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Research Article

Rayleigh Wave Propagation With Rotation, Two Temperature With Diffusion in Context to Dual Phase Lag Thermoelasticity

Bharti Thakur*[1], Sangeeta Kumari[1] and Ashwani Kumar[2]

[1] *Department of Mathematics, Chandigarh University, Gharuan, India*
[2] *CCET Degree Wing, Sector 26, Chandigarh, India*
***Corresponding author:** bharti.cgcgharuan@gmail.com

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Abstract. In the present paper, the theory of generalized thermoelasticity projected by Lord and Shulman (LS) [29] is used to originate the frequency equation for *Rayleigh Wave* (RW) through rotating isotropic *Dual Phase Lag* (DPL) with 2-temperature thermo-elastic medium with diffusion. The methodology of surface wave solution is deployed to solve these equation, further these equations are summarized to isotropic case in xz-plane. The characteristic equation related to speed of *Rayleigh Wave* (RW) is also obtained in context to the suitable boundary conditions and is solved for half space using programming. Some significant results are also concluded in the present study in a half space. The effects of various parameters in the present problem such as 2-temperature, rotation, dual phase lag, frequency and diffusion on the wave speed of Rayleigh wave is depicted by graph.

Keywords. Generalized thermoelasticity, Rayleigh Wave (RW), Rotation, 2-temperature, Diffusion, Wave speed, Dual-Phase Lag (DPL), Secular equation

Mathematics Subject Classification (2020). 74A

1. Introduction

Biot [3–5] formulated the thermo-elastic equation and gives general solutions of the equations of thermoelasticity, also the movement of considered stress waves through poro-elastic solid in case of low range frequency, further extended to that for high frequencies. Lockett [28] initiated the study of movement of Rayleigh's wave in semi infinite solid in context to linear theory of thermoelasticity. This above mentioned theory was again re-examined and studied by Chadwick [6] and Deresiewicz [12], respectively. Chadwick and Windle [7], again studied the consequences of heat conduction on the movement of Rayleigh wave with respect to two assumptions, *i.e.*, when constant temperature is maintained for the solid surface and when it is thermally insulating, also secular equation is calculated for Rayleigh wave. Lord and Shulman [29], and Green and Lindsay [18] continued the *Classical-Dynamical-Coupled-Theory* (CDCT) of thermoelasticity into generalized thermoelasticity theories. The concept of coupled thermoelasticity and generalized thermoelasticity was discussed, analysed and compared by various researchers like Kovalenko [25], McCarthy [30], Green [17], Ignaczak [24], Green and Naghdi [19], Chandrasekharaiah [8] and many more reviewed these theories of generalized thermoelasticity in detail.

Based on the theory given by Gurtin and Williams [20, 21] that it is useful to consider 2nd law of thermodynamics which includes two different temperatures which governed heat supply and entropy due to heat conduction, and based on above theory Chen and Gurtin [9] explains the new elementary equation of heat flux denoted by q and the thermodynamic temperature denoted by θ for non simple rigid heat conductor. Further Chen *et al*. [10, 11] developed the heat conduction theory subject to distorted bodies depending upon the 2-different temperatures ϕ and T named as conductive and thermodynamic temperature respectively. Considering the case that if steady state (time independent) situation is taken then it can be seen that the distinction between both temperatures is analogous to that of heat supply, but when the supply of heat is terminated then both the temperatures are same. In context to the above mentioned theories, Warren and Chen [40] explore two wave propagation problems and it is also noticed that the 2-different temperatures θ and ϕ along with strain can be represented like moving wave pulse also the prediction of velocity (travelling wave) is the major factor that separates the one-temperature and 2-temperature theory. Later, Ieşan [23] studied the theory(linear) of coupled thermoelasticity with 2-temperature for homogeneous isotropic solid and obtained various relevant theorems. The researchers who contributed in developing this theory of 2-temperature further are Quintanilla [32]. The researcher studied 2-temperature with thermoelasticity and its structural stability, it is also examined that in the absence of parameters whether the system reduces to usual thermoelasticity or not. Youssef [41, 42] obtained the uniqueness theorem and established 1-dimensional model of equations for 2-temperature generalized theory under magnetic field and 2-relaxation time in a medium which conduct electricity, state space approach was adopted to get solution. Kumari *et al*. [26] also study the outcomes of 2-Temperature with initial stress for rayleigh's wave in generalized thermo-elastic media under influence of magnetic field.

The concept of generalized thermoelasticity was further modified by proposing some changes in the classical fourier law though introducing 2-phase lag τ_T and τ_q to temperature gradient ∇T and heat flux $\vec{q}$ respectively and is named as *Dual Phase Lag Model* (DPLM) by Tzou [38, 39]. The related model was also proposed by Chandrasekharaiah [8] by taking $\tau_q = \tau_T = 0$. Quintanilla and Racke [33] checks the stability of DPLM so as to get the stable solution for the relative heat equation. Some important factors related to DPLM was also studied by various researchers such as Quintanilla and Racke [34], Ezzat *et al*. [14, 15], Singh [36], Fabrizio and Lazzari [16], El-Karamany and Ezzat [13], Singh *et al*. [37], Liu and Quintanilla [27], Mondal *et al*. [31], Ahmed *et al*. [2], Hobiny and Abbas [22], Ahmed *et al*. [1], Ahmed *et al*. [35]. DPLM is of great significance because of its various applications in the field of engineering as well as medical sciences.

In the present work we attempt to investigate propogation of Rayleigh wave in rotating isotropic DPL with 2-temperature thermoelastic medium with diffusion using the methodology of surface wave solution by applying suitable boundary conditions. Also, phase-speed and attenuation coefficient of propagating surface wave along with the secular equation of rayleigh wave for a real model, in order to analyze the problem in detail will be calculated. Some graphical representation is used to show the effects of a variety of parameters such as 2-temperature, rotation, DPL, frequency and diffusion on the Rayleigh wave's speed.

2. Basic Equations

Following Chen *et al*. [9, 10], and Sherief *et al*. [35], fundamental equations for linear isotropic, generalized thermo-diffusion, homogeneous, elastic solid having constant temperature T when body forces remain absent are:

(i) *correspondence of Strain-displacement*:

$$e_{ij} = \frac{(u_{i,j} + u_{j,i})}{2}, \tag{1}$$

(ii) *energy equation*:

$$-q_{i,i} = \rho T_0 \dot{S}, \tag{2}$$

(iii) *the modified Fourier law*:

$$-K_{ij}\Phi_{,j} = q_i + \tau_0 \dot{q}_i, \tag{3}$$

(iv) *equation expressing motion*:

$$\rho \ddot{u}_i = \mu u_{i,jj} - \beta_1 \Theta_{,i} + (\lambda + \mu) u_{j,ij} - \beta_2 C_{,i}, \tag{4}$$

(v) *heat conduction equation*:

$$(\dot{\Theta} + \tau_0 \ddot{\Theta})\rho c_E + a T_0 (\dot{C} + \tau_0 \ddot{C}) + \beta_1 T_0 (\dot{e} + \tau_0 \ddot{e}) = K \Phi_{,ii}, \tag{5}$$

(vi) *mass diffusion equation*:

$$D^* \beta_2 e_{,ii} + D^* a \Theta_{,ii} + \dot{C} + \tau \ddot{C} - D^* b C_{,ii} = 0, \tag{6}$$

(vii) *two temperature relation*:

$$\Phi - \Theta = a^* \Phi_{,ii}, \tag{7}$$

(viii) *the fundamental equations*:

$$2\mu e_{ij} + \delta_{ij}(\lambda e_{kk} - \beta_1 \Theta - \beta_2 C) = \sigma_{ij}, \tag{8}$$

$$-\beta_2 e_{kk} + bC - a\Theta = P, \tag{9}$$

$$\rho c_E \Theta + \beta_1 T_0 e_{kk} + a T_0 C = \rho T_0 S, \tag{10}$$

where

β_1	:	$(3\lambda + 2\mu)\alpha_t$
β_2	:	$(3\lambda + 2\mu)\alpha_c$
T	:	temperature (absolute)
T_0	:	natural state temperature (medium)
Φ	:	conductive temperature
σ_{ij}	:	Stress tensor (components)
e_{ij}	:	Strain tensor (components)
u_i	:	displacement vector (components)
S	:	$\frac{\text{Entropy}}{\text{Mass (unit)}}$
P	:	$\frac{\text{Chemical potential}}{\text{Mass (unit)}}$
C	:	concentration of mass
c_E	:	specific heat (when strain is constant)
K	:	thermal conductivity coefficient
D^*	:	constant of thermo-diffusion
a	:	estimate of thermo-diffusion outcome
b	:	estimate of diffusive outcome

The dots over variables denote the differentiation with regard to time:

- The DPL theory reduces to CT theory when $\tau_\theta = 0$, $\tau_q = 0$.
- It reduces to LS-theory when we replace$\tau_\theta = 0$ and τ_q by τ_θ.

3. Discussion of Problem and Results

Consider a linearly homogeneous, isotropic, diffusion, 2-temperature and thermo-elastic medium for an infinite stretch taking the system of Cartesian coordinates. The origin is considered on the level surface where as the axis of z- is taken normal to the medium, i.e., $z \geq 0$. We take, surface z as zero stress (free) as well as thermally (insulated). Present problem is confined only to x-z plane, taking vector (displacement) $\vec{u} = (u_1, 0, u_3)$, where

$$u_1 = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z}, \quad u_3 = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x}, \tag{11}$$

taking equations (7) and (11), thereby reducing (4) to (6):

$$\mu(\psi_{,11}+\psi_{,33})=\rho\left(\ddot{\psi}-\Omega^2\psi+2\Omega\frac{\partial\phi}{\partial t}\right), \tag{12}$$

$$(\lambda+2\mu)(\phi_{,11}+\phi_{,33})-\beta_1\Theta-\beta_2 C=\rho\left(\ddot{\phi}-\Omega^2\phi-2\Omega\frac{\partial\psi}{\partial t}\right), \tag{13}$$

$$\left(1+\tau_\theta\frac{\partial}{\partial t}\right)K(\Phi_{,11}+\Phi_{,33})=\left(1+\tau_q\frac{\partial}{\partial t}\right)\left(\rho c_E\tau_m\dot{\Theta}+\beta_1 T_0\tau_m\frac{\partial}{\partial t}\nabla^2\phi+aT_0\tau_m\dot{C}\right), \tag{14}$$

$$D^*\beta_2\nabla^2(\phi_{,11}+\phi_{,33})-D^*b(C_{,11}+C_{,33})+D^*a(\Theta_{,11}+\Theta_{,33})+\tau_n\dot{C}=0, \tag{15}$$

$$\Theta=\Phi-a^*(\Phi_{11}+\Phi_{33}), \tag{16}$$

where $\tau_n=1+\tau\frac{\partial}{\partial t}$ and $\tau_m=1+\tau_0\frac{\partial}{\partial t}$.

For propagation of thermoelastic *Rayleigh Surface Wave* (RSW) along x-axis, ϕ,Φ,C,ψ (potential functions)are represented as:

$$[\phi,\Phi,\psi,C]=[\widehat{\phi}(z),\widehat{\Phi}(z),\widehat{\psi}(z),\widehat{C}(z)]e^{\iota(\eta x-\chi t)}. \tag{17}$$

From above expression (17) and from expressions (12) to (16) the final relation gives us a homogeneous system having non trivial solution, i.e.,

$$D^8-LD^6+MD^4-ND^2+O=0, \tag{18}$$

where L,M,N,O are expressed as,

$$L=4\eta^2-\frac{A_2}{A_1}, \tag{19}$$

$$M=6\eta^4-3\eta^2\frac{A_2}{A_1}+\frac{A_3}{A_1}, \tag{20}$$

$$N=4\eta^6-3\eta^4\frac{A_2}{A_1}+2\eta^2\frac{A_3}{A_1}-\frac{A_4}{A_1}, \tag{21}$$

$$O=\eta^8-\eta^2\frac{A_2}{A_1}+\eta^4\frac{A_3}{A_1}-\eta^2\frac{A_4}{A_1}+\frac{A_5}{A_1}, \tag{22}$$

where

$$A_1=c_1^2[D^*b(\bar{K}-a^*)-\epsilon_2 a^*D^*a]-\epsilon_1 a^*(\bar{\beta}_1 D^*b+\bar{\beta}_2 D^*a), \tag{23}$$

$$\begin{aligned}A_2&=c_1^2c_2^2[D^*b-(\bar{K}-a^*)\chi^2\tau_n^*+\epsilon_2 D^*a]+c_2^2[\epsilon_1(\beta_1[D^*b+\chi^2a^*\tau_n^*]+\bar{\beta}_2D^*)\\&\quad-D^*\beta_2(\bar{\beta}_1\epsilon_2a^*\beta_2(\bar{K}-a^*))]+(\chi^2+\Omega^2)[(c_1^2+c_2^2)(D^*b(\bar{K}-a^*)-\epsilon_2a^*D^*a)\\&\quad-\epsilon_1a^*(\bar{\beta}_1D^*b+\bar{\beta}_2D^*)],\end{aligned} \tag{24}$$

$$\begin{aligned}A_3&=4\chi^2\Omega^2[\epsilon_2a^*D^*a-D*b(\bar{K}-a^*)-c_2^2[\chi^2\tau_n^*(c_1^2+\epsilon_1\bar{\beta}_1)-D^*\beta_2(\bar{\beta}_1\epsilon_2+\bar{\beta}_2)]\\&\quad-(\chi^2+\Omega^2)[-D^*b+(\bar{K}-a^*)\chi^2\tau_n^*-\epsilon_2D^*a](c_2^2+c_1^2)-\epsilon_1(\bar{\beta}_1D^*b+\bar{\beta}_2D^*a)\\&\quad+\bar{\beta}_1a^*(D^*\beta_2\epsilon_2-\epsilon_1\chi^2\tau_n^*)-D^*\beta_2\bar{\beta}_2(\bar{K}-a^*)-(\chi^2+\Omega^2)^2[-D^*b(\bar{K}-a^*)\\&\quad-\epsilon_2a^*D^*a],\end{aligned} \tag{25}$$

$$\begin{aligned}A_4&=4\chi^2\Omega^2[-D^*b+\chi^2\tau_n^*(\bar{K}-a^*)-\epsilon_2D^*a]-(\chi^2+\Omega^2)[\chi^2\tau_n^*({c_2}^2+{c_1}^2)\\&\quad+\bar{\beta}_1(\epsilon_1\chi^2\tau_n^*-D^*\beta_2\epsilon_2)+D^*\beta_2\bar{\beta}_2],\end{aligned} \tag{26}$$

$$A_5 = 4\chi^2\Omega^2\tau_n^* - (\chi^2+\Omega^2)^2\chi^2\tau_n^* \tag{27}$$

the equations for F_i, F_i^* and F_i^{**} with values of i ranging from 1 to 3.

$$F_i = \left[\frac{-2\Omega i\chi}{c_2^2(-\eta^2+m_i^2)+(\chi^2+\Omega^2)}\right], \tag{28}$$

$$F_i^* = \left[\frac{\eta^2\left[\epsilon_2\left[c_1^2\left(-1+\frac{m_i^2}{\eta^2}\right)+\frac{\chi^2}{\eta^2}+\frac{\Omega^2}{\eta^2}\right]\right]+\eta^2\left[\tau^*\bar{\beta}_2\epsilon_1\left(-1+\frac{m_i^2}{\eta^2}\right)+2\frac{\Omega}{\eta}\frac{\chi_i}{\eta}F_i\epsilon_2\right]}{-\bar{\beta}_2\left[\bar{K}\eta^2\left(-1+\frac{m_i^2}{\eta^2}\right)+\tau^*\left(1-a^*\eta^2\left(-1+\frac{m_i^2}{\eta^2}\right)\right)\right]-\bar{\beta}_2\bar{\beta}_1\epsilon_2\left[1-a^*\eta^2\left(-1+\frac{m_i^2}{\eta^2}\right)\right]}\right], \tag{29}$$

$$F_i^{**} = \frac{P}{Q}, \tag{30}$$

$$\begin{aligned}P &= [D^*\beta_2(-\eta^2+m_i^2)[\bar{K}(-\eta^2+m_i^2)+\tau^*(1-a^*(-\eta^2+m_i^2))] \\ &\quad - [\tau^*\epsilon_1(-\eta^2+m_i^2)^2D^*a(1-a^*(-\eta^2+m_i^2))], \\ Q &= \epsilon_2D^*a\left[1-a^*(-\eta^2+m_i^2)\right](-\eta^2+m_i^2) \\ &\quad + [\bar{K}(-\eta^2+m_i^2)+\tau^*(1-a^*(-\eta^2+m_i^2))][D^*(-\eta^2+m_i^2)+\chi^2\tau^*],\end{aligned}$$

where

$$\bar{K} = \frac{K_1}{\rho C_\epsilon\chi^2\tau_m},\quad c_1^2 = \frac{\lambda+2\mu}{\rho},\quad c_2^2 = \frac{\mu}{\rho},\quad \bar{\beta}_1 = \frac{\beta_1}{\rho},\quad \bar{\beta}_2 = \frac{\beta_2}{\rho},\quad \epsilon = \frac{\beta_1^2T_0}{\rho^2C_\epsilon},$$

$$\epsilon_1 = \frac{\epsilon\rho}{\beta_1},\quad \epsilon_2 = \frac{aT_0}{\rho C_\epsilon},\quad \tau_n^* = \frac{i\tau_n}{\chi},\quad \tau^* = \frac{\tau_q+\frac{i}{\chi}}{1-i\chi\tau_\theta},\quad \tau_m = 1+\tau_0\frac{\partial}{\partial t},\quad \tau_n = 1+\tau\frac{\partial}{\partial t}.$$

General results of expression (12) to (16):

$$\psi(z) = \sum_{i=1}^{4}[P_i\exp(-m_iz)+P_i'\exp(m_iz)]e^{\iota(\eta x-\chi t)}, \tag{31}$$

$$\phi(z) = \sum_{i=1}^{4}[Q_i\exp(-m_iz)+Q_i'\exp(m_iz)]e^{\iota(\eta x-\chi t)}, \tag{32}$$

$$\Phi(z) = \sum_{i=1}^{4}[R_i\exp(-m_iz)+R_i'\exp(m_iz)]e^{\iota(\eta x-\chi t)}, \tag{33}$$

$$C(z) = \sum_{i=1}^{4}[S_i\exp(-m_iz)+S_i'\exp(m_iz)]e^{\iota(\eta x-\chi t)}. \tag{34}$$

Here $P_i = F_iQ_i$, $R_i = F_i^*Q_i$ and $S_i = F_i^{**}Q_i$ and the expressions for F_i, F_i^* and F_i^{**}, with values of i ranging from 1 to 3 are expressed above. m_i^2 are roots from the equation

$$x^4 - Lx^3 + Mx^2 - Nx + O = 0. \tag{35}$$

In general the nature of roots, m_i, where with values of i ranging from 1 to 3 is complex, so taking $Re(m_i) > 0$ for solution of SW

$$\psi(z), \Phi(z), C(z), \phi(z) \to 0 \text{ when } z \to \infty, \tag{36}$$

the results (31) to (34) become

$$\phi(z) = \sum_{i=1}^{4} Q_i\exp(-m_iz)e^{\iota(\eta x-\chi t)}, \tag{37}$$

$$\psi(z) = \sum_{i=1}^{4} F_i Q_i \exp(-m_4 z) e^{\iota(\eta x - \chi t)}, \tag{38}$$

$$\Phi(z) = \sum_{i=1}^{4} F_i^* Q_i \exp(-m_i z) e^{\iota(\eta x - \chi t)}, \tag{39}$$

$$C(z) = \sum_{i=1}^{4} F_i^{**} Q_i \exp(-m_i z) e^{\iota(\eta x - \chi t)}. \tag{40}$$

4. Deduction of Secular Equation

At stress free surface the relevant boundary conditions, $z = 0$ are

$$\sigma_{zz} = 0, \quad \sigma_{zx} = 0, \quad \frac{\partial P}{\partial z} = 0, \quad \frac{\partial \Theta}{\partial z} + h\Theta = 0. \tag{41}$$

Using (37) to (40) in BC (41), thereby getting a homogeneous system of four expressions represented in terms of Q_1, Q_2, Q_3 and Q_4, which has non trivial solution if

$$\begin{aligned}
& S_1\left[-\frac{m_2}{\eta}\frac{F_2^*}{\eta^2}(X_3Y_4 - X_4Y_3) + \frac{m_3}{\eta}\frac{F_3^*}{\eta^2}(X_2Y_4 - X_4Y_2) - \frac{m_4}{\eta}\frac{F_4^*}{\eta^2}(X_2Y_3 - X_3Y_2)\right] \\
& + S_2\left[\frac{m_1}{\eta}\frac{F_1^*}{\eta^2}(X_3Y_4 - X_4Y_3) + \frac{m_3}{\eta}\frac{F_3^*}{\eta^2}(X_4Y_1 - X_1Y_4) - \frac{m_4}{\eta}\frac{F_4^*}{\eta^2}(X_3Y_1 - X_1Y_3)\right] \\
& + S_3\left[\frac{m_1}{\eta}\frac{F_1^*}{\eta^2}(X_4Y_2 - X_2Y_4) - \frac{m_2}{\eta}\frac{F_2^*}{\eta^2}(X_4Y_1 - X_1Y_4) - \frac{m_4}{\eta}\frac{F_4^*}{\eta^2}(X_1Y_2 - X_2Y_1)\right] \\
& + S_4\left[\frac{m_1}{\eta}\frac{F_1^*}{\eta^2}(X_2Y_3 - X_3Y_2) - \frac{m_2}{\eta}\frac{F_2^*}{\eta^2}(X_1Y_3 - X_3Y_1) - \frac{m_3}{\eta}\frac{F_3^*}{\eta^2}(X_1Y_2 - X_2Y_1)\right] \\
& = h\Bigg[-\frac{m_2}{\eta}\frac{F_2^*}{\eta^2}(X_3Y_4 - X_4Y_3) + \frac{m_3}{\eta}\frac{F_3^*}{\eta^2}(X_2Y_4 - X_4Y_2) - \frac{m_4}{\eta}\frac{F_4^*}{\eta^2}(X_2Y_3 - X_3Y_2) \\
& \quad + \frac{m_1}{\eta}\frac{F_1^*}{\eta^2}(X_3Y_4 - X_4Y_3) + \frac{m_3}{\eta}\frac{F_3^*}{\eta^2}(X_4Y_1 - X_1Y_4) - \frac{m_4}{\eta}\frac{F_4^*}{\eta^2}(X_3Y_1 - X_1Y_3) \\
& \quad + \frac{m_1}{\eta}\frac{F_1^*}{\eta^2}(X_4Y_2 - X_2Y_4) - \frac{m_2}{\eta}\frac{F_2^*}{\eta^2}(X_4Y_1 - X_1Y_4) - \frac{m_4}{\eta}\frac{F_4^*}{\eta^2}(X_1Y_2 - X_2Y_1) \\
& \quad + \frac{m_1}{\eta}\frac{F_1^*}{\eta^2}(X_2Y_3 - X_3Y_2) - \frac{m_2}{\eta}\frac{F_2^*}{\eta^2}(X_1Y_3 - X_3Y_1) - \frac{m_3}{\eta}\frac{F_3^*}{\eta^2}(X_1Y_2 - X_2Y_1)\Bigg],
\end{aligned} \tag{42}$$

where

$$X_i = -\lambda + (\lambda + 2\mu)\frac{m_i^2}{\eta^2} - \beta_1\left[1 - a^*\eta^2\left(-1 + \frac{m_i^2}{\eta^2}\right)\right]\frac{F_i^*}{\eta^2} - \beta_2\frac{F_i^{**}}{\eta^2} - 2\frac{\mu_i m_i}{\eta}F_i, \quad (i = 1,2,3),$$

$$Y_i = D^*\beta_2\left(1 + \frac{m_i^2}{\eta^2}\right) + D^* b\frac{F_i^{**}}{\eta^2} - D^* a\left[-1 + a^*\eta^2\left(-1 + \frac{m_i^2}{\eta^2}\right)\right]\frac{F_i^*}{\eta^2}, \quad (i = 1,2,3),$$

$$S_i = -\left(\frac{m_i^2}{\eta^2} + 1\right)F_i - 2\mu\iota\frac{m_i}{\eta}, \quad (i = 1,2,3).$$

Expression (42) is the secular equation for wave speed of RW in DPL thermo elasticity with a two-temperature rotation and diffusion in free surface.

5. Special Cases

(a) For thermally insulated case ($h \to 0$), the frequency equation (42):

$$\begin{aligned}
&S_1\left[-\frac{m_2}{\eta}\frac{F_2^*}{\eta^2}(X_3Y_4-X_4Y_3)+\frac{m_3}{\eta}\frac{F_3^*}{\eta^2}(X_2Y_4-X_4Y_2)-\frac{m_4}{\eta}\frac{F_4^*}{\eta^2}(X_2Y_3-X_3Y_2)\right]\\
&\quad+S_2\left[\frac{m_1}{\eta}\frac{F_1^*}{\eta^2}(X_3Y_4-X_4Y_3)+\frac{m_3}{\eta}\frac{F_3^*}{\eta^2}(X_4Y_1-X_1Y_4)-\frac{m_4}{\eta}\frac{F_4^*}{\eta^2}(X_3Y_1-X_1Y_3)\right]\\
&\quad+S_3\left[\frac{m_1}{\eta}\frac{F_1^*}{\eta^2}(X_4Y_2-X_2Y_4)-\frac{m_2}{\eta}\frac{F_2^*}{\eta^2}(X_4Y_1-X_1Y_4)-\frac{m_4}{\eta}\frac{F_4^*}{\eta^2}(X_1Y_2-X_2Y_1)\right]\\
&\quad+S_4\left[\frac{m_1}{\eta}\frac{F_1^*}{\eta^2}(X_2Y_3-X_3Y_2)-\frac{m_2}{\eta}\frac{F_2^*}{\eta^2}(X_1Y_3-X_3Y_1)-\frac{m_3}{\eta}\frac{F_3^*}{\eta^2}(X_1Y_2-X_2Y_1)\right]=0. \qquad (43)
\end{aligned}$$

(b) For isothermal case, ($h \to \infty$), the frequency equation (42):

$$\begin{aligned}
&\left[-\frac{m_2}{\eta}\frac{F_2^*}{\eta^2}(X_3Y_4-X_4Y_3)+\frac{m_3}{\eta}\frac{F_3^*}{\eta^2}(X_2Y_4-X_4Y_2)-\frac{m_4}{\eta}\frac{F_4^*}{\eta^2}(X_2Y_3-X_3Y_2)\right]\\
&\quad+\left[\frac{m_1}{\eta}\frac{F_1^*}{\eta^2}(X_3Y_4-X_4Y_3)+\frac{m_3}{\eta}\frac{F_3^*}{\eta^2}(X_4Y_1-X_1Y_4)-\frac{m_4}{\eta}\frac{F_4^*}{\eta^2}(X_3Y_1-X_1Y_3)\right]\\
&\quad+\left[\frac{m_1}{\eta}\frac{F_1^*}{\eta^2}(X_4Y_2-X_2Y_4)-\frac{m_2}{\eta}\frac{F_2^*}{\eta^2}(X_4Y_1-X_1Y_4)-\frac{m_4}{\eta}\frac{F_4^*}{\eta^2}(X_1Y_2-X_2Y_1)\right]\\
&\quad+\left[\frac{m_1}{\eta}\frac{F_1^*}{\eta^2}(X_2Y_3-X_3Y_2)-\frac{m_2}{\eta}\frac{F_2^*}{\eta^2}(X_1Y_3-X_3Y_1)-\frac{m_3}{\eta}\frac{F_3^*}{\eta^2}(X_1Y_2-X_2Y_1)\right]=0. \qquad (44)
\end{aligned}$$

(c) In absence of dual phase lag and rotation for thermally insulated case, the equation (42) reduces to

$$\begin{aligned}
&4\mu\frac{m_4}{\eta}\left[\frac{m_1}{\eta}\left(Y_2\frac{m_3}{\eta}\frac{F_3}{\eta^2}-Y_3\frac{m_2}{\eta}\frac{F_2}{\eta^2}\right)-\frac{m_2}{\eta}\left(Y_1\frac{m_3}{\eta}\frac{F_3}{\eta^2}-Y_3\frac{m_1}{\eta}\frac{F_1}{\eta^2}\right)\right.\\
&\quad\left.+\frac{m_3}{\eta}\left(Y_1\frac{m_2}{\eta}\frac{F_2}{\eta^2}-Y_2\frac{m_1}{\eta}\frac{F_1}{\eta^2}\right)\right]-\left(\frac{{m_4}^2}{\eta^2}+1\right)\left[X_1\left(Y_2\frac{m_3}{\eta}\frac{F_3}{\eta^2}-Y_3\frac{m_2}{\eta}\frac{F_2}{\eta^2}\right)\right.\\
&\quad\left.-X_2\left(Y_1\frac{m_3}{\eta}\frac{F_3}{\eta^2}-Y_3\frac{m_1}{\eta}\frac{F_1}{\eta^2}\right)+X_3\left(Y_1\frac{m_2}{\eta}\frac{F_2}{\eta^2}-Y_2\frac{m_1}{\eta}\frac{F_1}{\eta^2}\right)\right]=0, \qquad (45)
\end{aligned}$$

where the expressions for X_i, Y_i, $\frac{m_i}{\eta}$, $\frac{F_i}{\eta^2}$, $\frac{F_i^*}{\eta^2}$ and $\frac{F_i^{**}}{\eta^2}$ reduce accordingly.

(d) In absence of diffusion parameters, rotation and dual phase lag for the case of thermally isolated the frequency equation (42) reduces to

$$\begin{aligned}
&X_1\left(\frac{m_2}{\eta}\frac{m_3}{\eta}\frac{F_3}{\eta^2}-\frac{m_2}{\eta}\frac{m_3}{\eta}\frac{F_2}{\eta^2}\right)-X_2\left(\frac{m_3}{\eta}\frac{m_1}{\eta}\frac{F_3}{\eta^2}-\frac{m_1}{\eta}\frac{m_3}{\eta}\frac{F_1}{\eta^2}\right)\\
&\quad+X_3\left[\frac{m_1}{\eta}\frac{m_2}{\eta}\frac{F_2}{\eta^2}-\frac{m_2}{\eta}\frac{m_1}{\eta}\frac{F_1}{\eta^2}\right]=0, \qquad (46)
\end{aligned}$$

where the expressions for X_i, Y_i, $\frac{m_i}{\eta}$, $\frac{F_i}{\eta^2}$, $\frac{F_i^*}{\eta^2}$ and $\frac{F_i^{**}}{\eta^2}$ reduce accordingly.

6. Numerical Results Along With Discussion

To calculate the dimension less speed c^2/c_2^2 of Rayleigh wave, we considered some relevant physical constants subject to thermoelastic solid in presence of dual phase lag, diffusion and 2-temperature in half-space at $T_0 = 27$ C
$\lambda = 5.775 \times 10^{11}$ dyne cm^{-2}, $\mu = 1.89 \times 10^{11}$ dyne cm^{-2},
$\rho = 2.7$ gm cm^{-3}, $c_E = 2.361$ cal gm^{-1} C^{-1}, $K = 0.494$ cal cm^{-1} s^{-1} C^{-1},
$\tau_0 = 0.05$ s, values of a and b are 0.005 and 0.05, respectively.
Using above mentioned values, we solve (42) numerically also the dimension less speed c^2/c_2^2 of RW is calculated for various ranges of D^*, χ, a^* and τ.

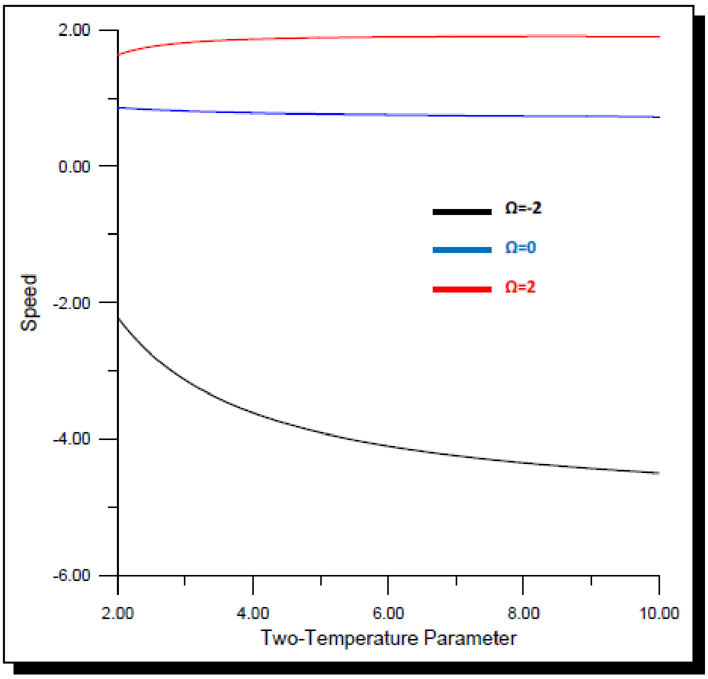

Figure 1. Dimensionless speed c^2/c_2^2 plotted against two-temperature parameter a^*, when rotation $\Omega = -2, 0, 2$

In Figure 1, the dimensionless speed c^2/c_2^2 of Rayleigh wave is plotted against the two-temperature parameter a^* varying from 2 to 10 when $\Omega = -2, 0, 2$, $\tau = 0.04$ s and $\chi = 1$ Hz. For $\Omega = -2$, value of the speed is -2.21542 at $a^* = 2$. It decreases slowly with increase of a^* and concludes for least value -4.49668 for $a^* = 10$ (Figure 1). The blue graph and red graph shows the variation for values of $\Omega = 0, 2$ and it decreases steadily with increase of two-temperature parameter. The comparison of the graph shows the outcomes of rotation at different value of two-temperature.

In Figure 2, the dimensionless speed c^2/c_2^2 of RW vis-a-vis D^* ranging 0.2 to 1 when a^* is 0.5, 0, 0.005, $\tau = 0.04$ s and $\chi = 1$ Hz. When a^* is 0.5 and $D^* = 0.2$, speed become 4.16843. It decrease slowly with a rise of value in a^* and conclude for least value 0.05340 for $D^* = 0.9$ (Figure 2). The blue graph and red graph shows the variation for the $a^* = 0$ and 0.005. The graph exhibits the effect of 2-temperature at different value of D^*.

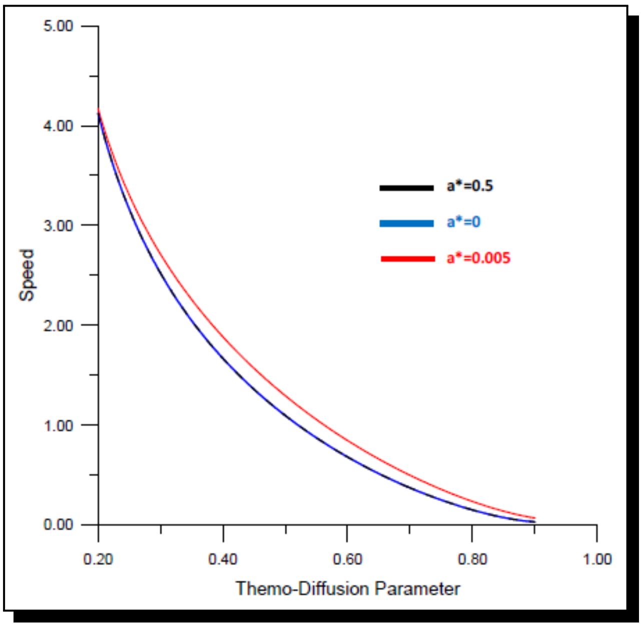

Figure 2. Dimensionless speed c^2/c_2^2 plotted against Thermo-Diffusion parameter D^* when $a^* = 0.5, 0$ and 0.005

In Figure 3, the dimensionless speed c^2/c_2^2 of RW vis-a-vis Ω having values 0.2 to 1, when a^* is 0.5, 0, 0.005, $\tau = 0.04$ s and $\chi = 1$ Hz. When $a^* = 0.5$, speed become 0.91056 for $\Omega = 0.2$. Firstly, black line remains constant for a while, then it starts decreasing and finally it further increases as shown in Figure 3 and minimum speed value attained is 0.83986 for $\Omega = 0.72$. This variation is shown by black graph (Figure 3). The blue graph and red graph exhibits variation for the $a^* = 0$ and 0.005. Figure 3 shows the effect of two-temperature at different value of rotation parameter.

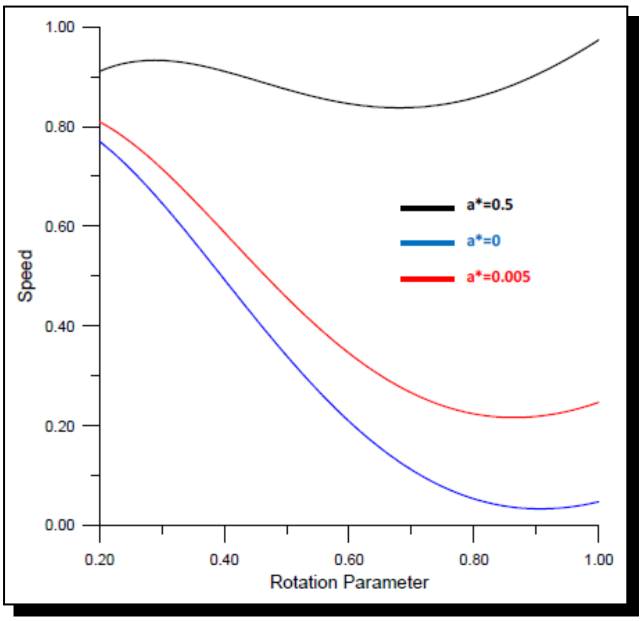

Figure 3. Dimensionless speed c^2/c_2^2 plotted against Rotation parameter Ω when $a^* = 0.5$, 0 and 0.005

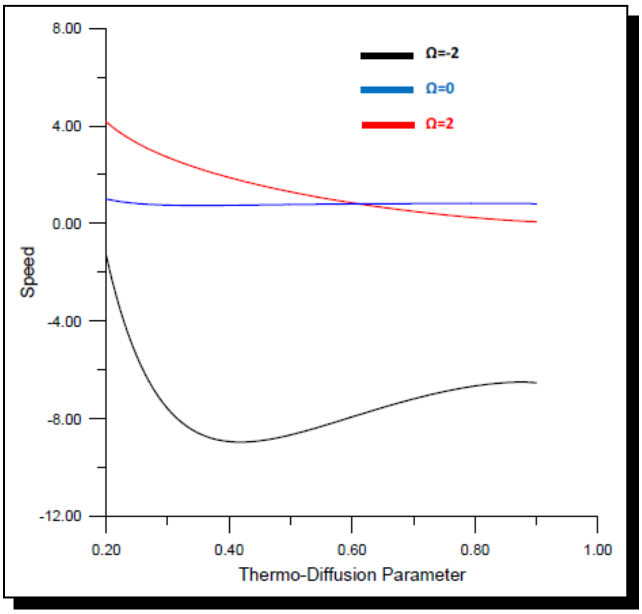

Figure 4. Dimensionless speed c^2/c_2^2 plotted against the thermo-diffusion parameter D^* when $\Omega = -2, 0$ and 2

In Figure 4, the dimensionless speed c^2/c_2^2 of RW vis-a-vis D^* ranging from 0.2 to 1 for $\Omega = -2, 0$ and 2, $\tau = 0.04$ s and $\chi = 1$ Hz. For $\Omega = -2$, value of the speed is -1.30435 at D^* as 0.2. Initially there is a sharp decrease and then it increases with the rise of thermo-diffusion parameter and finally concludes to a least value -8.95813 at $D^* = 0.4$ as in (Figure 4). The blue graph and red graph exhibits the variation for the $\Omega = 0, 2$. Figure 4 shows the effect of thermo-diffusion at different value of rotation parameter. Using Figures 1 to 4 it is concluded that material parameters affects the non dimensional speed of RW.

7. Conclusion

A secular equation for RW through rotating isotropic DPL with 2-temperature thermo-elastic medium with diffusion is obtained after applying the suitable set of boundary conditions. For a given set of values of the material (constant), the dimensionless speed of RW is generated and shown qualitatively. The theoretical and numerical results show the dependence of dimensionless speed on 2-temperature parameter, rotation parameter and thermo-diffusive parameter. This theoretically solved case may be of interest to researchers doing experimental work in seismology, extracting fossils from mines, geophysics, and non-destructive testing.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] E. A. A. Ahmed, M. S. Abou-Dina and A. F. Ghaleb, Magnetic field effect on piezo-thermoelastic wave propagation in a half-space within dual-phase-lag, *Indian Journal of Physics* **95** (2021), 1101 – 1111, DOI: 10.1007/s12648-020-01779-3.

[2] E. A. A. Ahmed, M. S. Abou-Dina and A. R. El Dhaba, Effect of gravity on piezo-thermoelasticity within the dual-phase-lag model, *Microsystem Technologies* **25** (2019), 1 – 10, DOI: 10.1007/s00542-018-3959-2.

[3] M. A. Biot, Theory of propagation of elastic waves in a fluid-saturated porous solid. I. Low-frequency range, *The Journal of the Acoustical Society of America* **28** (1956), 168 – 178, DOI: 10.1121/1.1908239.

[4] M. A Biot, Theory of propagation of elastic waves in a fluid-saturated porous solid. II. Higher frequency range, *The Journal of the Acoustical Society of America* **28** (1956), 179 – 191, DOI: 10.1121/1.1908241.

[5] M. A. Biot, Thermoelasticity and irreversible thermodynamics, *Journal of Applied Physics* **27** (1956), 240 – 253, DOI: 10.1063/1.1722351.

[6] P. Chadwick, Thermoelasticity, the dynamical theory, in: *Progress in Solid Mechanics*, 2nd edition, I. N. Sneddon and R. Hill (editors), Vol. **1**, North-Holland Publishing Co., Amsterdam, pp. 263 – 328 (1960).

[7] P. Chadwick and D. W. Windle, Propagation of Rayleigh waves along isothermal and insulated boundaries, *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences* **280** (1964), 47 – 71, DOI: 10.1098/rspa.1964.0130.

[8] D. S. Chandrasekharaiah, Hyperbolic thermoelasticity: A review of recent literature, *Applied Mechanics Reviews* **51** (1998), 705 – 729, DOI: 10.1115/1.3098984.

[9] P. J. Chen and M. E. Gurtin, On a theory of heat conduction involving two temperatures, *Zeitschrift für angewandte Mathematik und Physik ZAMP* **19** (1968), 614 – 627, DOI: 10.1007/BF01594969.

[10] P. J. Chen and W. O. Williams, A note on non-simple heat conduction, *Zeitschrift für angewandte Mathematik und Physik ZAMP* **19** (1968), 969 – 970, DOI: 10.1007/BF01602278.

[11] P. J. Chen, M. E. Gurtin and W. O. Williams, On the thermodynamics of non-simple elastic materials with two temperatures, *Zeitschrift für angewandte Mathematik und Physik ZAMP* **20** (1969), 107 – 112, DOI: 10.1007/BF01591120.

[12] H. Deresiewicz, A note on thermoelastic rayleigh waves, *Journal of the Mechanics and Physics of Solids* **9**(3) (1961), 191 – 195, DOI: 10.1016/0022-5096(61)90017-5.

[13] A. S. El-Karamany and M. A. Ezzat, On the dual-phase-lag thermoelasticity theory, *Meccanica* **49** (2014), 79 – 89, DOI: 10.1007/s11012-013-9774-z.

[14] M. A. Ezzat, A. S. El-Karamany and A. A. El-Bary, On dual-phase-lag thermoelasticity theory with memory-dependent derivative, *Mechanics of Advanced Materials and Structures* **24**(11) (2017), 908 – 916, DOI: 10.1080/15376494.2016.1196793.

[15] M. A. Ezzat, A. S. El-Karamany and S. M. Ezzat, Two-temperature theory in magneto-thermoelasticity with fractional order dual-phase-lag heat transfer, *Nuclear Engineering and Design* **252** (2012), 267 – 277, DOI: 10.1016/j.nucengdes.2012.06.012.

[16] M. Fabrizio and B. Lazzari, Stability and Second Law of Thermodynamics in dual-phase-lag heat conduction, *International Journal of Heat and Mass Transfer* **74** (2014), 484 – 489, DOI: 10.1016/j.ijheatmasstransfer.2014.02.027.

[17] A. E. Green, A note on linear thermoelasticity, *Mathematika* **19**(1) (1972), 69 – 75, DOI: 10.1112/S0025579300004952.

[18] A. E. Green and K. A. Lindsay, Thermoelasticity, *Journal of Elasticity* **2** (1972), 1 – 7, DOI: 10.1007/BF00045689.

[19] A. E. Green and P. M. Naghdi, Thermoelasticity without energy dissipation, *Journal of Elasticity* **31** (1993), 189 – 208, DOI: 10.1007/BF00044969.

[20] M. E. Gurtin and W. O. Williams, An axiomatic foundation for continuum thermodynamics, *Archive for Rational Mechanics and Analysis* **26** (1967), 83 – 117, DOI: 10.1007/BF00285676.

[21] M. E. Gurtin and W. O. Williams, On the Clausius-Duhem inequality, *Zeitschrift für angewandte Mathematik und Physik ZAMP* **17** (1966), 626 – 633, DOI: 10.1007/BF01597243.

[22] A. Hobiny and I. Abbas, Generalized thermoelastic interaction in a two-dimensional porous medium under dual phase lag model, *International Journal of Numerical Methods for Heat & Fluid Flow* **30**(11) (2020), 4865 – 4881, DOI: 10.1108/HFF-12-2019-0917.

[23] D. Ieşan, On the linear coupled thermoelasticity with two temperatures, *Zeitschrift für angewandte Mathematik und Physik ZAMP* **21** (1970), 583 – 591, DOI: 10.1007/BF01587687.

[24] J. Ignaczak, Uniqueness in generalized thermoelasticity, *Journal of Thermal Stresses* **2** (1979), 171 – 175, DOI: 10.1080/01495737908962399.

[25] A. D. Kovalenko, Fundamentals of thermoelasticity, *Strength of Materials* **3** (1971), 1134 – 1135, DOI: 10.1007/BF01527541.

[26] S. Kumari, Bharti and B. Singh, Effects of two-temperature on rayleigh wave in generalized magneto-thermoelastic media with hydrostatic initial stress, *ASME Journal of Heat and Mass Transfer* **141**(7) (2019), 072002, DOI: 10.1115/1.4043677.

[27] Z. Liu and R. Quintanilla, Time decay in dual-phase-lag thermoelasticity: Critical case, *Communications on Pure and Applied Analysis* **17**(1) (2018), 177 – 190, DOI: 10.3934/cpaa.2018011.

[28] F. J. Lockett, Effect of thermal properties of a solid on the velocity of Rayleigh waves, *Journal of the Mechanics and Physics of Solids* **7**(1) (1958), 71 – 75, DOI: 10.1016/0022-5096(58)90040-1.

[29] H. W. Lord and Y. Shulman, A generalized dynamical theory of thermoelasticity, *Journal of the Mechanics and Physics of Solids* **15** (1967), 299 – 309, DOI: 10.1016/0022-5096(67)90024-5.

[30] M. F. McCarthy, Wave propagation in generalized thermoelasticity, *International Journal of Engineering Science* **10**(7) (1972), 593 – 602, DOI: 10.1016/0020-7225(72)90085-7.

[31] S. Mondal, N. Sarkar and N. Sarkar, Waves in dual-phase-lag thermoelastic materials with voids based on Eringen's nonlocal elasticity, *Journal of Thermal Stresses* **42** (2019), 1035 – 1050, DOI: 10.1080/01495739.2019.1591249.

[32] R. Quintanilla, On existence, structural stability, convergence and spatial behavior in thermoelasticity with two temperatures, *Acta Mechanica* **168** (2004), 61 – 73, DOI: 10.1007/s00707-004-0073-6.

[33] R. Quintanilla and R. Racke, A note on stability in dual-phase-lag heat conduction, *International Journal of Heat and Mass Transfer* **49**(7-8) (2006), 1209 – 1213, DOI: 10.1016/j.ijheatmasstransfer.2005.10.016.

[34] R. Quintanilla and R. Racke, Qualitative aspects in dual-phase-lag thermoelasticity, *SIAM Journal on Applied Mathematics* **66**(3) (2006), 977 – 1001, DOI: 10.1137/05062860X.

[35] H. H. Sherief, F. A. Hamza and H. A. Saleh, The theory of generalized thermoelastic diffusion, *International Journal of Engineering Science* **42**(5-6) (2004), 591 – 608, DOI: 10.1016/j.ijengsci.2003.05.001.

[36] B. Singh, Wave propagation in dual-phase-lag anisotropic thermoelasticity, *Continuum Mechanics and Thermodynamics* **25** (2013), 675 – 683, DOI: 10.1007/s00161-012-0261-x.

[37] B. Singh, S. Kumari and J. Singh, Propagation of the rayleigh wave in an initially stressed transversely isotropic dual-phase-lag magnetothermoelastic half-space, *Journal of Engineering Physics and Thermophysics* **87** (2014), 1539 – 1547, DOI: 10.1007/s10891-014-1160-8.

[38] D. Y. Tzou, A unified field approach for heat conduction from macro- to micro-scales, *ASME Journal of Heat and Mass Transfer* **117**(1) (1995), 8 – 16, DOI: 10.1115/1.2822329.

[39] D. Y. Tzou, *Macro- to Microscale Heat Transfer: The Lagging Behavior*, John Wiley & Sons, Ltd., (2014), DOI: 10.1002/9781118818275.

[40] W. E. Warren and P. J. Chen, Wave propagation in the two temperature theory of thermoelasticity, *Acta Mechanica* **16** (1973), 21 – 33, DOI: 10.1007/BF01177123.

[41] H. M. Youssef, Theory of two-temperature thermoelasticity without energy dissipation, *Journal of Thermal Stresses* **34**(2) (2011), 138 – 146, DOI: 10.1080/01495739.2010.511941.

[42] H. M. Youssef, Theory of two-temperature-generalized thermoelasticity, *IMA Journal of Applied Mathematics* **71**(3) (2006), 383 – 390, DOI: 10.1093/imamat/hxh101.