langle .9,.3,.9\rangle},\frac{\gamma_{61}}{\langle .3,.1,.8\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{12}}{\langle .1,.3,.7\rangle},\frac{\gamma_{22}}{\langle .4,.4,.5\rangle},\frac{\gamma_{33}}{\langle .7,.4,.8\rangle},\frac{\gamma_{44}}{\langle .2,.6,.3\rangle},\frac{\gamma_{51}}{\langle .7,.4,.1\rangle},\frac{\gamma_{61}}{\langle .6,.4,.3\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .5,.4,.1\rangle},\frac{\gamma_{22}}{\langle .6,.3,.2\rangle},\frac{\gamma_{33}}{\langle .6,.9,.5\rangle},\frac{\gamma_{44}}{\langle .2,.8,.4\rangle},\frac{\gamma_{51}}{\langle .6,.2,.6\rangle},\frac{\gamma_{61}}{\langle .5,.6,.8\rangle}\Big)\Big\}\Big)$$

$$\Big((\gamma_{12},\gamma_{24},\gamma_{33},\gamma_{44},\gamma_{51},\gamma_{61}),$$

$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .2,.6,.1\rangle},\frac{\gamma_{24}}{\langle .4,.9,.1\rangle},\frac{\gamma_{33}}{\langle .6,.3,.5\rangle},\frac{\gamma_{44}}{\langle .3,.5,.7\rangle},\frac{\gamma_{51}}{\langle .3,.7,.5\rangle},\frac{\gamma_{61}}{\langle .3,.6,.1\rangle}\Big),$$

$$x_2,\Big(\frac{\gamma_{12}}{\langle .5,.4,.7\rangle},\frac{\gamma_{24}}{\langle .4,.3,.2\rangle},\frac{\gamma_{33}}{\langle .6,.8,.5\rangle},\frac{\gamma_{44}}{\langle .1,.4,.4\rangle},\frac{\gamma_{51}}{\langle .6,.2,.3\rangle},\frac{\gamma_{61}}{\langle .1,.4,.7\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{12}}{\langle .3,.2,.9\rangle},\frac{\gamma_{24}}{\langle .3,.6,.1\rangle},\frac{\gamma_{33}}{\langle .4,.9,.1\rangle},\frac{\gamma_{44}}{\langle .6,.3,.2\rangle},\frac{\gamma_{51}}{\langle .8,.4,.7\rangle},\frac{\gamma_{61}}{\langle .9,.4,.2\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .4,.9,.2\rangle},\frac{\gamma_{24}}{\langle .4,.1,.5\rangle},\frac{\gamma_{33}}{\langle .2,.1,.9\rangle},\frac{\gamma_{44}}{\langle .3,.8,.3\rangle},\frac{\gamma_{51}}{\langle .1,.9,.3\rangle},\frac{\gamma_{61}}{\langle .8,.4,.3\rangle}\Big)\Big\}\Big)$$

$$\Big((\gamma_{12},\gamma_{22},\gamma_{33},\gamma_{44},\gamma_{54},\gamma_{61}),$$

$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .2,.4,.7\rangle},\frac{\gamma_{22}}{\langle .1,.3,.6\rangle},\frac{\gamma_{33}}{\langle .9,.5,.4\rangle},\frac{\gamma_{44}}{\langle .3,.5,.9\rangle},\frac{\gamma_{54}}{\langle .2,.6,.3\rangle},\frac{\gamma_{61}}{\langle .1,.5,.8\rangle}\Big),$$

$$x_2,\Big(\frac{\gamma_{12}}{\langle .1,.4,.7\rangle},\frac{\gamma_{22}}{\langle .8,.7,.2\rangle},\frac{\gamma_{33}}{\langle .6,.3,.5\rangle},\frac{\gamma_{44}}{\langle .6,.1,.4\rangle},\frac{\gamma_{54}}{\langle .7,.8,.6\rangle},\frac{\gamma_{61}}{\langle .5,.3,.8\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{12}}{\langle .3,.6,.1\rangle},\frac{\gamma_{22}}{\langle .5,.3,.1\rangle},\frac{\gamma_{33}}{\langle .6,.3,.9\rangle},\frac{\gamma_{44}}{\langle .3,.1,.9\rangle},\frac{\gamma_{54}}{\langle .3,.6,.1\rangle},\frac{\gamma_{61}}{\langle .3,.4,.5\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .7,.4,.7\rangle},\frac{\gamma_{22}}{\langle .2,.3,.2\rangle},\frac{\gamma_{33}}{\langle .4,.1,.5\rangle},\frac{\gamma_{44}}{\langle .8,.8,.4\rangle},\frac{\gamma_{54}}{\langle .8,.2,.6\rangle},\frac{\gamma_{61}}{\langle .5,.4,.8\rangle}\Big)\Big\}\Big)$$

$$\Big((\gamma_{12},\gamma_{24},\gamma_{33},\gamma_{44},\gamma_{54},\gamma_{61}),$$

$$\Big\{x_1,\big(\frac{\gamma_{12}}{\langle .6,.3,.7\rangle},\frac{\gamma_{24}}{\langle .8,.4,.2},\frac{\gamma_{33}}{\langle .4,.3,.2\rangle},\frac{\gamma_{44}}{\langle .7,.4,.1\rangle},\frac{\gamma_{54}}{\langle .3,.5,.9\rangle},\frac{\gamma_{61}}{\langle .2,.3,.7\rangle}\Big),$$
$$x_2,\Big(\frac{\gamma_{12}}{\langle .2,.4,.3\rangle},\frac{\gamma_{24}}{\langle .8,.3,.1\rangle},\frac{\gamma_{33}}{\langle .6,.1,.9\rangle},\frac{\gamma_{44}}{\langle .6,.8,.3\rangle},\frac{\gamma_{54}}{\langle .7,.2,.2\rangle},\frac{\gamma_{61}}{\langle .5,.4,.7\rangle}\Big),$$
$$x_3,\Big(\frac{\gamma_{12}}{\langle .1,.9,.3\rangle},\frac{\gamma_{24}}{\langle .4,.9,.2\rangle},\frac{\gamma_{33}}{\langle .2,.5,.9\rangle},\frac{\gamma_{44}}{\langle .3,.6,.2\rangle},\frac{\gamma_{54}}{\langle .9,.3,.1\rangle},\frac{\gamma_{61}}{\langle .5,.3,.9\rangle}\Big),$$
$$x_4,\Big(\frac{\gamma_{12}}{\langle .3,.4,.3\rangle},\frac{\gamma_{24}}{\langle .8,.1,.2\rangle},\frac{\gamma_{33}}{\langle .6,.1,.3\rangle},\frac{\gamma_{44}}{\langle .6,.1,.4\rangle},\frac{\gamma_{54}}{\langle .7,.9,.6\rangle},\frac{\gamma_{61}}{\langle .5,.4,.2\rangle}\Big)\Big\}\Big)\Big\}.$$

Suppose that $\iota_1=\{\gamma_{12},\gamma_{13}\}$, $\iota_2=\{\gamma_{22},\gamma_{23},\gamma_{24}\}$, $\iota_3=\{\gamma_{32},\gamma_{34}\}$, $\iota_4=\{\gamma_{41},\gamma_{42},\gamma_{44},\gamma_{45}\}$, $\iota_5=\{\gamma_{51},\gamma_{54}\}$, $\iota_6=\{\gamma_{61},\gamma_{62}\}$ that is $\prod_{j=1}^{n}\iota_j\subseteq\prod_{j=1}^{n}\Delta_j$, $j=1,2,3$. Let the $\mathsf{Pn_{hss}}$,

$$\Big(\mathbb{N}_1,\prod_{j=1}^{3}\iota_i\Big)=\Big\{\Big((\gamma_{12},\gamma_{22},\gamma_{32},\gamma_{41},\gamma_{51},\gamma_{61}),$$
$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .3,.1,.9\rangle},\frac{\gamma_{22}}{\langle .3,.1,.8\rangle},\frac{\gamma_{32}}{\langle .3,.1,.9\rangle},\frac{\gamma_{41}}{\langle .4,.1,.8\rangle},\frac{\gamma_{51}}{\langle .3,.4,.2\rangle},\frac{\gamma_{61}}{\langle .6,.4,.3\rangle}\Big),$$
$$x_2,\Big(\frac{\gamma_{12}}{\langle .7,.3,.1\rangle},\frac{\gamma_{22}}{\langle .3,.4,.9\rangle},\frac{\gamma_{32}}{\langle .2,.6,.9\rangle},\frac{\gamma_{41}}{\langle .1,.4,.6\rangle},\frac{\gamma_{51}}{\langle .9,.3,.1\rangle},\frac{\gamma_{61}}{\langle .4,.3,.6\rangle}\Big),$$
$$x_3,\Big(\frac{\gamma_{12}}{\langle .3,.1,.7\rangle},\frac{\gamma_{22}}{\langle .3,.2,.6\rangle},\frac{\gamma_{32}}{\langle .3,.6,.1\rangle},\frac{\gamma_{41}}{\langle .4,.5,.3\rangle},\frac{\gamma_{51}}{\langle .7,.4,.2\rangle},\frac{\gamma_{61}}{\langle .7,.3,.2\rangle}\Big),$$
$$x_4,\Big(\frac{\gamma_{12}}{\langle .7,.4,.1\rangle},\frac{\gamma_{22}}{\langle .5,.3,.4\rangle},\frac{\gamma_{32}}{\langle .6,.3,.8\rangle},\frac{\gamma_{41}}{\langle .1,.3,.5\rangle},\frac{\gamma_{51}}{\langle .6,.4,.2\rangle},\frac{\gamma_{61}}{\langle .6,.4,.2\rangle}\Big)\Big\}\Big),$$
$$\Big((\gamma_{13},\gamma_{23},\gamma_{34},\gamma_{42},\gamma_{54},\gamma_{62}),$$
$$\Big\{x_2,\big(\frac{\gamma_{13}}{\langle .1,.4,.9\rangle},\frac{\gamma_{23}}{\langle .6,.3,.7\rangle},\frac{\gamma_{34}}{\langle .2,.9,.3\rangle},\frac{\gamma_{42}}{\langle .1,.6,.9\rangle},\frac{\gamma_{54}}{\langle .4,.2,.8\rangle},\frac{\gamma_{62}}{\langle .1,.6,.8\rangle}\Big),$$
$$x_3,\Big(\frac{\gamma_{13}}{\langle .1,.3,.8\rangle},\frac{\gamma_{23}}{\langle .5,.3,.2\rangle},\frac{\gamma_{34}}{\langle .4,.3,.9\rangle},\frac{\gamma_{42}}{\langle .3,.2,.1\rangle},\frac{\gamma_{54}}{\langle .3,.4,.8\rangle},\frac{\gamma_{62}}{\langle .2,.6,.7\rangle}\Big),$$
$$x_4,\Big(\frac{\gamma_{13}}{\langle .2,.4,.9\rangle},\frac{\gamma_{23}}{\langle .5,.4,.1\rangle},\frac{\gamma_{34}}{\langle .4,.6,.9\rangle},\frac{\gamma_{42}}{\langle .3,.5,.1\rangle},\frac{\gamma_{54}}{\langle .1,.3,.5\rangle},\frac{\gamma_{62}}{\langle .6,.4,.3\rangle}\Big)\Big\}\Big),$$
$$\Big((\gamma_{12},\gamma_{24},\gamma_{32},\gamma_{44},\gamma_{51},\gamma_{61}),$$
$$\Big\{x_3,\big(\frac{\gamma_{12}}{\langle .1,.3,.5\rangle},\frac{\gamma_{24}}{\langle .4,.9,.2\rangle},\frac{\gamma_{32}}{\langle .4,.3,.2\rangle},\frac{\gamma_{44}}{\langle .1,.4,.9\rangle},\frac{\gamma_{51}}{\langle .5,.4,.3\rangle},\frac{\gamma_{61}}{\langle .2,.4,.9\rangle}\Big)\Big\}\Big),$$
$$\Big((\gamma_{13},\gamma_{21},\gamma_{32},\gamma_{45},\gamma_{54},\gamma_{62}),$$
$$\Big\{x_1,\big(\frac{\gamma_{13}}{\langle .2,.4,.5\rangle},\frac{\gamma_{21}}{\langle .5,.2,.4\rangle},\frac{\gamma_{32}}{\langle .1,.4,.5\rangle},\frac{\gamma_{45}}{\langle .7,.4,.7\rangle},\frac{\gamma_{54}}{\langle .3,.4,.1\rangle},\frac{\gamma_{62}}{\langle .1,.5,.4\rangle}\Big),$$
$$x_3,\Big(\frac{\gamma_{13}}{\langle .2,.1,.4\rangle},\frac{\gamma_{21}}{\langle .5,.4,.1\rangle},\frac{\gamma_{32}}{\langle .7,.4,.1\rangle},\frac{\gamma_{45}}{\langle .5,.3,.6\rangle},\frac{\gamma_{54}}{\langle .2,.7,.2\rangle},\frac{\gamma_{62}}{\langle .4,.3,.3\rangle}\Big),$$
$$x_4,\Big(\frac{\gamma_{13}}{\langle .4,.1,.9\rangle},\frac{\gamma_{21}}{\langle .4,.2,.3\rangle},\frac{\gamma_{32}}{\langle .1,.3,.8\rangle},\frac{\gamma_{45}}{\langle .5,.3,.6\rangle},\frac{\gamma_{54}}{\langle .7,.1,.8\rangle},\frac{\gamma_{62}}{\langle .3,.2,.5\rangle}\Big)\Big\}\Big),$$
$$\Big((\gamma_{12},\gamma_{22},\gamma_{32},\gamma_{41},\gamma_{54},\gamma_{61}),$$
$$\Big\{x_1,\big(\frac{\gamma_{12}}{\langle .1,.3,.7\rangle},\frac{\gamma_{22}}{\langle .3,.7,.3\rangle},\frac{\gamma_{32}}{\langle .5,.3,.2\rangle},\frac{\gamma_{41}}{\langle .9,.4,.3\rangle},\frac{\gamma_{54}}{\langle .7,.8,.4\rangle},\frac{\gamma_{61}}{\langle .9,.4,.2\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .3,.2,.1\rangle},\frac{\gamma_{22}}{\langle .5,.4,.3\rangle},\frac{\gamma_{32}}{\langle .8,.3,.7\rangle},\frac{\gamma_{41}}{\langle .3,.4,.8\rangle},\frac{\gamma_{54}}{\langle .2,.3,.5\rangle},\frac{\gamma_{61}}{\langle .3,.5,.3\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{13},\gamma_{23},\gamma_{32},\gamma_{44},\gamma_{51},\gamma_{61}),$$

$$\Big\{x_2,(\frac{\gamma_{13}}{\langle .4,.3,.1\rangle},\frac{\gamma_{23}}{\langle .6,.3,.7\rangle},\frac{\gamma_{32}}{\langle .3,.4,.2\rangle},\frac{\gamma_{44}}{\langle .5,.7,.2\rangle},\frac{\gamma_{51}}{\langle .1,.5,.8\rangle},\frac{\gamma_{61}}{\langle .2,.4,.5\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{13}}{\langle .1,.4,.5\rangle},\frac{\gamma_{23}}{\langle .6,.3,.2\rangle},\frac{\gamma_{32}}{\langle .1,.3,.6\rangle},\frac{\gamma_{44}}{\langle .7,.3,.4\rangle},\frac{\gamma_{51}}{\langle .1,.5,.9\rangle},\frac{\gamma_{61}}{\langle .4,.3,.2\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{23},\gamma_{32},\gamma_{45},\gamma_{54},\gamma_{61}),$$

$$\Big\{x_1,(\frac{\gamma_{12}}{\langle .4,.2,.1\rangle},\frac{\gamma_{23}}{\langle .3,.6,.3\rangle},\frac{\gamma_{32}}{\langle .5,.3,.2\rangle},\frac{\gamma_{45}}{\langle .3,.5,.9\rangle},\frac{\gamma_{54}}{\langle .1,.3,.4\rangle},\frac{\gamma_{61}}{\langle .4,.1,.6\rangle}\Big),$$

$$x_2,\Big(\frac{\gamma_{12}}{\langle .3,.6,.1\rangle},\frac{\gamma_{23}}{\langle .6,.3,.6\rangle},\frac{\gamma_{32}}{\langle .1,.7,.5\rangle},\frac{\gamma_{45}}{\langle .8,.3,.4\rangle},\frac{\gamma_{54}}{\langle .3,.9,.6\rangle},\frac{\gamma_{61}}{\langle .5,.2,.8\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{12}}{\langle .2,.3,.9\rangle},\frac{\gamma_{23}}{\langle .6,.4,.1\rangle},\frac{\gamma_{32}}{\langle .4,.6,.1\rangle},\frac{\gamma_{45}}{\langle .4,.6,.3\rangle},\frac{\gamma_{54}}{\langle .9,.6,.1\rangle},\frac{\gamma_{61}}{\langle .4,.6,.2\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .4,.1,.7\rangle},\frac{\gamma_{23}}{\langle .7,.3,.2\rangle},\frac{\gamma_{32}}{\langle .6,.3,.5\rangle},\frac{\gamma_{45}}{\langle .1,.3,.4\rangle},\frac{\gamma_{54}}{\langle .6,.2,.6\rangle},\frac{\gamma_{61}}{\langle .5,.,8.3\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{24},\gamma_{33},\gamma_{44},\gamma_{51},\gamma_{61}),$$

$$\Big\{x_1,(\frac{\gamma_{12}}{\langle .3,.5,.2\rangle},\frac{\gamma_{24}}{\langle .7,.1,.4\rangle},\frac{\gamma_{33}}{\langle .7,.4,.8\rangle},\frac{\gamma_{44}}{\langle .2,.5,.8\rangle},\frac{\gamma_{51}}{\langle .6,.4,.3\rangle},\frac{\gamma_{61}}{\langle .8,.3,.1\rangle}\Big),$$

$$x_2,\Big(\frac{\gamma_{12}}{\langle .4,.3,.7\rangle},\frac{\gamma_{24}}{\langle .7,.2,.2\rangle},\frac{\gamma_{33}}{\langle .4,.3,.9\rangle},\frac{\gamma_{44}}{\langle .2,.5,.4\rangle},\frac{\gamma_{51}}{\langle .3,.2,.6\rangle},\frac{\gamma_{61}}{\langle .5,.4,.7\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{12}}{\langle .4,.1,.8\rangle},\frac{\gamma_{24}}{\langle .3,.2,.8\rangle},\frac{\gamma_{33}}{\langle .1,.3,.9\rangle},\frac{\gamma_{44}}{\langle .3,.6,.1\rangle},\frac{\gamma_{51}}{\langle .9,.5,.2\rangle},\frac{\gamma_{61}}{\langle .8,.3,.1\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .3,.6,.7\rangle},\frac{\gamma_{24}}{\langle .2,.3,.9\rangle},\frac{\gamma_{33}}{\langle .3,.6,.5\rangle},\frac{\gamma_{44}}{\langle .6,.2,.4\rangle},\frac{\gamma_{51}}{\langle .6,.2,.6\rangle},\frac{\gamma_{61}}{\langle .5,.4,.4\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{22},\gamma_{33},\gamma_{44},\gamma_{54},\gamma_{61}),$$

$$\Big\{x_1,(\frac{\gamma_{12}}{\langle .4,.1,.3\rangle},\frac{\gamma_{22}}{\langle .8,.4,.8\rangle},\frac{\gamma_{33}}{\langle .3,.9,.1\rangle},\frac{\gamma_{44}}{\langle .7,.4,.9\rangle},\frac{\gamma_{54}}{\langle .2,.5,.4\rangle},\frac{\gamma_{61}}{\langle .6,.4,.2\rangle}\Big),$$

$$x_2,\Big(\frac{\gamma_{12}}{\langle .3,.4,.7\rangle},\frac{\gamma_{22}}{\langle .8,.4,.2\rangle},\frac{\gamma_{33}}{\langle .8,.1,.5\rangle},\frac{\gamma_{44}}{\langle .1,.8,.4\rangle},\frac{\gamma_{54}}{\langle .4,.2,.6\rangle},\frac{\gamma_{61}}{\langle .5,.6,.8\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{12}}{\langle .6,.4,.3\rangle},\frac{\gamma_{22}}{\langle .4,.3,.2\rangle},\frac{\gamma_{33}}{\langle .7,.4,.3\rangle},\frac{\gamma_{44}}{\langle .6,.5,.3\rangle},\frac{\gamma_{54}}{\langle .7,.4,.1\rangle},\frac{\gamma_{61}}{\langle .6,.4,.3\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .3,.2,.7\rangle},\frac{\gamma_{22}}{\langle .6,.4,.2\rangle},\frac{\gamma_{33}}{\langle .1,.6,.8\rangle},\frac{\gamma_{44}}{\langle .6,.8,.4\rangle},\frac{\gamma_{54}}{\langle .2,.2,.6\rangle},\frac{\gamma_{61}}{\langle .5,.4,.9\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{24},\gamma_{33},\gamma_{44},\gamma_{54},\gamma_{61}),$$

$$\Big\{x_1,(\frac{\gamma_{12}}{\langle .4,.3,.1\rangle},\frac{\gamma_{24}}{\langle .4,.3,.4\rangle},\frac{\gamma_{33}}{\langle .5,.3,.1\rangle},\frac{\gamma_{44}}{\langle .3,.5,.9\rangle},\frac{\gamma_{54}}{\langle .3,.2,.4\rangle},\frac{\gamma_{61}}{\langle .5,.4,.3\rangle}\Big),$$

$$x_2,\Big(\frac{\gamma_{12}}{\langle .3,.4,.7\rangle},\frac{\gamma_{24}}{\langle .8,.3,.2\rangle},\frac{\gamma_{33}}{\langle .6,.2,.5\rangle},\frac{\gamma_{44}}{\langle .6,.1,.4\rangle},\frac{\gamma_{54}}{\langle .7,.2,.6\rangle},\frac{\gamma_{61}}{\langle .7,.4,.8\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{12}}{\langle .2,.1,.7\rangle},\frac{\gamma_{24}}{\langle .4,.3,.4\rangle},\frac{\gamma_{33}}{\langle .3,.4,.2\rangle},\frac{\gamma_{44}}{\langle .5,.4,.3\rangle},\frac{\gamma_{54}}{\langle .5,.3,.2\rangle},\frac{\gamma_{61}}{\langle .5,.4,.3\rangle}\Big),$$

$$x_4,\left(\frac{\gamma_{12}}{\langle .1,.5,.3\rangle},\frac{\gamma_{24}}{\langle .7,.2,.8\rangle},\frac{\gamma_{33}}{\langle .7,.1,.5\rangle},\frac{\gamma_{44}}{\langle .4,.8,.4\rangle},\frac{\gamma_{54}}{\langle .8,.2,.6\rangle},\frac{\gamma_{61}}{\langle .4,.2,.8\rangle}\right)\Big\}\Big)\Big\}.$$

Let K be a $\mathsf{Pn}_{\mathsf{hss}}$ defined as

$$K=\Big\{\Big((\gamma_{11},\gamma_{21},\gamma_{32},\gamma_{41},\gamma_{51},\gamma_{61}),$$

$$\Big\{x_2,\left(\frac{\gamma_{11}}{\langle .5,.3,.3\rangle},\frac{\gamma_{21}}{\langle .5,.3,.5\rangle},\frac{\gamma_{32}}{\langle .7,.5,.3\rangle},\frac{\gamma_{41}}{\langle .6,.4,.8\rangle},\frac{\gamma_{51}}{\langle .4,.3,.5\rangle},\frac{\gamma_{61}}{\langle .8,.5,.3\rangle}\right),$$

$$x_4,\left(\frac{\gamma_{11}}{\langle .1,.4,.7\rangle},\frac{\gamma_{21}}{\langle .4,.3,.9\rangle},\frac{\gamma_{32}}{\langle .3,.2,.5\rangle},\frac{\gamma_{41}}{\langle .8,.5,.3\rangle},\frac{\gamma_{51}}{\langle .6,.4,.3\rangle},\frac{\gamma_{61}}{\langle .9,.5,.3\rangle}\right)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{22},\gamma_{33},\gamma_{42},\gamma_{54},\gamma_{61}),$$

$$\Big\{x_1,\left(\frac{\gamma_{12}}{\langle .2,.4,.5\rangle},\frac{\gamma_{22}}{\langle .6,.4,.9\rangle},\frac{\gamma_{33}}{\langle .3,.4,.6\rangle},\frac{\gamma_{42}}{\langle .9,.4,.2\rangle},\frac{\gamma_{54}}{\langle .6,.4,.3\rangle},\frac{\gamma_{61}}{\langle .3,.4,.5\rangle}\right),$$

$$x_2,\left(\frac{\gamma_{12}}{\langle .8,.2,.6\rangle},\frac{\gamma_{22}}{\langle .2,.8,.5\rangle},\frac{\gamma_{33}}{\langle .5,.2,.4\rangle},\frac{\gamma_{42}}{\langle .8,.2,.6\rangle},\frac{\gamma_{54}}{\langle .8,.2,.6\rangle},\frac{\gamma_{61}}{\langle .8,.2,.6\rangle}\right),$$

$$x_3,\left(\frac{\gamma_{12}}{\langle .7,.5,.3\rangle},\frac{\gamma_{22}}{\langle .5,.7,.5\rangle},\frac{\gamma_{33}}{\langle .3,.5,.8\rangle},\frac{\gamma_{42}}{\langle .4,.3,.7\rangle},\frac{\gamma_{54}}{\langle .4,.5,.8\rangle},\frac{\gamma_{61}}{\langle .3,.5,.9\rangle}\right),$$

$$x_4,\left(\frac{\gamma_{12}}{\langle .8,.5,.3\rangle},\frac{\gamma_{22}}{\langle .5,.6,.4\rangle},\frac{\gamma_{33}}{\langle .9,.3,.2\rangle},\frac{\gamma_{42}}{\langle .6,.4,.8\rangle},\frac{\gamma_{54}}{\langle .2,.4,.8\rangle},\frac{\gamma_{61}}{\langle .4,.6,.7\rangle}\right)\Big\}\Big),$$

$$\Big((\gamma_{13},\gamma_{22},\gamma_{33},\gamma_{44},\gamma_{54},\gamma_{61}),$$

$$\Big\{x_3,\left(\frac{\gamma_{13}}{\langle .1,.3,.6\rangle},\frac{\gamma_{22}}{\langle .8,.6,.3\rangle},\frac{\gamma_{33}}{\langle .5,.3,.9\rangle},\frac{\gamma_{44}}{\langle .6,.4,.3\rangle},\frac{\gamma_{54}}{\langle .6,.7,.8\rangle},\frac{\gamma_{61}}{\langle .4,.1,.7\rangle}\right)\Big\}\Big),$$

$$((\gamma_{12},\gamma_{24},\gamma_{32},\gamma_{44},\gamma_{52},\gamma_{62}),$$

$$\Big\{x_1,\left(\frac{\gamma_{12}}{\langle .7,.5,.4\rangle},\frac{\gamma_{24}}{\langle .3,.6,.8\rangle},\frac{\gamma_{32}}{\langle .4,.6,.7\rangle},\frac{\gamma_{44}}{\langle .3,.6,.1\rangle},\frac{\gamma_{52}}{\langle .5,.7,.8\rangle},\frac{\gamma_{62}}{\langle .4,.6,.3\rangle}\right),$$

$$x_2,\left(\frac{\gamma_{12}}{\langle .3,.5,.7\rangle},\frac{\gamma_{24}}{\langle .4,.6,.8\rangle},\frac{\gamma_{32}}{\langle .1,.3,.6\rangle},\frac{\gamma_{44}}{\langle .4,.6,.8\rangle},\frac{\gamma_{52}}{\langle .3,.5,.8\rangle},\frac{\gamma_{62}}{\langle .3,.5,.7\rangle}\right),$$

$$x_3,\left(\frac{\gamma_{12}}{\langle .3,.5,.6\rangle},\frac{\gamma_{24}}{\langle .2,.4,.5\rangle},\frac{\gamma_{32}}{\langle .7,.4,.2\rangle},\frac{\gamma_{44}}{\langle .8,.5,.3\rangle},\frac{\gamma_{52}}{\langle .1,.3,.5\rangle},\frac{\gamma_{62}}{\langle .8,.2,.3\rangle}\right),$$

$$x_4,\left(\frac{\gamma_{12}}{\langle .3,.4,.2\rangle},\frac{\gamma_{24}}{\langle .4,.5,.6\rangle},\frac{\gamma_{32}}{\langle .7,.4,.3\rangle},\frac{\gamma_{44}}{\langle .7,.4,.5\rangle},\frac{\gamma_{52}}{\langle .6,.3,.7\rangle},\frac{\gamma_{62}}{\langle .3,.5,.6\rangle}\right)\Big\}\Big)$$

$$\Big((\gamma_{12},\gamma_{24},\gamma_{32},\gamma_{45},\gamma_{51},\gamma_{62}),$$

$$\Big\{x_1,\left(\frac{\gamma_{12}}{\langle .3,.5,.9\rangle},\frac{\gamma_{24}}{\langle .1,.3,.7\rangle},\frac{\gamma_{32}}{\langle .8,.3,.2\rangle},\frac{\gamma_{45}}{\langle .9,.1,.6\rangle},\frac{\gamma_{51}}{\langle .8,.3,.9\rangle},\frac{\gamma_{62}}{\langle .1,.5,.8\rangle}\right),$$

$$x_4,\left(\frac{\gamma_{12}}{\langle .1,.4,.8\rangle},\frac{\gamma_{24}}{\langle .6,.8,.3\rangle},\frac{\gamma_{32}}{\langle .8,.2,.9\rangle},\frac{\gamma_{45}}{\langle .1,.7,.9\rangle},\frac{\gamma_{51}}{\langle .4,.6,.1\rangle},\frac{\gamma_{62}}{\langle .7,.3,.9\rangle}\right)\Big\}\Big),$$

$$\Big((\gamma_{13},\gamma_{22},\gamma_{34},\gamma_{42},\gamma_{54},\gamma_{61}),$$

$$\Big\{x_2,\left(\frac{\gamma_{13}}{\langle .7,.9,.1\rangle},\frac{\gamma_{22}}{\langle .1,.5,.7\rangle},\frac{\gamma_{34}}{\langle .5,.8,.2\rangle},\frac{\gamma_{42}}{\langle .8,.5,.1\rangle},\frac{\gamma_{54}}{\langle .6,.4,.3\rangle},\frac{\gamma_{61}}{\langle .2,.4,.2\rangle}\right),$$

$$x_3,\left(\frac{\gamma_{13}}{\langle .2,.4,.2\rangle},\frac{\gamma_{22}}{\langle .4,.3,.3\rangle},\frac{\gamma_{34}}{\langle .3,.2,.2\rangle},\frac{\gamma_{42}}{\langle .6,.4,.3\rangle},\frac{\gamma_{54}}{\langle .3,.2,.6\rangle},\frac{\gamma_{61}}{\langle .3,.5,.2\rangle}\right)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{22},\gamma_{32},\gamma_{42},\gamma_{53},\gamma_{62}),$$
$$\Big\{x_2,\Big(\frac{\gamma_{12}}{\langle .5,.3,.7\rangle},\frac{\gamma_{22}}{\langle .8,.6,.8\rangle},\frac{\gamma_{32}}{\langle .9,.1,.6\rangle},\frac{\gamma_{42}}{\langle .8,.2,.8\rangle},\frac{\gamma_{53}}{\langle .5,.2,.7\rangle},\frac{\gamma_{62}}{\langle .8,.3,.2\rangle}\Big),$$
$$x_3,\Big(\frac{\gamma_{12}}{\langle .2,.3,.5\rangle},\frac{\gamma_{22}}{\langle .8,.6,.5\rangle},\frac{\gamma_{32}}{\langle .4,.5,.6\rangle},\frac{\gamma_{42}}{\langle .8,.6,.5\rangle},\frac{\gamma_{53}}{\langle .3,.4,.8\rangle},\frac{\gamma_{62}}{\langle .4,.3,.6\rangle}\Big),$$
$$x_4,\Big(\frac{\gamma_{12}}{\langle .5,.3,.4\rangle},\frac{\gamma_{22}}{\langle .5,.5,.6\rangle},\frac{\gamma_{32}}{\langle .4,.3,.5\rangle},\frac{\gamma_{42}}{\langle .6,.5,.4\rangle},\frac{\gamma_{53}}{\langle .3,.5,.6\rangle},\frac{\gamma_{62}}{\langle .4,.3,.2\rangle}\Big)\Big\}\Big),$$
$$\Big((\gamma_{13},\gamma_{23},\gamma_{34},\gamma_{43},\gamma_{53},\gamma_{61}),$$
$$\Big\{x_3,\Big(\frac{\gamma_{13}}{\langle .6,.4,.2\rangle},\frac{\gamma_{23}}{\langle .4,.7,.5\rangle},\frac{\gamma_{34}}{\langle .4,.5,.6\rangle},\frac{\gamma_{43}}{\langle .7,.4,.6\rangle},\frac{\gamma_{53}}{\langle .8,.5,.3\rangle},\frac{\gamma_{61}}{\langle .5,.4,.2\rangle}\Big)\Big\}\Big),$$
$$\Big((\gamma_{12},\gamma_{22},\gamma_{32},\gamma_{41},\gamma_{54},\gamma_{61}),$$
$$\Big\{x_1,(\frac{\gamma_{12}}{\langle .6,.5,.4\rangle},\frac{\gamma_{22}}{\langle .3,.2,.7\rangle},\frac{\gamma_{32}}{\langle .5,.4,.3\rangle},\frac{\gamma_{41}}{\langle .2,.3,.6\rangle},\frac{\gamma_{54}}{\langle .4,.1,.3\rangle},\frac{\gamma_{61}}{\langle .9,.7,.7\rangle}\Big),$$
$$x_4,\Big(\frac{\gamma_{12}}{\langle .4,.3,.4\rangle},\frac{\gamma_{22}}{\langle .4,.4,.3\rangle},\frac{\gamma_{32}}{\langle .1,.3,.7\rangle},\frac{\gamma_{41}}{\langle .4,.6,.4\rangle},\frac{\gamma_{54}}{\langle .2,.7,.9\rangle},\frac{\gamma_{61}}{\langle .4,.2,.4\rangle}\Big)\Big\}\Big),$$
$$\Big((\gamma_{13},\gamma_{23},\gamma_{32},\gamma_{44},\gamma_{51},\gamma_{61}),$$
$$\Big\{x_2,\Big(\frac{\gamma_{13}}{\langle .7,.1,.4\rangle},\frac{\gamma_{23}}{\langle .5,.6,.8\rangle},\frac{\gamma_{32}}{\langle .4,.5,.6\rangle},\frac{\gamma_{44}}{\langle .9,.8,.1\rangle},\frac{\gamma_{51}}{\langle .4,.3,.4\rangle},\frac{\gamma_{61}}{\langle .6,.7,.8\rangle}\Big),$$
$$x_3,\Big(\frac{\gamma_{13}}{\langle .2,.4,.8\rangle},\frac{\gamma_{23}}{\langle .5,.4,.2\rangle},\frac{\gamma_{32}}{\langle .9,.8,.3\rangle},\frac{\gamma_{44}}{\langle .6,.5,.4\rangle},\frac{\gamma_{51}}{\langle .7,.4,.2\rangle},\frac{\gamma_{61}}{\langle .3,.5,.6\rangle}\Big)\Big\}\Big),$$
$$\Big((\gamma_{12},\gamma_{22},\gamma_{33},\gamma_{44},\gamma_{51},\gamma_{61}),$$
$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .3,.2,.4\rangle},\frac{\gamma_{22}}{\langle .6,.7,.4\rangle},\frac{\gamma_{33}}{\langle .3,.5,.6\rangle},\frac{\gamma_{44}}{\langle .6,.4,.6\rangle},\frac{\gamma_{51}}{\langle .3,.4,.5\rangle},\frac{\gamma_{61}}{\langle .5,.4,.8\rangle}\Big),$$
$$x_2,\Big(\frac{\gamma_{12}}{\langle .1,.4,.6\rangle},\frac{\gamma_{22}}{\langle .3,.6,.2\rangle},\frac{\gamma_{33}}{\langle .1,.2,.5\rangle},\frac{\gamma_{44}}{\langle .8,.4,.4\rangle},\frac{\gamma_{51}}{\langle .2,.5,.6\rangle},\frac{\gamma_{61}}{\langle .1,.8,.4\rangle}\Big),$$
$$x_3,\Big(\frac{\gamma_{12}}{\langle .4,.5,.8\rangle},\frac{\gamma_{22}}{\langle .4,.5,.3\rangle},\frac{\gamma_{33}}{\langle .7,.5,.2\rangle},\frac{\gamma_{44}}{\langle .1,.3,.4\rangle},\frac{\gamma_{51}}{\langle .6,.4,.7\rangle},\frac{\gamma_{61}}{\langle .4,.6,.7\rangle}\Big),$$
$$x_4,\Big(\frac{\gamma_{12}}{\langle .5,.4,.2\rangle},\frac{\gamma_{22}}{\langle .6,.4,.9\rangle},\frac{\gamma_{33}}{\langle .2,.4,.8\rangle},\frac{\gamma_{44}}{\langle .5,.3,.1\rangle},\frac{\gamma_{51}}{\langle .8,.2,.6\rangle},\frac{\gamma_{61}}{\langle .1,.5,.8\rangle}\Big)\Big\}\Big),$$
$$\Big((\gamma_{12},\gamma_{24},\gamma_{33},\gamma_{44},\gamma_{51},\gamma_{61}),$$
$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .4,.3,.2\rangle},\frac{\gamma_{24}}{\langle .5,.3,.8\rangle},\frac{\gamma_{33}}{\langle .5,.3,.2\rangle},\frac{\gamma_{44}}{\langle .6,.5,.2\rangle},\frac{\gamma_{51}}{\langle .7,.4,.3\rangle},\frac{\gamma_{61}}{\langle .1,.3,.8\rangle}\Big),$$
$$x_2,\Big(\frac{\gamma_{12}}{\langle .6,.9,.7\rangle},\frac{\gamma_{24}}{\langle .1,.4,.9\rangle},\frac{\gamma_{33}}{\langle .6,.2,.1\rangle},\frac{\gamma_{44}}{\langle .8,.2,.1\rangle},\frac{\gamma_{51}}{\langle .3,.8,.6\rangle},\frac{\gamma_{61}}{\langle .2,.1,.6\rangle}\Big),$$
$$x_3,\Big(\frac{\gamma_{12}}{\langle .2,.3,.7\rangle},\frac{\gamma_{24}}{\langle .2,.7,.8\rangle},\frac{\gamma_{33}}{\langle .4,.6,.7\rangle},\frac{\gamma_{44}}{\langle .3,.7,.4\rangle},\frac{\gamma_{51}}{\langle .9,.7,.3\rangle},\frac{\gamma_{61}}{\langle .7,.5,.1\rangle}\Big),$$
$$x_4,\Big(\frac{\gamma_{12}}{\langle .1,.4,.3\rangle},\frac{\gamma_{24}}{\langle .7,.3,.2\rangle},\frac{\gamma_{33}}{\langle .3,.2,.9\rangle},\frac{\gamma_{44}}{\langle .2,.1,.5\rangle},\frac{\gamma_{51}}{\langle .7,.3,.2\rangle},\frac{\gamma_{61}}{\langle .7,.4,.8\rangle}\Big)\Big\}\Big),$$
$$\Big((\gamma_{12},\gamma_{22},\gamma_{33},\gamma_{44},\gamma_{54},\gamma_{61}),$$

$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .4,.3,.2\rangle},\frac{\gamma_{22}}{\langle .3,.2,.3\rangle},\frac{\gamma_{33}}{\langle .6,.4,.2\rangle},\frac{\gamma_{44}}{\langle .7,.1,.4\rangle},\frac{\gamma_{54}}{\langle .6,.4,.3\rangle},\frac{\gamma_{61}}{\langle .1,.3,.7\rangle}\Big),$$
$$x_2,\Big(\frac{\gamma_{12}}{\langle .2,.4,.7\rangle},\frac{\gamma_{22}}{\langle .8,.3,.2\rangle},\frac{\gamma_{33}}{\langle .6,.1,.5\rangle},\frac{\gamma_{44}}{\langle .6,.8,.4\rangle},\frac{\gamma_{54}}{\langle .7,.2,.6\rangle},\frac{\gamma_{61}}{\langle .5,.4,.8\rangle}\Big),$$
$$x_3,\Big(\frac{\gamma_{12}}{\langle .9,.7,.3\rangle},\frac{\gamma_{22}}{\langle .6,.4,.3\rangle},\frac{\gamma_{33}}{\langle .7,.5,.1\rangle},\frac{\gamma_{44}}{\langle .7,.5,.4\rangle},\frac{\gamma_{54}}{\langle .1,.6,.3\rangle},\frac{\gamma_{61}}{\langle .7,.5,.3\rangle}\Big),$$
$$x_4,\Big(\frac{\gamma_{12}}{\langle .3,.2,.7\rangle},\frac{\gamma_{22}}{\langle .6,.3,.2\rangle},\frac{\gamma_{33}}{\langle .6,.4,.5\rangle},\frac{\gamma_{44}}{\langle .6,.8,.8\rangle},\frac{\gamma_{54}}{\langle .1,.2,.6\rangle},\frac{\gamma_{61}}{\langle .1,.4,.8\rangle}\Big)\Big\}\Big),$$
$$\Big((\gamma_{12},\gamma_{24},\gamma_{33},\gamma_{44},\gamma_{54},\gamma_{61}),$$
$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .3,.2,.1\rangle},\frac{\gamma_{24}}{\langle .3,.5,.8\rangle},\frac{\gamma_{33}}{\langle .4,.5,.6\rangle},\frac{\gamma_{44}}{\langle .1,.4,.7\rangle},\frac{\gamma_{54}}{\langle .5,.4,.3\rangle},\frac{\gamma_{61}}{\langle .2,.4,.5\rangle}\Big),$$
$$x_2,\Big(\frac{\gamma_{12}}{\langle .3,.4,.7\rangle},\frac{\gamma_{24}}{\langle .6,.4,.2\rangle},\frac{\gamma_{33}}{\langle .3,.9,.5\rangle},\frac{\gamma_{44}}{\langle .3,.2,.1\rangle},\frac{\gamma_{54}}{\langle .9,.4,.6\rangle},\frac{\gamma_{61}}{\langle .5,.6,.8\rangle}\Big),$$
$$x_3,\Big(\frac{\gamma_{12}}{\langle .6,.4,.8\rangle},\frac{\gamma_{24}}{\langle .4,.5,.3\rangle},\frac{\gamma_{33}}{\langle .1,.4,.6\rangle},\frac{\gamma_{44}}{\langle .4,.7,.4\rangle},\frac{\gamma_{54}}{\langle .5,.2,.9\rangle},\frac{\gamma_{61}}{\langle .9,.6,.1\rangle}\Big),$$
$$x_4,\Big(\frac{\gamma_{12}}{\langle .3,.1,.8\rangle},\frac{\gamma_{24}}{\langle .7,.4,.2\rangle},\frac{\gamma_{33}}{\langle .8,.3,.1\rangle},\frac{\gamma_{44}}{\langle .2,.4,.8\rangle},\frac{\gamma_{54}}{\langle .2,.8,.6\rangle},\frac{\gamma_{61}}{\langle .3,.8,.8\rangle}\Big)\Big\}\Big)\Big\}.$$

The *lower* and *upper* $\mathsf{Pn}_{\mathsf{hss}}$ approximation of K are calculated as

$$\underline{\mathsf{Pn}_{\mathsf{hss}}}(K)=\Big\{\Big((\gamma_{12},\gamma_{22},\gamma_{33},\gamma_{44},\gamma_{51},\gamma_{61}),$$
$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .2,.2,.9\rangle},\frac{\gamma_{22}}{\langle .1,.3,.9\rangle},\frac{\gamma_{33}}{\langle .1,.2,.6\rangle},\frac{\gamma_{44}}{\langle .1,.2,.9\rangle},\frac{\gamma_{51}}{\langle .1,.3,.9\rangle},\frac{\gamma_{61}}{\langle .1,.1,.9\rangle}\Big),$$
$$x_2,\Big(\frac{\gamma_{12}}{\langle .1,.1,.8\rangle},\frac{\gamma_{22}}{\langle .3,.1,.8\rangle},\frac{\gamma_{33}}{\langle .1,.2,.7\rangle},\frac{\gamma_{44}}{\langle .1,.3,.4\rangle},\frac{\gamma_{51}}{\langle .2,.3,.9\rangle},\frac{\gamma_{61}}{\langle .1,.1,.8\rangle}\Big),$$
$$x_3,\Big(\frac{\gamma_{12}}{\langle .1,.1,.9\rangle},\frac{\gamma_{22}}{\langle .3,.3,.5\rangle},\frac{\gamma_{33}}{\langle .4,.1,.8\rangle},\frac{\gamma_{44}}{\langle .2,.3,.4\rangle},\frac{\gamma_{51}}{\langle .1,.2,.9\rangle},\frac{\gamma_{61}}{\langle .1,.4,.9\rangle}\Big),$$
$$x_4,\Big(\frac{\gamma_{12}}{\langle .3,.1,.8\rangle},\frac{\gamma_{22}}{\langle .1,.2,.9\rangle},\frac{\gamma_{33}}{\langle .2,.3,.9\rangle},\frac{\gamma_{44}}{\langle .1,.3,.8\rangle},\frac{\gamma_{51}}{\langle .1,.2,.6\rangle},\frac{\gamma_{61}}{\langle .1,.3,.8\rangle}\Big)\Big\}\Big),$$
$$\Big((\gamma_{12},\gamma_{24},\gamma_{33},\gamma_{44},\gamma_{51},\gamma_{61}),$$
$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .1,.1,.6\rangle},\frac{\gamma_{24}}{\langle .4,.1,.9\rangle},\frac{\gamma_{33}}{\langle .2,.3,.8\rangle},\frac{\gamma_{44}}{\langle .2,.5,.8\rangle},\frac{\gamma_{51}}{\langle .1,.2,.8\rangle},\frac{\gamma_{61}}{\langle .1,.3,.8\rangle}\Big),$$
$$x_2,\Big(\frac{\gamma_{12}}{\langle .3,.2,.7\rangle},\frac{\gamma_{24}}{\langle .1,.1,.9\rangle},\frac{\gamma_{33}}{\langle .4,.1,.9\rangle},\frac{\gamma_{44}}{\langle .1,.2,.9\rangle},\frac{\gamma_{51}}{\langle .3,.2,.9\rangle},\frac{\gamma_{61}}{\langle .1,.1,.8\rangle}\Big),$$
$$x_3,\Big(\frac{\gamma_{12}}{\langle .1,.1,.9\rangle},\frac{\gamma_{24}}{\langle .2,.2,.9\rangle},\frac{\gamma_{33}}{\langle .1,.3,.9\rangle},\frac{\gamma_{44}}{\langle .3,.2,.8\rangle},\frac{\gamma_{51}}{\langle .2,.1,.7\rangle},\frac{\gamma_{61}}{\langle .3,.1,.8\rangle}\Big),$$
$$x_4,\Big(\frac{\gamma_{12}}{\langle .1,.4,.7\rangle},\frac{\gamma_{24}}{\langle .2,.1,.9\rangle},\frac{\gamma_{33}}{\langle .2,.1,.9\rangle},\frac{\gamma_{44}}{\langle .1,.1,.5\rangle},\frac{\gamma_{51}}{\langle .1,.2,.8\rangle},\frac{\gamma_{61}}{\langle .4,.2,.9\rangle}\Big)\Big\}\Big),$$
$$\Big((\gamma_{12},\gamma_{22},\gamma_{33},\gamma_{44},\gamma_{54},\gamma_{61}),$$
$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .2,.1,.7\rangle},\frac{\gamma_{22}}{\langle .1,.1,.8\rangle},\frac{\gamma_{33}}{\langle .3,.4,.9\rangle},\frac{\gamma_{44}}{\langle .1,.1,.9\rangle},\frac{\gamma_{54}}{\langle .2,.1,.9\rangle},\frac{\gamma_{61}}{\langle .1,.3,.9\rangle}\Big),$$
$$x_2,\Big(\frac{\gamma_{12}}{\langle .1,.2,.9\rangle},\frac{\gamma_{22}}{\langle .1,.2,.9\rangle},\frac{\gamma_{33}}{\langle .3,.1,.7\rangle},\frac{\gamma_{44}}{\langle .1,.1,.7\rangle},\frac{\gamma_{54}}{\langle .3,.1,.9\rangle},\frac{\gamma_{61}}{\langle .2,.2,.8\rangle}\Big),$$

$$x_3,\left(\frac{\gamma_{12}}{\langle .1,.2,.8\rangle},\frac{\gamma_{22}}{\langle .3,.3,.8\rangle},\frac{\gamma_{33}}{\langle .6,.1,.9\rangle},\frac{\gamma_{44}}{\langle .3,.1,.9\rangle},\frac{\gamma_{54}}{\langle .1,.3,.8\rangle},\frac{\gamma_{61}}{\langle .2,.3,.9\rangle}\right),$$
$$x_4,\left(\frac{\gamma_{12}}{\langle .1,.1,.7\rangle},\frac{\gamma_{22}}{\langle .1,.3,.4\rangle},\frac{\gamma_{33}}{\langle .1,.1,.8\rangle},\frac{\gamma_{44}}{\langle .1,.4,.9\rangle},\frac{\gamma_{54}}{\langle .1,.2,.6\rangle},\frac{\gamma_{61}}{\langle .1,.2,.9\rangle}\right)\Big\}\Big),$$
$$\Big((\gamma_{12},\gamma_{24},\gamma_{33},\gamma_{44},\gamma_{54},\gamma_{61}),$$
$$\Big\{x_1,\left(\frac{\gamma_{12}}{\langle .2,.2,.8\rangle},\frac{\gamma_{24}}{\langle .3,.3,.9\rangle},\frac{\gamma_{33}}{\langle .1,.3,.9\rangle},\frac{\gamma_{44}}{\langle .1,.4,.9\rangle},\frac{\gamma_{54}}{\langle .3,.3,.9\rangle},\frac{\gamma_{61}}{\langle .1,.3,.7\rangle}\right),$$
$$x_2,\left(\frac{\gamma_{12}}{\langle .2,.2,.8\rangle},\frac{\gamma_{24}}{\langle .5,.3,.9\rangle},\frac{\gamma_{33}}{\langle .3,.1,.9\rangle},\frac{\gamma_{44}}{\langle .3,.1,.9\rangle},\frac{\gamma_{54}}{\langle .2,.2,.6\rangle},\frac{\gamma_{61}}{\langle .5,.4,.8\rangle}\right),$$
$$x_3,\left(\frac{\gamma_{12}}{\langle .1,.1,.8\rangle},\frac{\gamma_{24}}{\langle .3,.3,.4\rangle},\frac{\gamma_{33}}{\langle .1,.3,.9\rangle},\frac{\gamma_{44}}{\langle .3,.1,.9\rangle},\frac{\gamma_{54}}{\langle .2,.2,.9\rangle},\frac{\gamma_{61}}{\langle .1,.1,.9\rangle}\right),$$
$$x_4,\left(\frac{\gamma_{12}}{\langle .1,.1,.8\rangle},\frac{\gamma_{24}}{\langle .6,.1,.9\rangle},\frac{\gamma_{33}}{\langle .3,.1,.8\rangle},\frac{\gamma_{44}}{\langle .2,.1,.9\rangle},\frac{\gamma_{54}}{\langle .2,.2,.6\rangle},\frac{\gamma_{61}}{\langle .1,.2,.8\rangle}\right)\Big\}\Big)\Big\}.$$

$$\overline{\mathsf{Pn}_{\mathsf{hss}}}(K)=\Big\{\Big((\gamma_{11},\gamma_{21},\gamma_{32},\gamma_{41},\gamma_{51},\gamma_{61}),$$
$$\Big\{x_2,\left(\frac{\gamma_{11}}{\langle .5,.3,.3\rangle},\frac{\gamma_{21}}{\langle .5,.3,.5\rangle},\frac{\gamma_{32}}{\langle .7,.5,.3\rangle},\frac{\gamma_{41}}{\langle .6,.4,.8\rangle},\frac{\gamma_{51}}{\langle .4,.3,.5\rangle},\frac{\gamma_{61}}{\langle .8,.5,.3\rangle}\right),$$
$$x_4,\left(\frac{\gamma_{11}}{\langle .1,.4,.7\rangle},\frac{\gamma_{21}}{\langle .4,.3,.9\rangle},\frac{\gamma_{32}}{\langle .3,.2,.5\rangle},\frac{\gamma_{41}}{\langle .8,.5,.3\rangle},\frac{\gamma_{51}}{\langle .6,.4,.3\rangle},\frac{\gamma_{61}}{\langle .9,.5,.3\rangle}\right)\Big\}\Big),$$
$$\Big((\gamma_{12},\gamma_{22},\gamma_{33},\gamma_{42},\gamma_{54},\gamma_{61}),$$
$$\Big\{x_1,\left(\frac{\gamma_{12}}{\langle .2,.4,.5\rangle},\frac{\gamma_{22}}{\langle .6,.4,.9\rangle},\frac{\gamma_{33}}{\langle .3,.4,.6\rangle},\frac{\gamma_{42}}{\langle .9,.4,.2\rangle},\frac{\gamma_{54}}{\langle .6,.4,.3\rangle},\frac{\gamma_{61}}{\langle .3,.4,.5\rangle}\right),$$
$$x_2,\left(\frac{\gamma_{12}}{\langle .8,.2,.6\rangle},\frac{\gamma_{22}}{\langle .2,.8,.5\rangle},\frac{\gamma_{33}}{\langle .5,.2,.4\rangle},\frac{\gamma_{42}}{\langle .8,.2,.6\rangle},\frac{\gamma_{54}}{\langle .8,.2,.6\rangle},\frac{\gamma_{61}}{\langle .8,.2,.6\rangle}\right),$$
$$x_3,\left(\frac{\gamma_{12}}{\langle .7,.5,.3\rangle},\frac{\gamma_{22}}{\langle .5,.7,.5\rangle},\frac{\gamma_{33}}{\langle .3,.5,.8\rangle},\frac{\gamma_{42}}{\langle .4,.3,.7\rangle},\frac{\gamma_{54}}{\langle .4,.5,.8\rangle},\frac{\gamma_{61}}{\langle .3,.5,.9\rangle}\right),$$
$$x_4,\left(\frac{\gamma_{12}}{\langle .8,.5,.3\rangle},\frac{\gamma_{22}}{\langle .5,.6,.4\rangle},\frac{\gamma_{33}}{\langle .9,.3,.2\rangle},\frac{\gamma_{42}}{\langle .6,.4,.8\rangle},\frac{\gamma_{54}}{\langle .2,.4,.8\rangle},\frac{\gamma_{61}}{\langle .4,.6,.7\rangle}\right)\Big\}\Big),$$
$$\Big((\gamma_{13},\gamma_{22},\gamma_{33},\gamma_{44},\gamma_{54},\gamma_{61}),$$
$$\Big\{x_3,\left(\frac{\gamma_{13}}{\langle .1,.3,.6\rangle},\frac{\gamma_{22}}{\langle .8,.6,.3\rangle},\frac{\gamma_{33}}{\langle .5,.3,.9\rangle},\frac{\gamma_{44}}{\langle .6,.4,.3\rangle},\frac{\gamma_{54}}{\langle .6,.7,.8\rangle},\frac{\gamma_{61}}{\langle .4,.1,.7\rangle}\right)\Big\}\Big),$$
$$\Big((\gamma_{12},\gamma_{24},\gamma_{32},\gamma_{44},\gamma_{52},\gamma_{62}),$$
$$\Big\{x_1,\left(\frac{\gamma_{12}}{\langle .7,.5,.4\rangle},\frac{\gamma_{24}}{\langle .3,.6,.8\rangle},\frac{\gamma_{32}}{\langle .4,.6,.7\rangle},\frac{\gamma_{44}}{\langle .3,.6,.1\rangle},\frac{\gamma_{52}}{\langle .5,.7,.8\rangle},\frac{\gamma_{62}}{\langle .4,.6,.3\rangle}\right),$$
$$x_2,\left(\frac{\gamma_{12}}{\langle .3,.5,.7\rangle},\frac{\gamma_{24}}{\langle .4,.6,.8\rangle},\frac{\gamma_{32}}{\langle .1,.3,.6\rangle},\frac{\gamma_{44}}{\langle .4,.6,.8\rangle},\frac{\gamma_{52}}{\langle .3,.5,.8\rangle},\frac{\gamma_{62}}{\langle .3,.5,.7\rangle}\right),$$
$$x_3,\left(\frac{\gamma_{12}}{\langle .3,.5,.6\rangle},\frac{\gamma_{24}}{\langle .2,.4,.5\rangle},\frac{\gamma_{32}}{\langle .7,.4,.2\rangle},\frac{\gamma_{44}}{\langle .8,.5,.3\rangle},\frac{\gamma_{52}}{\langle .1,.3,.5\rangle},\frac{\gamma_{62}}{\langle .8,.2,.3\rangle}\right),$$
$$x_4,\left(\frac{\gamma_{12}}{\langle .3,.4,.2\rangle},\frac{\gamma_{24}}{\langle .4,.5,.6\rangle},\frac{\gamma_{32}}{\langle .7,.4,.3\rangle},\frac{\gamma_{44}}{\langle .7,.4,.5\rangle},\frac{\gamma_{52}}{\langle .6,.3,.7\rangle},\frac{\gamma_{62}}{\langle .3,.5,.6\rangle}\right)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{24},\gamma_{32},\gamma_{45},\gamma_{51},\gamma_{62}),$$

$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .3,.5,.9\rangle},\frac{\gamma_{24}}{\langle .1,.3,.7\rangle},\frac{\gamma_{32}}{\langle .8,.3,.2\rangle},\frac{\gamma_{45}}{\langle .9,.1,.6\rangle},\frac{\gamma_{51}}{\langle .8,.3,.9\rangle},\frac{\gamma_{62}}{\langle .1,.5,.8\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .1,.4,.8\rangle},\frac{\gamma_{24}}{\langle .6,.8,.3\rangle},\frac{\gamma_{32}}{\langle .8,.2,.9\rangle},\frac{\gamma_{45}}{\langle .1,.7,.9\rangle},\frac{\gamma_{51}}{\langle .4,.6,.1\rangle},\frac{\gamma_{62}}{\langle .7,.3,.9\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{13},\gamma_{22},\gamma_{34},\gamma_{42},\gamma_{54},\gamma_{61}),$$

$$\Big\{x_2,\Big(\frac{\gamma_{13}}{\langle .7,.9,.1\rangle},\frac{\gamma_{22}}{\langle .1,.5,.7\rangle},\frac{\gamma_{34}}{\langle .5,.8,.2\rangle},\frac{\gamma_{42}}{\langle .8,.5,.1\rangle},\frac{\gamma_{54}}{\langle .6,.4,.3\rangle},\frac{\gamma_{61}}{\langle .2,.4,.2\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{13}}{\langle .2,.4,.2\rangle},\frac{\gamma_{22}}{\langle .4,.3,.3\rangle},\frac{\gamma_{34}}{\langle .3,.2,.2\rangle},\frac{\gamma_{42}}{\langle .6,.4,.3\rangle},\frac{\gamma_{54}}{\langle .3,.2,.6\rangle},\frac{\gamma_{61}}{\langle .3,.5,.2\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{22},\gamma_{32},\gamma_{42},\gamma_{53},\gamma_{62}),$$

$$\Big\{x_2,\Big(\frac{\gamma_{12}}{\langle .5,.3,.7\rangle},\frac{\gamma_{22}}{\langle .8,.6,.8\rangle},\frac{\gamma_{32}}{\langle .9,.1,.6\rangle},\frac{\gamma_{42}}{\langle .8,.2,.8\rangle},\frac{\gamma_{53}}{\langle .5,.2,.7\rangle},\frac{\gamma_{62}}{\langle .8,.3,.2\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{12}}{\langle .2,.3,.5\rangle},\frac{\gamma_{22}}{\langle .8,.6,.5\rangle},\frac{\gamma_{32}}{\langle .4,.5,.6\rangle},\frac{\gamma_{42}}{\langle .8,.6,.5\rangle},\frac{\gamma_{53}}{\langle .3,.4,.8\rangle},\frac{\gamma_{62}}{\langle .4,.3,.6\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .5,.3,.4\rangle},\frac{\gamma_{22}}{\langle .5,.5,.6\rangle},\frac{\gamma_{32}}{\langle .4,.3,.5\rangle},\frac{\gamma_{42}}{\langle .6,.5,.4\rangle},\frac{\gamma_{53}}{\langle .3,.5,.6\rangle},\frac{\gamma_{62}}{\langle .4,.3,.2\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{13},\gamma_{23},\gamma_{34},\gamma_{43},\gamma_{53},\gamma_{61}),$$

$$\Big\{x_3,\Big(\frac{\gamma_{13}}{\langle .6,.4,.2\rangle},\frac{\gamma_{23}}{\langle .4,.7,.5\rangle},\frac{\gamma_{34}}{\langle .4,.5,.6\rangle},\frac{\gamma_{43}}{\langle .7,.4,.6\rangle},\frac{\gamma_{53}}{\langle .8,.5,.3\rangle},\frac{\gamma_{61}}{\langle .5,.4,.2\rangle})\}\Big),$$

$$\Big((\gamma_{12},\gamma_{22},\gamma_{32},\gamma_{41},\gamma_{54},\gamma_{61}),$$

$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .6,.5,.4\rangle},\frac{\gamma_{22}}{\langle .3,.2,.7\rangle},\frac{\gamma_{32}}{\langle .5,.4,.3\rangle},\frac{\gamma_{41}}{\langle .2,.3,.6\rangle},\frac{\gamma_{54}}{\langle .4,.1,.3\rangle},\frac{\gamma_{61}}{\langle .9,.7,.7\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .4,.3,.4\rangle},\frac{\gamma_{22}}{\langle .4,.4,.3\rangle},\frac{\gamma_{32}}{\langle .1,.3,.7\rangle},\frac{\gamma_{41}}{\langle .4,.6,.4\rangle},\frac{\gamma_{54}}{\langle .2,.7,.9\rangle},\frac{\gamma_{61}}{\langle .4,.2,.4\rangle}\Big)\Big\}\Big),$$

$$((\gamma_{13},\gamma_{23},\gamma_{32},\gamma_{44},\gamma_{51},\gamma_{61}),$$

$$\Big\{x_2,\Big(\frac{\gamma_{13}}{\langle .7,.1,.4\rangle},\frac{\gamma_{23}}{\langle .5,.6,.8\rangle},\frac{\gamma_{32}}{\langle .4,.5,.6\rangle},\frac{\gamma_{44}}{\langle .9,.8,.1\rangle},\frac{\gamma_{51}}{\langle .4,.3,.4\rangle},\frac{\gamma_{61}}{\langle .6,.7,.8\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{13}}{\langle .2,.4,.8\rangle},\frac{\gamma_{23}}{\langle .5,.4,.2\rangle},\frac{\gamma_{32}}{\langle .9,.8,.3\rangle},\frac{\gamma_{44}}{\langle .6,.5,.4\rangle},\frac{\gamma_{51}}{\langle .7,.4,.2\rangle},\frac{\gamma_{61}}{\langle .3,.5,.6\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{22},\gamma_{33},\gamma_{44},\gamma_{51},\gamma_{61}),$$

$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .5,.9,.1\rangle},\frac{\gamma_{22}}{\langle .8,.7,.1\rangle},\frac{\gamma_{33}}{\langle .9,.9,.1\rangle},\frac{\gamma_{44}}{\langle .6,.7,.7\rangle},\frac{\gamma_{51}}{\langle .9,.5,.4\rangle},\frac{\gamma_{61}}{\langle .9,.5,.2\rangle}\Big),$$

$$x_2,\Big(\frac{\gamma_{12}}{\langle .6,.9,.1\rangle},\frac{\gamma_{22}}{\langle .9,.9,.1\rangle},\frac{\gamma_{33}}{\langle .8,.7,.3\rangle},\frac{\gamma_{44}}{\langle .8,.8,.2\rangle},\frac{\gamma_{51}}{\langle .9,.9,.2\rangle},\frac{\gamma_{61}}{\langle .9,.8,.1\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{12}}{\langle .6,.7,.1\rangle},\frac{\gamma_{22}}{\langle .8,.8,.1\rangle},\frac{\gamma_{33}}{\langle .7,.9,.1\rangle},\frac{\gamma_{44}}{\langle .7,.8,.2\rangle},\frac{\gamma_{51}}{\langle .9,.6,.1\rangle},\frac{\gamma_{61}}{\langle .6,.8,.2\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .6,.9,.1\rangle},\frac{\gamma_{22}}{\langle .9,.6,.3\rangle},\frac{\gamma_{33}}{\langle .8,.9,.1\rangle},\frac{\gamma_{44}}{\langle .7,.8,.1\rangle},\frac{\gamma_{51}}{\langle .8,.8,.2\rangle},\frac{\gamma_{61}}{\langle .8,.8,.3\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{24},\gamma_{33},\gamma_{44},\gamma_{51},\gamma_{61}),$$

$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .3,.8,.1\rangle},\frac{\gamma_{24}}{\langle .7,.9,.1\rangle},\frac{\gamma_{33}}{\langle .9,.6,.1\rangle},\frac{\gamma_{44}}{\langle .9,.8,.1\rangle},\frac{\gamma_{51}}{\langle .9,.7,.5\rangle},\frac{\gamma_{61}}{\langle .8,.9,.1\rangle}\Big),$$

$$x_2,\Big(\frac{\gamma_{12}}{\langle .6,.9,.1\rangle},\frac{\gamma_{24}}{\langle .6,.8,.2\rangle},\frac{\gamma_{33}}{\langle .6,.8,.1\rangle},\frac{\gamma_{44}}{\langle .9,.5,.1\rangle},\frac{\gamma_{51}}{\langle .6,.8,.3\rangle},\frac{\gamma_{61}}{\langle .5,.8,.2\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{12}}{\langle .4,.7,.1\rangle},\frac{\gamma_{24}}{\langle .6,.8,.1\rangle},\frac{\gamma_{33}}{\langle .7,.9,.1\rangle},\frac{\gamma_{44}}{\langle .9,.7,.1\rangle},\frac{\gamma_{51}}{\langle .9,.8,.1\rangle},\frac{\gamma_{61}}{\langle .9,.8,.1\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .6,.9,.2\rangle},\frac{\gamma_{24}}{\langle .7,.5,.2\rangle},\frac{\gamma_{33}}{\langle .9,.8,.2\rangle},\frac{\gamma_{44}}{\langle .8,.9,.1\rangle},\frac{\gamma_{51}}{\langle .8,.9,.3\rangle},\frac{\gamma_{61}}{\langle .8,.7,.1\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{22},\gamma_{33},\gamma_{44},\gamma_{54},\gamma_{61}),$$

$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .6,.8,.3\rangle},\frac{\gamma_{22}}{\langle .8,.5,.2\rangle},\frac{\gamma_{33}}{\langle .9,.9,.1\rangle},\frac{\gamma_{44}}{\langle .8,.6,.5\rangle},\frac{\gamma_{54}}{\langle .9,.6,.3\rangle},\frac{\gamma_{61}}{\langle .6,.8,.3\rangle}\Big),$$

$$x_2,\Big(\frac{\gamma_{12}}{\langle .6,.5,.1\rangle},\frac{\gamma_{22}}{\langle .8,.7,.2\rangle},\frac{\gamma_{33}}{\langle .8,.6,.1\rangle},\frac{\gamma_{44}}{\langle .9,.8,.1\rangle},\frac{\gamma_{54}}{\langle .7,.9,.3\rangle},\frac{\gamma_{61}}{\langle .7,.8,.3\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{12}}{\langle .9,.7,.1\rangle},\frac{\gamma_{22}}{\langle .8,.7,.1\rangle},\frac{\gamma_{33}}{\langle .9,.5,.1\rangle},\frac{\gamma_{44}}{\langle .7,.8,.3\rangle},\frac{\gamma_{54}}{\langle .8,.6,.1\rangle},\frac{\gamma_{61}}{\langle .8,.9,.1\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .7,.6,.3\rangle},\frac{\gamma_{22}}{\langle .8,.9,.1\rangle},\frac{\gamma_{33}}{\langle .6,.6,.5\rangle},\frac{\gamma_{44}}{\langle .9,.8,.3\rangle},\frac{\gamma_{54}}{\langle .8,.6,.1\rangle},\frac{\gamma_{61}}{\langle .9,.4,.1\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{24},\gamma_{33},\gamma_{44},\gamma_{54},\gamma_{61}),$$

$$\Big\{x_1,(\frac{\gamma_{12}}{\langle .8,.5,.1\rangle},\frac{\gamma_{24}}{\langle .8,.5,.2\rangle},\frac{\gamma_{33}}{\langle .5,.6,.1\rangle},\frac{\gamma_{44}}{\langle .8,.7,.1\rangle},\frac{\gamma_{54}}{\langle .8,.8,.1\rangle},\frac{\gamma_{61}}{\langle .9,.6,.1\rangle}\Big),$$

$$x_2,\Big(\frac{\gamma_{12}}{\langle .3,.8,.2\rangle},\frac{\gamma_{24}}{\langle .8,.5,.1\rangle},\frac{\gamma_{33}}{\langle .8,.9,.1\rangle},\frac{\gamma_{44}}{\langle .8,.8,.3\rangle},\frac{\gamma_{54}}{\langle .9,.7,.1\rangle},\frac{\gamma_{61}}{\langle .9,.9,.2\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{12}}{\langle .8,.9,.3\rangle},\frac{\gamma_{24}}{\langle .7,.9,.1\rangle},\frac{\gamma_{33}}{\langle .9,.4,.1\rangle},\frac{\gamma_{44}}{\langle .4,.7,.2\rangle},\frac{\gamma_{54}}{\langle .9,.7,.1\rangle},\frac{\gamma_{61}}{\langle .9,.6,.1\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .4,8,.2\rangle},\frac{\gamma_{24}}{\langle .8,.5,.1\rangle},\frac{\gamma_{33}}{\langle .9,.5,.1\rangle},\frac{\gamma_{44}}{\langle .6,.8,.1\rangle},\frac{\gamma_{54}}{\langle .9,.9,.2\rangle},\frac{\gamma_{61}}{\langle .6,.8,.1\rangle}\Big)\Big\}\Big)\Big\}.$$

Step 2: By Definition 3.2(iii), we have

$$\underline{\mathsf{Pn}_{\mathsf{hss}}}(K)\oplus\overline{\mathsf{Pn}_{\mathsf{hss}}}(K)$$

$$=\Big\{\Big((\gamma_{12},\gamma_{22},\gamma_{33},\gamma_{44},\gamma_{51},\gamma_{61}),$$

$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .6,.18,.09\rangle},\frac{\gamma_{22}}{\langle .82,.24,.09\rangle},\frac{\gamma_{33}}{\langle .91,.14,.42\rangle},\frac{\gamma_{44}}{\langle .64,.1,.36\rangle},\frac{\gamma_{51}}{\langle .91,.15,.36\rangle},\frac{\gamma_{61}}{\langle .91,.05,.18\rangle}\Big),$$

$$x_2,\Big(\frac{\gamma_{12}}{\langle .64,.09,.08\rangle},\frac{\gamma_{22}}{\langle .93,.09,.08\rangle},\frac{\gamma_{33}}{\langle .82,.14,.21\rangle},\frac{\gamma_{44}}{\langle .82,.24,.08\rangle},\frac{\gamma_{51}}{\langle .92,.27,.18\rangle},\frac{\gamma_{61}}{\langle .91,.08,.08\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{12}}{\langle .64,.07,.09\rangle},\frac{\gamma_{22}}{\langle .86,.24,.05\rangle},\frac{\gamma_{33}}{\langle .82,.09,.08\rangle},\frac{\gamma_{44}}{\langle .76,.24,.08\rangle},\frac{\gamma_{51}}{\langle .91,.12,.09\rangle},\frac{\gamma_{61}}{\langle .64,.32,.18\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .72,.09,.08\rangle},\frac{\gamma_{22}}{\langle .91,.12,.27\rangle},\frac{\gamma_{33}}{\langle .84,.27,.09\rangle},\frac{\gamma_{44}}{\langle .73,.24,.08\rangle},\frac{\gamma_{51}}{\langle .82,.16,.12\rangle},\frac{\gamma_{61}}{\langle .82,.24,.24\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{24},\gamma_{33},\gamma_{44},\gamma_{51},\gamma_{61}),$$

$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .37,.08,.06\rangle},\frac{\gamma_{24}}{\langle .82,.09,.09\rangle},\frac{\gamma_{33}}{\langle .92,.18,.08\rangle},\frac{\gamma_{44}}{\langle .92,.4,.08\rangle},\frac{\gamma_{51}}{\langle .91,.14,.4\rangle},\frac{\gamma_{61}}{\langle .82,.27,.08\rangle}\Big),$$

$$x_2,\Big(\frac{\gamma_{12}}{\langle .72,.18,.07\rangle},\frac{\gamma_{24}}{\langle .64,.08,.18\rangle},\frac{\gamma_{33}}{\langle .76,.08,.09\rangle},\frac{\gamma_{44}}{\langle .91,.1,.09\rangle},\frac{\gamma_{51}}{\langle .72,.16,.27\rangle},\frac{\gamma_{61}}{\langle .55,.08,.16\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{12}}{\langle .46,.07,.09\rangle},\frac{\gamma_{24}}{\langle .84,.16,.09\rangle},\frac{\gamma_{33}}{\langle .73,.27,.09\rangle},\frac{\gamma_{44}}{\langle .79,.14,.08\rangle},\frac{\gamma_{51}}{\langle .92,.08,.07\rangle},\frac{\gamma_{61}}{\langle .72,.08,.08\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .64,.36,.14\rangle},\frac{\gamma_{24}}{\langle .76,.05,.18\rangle},\frac{\gamma_{33}}{\langle .92,.08,.18\rangle},\frac{\gamma_{44}}{\langle .82,.09,.05\rangle},\frac{\gamma_{51}}{\langle .82,.18,.24\rangle},\frac{\gamma_{61}}{\langle .88,.14,.09\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{22},\gamma_{33},\gamma_{44},\gamma_{54},\gamma_{61}),$$

$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .68,.08,.21\rangle},\frac{\gamma_{22}}{\langle .82,.05,.16\rangle},\frac{\gamma_{33}}{\langle .93,.36,.09\rangle},\frac{\gamma_{44}}{\langle .82,.06,.45\rangle},\frac{\gamma_{54}}{\langle .92,.06,.27\rangle},\frac{\gamma_{61}}{\langle .64,.24,.27\rangle}\Big),$$

$$x_2,\Big(\frac{\gamma_{12}}{\langle .64,.1,.09\rangle},\frac{\gamma_{22}}{\langle .82,.14,.18\rangle},\frac{\gamma_{33}}{\langle .86,.06,.07\rangle},\frac{\gamma_{44}}{\langle .91,.08,.07\rangle},\frac{\gamma_{54}}{\langle .79,.09,.27\rangle},\frac{\gamma_{61}}{\langle .76,.16,.24\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{12}}{\langle .91,.14,.08\rangle},\frac{\gamma_{22}}{\langle .86,.21,.08\rangle},\frac{\gamma_{33}}{\langle .96,.05,.09\rangle},\frac{\gamma_{44}}{\langle .79,.08,.27\rangle},\frac{\gamma_{54}}{\langle .82,.18,.08\rangle},\frac{\gamma_{61}}{\langle .84,.27,.09\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .73,.06,.21\rangle},\frac{\gamma_{22}}{\langle .82,.27,.04\rangle},\frac{\gamma_{33}}{\langle .64,.06,.4\rangle},\frac{\gamma_{44}}{\langle .91,.32,.27\rangle},\frac{\gamma_{54}}{\langle .82,.12,.06\rangle},\frac{\gamma_{61}}{\langle .91,.08,.09\rangle}\Big)\Big\}\Big),$$

$$\Big((\gamma_{12},\gamma_{24},\gamma_{33},\gamma_{44},\gamma_{54},\gamma_{61}),$$

$$\Big\{x_1,\Big(\frac{\gamma_{12}}{\langle .84,.1,.08\rangle},\frac{\gamma_{24}}{\langle .86,.15,.18\rangle},\frac{\gamma_{33}}{\langle .55,.18,.09\rangle},\frac{\gamma_{44}}{\langle .82,.28,.09\rangle},\frac{\gamma_{54}}{\langle .86,.24,.09\rangle},\frac{\gamma_{61}}{\langle .91,.18,.07\rangle}\Big),$$

$$x_2,\Big(\frac{\gamma_{12}}{\langle .44,.16,.16\rangle},\frac{\gamma_{24}}{\langle .9,.15,.09\rangle},\frac{\gamma_{33}}{\langle .86,.09,.09\rangle},\frac{\gamma_{44}}{\langle .86,.08,.27\rangle},\frac{\gamma_{54}}{\langle .92,.14,.06\rangle},\frac{\gamma_{61}}{\langle .95,.36,.16\rangle}\Big),$$

$$x_3,\Big(\frac{\gamma_{12}}{\langle .82,.09,.24\rangle},\frac{\gamma_{24}}{\langle .79,.27,.04\rangle},\frac{\gamma_{33}}{\langle .91,.12,.09\rangle},\frac{\gamma_{44}}{\langle .58,.07,.18\rangle},\frac{\gamma_{54}}{\langle .92,.14,.09\rangle},\frac{\gamma_{61}}{\langle .91,.06,.09\rangle}\Big),$$

$$x_4,\Big(\frac{\gamma_{12}}{\langle .46,.08,.16\rangle},\frac{\gamma_{24}}{\langle .92,.05,.09\rangle},\frac{\gamma_{33}}{\langle .93,.05,.08\rangle},\frac{\gamma_{44}}{\langle .68,.08,.09\rangle},\frac{\gamma_{54}}{\langle .92,.18,.12\rangle},\frac{\gamma_{61}}{\langle .94,.16,.08\rangle}\Big)\Big\}\Big)\Big\}.$$

Step 3: By Definition 3.3, we compute the cosine similarity measure between each $\mathsf{Pn}_{\mathsf{hss}}$ element and the ideal $\mathsf{Pn}_{\mathsf{hss}}$ element (Figure 2) as follows,

$$COS_{Pn_{hss}}(\Omega,\Omega^*)$$
$$=\begin{cases}((\gamma_{12},\gamma_{22},\gamma_{33},\gamma_{44},\gamma_{51},\gamma_{61}),\{x_1,0.9284,x_2,0.9696,x_3,0.9577,x_4,0.9722\})\rightarrow x_4>x_2>x_3>x_1=x_4\\((\gamma_{12},\gamma_{24},\gamma_{33},\gamma_{44},\gamma_{51},\gamma_{61}),\{x_1,0.9462,x_2,0.9623,x_3,0.9741,x_4,0.9553\})\rightarrow x_3>x_2>x_4>x_1=x_3\\((\gamma_{12},\gamma_{22},\gamma_{33},\gamma_{44},\gamma_{54},\gamma_{61}),\{x_1,0.9272,x_2,0.9622,x_3,0.9686,x_4,0.9408\})\rightarrow x_3>x_2>x_4>x_1=x_3\\((\gamma_{12},\gamma_{24},\gamma_{33},\gamma_{44},\gamma_{54},\gamma_{61}),\{x_1,0.9630,x_2,0.9597,x_3,0.9704,x_4,0.9804\})\rightarrow x_4>x_3>x_2>x_1=x_4\end{cases}$$

Step 4: In *Step* 3, we cannot find the proper decision so *Average of* $COS_{Pn_{hss}}(\Omega,\Omega^*)$ are considered to calculate the optimal solution. It is given as follows.

Average of $COS_{Pn_{hss}}(\Omega,\Omega^*)=x_1,0.9412,x_2,0.9635,x_3,0.9675,x_4,0.9622=x_3>x_2>x_4>x_1=x_3$.

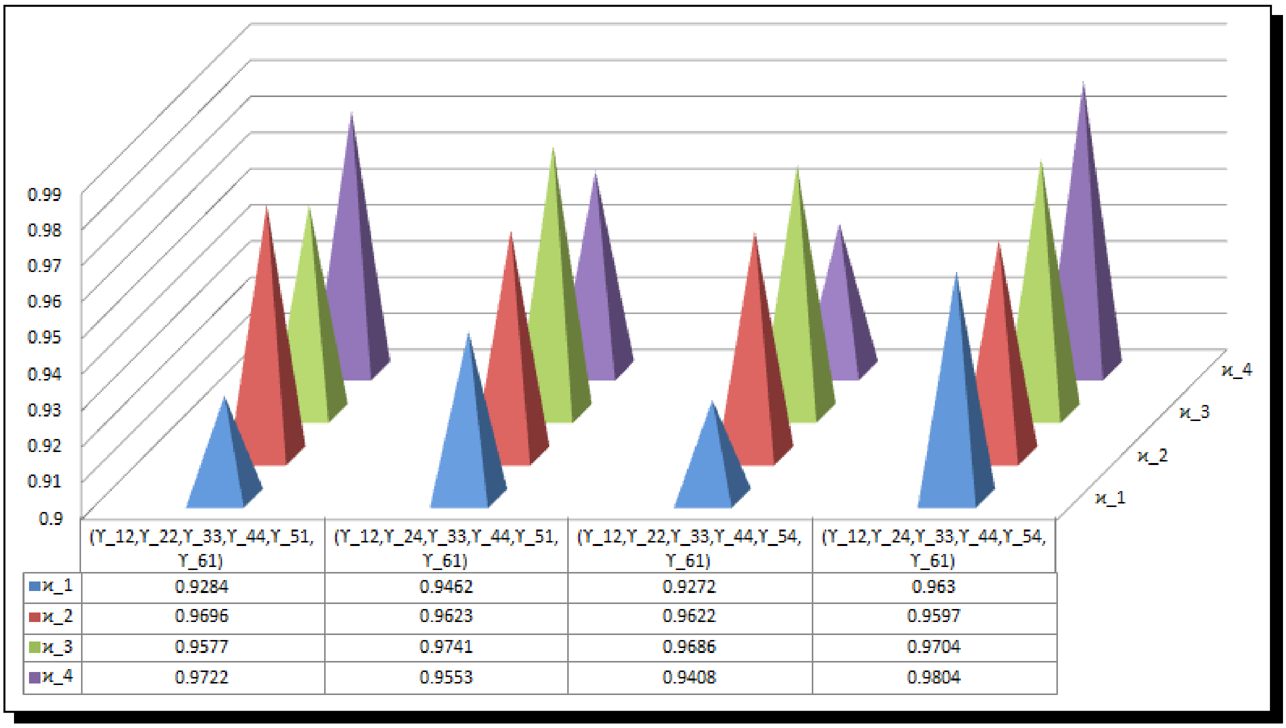

Figure 2. $COS_{Pn_{hss}}(\Omega,\Omega^*)$

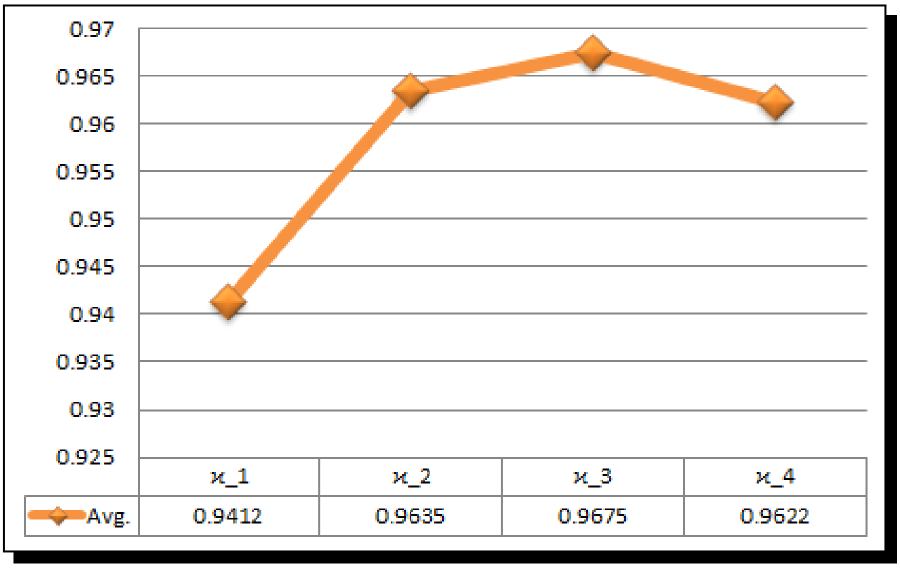

Figure 3. *Average of* $COS_{Pn_{hss}}(\Omega,\Omega^*)$

The optimal decision is to select x_3 (Figure 3). Therefore, we can conclude that is the best bakery in Chidambaram.

5. Conclusion

This article defines the plithogenic neutrosophic hypersoft rough set explicitly by combining three theories: neutrosophic, plithogenic hypersoft sets, and rough set theory. In addition, we provide an algorithm to handle decision making problem in bakery industries based on plithogenic neutrosophic hypersoft rough sets. Finally, a numerical example is employed to demonstrate the validness of the proposed plithogenic neutrosophic hypersoft rough sets.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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