Communications in Mathematics and Applications
Vol. 15, No. 1, pp. 359–366, 2024
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications
DOI: 10.26713/cma.v15i1.2417

http://www.rgnpublications.com

Research Article

Even Intensity and Implementation of Fuzzy Intrinsic Edge-Magic Graphs

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Received: September 18, 2023 **Accepted:** January 1, 2024

Abstract. A graph labelling is an assignment of integers to the vertices or edges, or both under certain restrictions. A fuzzy graph G is said to be intrinsic edge-magic if it satisfies the intrinsic edge-magic labelling with intrinsic constant $\lambda_c = \sigma(v_i) + \mu(v_i v_j) + \sigma(v_j)$, for all $v_i, v_j \in V$. In this article, we introduce the even intensity of intrinsic edge magic graph and the application of fuzzy intrinsic edge magic graph is illustrated with suitable example.

Keywords. Fuzzy intrinsic edge-magic labelling, Fuzzy intrinsic edge-magic graph, Star graph, Paw graph, Secure, Supreme and subordinate intrinsic edge magic graph

Mathematics Subject Classification (2020). 05C72, 05C78

1. Introduction

Fuzzy set was initially introduced by L.A. Zadeh [10]. Later various researches added productive concepts to develop fuzzy sets theory like Fuzzy graphs. A fuzzy graph contains many properties similar to crisp graph due to generalization of crisp graphs but it diverges at many places. A crisp graph G is an order pair of vertex-set V and edge set E such that $E \subseteq V \times V$. In addition $v = |V|$ is called order and $e = |E|$, size of the graph G, respectively. In a crisp graph, a bijective function $\rho : V \cup E \to N$ that produces a unique positive integer (to each vertex and/or edge) is termed labelling in 'Some results on magic graphs'. Having introduced the notion of magic graph where the labels vertices and edges are natural numbers from 1 to $|V|+|E|$ and the sum of the same must be constant in entire graph, 'Super edge magic graphs'. Enomoto *et al*. [2] extended the concept of magic graph adding a property that vertices always get smaller labels than edges which is named super edge magic labelling. Numerous other authors have explored diverse types of different magic graphs. The subject of edge-magic labelling of graphs had its origin in Kotzig and Rosa's work [7] on magic valuations of graphs. These labelling are currently referred to as either edge-magic labelling or edge-magic total labelling. Fuzzy graphs are generalization of graphs. In graphs two vertices are either related or unrelated to each other. Mathematically, the degree of relationship is either 0 or 1. In fuzzy graphs, the degree of relationship takes values from $[0,1]$. A fuzzy graph has ability to solve uncertain problems in a wide range of fields. The first definition of a fuzzy graph was introduced by Kaufmann [6]. Rosenfeld [8] developed the structure of fuzzy graphs and obtained analogs of several graph theoretical concepts in 1975. Gani *et al*. [3,4] introduced the concepts of order and size of fuzzy graphs and fuzzy labelling graphs, fuzzy magic graphs. Sobha *et al*. [9] discussed the concept of fuzzy magic graphs. Kaliraja and Sasikala [5] have discussed the fuzzy intrinsic edge-magic graphs.

2. Preliminaries

Definition 2.1 ([3])**.** A fuzzy graph $G=(\sigma,\mu)$ is a pair of functions $\sigma: V \to [0,1]$ and $\mu: V\times V \to [0,1]$ where for all $u,v \in V$, we have $\mu(u,v) \le \sigma(u)\Lambda\sigma(v)$.

Definition 2.2 ([3])**.** A bijection ω is a function from the set of all nodes and edges of to $[0,1]$ which assign each nodes $\sigma^{\omega}(a)$, $\sigma^{\omega}(b)$ and edge $\mu^{\omega}(a,b)$ a membership value such that $\mu^{\omega}(a,b) \le \sigma^{\omega}(a)\Lambda\sigma^{\omega}(b)$ for all $a,b \in V$ is called fuzzy labelling. A graph is said to be fuzzy labelling graph if it has a fuzzy labelling and it is denoted by G^{ω}.

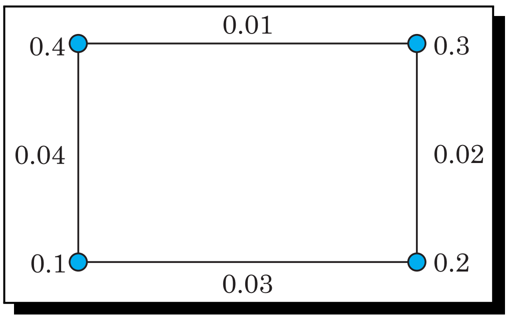

Figure 1. Fuzzy labelling graph

Definition 2.3 ([5])**.** A fuzzy labelling graph G is said to be fuzzy intrinsic labelling if $\sigma: V \to [0,1]$ and $\mu: V\times V \to [0,1]$ is bijective such that the membership values of edges and vertices are intrinsic labelling if $\{z,2z,3z,\dots,Nz\}$ without any repetition where N is the total number of vertices and edges and let $z=0.1$ for $N<6$ and $z=0.01$ for $N\ge 6$.

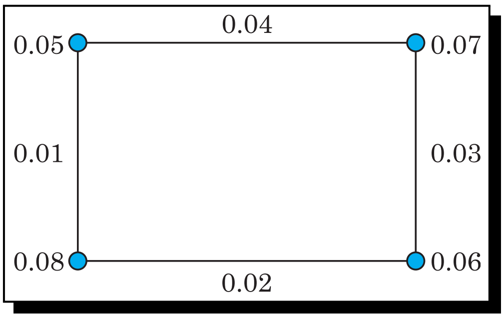

Figure 2. Fuzzy intrinsic labelling graph

Definition 2.4. A fuzzy graph is said to be intrinsic graph if it satisfies the intrinsic labelling. For example, the above graph is an intrinsic graph.

Definition 2.5 ([5])**.** A fuzzy intrinsic labelling said to be an edge-magic labelling if it has an intrinsic constant $\lambda_c = \sigma(v_i) + \mu(v_i v_j) + \sigma(v_j)$, for all $v_i, v_j \in V$.

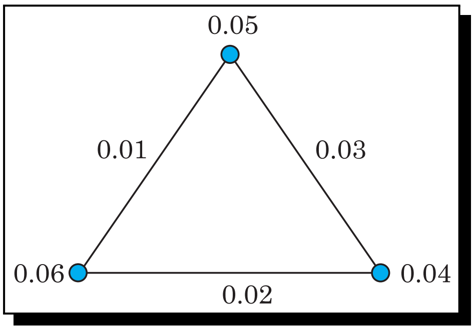

Figure 3. Fuzzy intrinsic edge magic graph

Definition 2.6 ([5])**.** A fuzzy graph G is said to be intrinsic edge-magic if it satisfies the intrinsic edge-magic labelling with intrinsic constant λ_c.

Example. Figure 4 shows the fuzzy intrinsic edge magic graph.

Definition 2.7 ([4])**.** A fuzzy graph is called a fuzzy star graph if there are two vertex sets X and Y with $|X| = 1$ and $|Y| > 1$, such that $\beta(x_j, x_{j+1}) = 0$, $1 \le j \le n$ and denoted by $S_{1,n}$.

Definition 2.8 ([5])**.** A fuzzy star graph is said to be fuzzy intrinsic edge magic star graph if it satisfies the fuzzy intrinsic edge magic labelling with $\lambda_c = \sigma(v_i) + \mu(v_i v_j) + \sigma(v_j)$, for all i and j.

Definition 2.9 ([5])**.** The pan graph is the graph obtained by joining a cycle graph to a singleton graph with a bridge. The 3-pan graph is some times known as the paw graph.

Definition 2.10 ([5])**.** A fuzzy cycle graph is said to be fuzzy intrinsic edge magic cycle if it satisfies the fuzzy intrinsic edge magic labelling with λ_c.

Definition 2.11 ([5])**.** A fuzzy path graph with intrinsic edge magic labelling with intrinsic constant is called the intrinsic edge magic path graph.

3. Focus on Even Intensity of FIEMG

Definition 3.1. Let G be a fuzzy intrinsic edge magic graph. Let $N = \upsilon + \varepsilon$ be the total number of vertices and edges in G, then, the even intensity of G is denoted by $E_I(G)$ and is defined as $E_I(G) = 2(\upsilon + \varepsilon)z = 2Nz$ where $z = 0.1$ for $N < 6$ and $z = 0.01$ for $N \geq 6$.

Example. Consider the following cycle graph with three vertices:

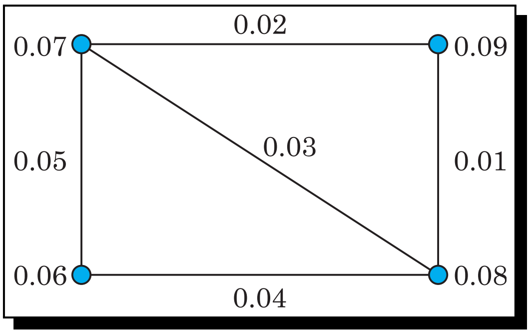

Figure 4. $E_I(G) = 0.18$

$$\upsilon = n \text{ (vertices)} \quad \text{and} \quad \varepsilon = n+1 \text{ (edges)};$$
$$E_I(G) = 2(n+n+1) = 2(2n+1) = 4n+2, \quad \text{(here } n = 4\text{)};$$
$$E_I(G) = 0.18.$$

Definition 3.2. Let G be a fuzzy intrinsic edge magic graph. Let $E_I(G)$ be the even intensity of G and $\lambda_c = \sigma(v_i) + \mu(v_i v_j) + \sigma(v_j)$ for all i and j be the intrinsic edge magic constant of G. Then, G is said to be

(i) secure intensity intrinsic edge magic if $E_I(G) = \lambda_c(G)$,

(ii) supreme intensity intrinsic edge magic if $E_I(G) > \lambda_c(G)$,

(iii) subordinate intensity intrinsic edge magic if $E_I(G) < \lambda_c(G)$.

Definition 3.3. A fuzzy paw graph is said to be intrinsic edge magic paw graph if it satisfies the fuzzy intrinsic edge magic labelling with intrinsic constant λ_c.

Definition 3.4. A fuzzy cycle graph with fuzzy intrinsic edge magic labelling with intrinsic constant λ_c is called a fuzzy intrinsic edge magic cycle graph.

Theorem 3.5. *A fuzzy intrinsic edge magic paw graph (n-pan graph) always a secure intensity intrinsic edge magic if $n = 3$ only.*

Proof. Let G be a fuzzy intrinsic edge magic paw graph. It satisfies the intrinsic edge magic labelling with intrinsic constant λ_c. To find the intrinsic constant of the n-pan graph:

$$\begin{aligned}
\lambda(n\text{-pan graph}) &= \sigma(v_i)+\mu(v_iv_j)+\sigma(v_j)\\
&= \begin{cases} (2n+2-i)z+iz+(2n+4-j)z, & \text{if } i=n-2 \text{ and } j=n-1,\\ (2n+4-i)z+iz+(2n+2-j)z, & \text{if } i=n-1 \text{ and } j=n-1,\\ (2n+4-i)z+iz+(2n+3-j)z, & \text{if } i=n-1 \text{ and } j=n+1,\\ (2n+3-j)z+(n+1)z+(2n+2-j)z, & \text{if } i=n+1 \text{ and } j=n-2 \end{cases}\\
&= (4n+6-i-j)z\\
&= (4n+6-(i+j)z\\
&= (4n+4)z, \quad \text{for } i,j=n-2,\\
v &= n+1 \text{ and } \varepsilon = n+1, \quad \text{if } n=3,\\
E_I(n\text{-pan graph}) &= [2(n+1+n+1)]z = 2(2n+2)z=(4n+4)z, \quad (\text{here } n=3),\\
E_I(n\text{-pan graph}) &= \lambda_c(n\text{-pan graph}) = (4n+4)z, \quad (\text{here } n=3).
\end{aligned}$$

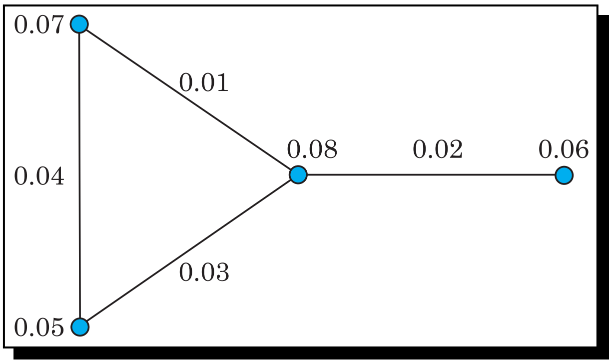

Figure 5. Fuzzy paw graph

By Definition 3.2(i), for $n=3$,

$$E_I(G)=\lambda_c(G)=(4n+4)z=0.16. \tag{3.1}$$

For $n>3$, it satisfies only the intrinsic edge magic labelling without intrinsic edge magic constant.

From the above observation, fuzzy intrinsic edge magic paw graph (FIEMPAW) graph is always a secure intensity intrinsic edge magic for $n=3$.

Theorem 3.6. *A fuzzy intrinsic edge magic cycle graph C_n always a secure intensity intrinsic edge magic iff* $n=3$.

Proof. Let C_n be a fuzzy intrinsic edge magic cycle graph with n vertices and n edges. It satisfies intrinsic edge magic labelling with intrinsic constant λ_c.

$$\begin{aligned}
\lambda(C_n) &= \sigma(v_{2i})+\mu(v_iv_{i+1})+\sigma(v_{2i-1})+\mu(v_nv_1)\\
&= (2n+1-i)z+iz+(n+3-i)z+nz
\end{aligned}$$

$$= (4n+4-i)z, \quad \text{for } i = n+1$$
$$= 4nz,$$
$$v = n \text{ and } \varepsilon = n \quad (\text{here } n = 3),$$
$$E_I(C_n) = 2(n+n)z = 2(2n) = 4nz \quad (\text{here } n = 3),$$
$$E_I(C_n) = \lambda_c(C_n) = 4nz \quad (\text{here } n = 3).$$

By Definition 3.2(i), for $n = 3$,

$$E_I(G) = \lambda_c(G) = 4nz = 0.12. \tag{3.2}$$

For $n > 3$, it satisfies only the intrinsic edge magic labelling without intrinsic edge magic constant. So, the given graph is not fuzzy intrinsic edge magic for $n > 3$.
From (3.2), fuzzy intrinsic edge (FIEM) cycle graph is always a secure intensity intrinsic edge magic for $n = 3$. □

Theorem 3.7. *A fuzzy intrinsic edge magic path graph P_n is always a secure intensity intrinsic edge magic for $n = 3$.*

Theorem 3.8. *A fuzzy intrinsic edge magic path graph P_n always a supreme intensity intrinsic edge magic for $n > 3$.*

Theorem 3.9. *A fuzzy star graph $K_{1,n}$ which is always an intrinsic edge magic for $n \geq 2$.*

Proof. Let $K_{1,n}$ be a fuzzy star graph. Obviously, it satisfies fuzzy labelling.
Now, we apply fuzzy intrinsic edge magic labelling with $n+1$ vertices and n edges. Here

$$\lambda(K_{1,n}) = \sigma(v_i) + \mu(v_i v_{i+1}) + \sigma(v_{i+1})$$
$$= (2n+2-i)z + iz + (2n+1-i)z$$
$$= (4n+3-i)z \qquad (\text{for } i = n-2)$$
$$= (4n+2)z.$$

From the above discussion, the mentioned graph is always a fuzzy intrinsic edge magic for all values of $n \geq 2$. □

Theorem 3.10. *A fuzzy intrinsic edge magic star graph $K_{1,n}$ always a secure intensity intrinsic edge magic for $n \geq 2$.*

Proof. By Theorem 3.10,

$$\lambda_c(K_{1,n}) = (4n+2)z,$$
$$v = n+1 \text{ (vertices) and } \varepsilon = n \text{ (edges)},$$
$$E_I(K_{1,n}) = 2(n+1+n)z = 2(2n+1)z = (4n+2)z,$$
$$E_I(K_{1,n}) = \lambda_c(K_{1,n}) = (4n+2)z, \quad \text{for } n \geq 2.$$

Hence by Definition 3.2(i), a fuzzy intrinsic edge magic star graph $K_{1,n}$ always a secure intensity intrinsic edge magic for $n \geq 2$. □

4. Implementation of Fuzzy Intrinsic Edge Magic Graph

Let a company has five automobile manufacturing departments in various cities say A, A_1, A_2, A_3, and A_4. Department in city 'A' coordinates with rest of all departments in other cities. Company gets a project and wants to complete it within a specific time period. Due to this company wishes to allocate work load according to individuals working abilities and size of departments with equalities. To estimate individual working ability based on qualities it is difficult. But apply fuzzy conditions any person or department working ability must always lie between $[0,1]$. Therefore fuzzy intrinsic edge magic graph facilitate to allocate the work load between departments for achievement to complete the goal within given time frame. We used star graph in this example, here vertices represent the departmental work load in cities and edges represent the share work load between coordinator cities and other departments in remaining cities. For completion of task, total work done by each department in cities must be 100%.

The graph corresponding to the given circumstance is given in Figure 6:

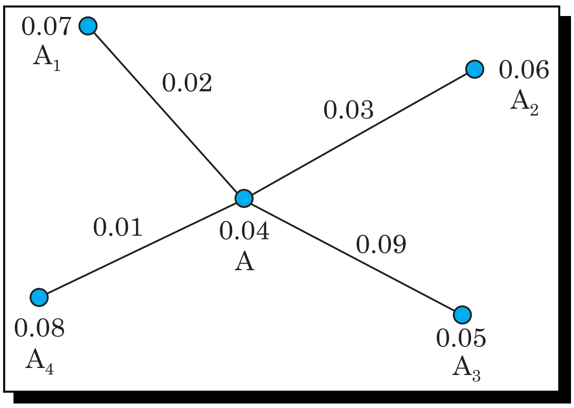

Figure 6. Fuzzy intrinsic edge magic star graph

Table 1. Percentage of work distributed between departments in various cities

Manufacturing departments (vertices)	A	A_1	A_2	A_3	A_4
Department's work in various cities	50%	38.89%	33.33%	27.78%	44.44%
Sharing work (edges)		S_1	S_2	S_3	S_4
Share work between departments in various cities		11.11%	16.67%	22.22%	5.56%

The above table shows the departments individual work in various cities (in percentage) and share work in percentage between departments in various cities.

5. Conclusion

Fuzzy graph theory may be used to solve the problems where uncertainities exist. Utilize it to model real systems in which involve uncertainity in different levels. In this article we focussed the even intensity, secure, supreme and subordinate intensity intrinsic edge magic graphs and implementation of fuzzy intrinsic edge magic graph. In future, we try to solve more problems using the above mentioned graph.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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