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Research Article

Optimizing a Fuzzy Multi-Item Inventory System and Ordering Cost Depletion Contingent on Lead Time With Carbon Emission Cost

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Abstract. Multi-item integrated inventory system and ordering cost depletion liable scheduled lead time with carbon emission cost is established in a fuzzy situation. Multiple items can considerably drop total inventory costs for hiring orders aimed at multiple items in single refill demand would drop ordering costs. Owing to the inaccuracy of various parameters and objective is imprecise in the environment. As the development of fuzzy objective is uncertain, the model is formulated as multi-item problems were confident/suspicious profit of the objective with some uncertainty. The model is solved via the graded mean technique with the addition of the Kuhn-Tucker method when the fuzzy equivalent of the problem remains available. An algorithm is established to attain optimal order quantity for each item and then find the minimum integrated total cost for a multi-item inventory system. The evaluation of a fuzzy multi-item inventory system through the crisp multi-item inventory system is completed over mathematical illustrations. Lastly, the graphical demonstration remains offered toward establishing the suggested system. An ending outcome demonstrates that this fuzzy multi-item system is perhaps moderately suitable defining optimal order quantity for each item and then the minimum integrated total cost for the multi-item technique when the lead time is assessed.

Keywords. Fuzzy multi-item inventory model, Graded mean integration technique, Minimum integrated total cost for multi-item, Optimal order quantity for each item, Kuhn-Tucker method

Mathematics Subject Classification (2020). 94D05, 35Q93

1. Introduction

Maximum of inventory concludes optimum strategies aimed at single items, supposing a certain inventory strategy aimed at solitary items ensures not impact the price on inventory and also the income of the organization. As an alternative to a single item, various firms or retailers, or enterprises are encouraged to stock various items in their factories for further gainful commercial circumstances. An additional source of their stimulus is to fascinate the consumers to acquisition more than a few items in single vendor. Multiple article inventory was initially presented by Federgruen *et al*. [5] who evaluated that synchronized replacements for multiple articles can pointedly decrease entire inventory prices as hiring orders for multiple articles in single top-up order exert modest set up prices. Moon and Silver [14] dispensed through a multi-item news seller problematic by allowing for the restricted whole expenditure and then resolved the typical by normal distribution, measured the min-max distribution free methodology. Shin *et al*. [19] considered dual dissimilar systems arranged the origin of principal period request supply then measured facility range constraints besides using transportation reductions. Designed for advanced learning in the multi-items typical, person who reads can grasp Cárdenas-Barrón *et al*. [2]. Vithyadevi and Annadurai [22] considered a combined inventory typical with ordering charge decrease reliant on lead time happening in fuzzy situation by hiring trapezoidal fuzzy quantity. Articles are getting worse at a stable rate and retailed from varies exits in the town under an individual organization presented by Maiti [11].

Due to the universal supply series, the shipping through consignment of goods converts a foremost experiment surrounded by altogether companies of supply series. Owed toward this difficulty, transport costs ought to be incorporated into the whole price to estimate the complete supply series price. Trendy the simple supply series typical, the transport price is involved inside the setup price or ordering price, then nowadays, worldwide supply series systems custom a single ordering to multiple delivery strategy of transport as an alternative of single ordering to single delivery. Through the single ordering to single delivery strategy, all goods are ordered in one order and shipped to the vendor in one supply, then owing to the single-ordering-multi-delivery strategy, all goods are ordered at one order, however it transports to the seller in several transfers. For instance of an outcome, the shipping count rises. Accordingly, stable shipping costs beside through inconstant shipping costs are additionally more to the typical to create additional faithful. The profit of consuming a single-ordering-multi-delivery strategy is that it can save the ordering price of the buyer. If the synchronization is dualistic or more, then the shipping price shows a significant part. Because of the single-ordering-multi-delivery strategy, the shipping counts rises, this affects the climate through the substance of carbon emission. For instance alike per shipping costs, the shipping counts rises which indicates a growing ratio of carbon emissions. Therefore, the flexible and static carbon radiation costs are additional to the total price. Malleeswaran and Uthayakumar [13] considered a combined seller-purchaser supply sequence typical on behalf of backorder amount deduction and cost-related demand consuming provision level restrictions and carbon discharge rate.

The lead time generally contains for the subsequent modules: dealer lead time, order planning, transport time, order shipment, and arrangement period. Malik and Sarkar [12] measured multi-item unremitting assessment inventory system and indeterminate request, eminence enhancement, structure rate drop in addition disparity resistor in principal period. Tiwari *et al*. [21] examined ecological inventory organization with worsening and defective feature matters allowing for carbon discharge. Li *et al*. [9] revealed a supply series holding a retailer besides a customer through manageable principal period. It handled dual circumstances such as whole evidence and inadequate evidence around the consumer. Kamble [7] deliberated the perception of pentagonal fuzzy numbers. Canonical pentagonal fuzzy numbers are measured via inner calculation processes through consuming alpha-cut processes. Fuzzy model declining inventory articles using time changing demand and shortages in entirely backlogged conditions remains framed by Nagar and Surana [15].

Taha [20] provided the Khun-Tucker technique used to resolve indecision issues by means of stated in operations research. Pan and Yang [17] considered delivering a lower total cost and smaller lead time compared to previous inventory problems. A merged inventory typical to minimalize the entire price by enhancing lead time, order size, and the amount of distributions is presented by Yang and Pan [24]. Fuzzy set concept presented by Zimmerman [25] concentrated on ambiguous groups in operational research. Chen [3] deliberated arithmetic processes in fuzzy numbers through the utility code. Maheswari *et al*. [10] handled a multi-product inventory system for an industrial unit outlet in crisp and fuzzy situations framed with storage planetary in one constraint. Das [4] developed a deteriorated multi-object inventory system in a fuzzy situation. Here the demand frequency is persistent. Ali *et al*. [1] studied the supply chain outline that grips perishability disputes in manufacture and dissemination. Investigators suggested a multi-objective mixed-integer non-linear supply chain synchronization model in indeterminate atmospheres to diminish the price. Joviani *et al*. [6] developed a multi-item inventory system in three inventory models along stable deterioration, and partial backlogging, with different demand functions. Nasseri *et al*. [16] presented a technique for ranking fuzzy quantities based on the angle between the reference functions of the fuzzy numbers. San-José *et al*. [18] established an inventory system that depends on the demands of objects is time-dependent and tracks the power arrangements system. Lacks are permissible and entirely backlogged. Kumar and Uthayakumar [8] investigated a two level supply chain with one producer and one seller is established for multi goods. The seller handled with the indeterminate demand for all goods which tracks a normal distribution.

The paper is structured by this manner: In Section 2, the notations, assumptions are familiarized. Section 3 treaties by a mathematical system towards optimize the total cost for multi-item, optimal order for each item. In Section 4, graded mean technique, fuzzy multiitem inventory system designed and an algorithm framed towards determine optimum solution for multi-item. In Section 5, arithmetical illustrations then graphical representation are offered toward establish crisp then fuzzy multi-item inventory model. Section 6 obtains a relative evaluation. In Section 7, the conclusion is tracked.

2. Notations and Assumptions

The succeeding notations are presented in this inventory system.

2.1 Notations

Q_i	– Order quantity for i-th item for the buyer,
L_i	– Lead time span for i-th item of the buyer,
A_i	– Buyer's ordering cost for i-th item of per order,
m_i	– Lots quantity for i-th item it's manufactured goods is supplied from the vendor to the buyer in single manufacture phase,
D_i	– Average demand for i-th item of per unit time on the buyer,
P_i	– Manufacture rate for i-th item of the vendor $P_i > D_i$,
S_i	– Vendor's setup cost for i-th item of per arrangement,
C_{vi}	– Production cost for i-th item of funded through vendor $C_{vi} < C_{bi}$,
C_{bi}	– Buying cost for i-th item of funded by the buyer,
r_i	– Yearly inventory holding cost for i-th item for each dollar capitalized in stocks,
R_i	– Reorder point for i-th item of the buyer,
VEC_{vi}	– Vendor's flexible carbon emission cost for i-th item,
FEC_{vi}	– Vendor's stable carbon emission cost for i-th item,
FTC_{vi}	– Vendor's stable transportation cost for i-th item,
VTC_{vi}	– Vendor's flexible transportation cost for i-th item,
$ITCMI(Q_i, L_i, m_i)$	– Integrated total cost for crisp multi-item inventory system,
$P(I\tilde{T}CMI(\tilde{Q}_i, L_i, m_i))$	– Integrated total cost for fuzzy multi-item inventory system.

2.2 Assumptions

The system is improved by implementing successful assumptions.

(i) The coordination comprises for single-vendor with single-buyer aimed at multi-item inventory system.

(ii) Buyer's order size Q_i and the vendor makings m_iQ_i using a limited manufacture ratio P_i ($P_i > D_i$) at single setup but transports quantity Q_i towards the buyer over m_i times. The vendor sustains a i-th item of set up cost S_i for each manufacture run and the buyer sustains a i-th item of an ordering cost A_i for every order of quantity Q_i.

(iii) The demand of i-th item X_i throughout lead time of i-th item L_i charts a normal distribution with mean μL_i, standard deviation $\sigma\sqrt{L_i}$.

(iv) The inventory is unceasingly studied. Upon reaching the reorder point R_i, the buyer needs the order.

(v) The reorder point equivalents the summation of the expected demand for the period of safety stock and lead time. The reorder point R_i = expected demand for the period of lead time for i-th item + safety stock, therefore $R_i = D_iL_i + k\sigma\sqrt{L_i}$ where k is safety factor.

(vi) The lead time for all items is similar and it involves of n_i mutually independent modules. The z-th module has a normal duration b_{iz}, least period a_{iz}, and crashing cost per unit time c_{iz}. For suitability, c_{iz} is organized in that way $c_{i1} < c_{i2} < c_{i3} < \ldots < c_{in}$.

(vii) The modules of lead time are crashed unique on a period beginning since the principal module for the situation takes the least unit crashing cost also the subsequent module, similar we get next value.

(viii) Let $L_{i0} = \sum_{j=1}^{n} b_{ij}$, and L_{iz} be the span of lead time using modules $1,2,3,\ldots,z$ crashed to their least period, then L_{iz} can be expelled as $L_{iz} = L_{i0} - \sum_{j=1}^{z}(b_{ij} - a_{ij})$, $z = 1,2,3,\ldots,n$; and the lead time crashing cost per cycle $R(L_i)$ is given by $R(L_i) = c_{iz}(L_{i(z-1)} - L_i) + \sum_{j=1}^{z-1} c_{ij}(b_{ij} - a_{ij})$, $L_i \in [L_{iz}, L_{i(z-1)}]$. Furthermore, the distance of lead time is equivalent of whole transport rotations, and the lead time crashing cost arises in every transport rotation. The association among crashing cost and lead time is exposed in Figure 1.

(ix) The decrease of lead time L_{iz} attends condensed ordering cost A_i and A_i is resolutely the concave function of L_{iz}, i.e., $A_i'(L_{iz}) > 0$ and $A_i''(L_{iz}) < 0$.

(x) If additional charges remain sustained through the vendor, it will be fully shifted towards the buyer after reduced lead time is mandatory.

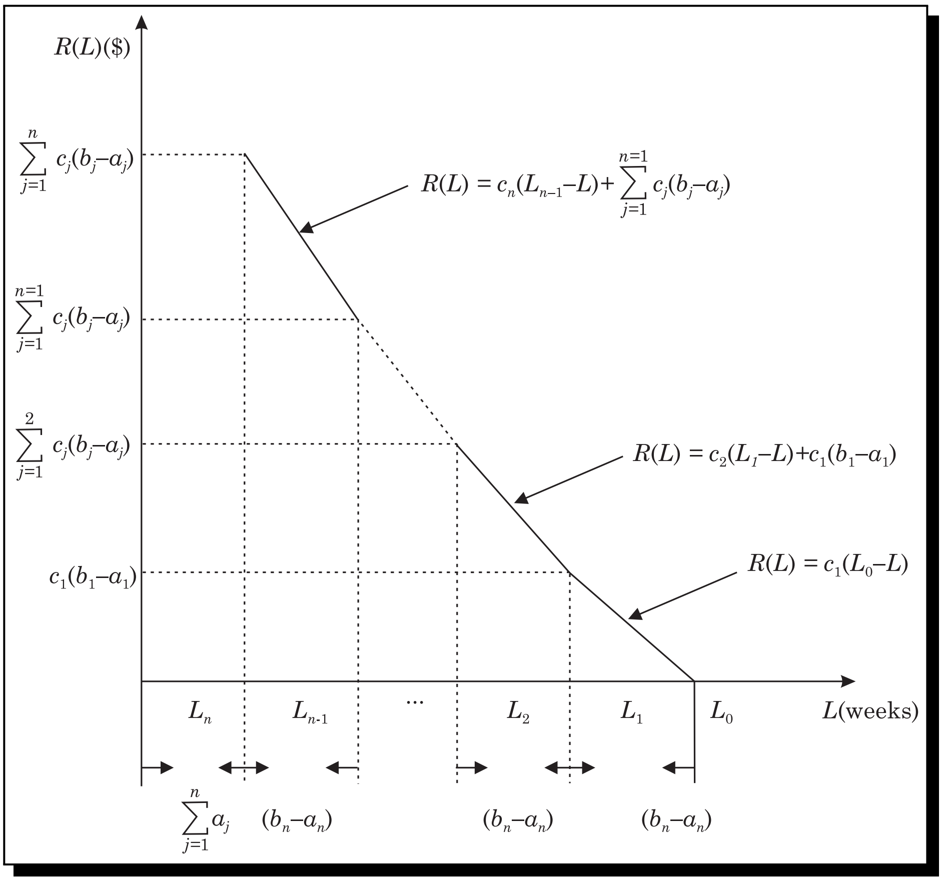


Figure 1. Relationship among crashing cost and lead time

3. Mathematical System

3.1 Crisp Multi-item Inventory System

Integrated Total Cost of Multi-Item (ITCMI). Integrated total cost of multi-item each unit time derived here and summation for succeeding components,

$$\text{Ordering cost for } i\text{-th item per unit time} = \frac{A_i}{Q_i/D_i} = \frac{A_iD_i}{Q_i}, \tag{1}$$

$$\text{Buyer's holding cost for } i\text{-th item per unit time is} = \left(\frac{Q_i}{2} + k\sigma\sqrt{L_i}\right) r_i C_{b_i}, \tag{2}$$

$$\text{Lead time crashing cost for } i\text{-th item per unit time} = \left(\frac{D_i}{Q_i}\right) R(L_i), \tag{3}$$

$$\text{Vendor setup cost for } i\text{-th item per year} = \left(\frac{D_i}{m_iQ_i}\right) S_i, \tag{4}$$

Vendor's average inventory for i-th item (see Figure 2)

$$= \left\{\left[m_iQ_i\left(\frac{Q_i}{P_i} + (m_i-1)\frac{Q_i}{D_i}\right) - \frac{m_i^2Q_i^2}{2P_i}\right] - \left[\frac{Q_i^2}{D_i}(1+2+\ldots+(m_i-1))\right]\right\}\frac{D_i}{m_iQ_i}$$
$$= \frac{Q_i}{2}\left[m_i\left(1-\frac{D_i}{P_i}\right) - 1 + \frac{2D_i}{P_i}\right].$$

So

$$\text{Vendor's holding cost for } i\text{-th item per unit time is} = \frac{Q_i}{2}\left[m_i\left(1-\frac{D_i}{P_i}\right) - 1 + \frac{2D_i}{P_i}\right] r_i C_{v_i}, \tag{5}$$

$$\text{Vendor annual transportation cost for } i\text{-th item} = m_i(FTC_{vi} + VTC_{vi}), \tag{6}$$

$$\text{Annual carbon emission cost for } i\text{-th item} = m_iFEC_{vi} + Q_iVEC_{vi}. \tag{7}$$

Affording to our assumptions and the eqs. (1) to (7) defined above, the integrated total cost for multi-item per unit time which is the collection of above mentioned costs and then expressed as

$$\begin{aligned} ITCMI(Q_i,L_i,m_i) = \sum_{i=1}^{n}\Bigg[&\frac{D_i}{Q_i}\left(A_i + \frac{S_i}{m_i} + R(L_i)\right) - \frac{Q_ir_iC_{v_i}}{2}\left(\frac{m_iD_i}{P_i}+1\right) \\ &+ \frac{Q_ir_i}{2}\left(\left(m_i + \frac{2D_i}{P_i}\right)C_{v_i} + C_{b_i}\right) + r_iC_{b_i}k\sigma\sqrt{L_i} \\ &+ Q_iVEC_{v_i} + m_i(FTC_{v_i} + VTC_{v_i} + FEC_{v_i})\Bigg]. \end{aligned} \tag{8}$$

If a specific rate of m_i and L_i the integrated total cost multi-item is $ITCMI(Q_i,L_i,m_i)$, then optimal order quantity Q_i obtained while integrated total cost of multi-item $ITCMI(Q_i,L_i,m_i)$ is minimum. Now directive to obtain minimization of $ITCMI(Q_i,L_i,m_i)$ we find the partial derivative of $ITCMI(Q_i,L_i,m_i)$ with Q_i and equate to zero, then we have

$$-\frac{D_i}{{Q_i}^2}\left(A_i + \frac{S_i}{m_i} + R(L_i)\right) - \frac{r_iC_{v_i}}{2}\left(\frac{m_iD_i}{P_i}+1\right) + \frac{r_i}{2}\left(\left(m_i + \frac{2D_i}{P_i}\right)C_{v_i} + C_{b_i}\right) + VEC_{vi} = 0. \tag{9}$$

For a static m_i and L_i, the integrated total cost of multi-item $ITCMI(Q_i,L_i,m_i)$ is positive definite on the point Q_i. Through inspecting the sufficient situations to get minimum value of $ITCMI(Q_i,L_i,m_i)$ second order partial derivatives of $ITCMI(Q_i,L_i,m_i)$ with respect to Q_i and obtain

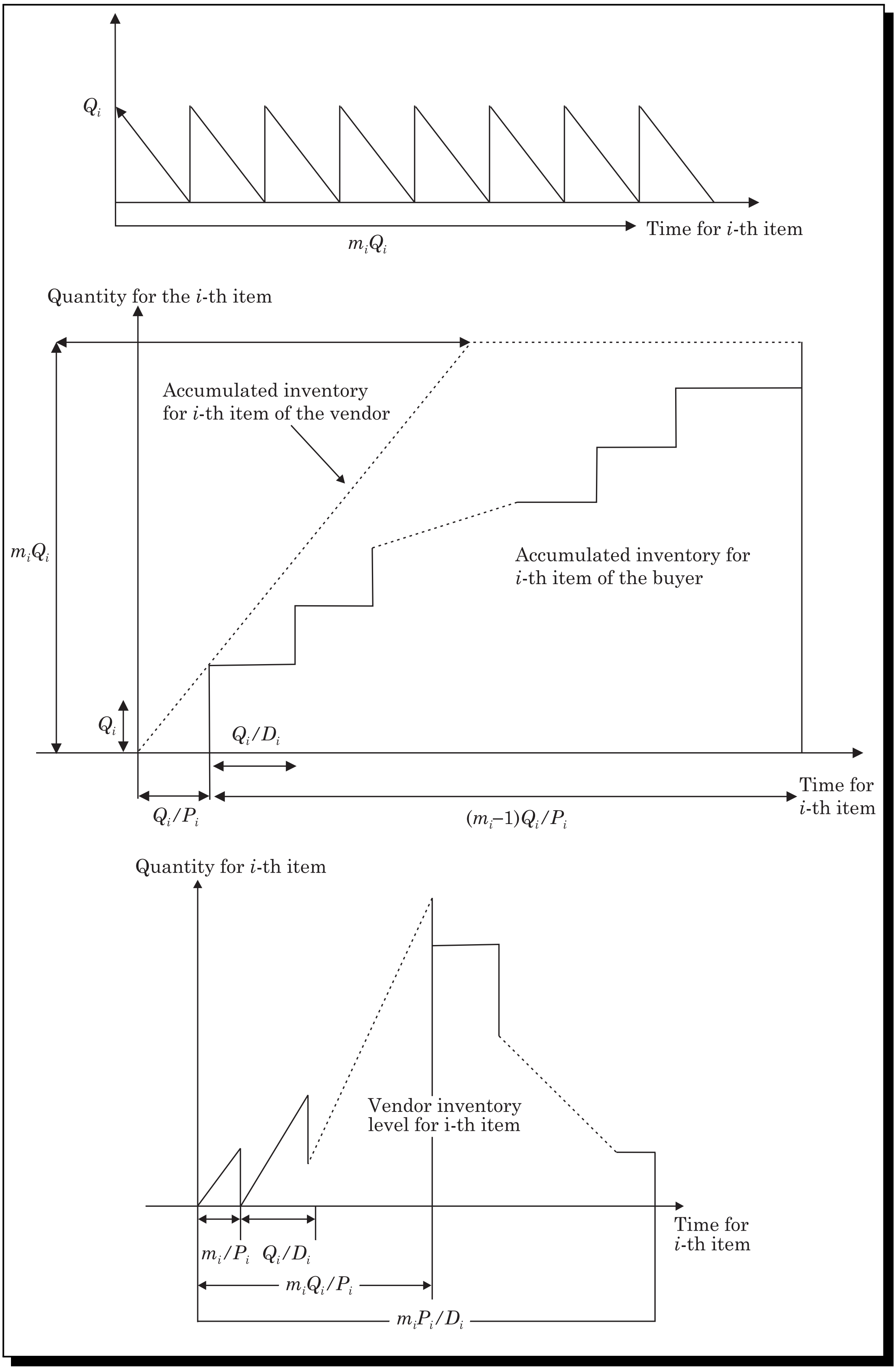


Figure 2. The inventory pattern for i-th item of the buyer and vendor

$$\frac{\partial^2 ITCMI(Q_i,L_i,m_i)}{\partial Q_i^2} = \frac{2D_i}{Q_i^3}\left(A_i + \frac{S_i}{m_i} + R(L_i)\right) > 0. \tag{10}$$

Therefore, $ITCMI(Q_i,L_i,m_i)$ is convex in Q_i, for a static m_i andL_i.

Consequently, observe aimed at optimal derivatives Q_i^* decrease towards obtain a local minimum. Henceforth, we find the optimal order quantity Q_i^* using eq. (9) is,

$$Q_i^* = Q_i = \sqrt{\frac{2D_i\left(A_i + \frac{S_i}{m_i} + R(L_i)\right)}{r_i\left(\left(m_i\left(1-\frac{D_i}{P_i}\right) - 1 + \frac{2D_i}{P_i}\right)C_{v_i} + C_{b_i}\right) + 2VEC_{v_i}}}. \tag{11}$$

4. Fuzzy Inventory System

4.1 Pentagonal Fuzzy Number By Graded Mean Integration Technique (Nagar and Surana [15])

In Figure 3, the graded mean integration technique for $\tilde{\alpha}$ is defined by $\tilde{\alpha} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5)$ as a pentagonal fuzzy number. Then the defuzzification

$$\begin{aligned} P(\tilde{\alpha}) &= \frac{1}{2}\frac{\int_0^1 \frac{h}{2}[\alpha_1 + \alpha_2 + (\alpha_3-\alpha_1)h + \alpha_4 + \alpha_5 - (\alpha_5-\alpha_3)h]dh}{\int_0^1 h dh} \\ &= \frac{1}{12}(\alpha_1 + 3\alpha_2 + 4\alpha_3 + 3\alpha_4 + \alpha_5). \end{aligned} \tag{12}$$

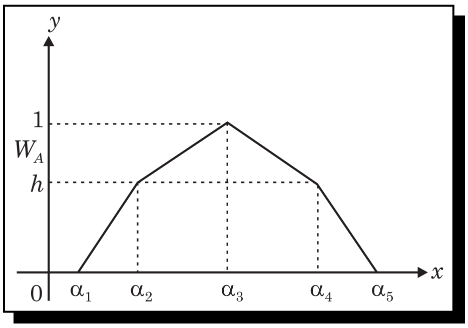


Figure 3. Pentagonal fuzzy number

4.2 Fuzzy Multi-item Inventory System

All over this paper, subsequent parameters and decision variable are utilized in order to shorten the act of fuzzy multi-item inventory system. Take $\tilde{D}_i$, $\tilde{A}_i$, $\tilde{S}_i$, $\tilde{r}_i$, $\tilde{P}_i$, $\tilde{C}_{vi}$, $\tilde{C}_{bi}$, $V\tilde{E}C_{bi}$, and $V\tilde{T}C_{bi}$ are fuzzy quantities. Currently, fuzzy multi-item inventory system is acquaint together fuzzy parameters and fuzzy optimal order quantity $\tilde{Q}_i$.

The fuzzy integrated total cost of multi-item (Chen [3]) is

$$\begin{aligned} I\tilde{T}CMI(\tilde{Q}_i,L_i,m_i) = \sum_{i=1}^{n} &[(\tilde{D}_i \oslash \tilde{Q}_i) \otimes (\tilde{A}_i \oplus (\tilde{S}_i \oslash m_i) \oplus R(L_i))] \\ &\ominus [[(\tilde{Q}_i \otimes \tilde{r}_i \otimes \tilde{C}_{vi}) \oslash 2] \otimes [(m_i \otimes \tilde{D}_i \oslash \tilde{P}_i) + 1] \\ &+ m_i \otimes (FTC_{v_i} + V\tilde{T}C_{v_i} + (m_i \otimes FEC_{v_i} + \tilde{Q}_i \otimes V\tilde{E}C_{v_i}))]. \end{aligned} \tag{13}$$

Assume $\tilde{D}_i = (D_{i_1}, D_{i_2}, D_{i_3}, D_{i_4}, D_{i_5})$, $\tilde{A}_i = (A_{i_1}, A_{i_2}, A_{i_3}, A_{i_4}, A_{i_5})$, $\tilde{r}_i = (r_{i_1}, r_{i_2}, r_{i_3}, r_{i_4}, r_{i_5})$, $\tilde{S}_i = (S_{i_1}, S_{i_2}, S_{i_3}, S_{i_4}, S_{i_5})$, $\tilde{P}_i = (P_{i_1}, P_{i_2}, P_{i_3}, P_{i_4}, P_{i_5})$, $\tilde{C}_{vi} = (C_{vi1}, C_{vi2}, C_{vi3}, C_{vi4}, C_{vi5})$, $\tilde{C}_{bi} = (C_{bi1}, C_{bi2}, C_{bi3}, C_{bi4}, C_{bi5})$, $V\tilde{E}C_{bi} = (VEC_{bi1}, VEC_{bi2}, VEC_{bi3}, VEC_{bi4}, VEC_{bi5})$, and $V\tilde{T}C_{bi} = (VTC_{bi1}, VTC_{bi2}, VTC_{bi3}, VTC_{bi4}, VTC_{bi5})$ are positive pentagonal fuzzy numbers. Furthermore we adopt the decision variable which is fuzzified affording to the pentagonal rule as: $\tilde{Q}_i = (Q_{i1}, Q_{i2}, Q_{i3}, Q_{i4}, Q_{i5})$.

Let us initiate by, fuzzy integrated total cost for multi-item $I\tilde{T}CMI(\tilde{Q}_i, L_i, m_i)$ which is given by eq. (13), that is

$$
\begin{aligned}
&I\tilde{T}CMI(\tilde{Q}_i, L_i, m_i)\\
&= \sum_{i=1}^{n} \Bigg[\Bigg(\frac{D_{i1}}{Q_{i5}} \Big(A_{i1} + \frac{S_{i1}}{m_i} + R(L_i) \Big) - \frac{Q_{i5} r_{i5} C_{vi5}}{2} \Big(\frac{m_i D_{i5}}{P_{i1}} + 1 \Big) + \frac{Q_{i1} r_{i1}}{2} \Big(\Big(m_i + \frac{2D_{i1}}{P_{i5}} \Big) C_{vi1} + C_{bi1} \Big) \\
&\quad + r_{i1} C_{bi1} k\sigma \sqrt{L_{i1}} + Q_{i1} VEC_{vi1} + m_i (FTC_{vi} + VTC_{vi1} + FEC_{vi}) \Bigg) \\
&\quad \cdot \Bigg(\frac{D_{i2}}{Q_{i4}} \Big(A_{i2} + \frac{S_{i2}}{m_i} + R(L_i) \Big) - \frac{Q_{i4} r_{i4} C_{vi4}}{2} \Big(\frac{m_i D_{i4}}{P_{i2}} + 1 \Big) + \frac{Q_{i2} r_{i2}}{2} \Big(\Big(m_i + \frac{2D_{i2}}{P_{i4}} \Big) C_{vi2} + C_{bi2} \Big) \\
&\quad + r_{i2} C_{bi2} k\sigma \sqrt{L_i} + Q_i VEC_{vi} + m_i (FTC_{vi} + VTC_{vi} + FEC_{vi}) \Bigg) \\
&\quad \cdot \Bigg(\frac{D_{i3}}{Q_{i3}} \Big(A_{i3} + \frac{S_{i3}}{m_i} + R(L_i) \Big) - \frac{Q_{i3} r_{i3} C_{vi3}}{2} \Big(\frac{m_i D_{i3}}{P_{i3}} + 1 \Big) + \frac{Q_{i3} r_{i3}}{2} \Big(\Big(m_i + \frac{2D_{i3}}{P_{i3}} \Big) C_{vi3} + C_{bi3} \Big) \\
&\quad + r_{i3} C_{bi3} k\sigma \sqrt{L_i} + Q_{i3} VEC_{vi3} + m_i (FTC_{vi} + VTC_{vi3} + FEC_{vi}) \Bigg) \\
&\quad \cdot \Bigg(\frac{D_{i4}}{Q_{i2}} \Big(A_{i4} + \frac{S_{i4}}{m_i} + R(L_i) \Big) - \frac{Q_{i2} r_{i2} C_{vi2}}{2} \Big(\frac{m_i D_{i2}}{P_{i4}} + 1 \Big) + \frac{Q_{i4} r_{i4}}{2} \Big(\Big(m_i + \frac{2D_{i4}}{P_{i2}} \Big) C_{vi4} + C_{bi4} \Big) \\
&\quad + r_{i4} C_{bi4} k\sigma \sqrt{L_i} + Q_{i4} VEC_{vi4} + m_i (FTC_{vi} + VTC_{vi4} + FEC_{vi}) \Bigg) \\
&\quad \cdot \Bigg(\frac{D_{i5}}{Q_{i1}} \Big(A_{i5} + \frac{S_{i5}}{m_i} + R(L_i) \Big) - \frac{Q_{i1} r_{i1} C_{vi1}}{2} \Big(\frac{m_i D_{i1}}{P_{i5}} + 1 \Big) + \frac{Q_{i5} r_{i5}}{2} \Big(\Big(m_i + \frac{2D_{i5}}{P_{i2}} \Big) C_{vi5} + C_{bi5} \Big) \\
&\quad + r_{i5} C_{bi5} k\sigma \sqrt{L_i} + Q_{i5} VEC_{vi5} + m_i (FTC_{vi} + VTC_{vi5} + FEC_{vi}) \Bigg) \Bigg]. \qquad (14)
\end{aligned}
$$

Also, the Graded mean integration representation of $I\tilde{T}CMI(\tilde{Q}_i, L_i, m_i)$ is obtained by eq. (12) as

$$
\begin{aligned}
&P(I\tilde{T}CMI(\tilde{Q}_i, L_i, m_i))\\
&= \sum_{i=1}^{n} \Bigg[\frac{1}{2} \Bigg(\frac{D_{i1}}{Q_{i5}} \Big(A_{i1} + \frac{S_{i1}}{m_i} + R(L_i) \Big) - \frac{Q_{i5} r_{i5} C_{vi5}}{2} \Big(\frac{m_i D_{i5}}{P_{i1}} + 1 \Big) + \frac{Q_{i1} r_{i1}}{2} \Big(\Big(m_i + \frac{2D_{i1}}{P_{i5}} \Big) C_{vi1} + C_{bi1} \Big) \\
&\quad + r_{i1} C_{bi1} k\sigma \sqrt{L_{i1}} + Q_{i1} VEC_{vi1} + m_i (FTC_{vi} + VTC_{vi1} + FEC_{vi}) \Bigg) \\
&\quad + \frac{3}{12} \Bigg(\frac{D_{i2}}{Q_{i4}} \Big(A_{i2} + \frac{S_{i2}}{m_i} + R(L_i) \Big) - \frac{Q_{i4} r_{i4} C_{vi4}}{2} \Big(\frac{m_i D_{i4}}{P_{i2}} + 1 \Big) + \frac{Q_{i2} r_{i2}}{2} \Big(\Big(m_i + \frac{2D_{i2}}{P_{i4}} \Big) C_{vi2} + C_{bi2} \Big) \\
&\quad + r_{i2} C_{bi2} k\sigma \sqrt{L_i} + Q_i VEC_{vi} + m_i (FTC_{vi} + VTC_{vi} + FEC_{vi}) \Bigg) \\
&\quad + \frac{4}{12} \Bigg(\frac{D_{i3}}{Q_{i3}} \Big(A_{i3} + \frac{S_{i3}}{m_i} + R(L_i) \Big) - \frac{Q_{i3} r_{i3} C_{vi3}}{2} \Big(\frac{m_i D_{i3}}{P_{i3}} + 1 \Big) + \frac{Q_{i3} r_{i3}}{2} \Big(\Big(m_i + \frac{2D_{i3}}{P_{i3}} \Big) C_{vi3} + C_{bi3} \Big)
\end{aligned}
$$

$$+r_{i3}C_{bi3}k\sigma\sqrt{L_i}+Q_{i3}VEC_{vi3}+m_i(FTC_{vi}+VTC_{vi3}+FEC_{vi})\Big)$$
$$+\frac{3}{12}\Big(\frac{D_{i4}}{Q_{i2}}\Big(A_{i4}+\frac{S_{i4}}{m_i}+R(L_i)\Big)-\frac{Q_{i2}r_{i2}C_{vi2}}{2}\Big(\frac{m_iD_{i2}}{P_{i4}}+1\Big)+\frac{Q_{i4}r_{i4}}{2}\Big(\Big(m_i+\frac{2D_{i4}}{P_{i2}}\Big)C_{vi4}+C_{bi4}\Big)$$
$$+r_{i4}C_{bi4}k\sigma\sqrt{L_i}+Q_{i4}VEC_{vi4}+m_i(FTC_{vi}+VTC_{vi4}+FEC_{vi})\Big)$$
$$+\frac{1}{12}\Big(\frac{D_{i5}}{Q_{i1}}\Big(A_{i5}+\frac{S_{i5}}{m_i}+R(L_i)\Big)-\frac{Q_{i1}r_{i1}C_{vi1}}{2}\Big(\frac{m_iD_{i1}}{P_{i5}}+1\Big)+\frac{Q_{i5}r_{i5}}{2}\Big(\Big(m_i+\frac{2D_{i5}}{P_{i2}}\Big)C_{vi5}+C_{bi5}\Big)$$
$$+r_{i5}C_{bi5}k\sigma\sqrt{L_i}+Q_{i5}VEC_{vi5}+m_i(FTC_{vi}+VTC_{vi5}+FEC_{vi})\Big)\Big]. \tag{15}$$

with $0<Q_{i_1}\le Q_{i_2}\le Q_{i_3}\le Q_{i_4}\le Q_{i_5}$. Exchange the inequality condition $0<Q_{i_1}\le Q_{i_2}\le Q_{i_3}\le Q_{i_4}\le Q_{i_5}$ to $Q_{i2}-Q_{i1}\ge 0$, $Q_{i3}-Q_{i2}\ge 0$, $Q_{i4}-Q_{i3}\ge 0$, $Q_{i5}-Q_{i4}\ge 0$ and $Q_{i1}>0$, eq. (15) will remain the same.

In the resulting steps, addition of Kuhn-Tucker process is used to get $Q_{i1},Q_{i2},Q_{i3},Q_{i4}$, and Q_{i5} to minimize $P[I\tilde{T}CMI(\tilde{Q}_i,L_i,m_i)]$ in eq. (15). Then we resolve the unconstraint system in order to find the minimization of $P[I\tilde{T}CMI(\tilde{Q}_i,L_i,m_i)]$. We find the partial derivatives of $P[I\tilde{T}CMI(\tilde{Q}_i,L_i,m_i)]$ with respect to $Q_{i1},Q_{i2},Q_{i3},Q_{i4}$, and Q_{i5} are equate to zero as follows:

$$\frac{\partial P[I\tilde{T}CMI(\tilde{Q}_i,L_i,m_i)]}{\partial Q_{i1}}=0,$$
$$\frac{\partial P[I\tilde{T}CMI(\tilde{Q}_i,L_i,m_i)]}{\partial Q_{i2}}=0,$$
$$\frac{\partial P[I\tilde{T}CMI(\tilde{Q}_i,L_i,m_i)]}{\partial Q_{i3}}=0,$$
$$\frac{\partial P[I\tilde{T}CMI(\tilde{Q}_i,L_i,m_i)]}{\partial Q_{i4}}=0,\ \text{and}$$
$$\frac{\partial P[I\tilde{T}CMI(\tilde{Q}_i,L_i,m_i)]}{\partial Q_{i5}}=0,$$

then

$$\frac{1}{12}\Big[-\frac{D_{i5}}{Q_{i1}^2}\Big(A_{i5}+\frac{S_{i5}}{m_i}+R(L_i)\Big)-\frac{r_{i1}C_{vi1}}{2}\Big(\frac{m_iD_{i1}}{P_{i5}}+1\Big)$$
$$+\frac{r_{i1}}{2}\Big(\Big(m_i+\frac{2D_{i1}}{P_{i5}}\Big)C_{vi1}+C_{bi1}\Big)+VEC_{vi1}\Big]=0, \tag{16}$$

$$\frac{3}{12}\Big[-\frac{D_{i4}}{Q_{i2}^2}\Big(A_{i4}+\frac{S_{i4}}{m_i}+R(L_i)\Big)-\frac{r_{i2}C_{vi2}}{2}\Big(\frac{m_iD_{i2}}{P_{i4}}+1\Big)$$
$$+\frac{r_{i2}}{2}\Big(\Big(m_i+\frac{2D_{i2}}{P_{i4}}\Big)C_{vi2}+C_{bi2}\Big)+VEC_{vi2}\Big]=0, \tag{17}$$

$$\frac{4}{12}\Big[-\frac{D_{i3}}{Q_{i3}^2}\Big(A_{i3}+\frac{S_{i3}}{m_i}+R(L_i)\Big)-\frac{r_{i3}C_{vi3}}{2}\Big(\frac{m_iD_{i3}}{P_{i3}}+1\Big)$$
$$+\frac{r_{i3}}{2}\Big(\Big(m_i+\frac{2D_{i3}}{P_{i3}}\Big)C_{vi3}+C_{bi3}\Big)+VEC_{vi3}\Big]=0, \tag{18}$$

$$\frac{3}{12}\Big[-\frac{D_{i2}}{Q_{i4}^2}\Big(A_{i2}+\frac{S_{i2}}{m_i}+R(L_i)\Big)-\frac{r_{i4}C_{vi4}}{2}\Big(\frac{m_iD_{i4}}{P_{i2}}+1\Big)$$

$$+\frac{r_{i4}}{2}\Big(\Big(m_i+\frac{2D_{i4}}{P_{i2}}\Big)C_{vi4}+C_{bi4}\Big)+VEC_{vi4}\Big]=0, \tag{19}$$

$$\frac{1}{12}\Big[-\frac{D_{i1}}{Q_{i5}^2}\Big(A_{i1}+\frac{S_{i1}}{m_i}+R(L_i)\Big)-\frac{r_{i5}C_{vi5}}{2}\Big(\frac{m_iD_{i5}}{P_{i1}}+1\Big)$$
$$+\frac{r_{i5}}{2}\Big(\Big(m_i+\frac{2D_{i5}}{P_{i1}}\Big)C_{vi5}+C_{bi5}\Big)+VEC_{vi5}\Big]=0. \tag{20}$$

By solving eqs. (16) to (20), we obtain the optimal order quantity for $Q_{i1}, Q_{i2}, Q_{i3}, Q_{i4}$, and Q_{i5}. They are

$$Q_{i1}=\sqrt{\frac{2D_{i5}\Big(A_{i5}+\frac{S_{i5}}{m_i}+R(L_i)\Big)}{r_{i1}\Big(\Big(m_i\Big(1-\frac{D_{i1}}{P_{i5}}\Big)-1+\frac{2D_{i1}}{P_{i5}}\Big)C_{vi1}+C_{bi1}\Big)+2VEC_{vi1}}}, \tag{21}$$

$$Q_{i2}=\sqrt{\frac{6D_{i4}\Big(A_{i4}+\frac{S_{i4}}{m_i}+R(L_i)\Big)}{3\Big[r_{i2}\Big(\Big(m_i\Big(1-\frac{D_{i2}}{P_{i4}}\Big)-1+\frac{2D_{i2}}{P_{i4}}\Big)C_{vi2}+C_{bi2}\Big)+2VEC_{vi2}\Big]}}, \tag{22}$$

$$Q_{i3}=\sqrt{\frac{8D_{i3}\Big(A_{i3}+\frac{S_{i3}}{m_i}+R(L_i)\Big)}{4\Big[r_{i3}\Big(\Big(m_i\Big(1-\frac{D_{i3}}{P_{i3}}\Big)-1+\frac{2D_{i3}}{P_{i3}}\Big)C_{vi3}+C_{bi3}\Big)+2VEC_{vi3}\Big]}}, \tag{23}$$

$$Q_{i4}=\sqrt{\frac{6D_{i2}\Big(A_{i2}+\frac{S_{i2}}{m_i}+R(L_i)\Big)}{3\Big[r_{i4}\Big(\Big(m_i\Big(1-\frac{D_{i4}}{P_{i2}}\Big)-1+\frac{2D_{i4}}{P_{i2}}\Big)C_{vi4}+C_{bi4}\Big)+2VEC_{vi4}\Big]}}, \tag{24}$$

$$Q_{i5}=\sqrt{\frac{2D_{i1}\Big(A_{i1}+\frac{S_{i1}}{m_i}+R(L_i)\Big)}{\Big[r_{i5}\Big(\Big(m_i\Big(1-\frac{D_{i5}}{P_{i1}}\Big)-1+\frac{2D_{i5}}{P_{i1}}\Big)C_{vi5}+C_{bi5}\Big)+2VEC_{vi5}\Big]}}, \tag{25}$$

with $Q_{i5} \ge Q_{i4} \ge Q_{i3} \ge Q_{i2} \ge Q_{i1} > 0$. Thus, the optimal solution of given in eq. (15), subject to the following inequality constraints: $Q_{i1}-Q_{i2} \le 0$, $Q_{i2}-Q_{i3} \le 0$, $Q_{i3}-Q_{i4} \le 0$, $Q_{i4}-Q_{i5} \le 0$, $-Q_{i1} < 0$. An optimal solution to $P[I\tilde{T}CMI(\tilde{Q}_i, L_i, m_i)]$ be found by applying the Kuhn-Tucker conditions (refer Taha [20]) subject to five inequalities as imposed situations. The conditions are as follows:

$$\nabla P([I\tilde{T}CMI(\tilde{Q}_i, L_i, m_i)] - \lambda \nabla E(\tilde{Q}_i, L_i, m_i)] = 0,$$
$$\lambda E[(\tilde{Q}_i, L_i, m_i)] = 0, \ \ E[(\tilde{Q}_i, L_i, m_i)] \le 0, \text{ and } \lambda \ge 0.$$

The conditions shorten towards succeeding $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5 \ge 0$,

$$\nabla P[I\tilde{T}CMI(\tilde{Q}_i, L_i, m_i)] - \lambda_1(Q_{i1}-Q_{i2}) - \lambda_2(Q_{i2}-Q_{i3}) - \lambda_3(Q_{i3}-Q_{i4})$$
$$-\lambda_4(Q_{i4}-Q_{i5}) - \lambda_5(-Q_{i1}) = 0, \tag{26}$$

$$\frac{1}{12}\Big[-\frac{D_{i5}}{Q_{i1}^2}\Big(A_{i5}+\frac{S_{i5}}{m_i}+R(L_i)\Big)-\frac{r_{i1}C_{vi1}}{2}\Big(\frac{m_iD_{i1}}{P_{i5}}+1\Big)$$

$$+\frac{r_{i1}}{2}\left(\left(m_i+\frac{2D_{i1}}{P_{i5}}\right)C_{vi1}+C_{bi1}\right)+VEC_{vi1}\Big]-\lambda_1+\lambda_5=0, \tag{27}$$

$$\frac{3}{12}\Big[-\frac{D_{i4}}{Q_{i2}^2}\left(A_{i4}+\frac{S_{i4}}{m_i}+R(L_i)\right)-\frac{r_{i2}C_{vi2}}{2}\left(\frac{m_iD_{i2}}{P_{i4}}+1\right)$$

$$+\frac{r_{i2}}{2}\left(\left(m_i+\frac{2D_{i2}}{P_{i4}}\right)C_{vi2}+C_{bi2}\right)+VEC_{vi2}\Big]-\lambda_2+\lambda_1=0, \tag{28}$$

$$\frac{4}{12}\Big[-\frac{D_{i3}}{Q_{i3}^2}\left(A_{i3}+\frac{S_{i3}}{m_i}+R(L_i)\right)-\frac{r_{i3}C_{vi3}}{2}\left(\frac{m_iD_{i3}}{P_{i3}}+1\right)$$

$$+\frac{r_{i3}}{2}\left(\left(m_i+\frac{2D_{i3}}{P_{i3}}\right)C_{vi3}+C_{bi3}\right)+VEC_{vi3}\Big]-\lambda_3+\lambda_2=0, \tag{29}$$

$$\frac{3}{12}\Big[-\frac{D_{i2}}{Q_{i4}^2}\left(A_{i2}+\frac{S_{i2}}{m_i}+R(L_i)\right)-\frac{r_{i4}C_{vi4}}{2}\left(\frac{m_iD_{i4}}{P_{i2}}+1\right)$$

$$+\frac{r_{i4}}{2}\left(\left(m_i+\frac{2D_{i4}}{P_{i2}}\right)C_{vi4}+C_{bi4}\right)+VEC_{vi4}\Big]-\lambda_4+\lambda_3=0, \tag{30}$$

$$\frac{1}{12}\Big[-\frac{D_{i1}}{Q_{i5}^2}\left(A_{i1}+\frac{S_{i1}}{m_i}+R(L_i)\right)-\frac{r_{i5}C_{vi5}}{2}\left(\frac{m_iD_{i5}}{P_{i1}}+1\right)$$

$$+\frac{r_{i5}}{2}\left(\left(m_i+\frac{2D_{i5}}{P_{i1}}\right)C_{vi5}+C_{bi5}\right)+VEC_{vi5}\Big]+\lambda_4=0, \tag{31}$$

$$Q_{ij}-Q_{i(j+1)}\le 0, \quad j=1,2,3,4, \tag{32}$$

$$-Q_{i1}<0, \tag{33}$$

$$\lambda_j(Q_{ij}-Q_{i(j+1)})=0, \quad j=1,2,3,4, \tag{34}$$

$$\lambda_5(-Q_{i1})=0, \tag{35}$$

$$Q_{ij}\ge 0, \quad j=1,2,3,4,5,\ i=1,2,\dots,5 \text{ and } \lambda_j\ge 0. \tag{36}$$

Because $Q_{i1}>0$, and $\lambda_5Q_{i1}=0$, then $\lambda_5=0$. If $\lambda_1=\lambda_2=\lambda_3=\lambda_4=0$, then $Q_{i5}<Q_{i4}<Q_{i3}<Q_{i2}<Q_{i1}$, it does not satisfy the constraints $0<Q_{i1}\le Q_{i2}\le Q_{i3}\le Q_{i4}\le Q_{i5}$. Therefore, $Q_{i1}=Q_{i2}, Q_{i2}=Q_{i3}, Q_{i3}=Q_{i4}, Q_{i4}=Q_{i5}$, that is $Q_{i1}=Q_{i2}=Q_{i3}=Q_{i4}=Q_{i5}=\tilde{Q}_i^*$. Hence, from eqs. (27)-(36), we obtain the fuzzy i-th item's optimal order quantity $\tilde{Q}_i^*$ follows:

$$\tilde{Q}_i^*=\sqrt{\frac{\begin{pmatrix}2D_{i5}\left(A_{i5}+\frac{S_{i5}}{m_i}+R(L_i)\right)+6D_{i4}\left(A_{i4}+\frac{S_{i4}}{m_i}+R(L_i)\right)\\ +8D_{i3}\left(A_{i3}+\frac{S_{i3}}{m_i}+R(L_i)\right)+6D_{i2}\left(A_{i2}+\frac{S_{i2}}{m_i}+R(L_i)\right)\\ +2D_{i1}\left(A_{i1}+\frac{S_{i1}}{m_i}+R(L_i)\right)\end{pmatrix}}{\begin{pmatrix}\left[r_{i1}\left(\left(m_i\left(1-\frac{D_{i1}}{P_{i5}}\right)-1+\frac{2D_{i1}}{P_{i5}}\right)C_{vi1}+C_{bi1}\right)+2VEC_{vi1}\right]\\ +3\left[r_{i2}\left(\left(m_i\left(1-\frac{D_{i2}}{P_{i4}}\right)-1+\frac{2D_{i2}}{P_{i4}}\right)C_{vi2}+C_{bi2}\right)+2VEC_{vi2}\right]\\ \cdot 4\left[r_{i3}\left(\left(m_i\left(1-\frac{D_{i3}}{P_{i3}}\right)-1+\frac{2D_{i3}}{P_{i3}}\right)C_{vi3}+C_{bi3}\right)+2VEC_{vi3}\right]\\ +3\left[r_{i4}\left(\left(m_i\left(1-\frac{D_{i4}}{P_{i2}}\right)-1+\frac{2D_{i4}}{P_{i2}}\right)C_{vi4}+C_{bi4}\right)+2VEC_{vi4}\right]\\ +\left[r_{i5}\left(\left(m_i\left(1-\frac{D_{i5}}{P_{i1}}\right)-1+\frac{2D_{i5}}{P_{i1}}\right)C_{vi5}+C_{bi5}\right)+2VEC_{vi5}\right]\end{pmatrix}}}. \tag{37}$$

The optimum fuzzy integrated total cost for multi-item $P[I\tilde{T}CMI(\tilde{Q}_i, L_i, m_i)]$ is obtained by direct substitution of eq. (7) into eq. (15).

It is eminent that when the input parameters $D_i, A_i, S_i, r_i, P_i, C_{vi}, C_{bi}, VEC_{vi}$, and VTC_{vi} are real numbers, that is $D_{i_1} = D_{i_2} = D_{i_3} = D_{i_4} = D_{i_5} = D_i$, $A_{i_1} = A_{i_2} = A_{i_3} = A_{i_4} = A_{i_5} = A_i$, $r_{i_1} = r_{i_2} = r_{i_3} = r_{i_4} = r_{i_5} = r_i$, $S_{i_1} = S_{i_2} = S_{i_3} = S_{i_4} = S_{i_5} = S_i$, $P_{i_1} = P_{i_2} = P_{i_3} = P_{i_4} = P_{i_5} = P_i$, $C_{vi1} = C_{vi2} = C_{vi3} = C_{vi4} = C_{vi5} = C_{vi}$, $C_{bi1} = C_{bi2} = C_{bi3} = C_{bi4} = C_{bi5} = C_{bi}$, $VEC_{vi1} = VEC_{vi2} = VEC_{vi3} = VEC_{vi4} = VEC_{vi5} = VEC_{vi}$, and $VTC_{vi1} = VTC_{vi2} = VTC_{vi3} = VTC_{vi4} = VTC_{vi5} = VTC_{vi}$ Also assume that the decision variable Q_i is real number, $Q_{i1} = Q_{i2} = Q_{i3} = Q_{i4} = Q_{i5} = Q_i$. Then eq. (37) condensed as eq. (11) as

$$Q_i^* = Q_i = \sqrt{\frac{2D_i\left(A_i + \frac{S_i}{m_i} + R(L_i)\right)}{r_i\left(\left(m_i\left(1 - \frac{D_i}{P_i}\right) - 1 + \frac{2D_i}{P_i}\right)C_{v_i} + C_{b_i}\right) + 2VEC_{v_i}}}. \tag{38}$$

4.3 Algorithm for Inventory Systems

Multi-item order quantities are calculated using the subsequent algorithm to determine the optimal order quantity each item and then the minimum integrated total cost for multi-item. The optimal order quantity per item Q_i^* for the crisp situation and the minimum integrated total cost $ITCMI(Q_i, L_i, m_i)$ for the multi-item set are obtained using eqs. (11) and (8) individually. We obtain the optimal order quantity for each item $\tilde{Q}_i^*$ and the minimum integrated total cost for multi-item $P[I\tilde{T}CMI(\tilde{Q}_i, L_i, m_i)]$ in fuzzy situations based on eqs. (37) and (15) separately. Additionally, the relationships are specified aimed at together crisp system and fuzzy system.

Algorithm

Step 1: Compute optimal order quantity for each item and then find minimum integrated total cost for multi-item in the crisp system for the specified crisp standards of $D_i, P_i, C_{iv}, k, r_i, A_i, S_i, \sigma, L_i, R_i(L), m_i, FEC_{vi}, FTC_{vi}, VEC_{vi}$, and VTC_{vi}. At that point crisp optimal order quantity Q_i^* for each item and crisp minimum integrated total cost for multi-item $ITCMI(Q_i, L_i, m_i)$ are achieved.

Step 2: Obtain fuzzy minimum integrated total cost for multi-item utilizing fuzzy arithmetic processes on fuzzy buyer and vendor ordering cost, fuzzy inventory holding cost, fuzzy setup cost, fuzzy lead time crashing cost, fuzzy transportation cost and fuzzy carbon emission cost which is taken as pentagonal fuzzy number.

Step 3: For multi-item orders, defuzzify the integrated total cost $I\tilde{T}CMI(\tilde{Q}_i, L_i, m_i)$ using graded mean integration to determine the order quantity $\tilde{Q}_i^*$ which able to acquire by setting the first derivative of $P[I\tilde{T}CMI(\tilde{Q}_i, L_i, m_i)]$ is equal to zero.

Step 4: Utilize the Khun-Tucker technique in the direction of obtain the optimal order quantity for each item $\tilde{Q}_i^* = (Q_{i1}^*, Q_{i2}^*, Q_{i3}^*, Q_{i4}^*, Q_{i5}^*)$ in fuzzy sense, which is the distinct method for pentagonal fuzzy number. The fuzzy optimal order quantity for i-th item's $\tilde{Q}_i^* = (Q_{i1}^*, Q_{i2}^*, Q_{i3}^*, Q_{i4}^*, Q_{i5}^*)$ is attained by applying the first derivative of $P[I\tilde{T}CMI(\tilde{Q}_i, L_i, m_i)]$ that is equal to zero.

Step 5: Determine the crisp optimal order quantity for i-th item's Q_i^* obtained by derivative method and to attain the fuzzy optimal order quantity for i-th item's $\tilde{Q}_i^*$ by using Graded Mean Integration and addition of Kuhn-Tucker method.

Step 6: Analyze the integrated total cost for multi-item and optimal order quantity for each item in crisp and fuzzy system. If $I\tilde{T}CMI(\tilde{Q}_i, L_i, m_i) > P[I\tilde{T}CMI(\tilde{Q}_i, L_i, m_i)]$ and $Q_i^* > \tilde{Q}_i^*$ then the proposed fuzzy multi-item system is optimum to find the optimal order quantity for each item and minimum integrated total cost for multi-item, else $Q_i^* < \tilde{Q}_i^*$ and $I\tilde{T}CMI(\tilde{Q}_i, L_i, m_i) < P[I\tilde{T}CMI(\tilde{Q}_i, L_i, m_i)]$ then the crisp multi-item model is the premium to find the optimal order quantity for each item and minimum integrated total cost for multi-item.

Step 7: Compare the minimum integrated total cost for multi-item, optimal order quantity for each item obtained from both fuzzy, crisp multi-item inventory system and with their profit percentages.

5. Numerical Example

Numerical cases are specified towards establish the outcome technique utilizing the suggested algorithm. Subsequently relating that, the finest multi-item inventory system is recognised. The results achieved through Matlab software and then suggested fuzzy multi-item inventory system can be used in businesses such as vehicles, tires, healthcare products, computer hardware, textiles, home appliance things (refrigerators, televisions, air conditioners, and washing machines), massive objects like produced trip panels, and cell phones, so on. The projected integrated multi-item inventory system is extra effective aimed at the supply chain business progression of vendor-buyer administration.

5.1 Multi-item's Crisp Inventory System

Example 5.1. The result demonstrate to crisp model, now we study the system with initial input taken in Pan and Yang [17], and remaining input is made-up permitting to the problem. Number of item $i = 4$; $P_i = (3200, 3520, 3840, 4160)$ units/year, $D_i = (1000, 1100, 1200, 1300)$ units/year, $r_i = (0.2, 0.22, 0.24, 0.26)$, $k = 2.33$, $S_i = \$(400, 440, 480, 520)$/setup, $C_{vi} = (20, 22, 24, 26)$/units, $C_{bi} = (25, 27.5, 30, 32.5)$/units, $\sigma = 7$ units/week, $FEC_{vi} = \$0.2$/shipment, $FTC_{vi} = \$0.2$/shipment, $VEC_{vi} = \$(0.1, 0.11, 0.12, 0.13)$/units and $VTC_v = \$(0.5, 0.55, 0.6, 0.65)$/units. In extension, we consider $A_i = \$25.00$/order, \$23.75/order, \$22.50/order and \$21.87/order, $L_i = 3, 4, 6$, and 8 weeks, $m_i = 3, 4, 5$, and $R(L_i) = \$53.2, \$18.2, \$1.4$, and \$0. Using eqs. (11) and (8) respectively, optimal order quantity for each item Q_i^* and minimum integrated total cost for multi-item $ITCMI(Q_i, L_i, m_i)$ are achieved. The outcomes are presented in Table 1. The optimal standards for crisp optimal order quantity for each item $Q_i^* = (109.46, 108.33, 107.38, 106.57)$ units and crisp minimum integrated total cost for multi-item $ITCMI(Q_i, L_i, m_i) = \$11059.0$ when lead time $L_i = 6$ weeks.

5.2 Multi-item's Fuzzy Inventory System

Example 5.2. The input exists similar for Example 5.1, but the fuzzy inputs are $D_{i1} = (900, 990, 1080, 1170)$ units/year, $D_{i2} = (950, 1045, 1140, 1235)$ units/year, $D_{i3} = (1000, 1100, 1200, 1300)$ units/year, $D_{i4} = (1050, 1155, 1260, 1365)$ units/year, $D_{i5} = (1100, 1210, 1320, 1430)$ units/year,

$P_{i1} = (2880, 3168, 3456, 3744)$ units/year, $P_{i2} = (3040, 3344, 3648, 3952)$ units/year, $P_{i3} = (3200, 3520, 3840, 4160)$ units/year, $P_{i4} = (3360, 3696, 4032, 4368)$ units/year, $P_{i5} = (3520, 3872, 4224, 4576)$ units/year, $C_{vi1} = (18, 19.8, 21.6, 23.4)$/units, $C_{vi2} = (19, 20.9, 22.8, 24.7)$/units, $C_{vi3} = (20, 22, 24, 26)$/ units, $C_{vi4} = (21, 23.1, 25.2, 27.3)$/units, $C_{vi5} = (22, 24.2, 26.4, 28.6)$/units, $r_{i1} = (0.18, 0.198, 0.216, 0.234)$, $r_{i2} = (0.19, 0.209, 0.228, 0.247)$, $r_{i3} = (0.2, 0.22, 0.24, 0.26)$, $r_{i4} = (0.21, 0.231, 0.252, 0.273)$, $r_{i5} = (0.22, 0.242, 0.264, 0.286)$, $S_{i1} = \$(360, 396, 432, 468)$/setup, $S_{i2} = \$(380, 418, 456, 494)$/setup, $S_{i3} = \$(400, 440, 480, 520)$/setup, $S_{i4} = \$(420, 462, 504, 546)$/setup, $S_{i5} = \$(440, 484, 528, 572)$/setup, $C_{bi1} = (22.5, 24.75, 27, 29.25)$/units, $C_{bi2} = (23.75, 26.125, 28.5, 30.875)$/units, $C_{bi3} = (25, 27.5, 30, 32.5)$/units, $C_{bi4} = (26.25, 28.875, 31.5, 34.125)$/units, $C_{bi5} = (27.5, 30.25, 33, 35.75)$/units, $VTC_{vi1} = \$(0.45, 0.495, 0.54, 0.585)$/units, $VTC_{vi2} = \$(0.475, 0.5225, 0.57, 0.6175)$/units, $VTC_{vi3} = \$(0.5, 0.55, 0.6, 0.65)$/units, $VTC_{vi4} = \$(0.525, 0.5775, 0.63, 0.6825)$/units, $VTC_{vi5} = \$(0.55, 0.605, 0.66, 0.715)$/units, $VEC_{vi1} = \$(0.09, 0.099, 0.108, 0.117)$/units, $VEC_{vi2} = \$(0.095, 0.1045, 0.114, 0.1235)$/ units, $VEC_{vi3} = \$(0.1, 0.11, 0.12, 0.13)$/units, $VEC_{vi4} = \$(0.105, 0.1155, 0.126, 0.1365)$/units, $VEC_{vi5} = \$(0.11, 0.121, 0.132, 0.143)$/units.

The proposed algorithm produces the outcome as presented in Table 1. Fuzzy order quantity for each item $\tilde{Q}_i = (Q_{i1}, Q_{i2}, Q_{i3}, Q_{i4}, Q_{i5})$ with $0 < Q_{i1} \le Q_{i2} \le Q_{i3} \le Q_{i4} \le Q_{i5}$ using eqs. (37) and (15) respectively, fuzzy optimal order quantity for each item $\tilde{Q}_i^*$ and fuzzy minimum integrated total cost for multi-item $P[I\tilde{TCMI}(\tilde{Q}_i, L_i, m_i)]$ are obtained. The optimal values for fuzzy optimal order quantity for each item $\tilde{Q}_i^* = (109.03, 107.96, 107.06, 106.29)$ units and fuzzy minimum integrated total cost for multi-item $P[I\tilde{TCMI}(\tilde{Q}_i, L_i, m_i)] = \$11026.0$ when lead time $L_i = 6$ weeks.

Table 1 displays crisp situation, while $L_i = 3, 4, 6$, and 8 weeks and varies ordering cost $A_1 = [21.87, 22.5, 23.75, 25]$/order, $A_2 = [21.09, 21.87, 23.46, 24.99]$/order, $A_3 = [19.79, 20.83, 22.91, 24.99]$/order, $A_4 = [17.25, 18.81, 21.93, 25.05]$/order, we get the optimal order quantity for each item Q_i^* and minimum integrated total cost for multi-item $ITCMI(Q_i, L_i, m_i)$ which ranges from $(188.34, 185.23, 182.60, 180.35)$ units to $(109.46, 108.33, 107.38, 106.57)$ units and from \$12368.0 to \$11059.0 correspondingly. The outcomes aimed at Table 1 demonstrate while lead time raises optimal order quantity for each item Q_i^* decreases and terminal stage slightly increases and then minimum integrated total cost for multi-item $ITCMI(Q_i, L_i, m_i)$ primarily drops and raises future.

Also, Table 1 shows fuzzy situation while $L_i = 3, 4, 6$, and 8 weeks varies ordering costs of A_1, A_2, A_3, A_4. We get the fuzzy optimal order quantity for each item $\tilde{Q}_i^*$ decreases and fuzzy minimum integrated total cost for multi-item $P[I\tilde{TCMI}(\tilde{Q}_i, L_i, m_i)]$ which ranges from $(186.87, 183.93, 181.43, 179.29)$ units to $(109.03, 107.96, 107.06, 106.29)$ units and from \$12336.0 to \$11026.0 correspondingly. The outcomes prove that fuzzy order $\tilde{Q}_i^*$ and total cost $P[I\tilde{TCMI}(\tilde{Q}_i, L_i, m_i)]$ primarily drop and rises far ahead after lead time rises. Also abbreviated minimum integrated total cost for multi-item and optimal order quantity for each item is presented in Table 2 and that remains savings whereas consuming the fuzzy multi-item inventory system ranges from 0.26% to 0.30% and from $(0.26, 0.31, 0.26, 0.26)$% to $(0.78, 0.71, 0.64, 0.59)$%, respectively. Our results specify the decision variable and total cost results of fuzzy multi-item situation marginally vary from the results of the crisp multi-item situation and this is presented.

Table 1. Crisp and fuzzy multi-item optimal solutions

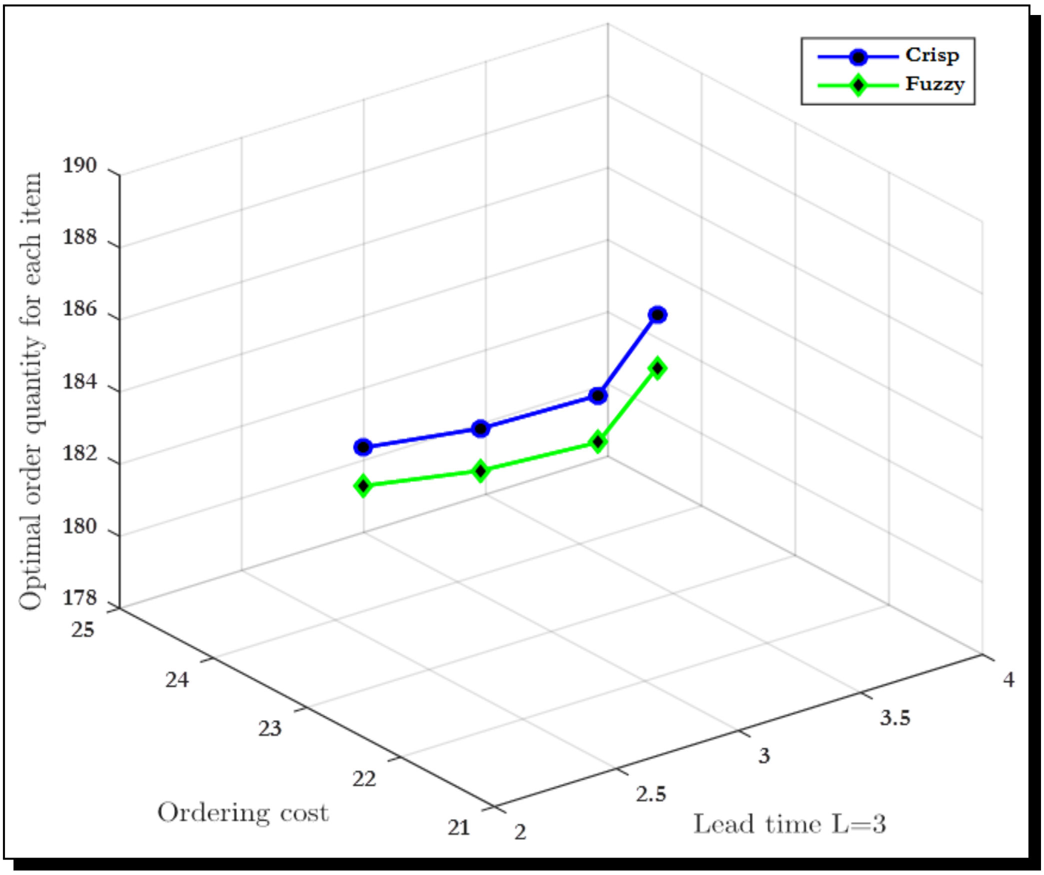
Crisp parameter	Fuzzy parameter	L_i	Q_i^*	Crisp minimum integrated total cost for multi-item	$\tilde{Q}_i^*$	Fuzzy minimum integrated total cost for multi-item
If $m_i = [3, 4, 5, 5]$, $A_1 = [21.87, 22.5, 23.75, 25]$	If $A_{i1} = [19.683, 20.25, 21.375, 22.5]$, $A_{i2} = [20.777, 21.375, 22.563, 23.75]$, $A_{i3} = [21.87, 22.5, 23.75, 25]$, $A_{i4} = [22.964, 23.625, 24.938, 26.25]$, $A_{i5} = [24.057, 24.75, 26.125, 27.5]$	3	(188.34, 185.23, 182.60, 180.35)	12368.0	(186.87, 183.93, 181.43, 179.29)	12336.0
		4	(139.31, 137.47, 135.91, 134.58)	11517.0	(138.54, 136.79, 135.32, 134.05)	11484.0
		6	(110.42, 109.21, 108.19, 107.33)	11136.0	(109.99, 108.84, 107.88, 107.05)	11103.0
		8	(110.34, 109.14, 108.13, 107.26)	11295.0	(109.91, 108.77, 107.81, 106.99)	11262.0
If $m_i = [3, 4, 5, 5]$, $A_2 = [21.09, 21.87, 23.46, 24.99]$	If $A_{i1} = [18.981, 19.683, 21.114, 22.491]$, $A_{i2} = [20.036, 20.777, 22.287, 23.741]$, $A_{i3} = [21.09, 21.87, 23.46, 24.99]$, $A_{i4} = [22.145, 22.964, 24.633, 26.24]$, $A_{i5} = [23.199, 24.057, 25.806, 27.489]$	3	(187.99, 184.91, 182.30, 180.06)	12348.0	(186.52, 183.60, 181.13, 179.01)	12316.0
		4	(139.00, 137.18, 135.64, 134.33)	11495.0	(138.23, 136.51, 135.05, 133.81)	11463.0
		6	(110.26, 109.07, 108.06, 107.20)	11124.0	(109.84, 108.70, 107.75, 106.93)	11091.0
		8	(110.33, 109.13, 108.12, 107.26)	11295.0	(109.91, 108.77, 107.81, 106.99)	11262.0

Table *Contd.*

Crisp parameter	Fuzzy parameter	L_i	Q_i^*	Crisp minimum integrated total cost for multi-item	$\tilde{Q}_i^*$	Fuzzy minimum integrated total cost for multi-item
If $m_i = [3, 4, 5, 5]$, $A_3 = [19.79, 20.83, 22.91, 24.99]$	If $A_{i1} = [17.811, 18.747, 20.619, 22.491]$, $A_{i2} = [18.801, 19.789, 21.765, 23.741]$, $A_{i3} = [19.79, 20.83, 22.91, 24.99]$, $A_{i4} = [20.78, 21.872, 24.056, 26.24]$, $A_{i5} = [21.769, 22.913, 25.201, 27.489]$	3	(187.40, 184.36, 181.79, 179.59)	12315.0	(185.93, 183.06, 180.63, 178.54)	12283.0
		4	(138.48, 136.70, 135.20, 133.92)	11460.0	(137.72, 136.03, 134.61, 133.40)	11428.0
		6	(109.97, 108.80, 107.82, 106.98)	11101.0	(109.55, 108.44, 107.50, 106.70)	11068.0
		8	(110.33, 109.13, 108.12, 107.26)	11295.0	(109.91, 108.77, 107.81, 106.99)	11262.0
If $m_i = [3, 4, 5, 5]$, $A_4 = [17.25, 18.81, 21.93, 25.05]$	If $A_{i1} = [15.525, 16.929, 19.737, 22.545]$, $A_{i2} = [16.388, 17.87, 20.834, 23.798]$, $A_{i3} = [17.25, 18.81, 21.93, 25.05]$, $A_{i4} = [18.113, 19.751, 23.027, 26.303]$, $A_{i5} = [18.975, 20.691, 24.123, 27.555]$	3	(186.24, 183.29, 180.80, 178.66)	12251.0	(184.78, 182.00, 179.64, 177.61)	12219.0
		4	(137.47, 135.77, 134.34, 133.12)	11391.0	(136.71, 135.11, 133.75, 132.60)	11359.0
		6	**(109.46, 108.33, 107.38, 106.57)**	**11059.0**	**(109.03, 107.96, 107.06, 106.29)**	**11026.0**
		8	(110.36, 109.16, 108.15, 107.28)	11297.0	(109.94, 108.80, 107.83, 107.01)	11265.0

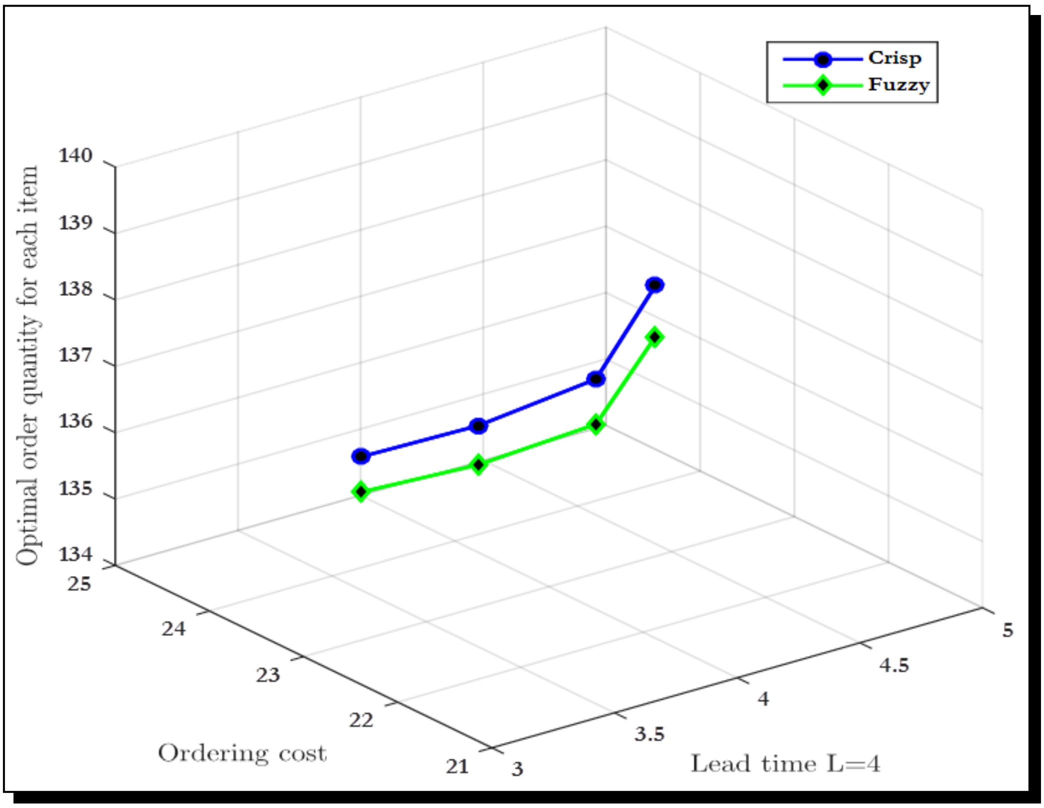
5.3 Graphical Representation

The optimal order quantity for each item dissimilar values of lead time and the ordering cost is related mutually in the fuzzy and crisp multi-item systems, as shown in the graphical representation of Figure 4(a)-(p). It is vibrant that optimal order quantity for each item Q_i^* and $\tilde{Q}_i^*$ decrease and terminal stage slightly increase while the lead time rises. That one stays perceived the optimal order quantity each item is efficiently enhanced happening in fuzzy multi-item system after related to the crisp multi-item system. The subsequent graphical representation of integrated total cost for multi-item beside by the lead time and various ordering costs are related both in the fuzzy and crisp multi-item system as exposed in Figure 5(a)-(d). The both crisp and fuzzy integrated total cost for multi-items $ITCMI(Q_i, L_i, m_i)$ and $P[I\tilde{T}CMI(\tilde{Q}_i, L_i, m_i)]$ is decline primarily and then starts toward raise future while the lead time rises. That one remains observed the integrated total cost for multi-item stays successfully decreased in the fuzzy multi-item system after related toward the crisp multi-item system.

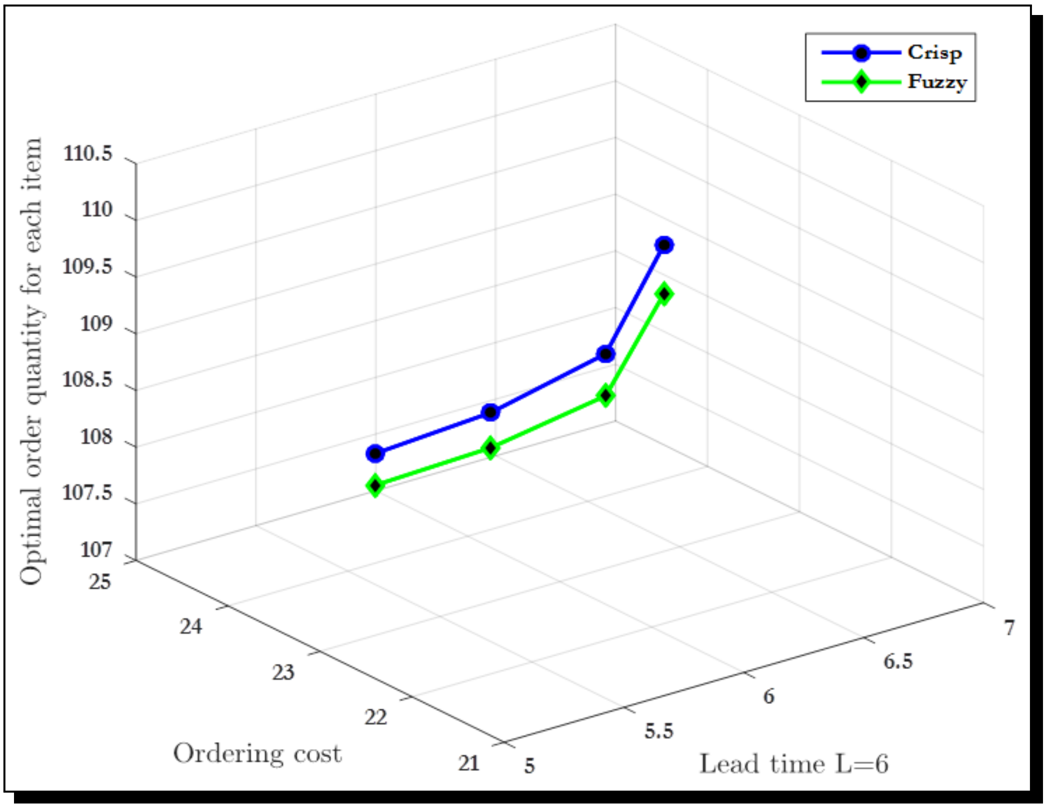


(a) Lead time $L = 3$

Figure *Contd.*

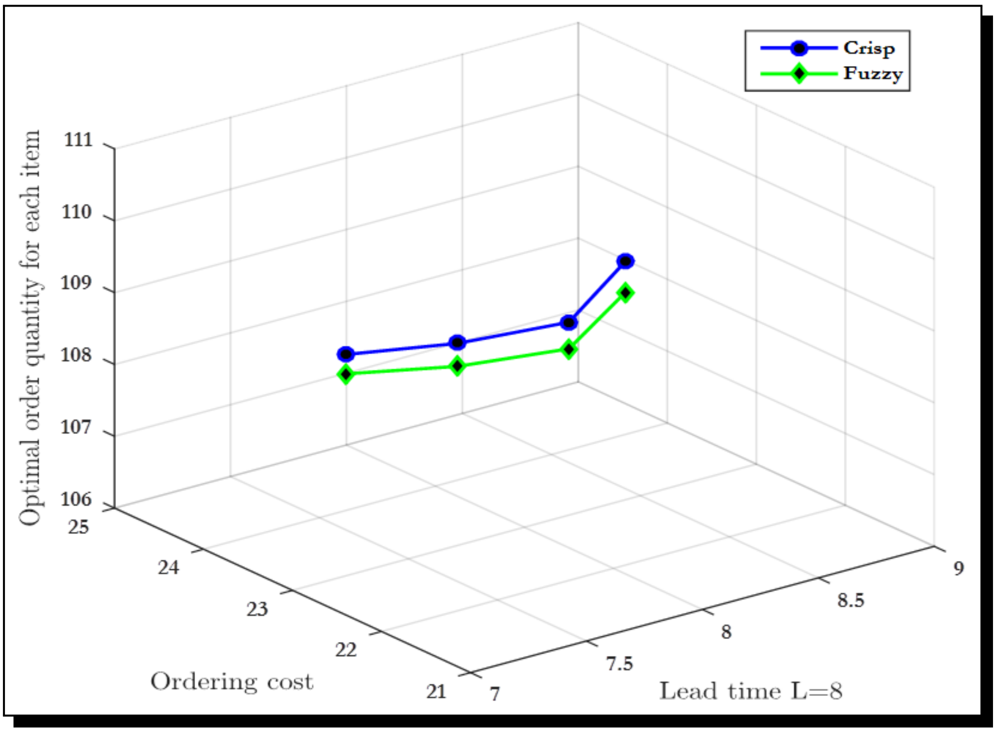


(b) Lead time $L = 4$



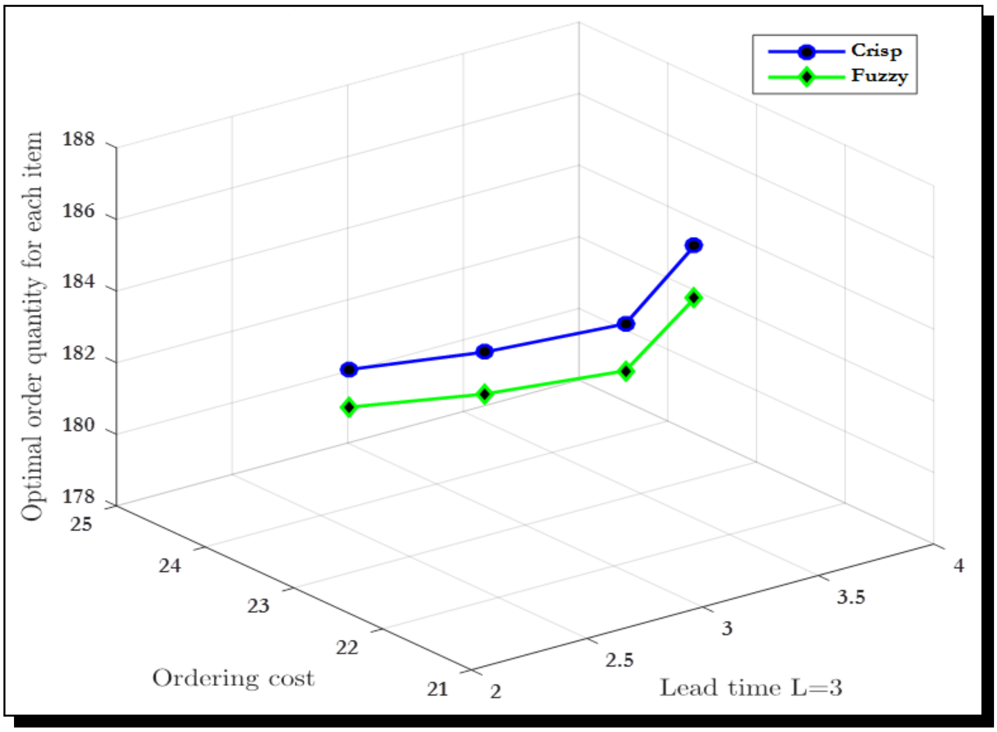
(c) Lead time $L = 6$

Figure *Contd.*



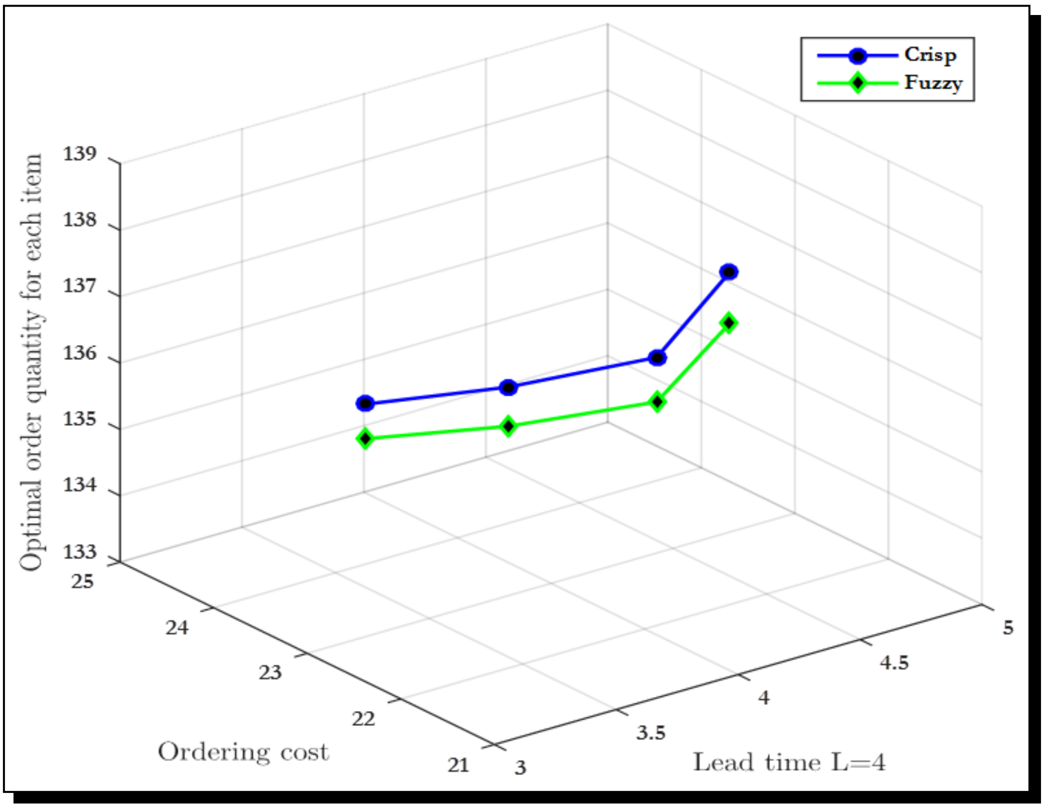
(d) Lead time $L = 8$

Figure 4. (a)-(d): Graphical representation of optimal order quantity for each item versus ordering cost $A_1 = [21.87, 22.5, 23.75, 25]$ and lead time

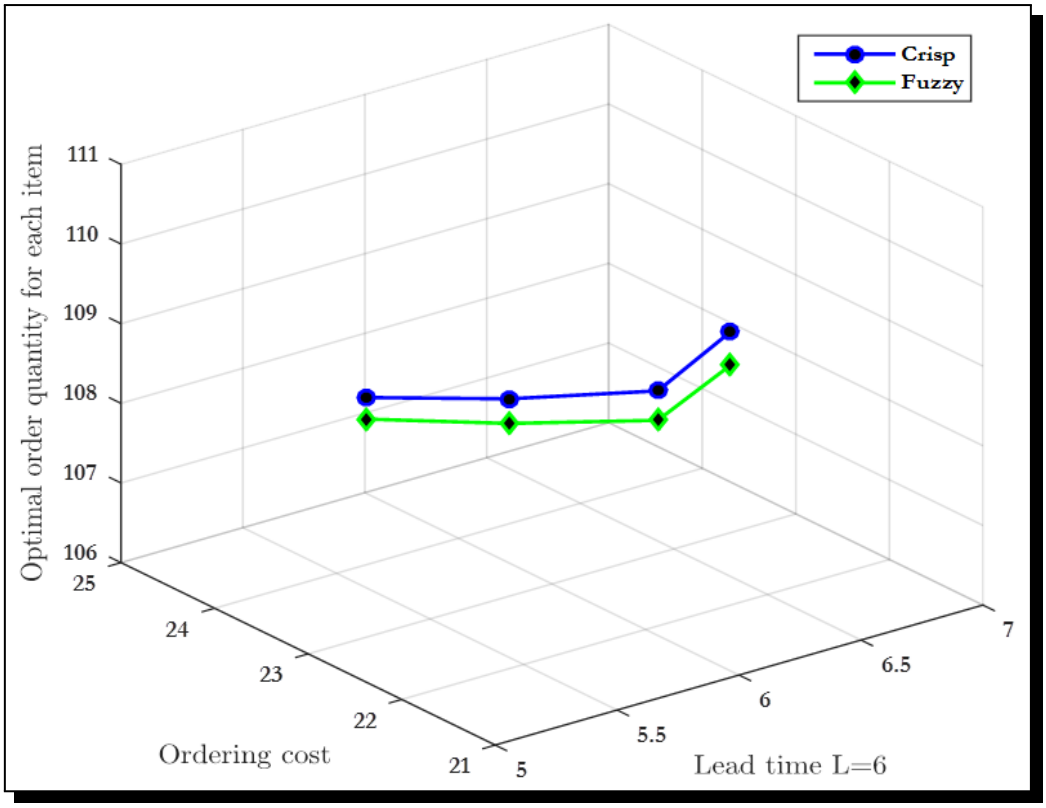


(e) Lead time $L = 3$

Figure *Contd.*

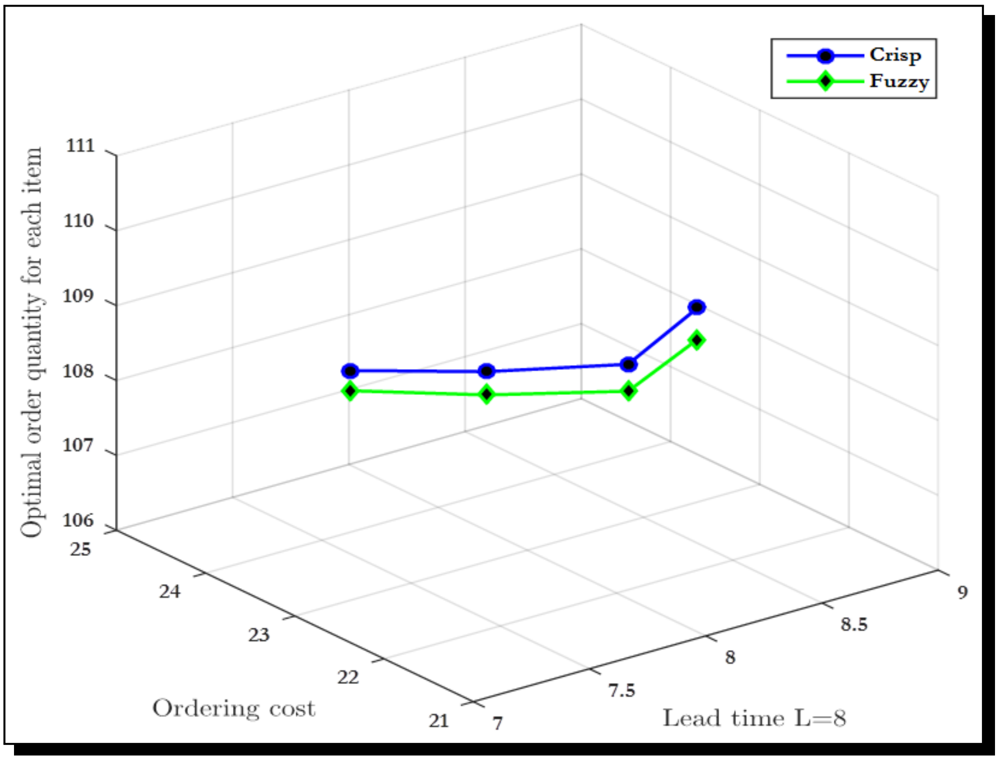


(f) Lead time $L = 4$



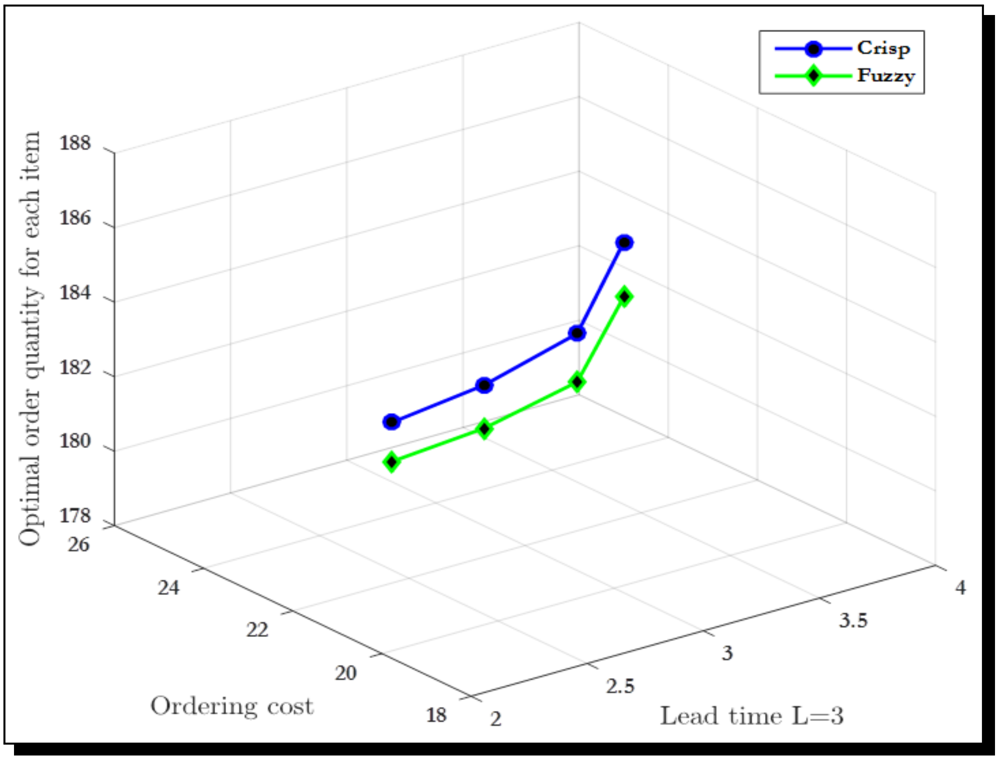
(g) Lead time $L = 6$

Figure *Contd.*



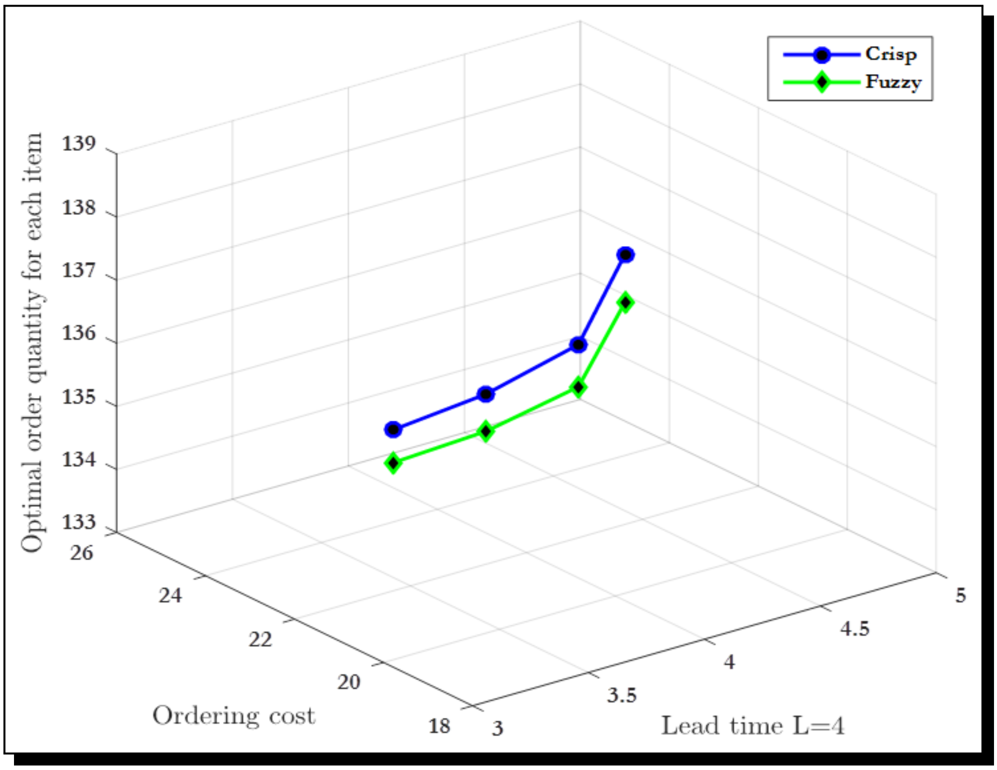
(h) Lead time $L = 8$

Figure 4. (e)-(h): Graphical representation of optimal order quantity for each item versus ordering cost $A_2 = [21.09, 21.87, 23.46, 24.99]$ and lead time

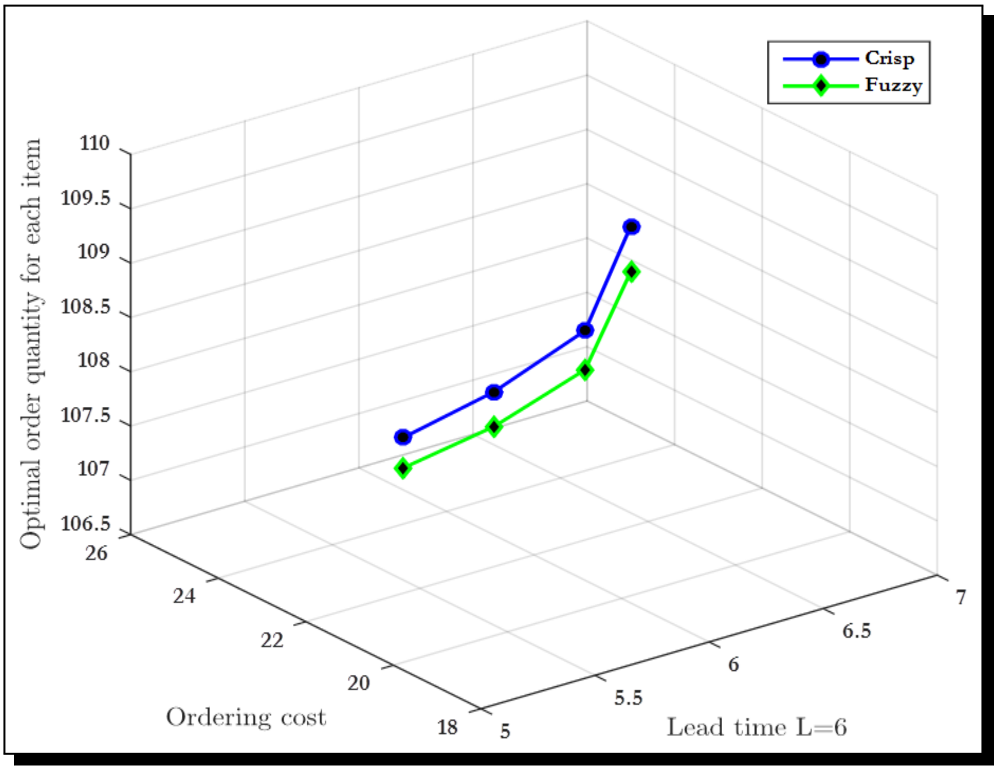


(i) Lead time $L = 3$

Figure *Contd.*

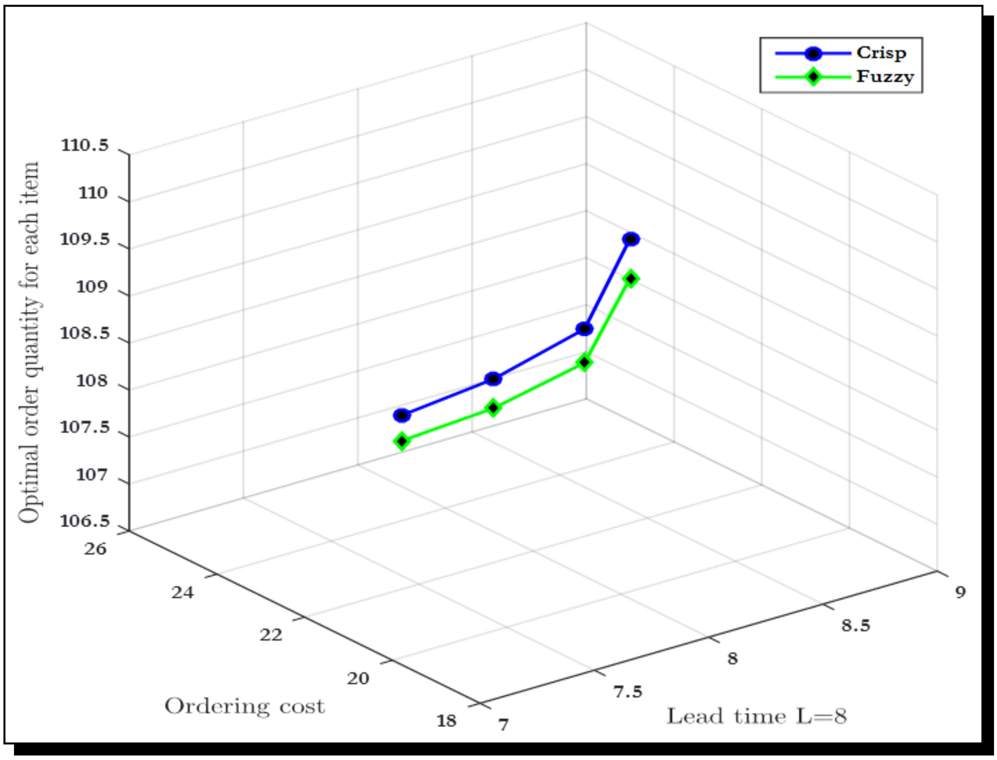


(j) Lead time $L = 4$



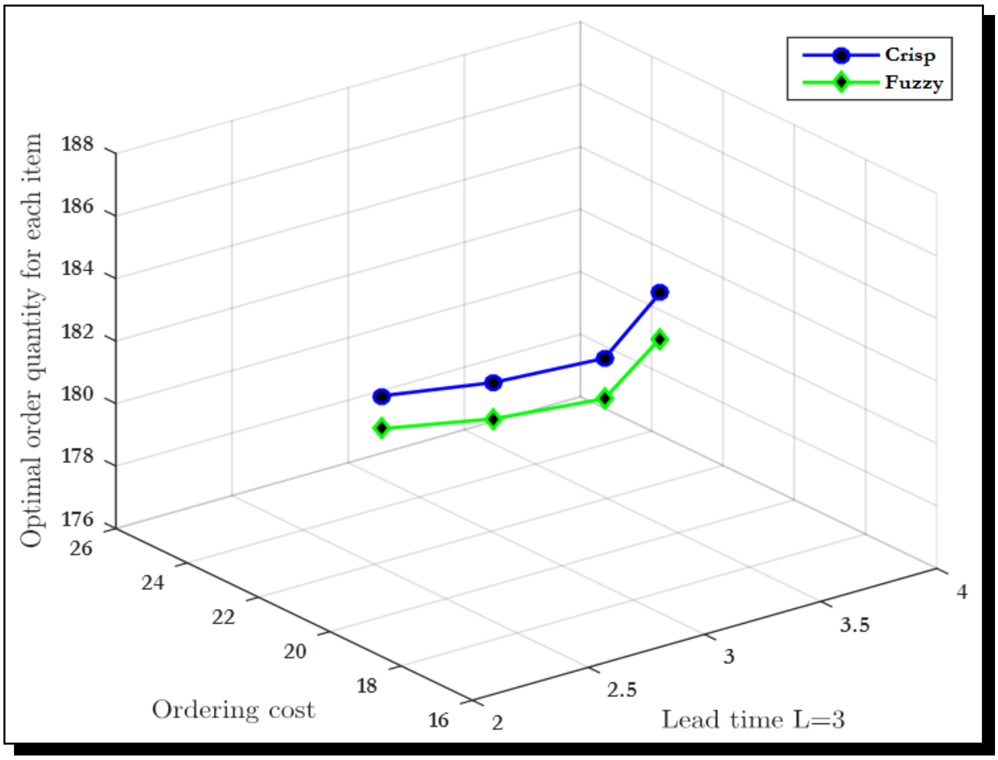
(k) Lead time $L = 6$

Figure *Contd*.



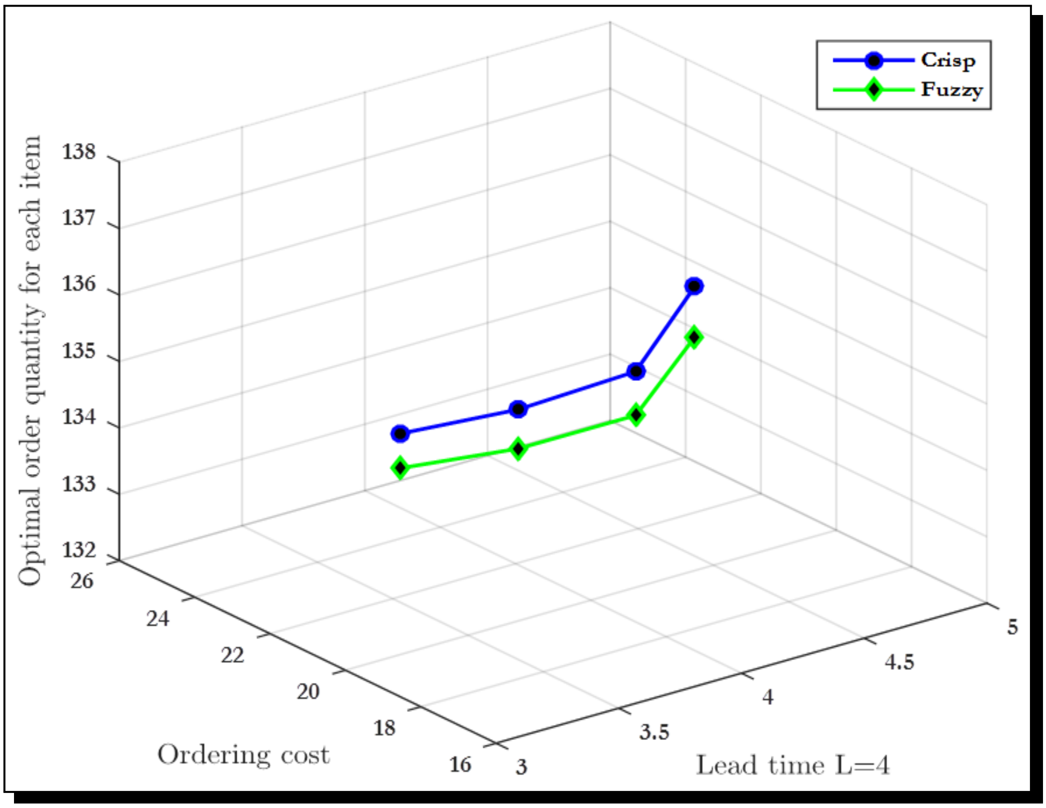
(l) Lead time $L = 8$

Figure 4. (i)-(l): Graphical representation of optimal order quantity for each item versus ordering cost $A_3 = [19.79, 20.83, 22.91, 24.99]$ and lead time

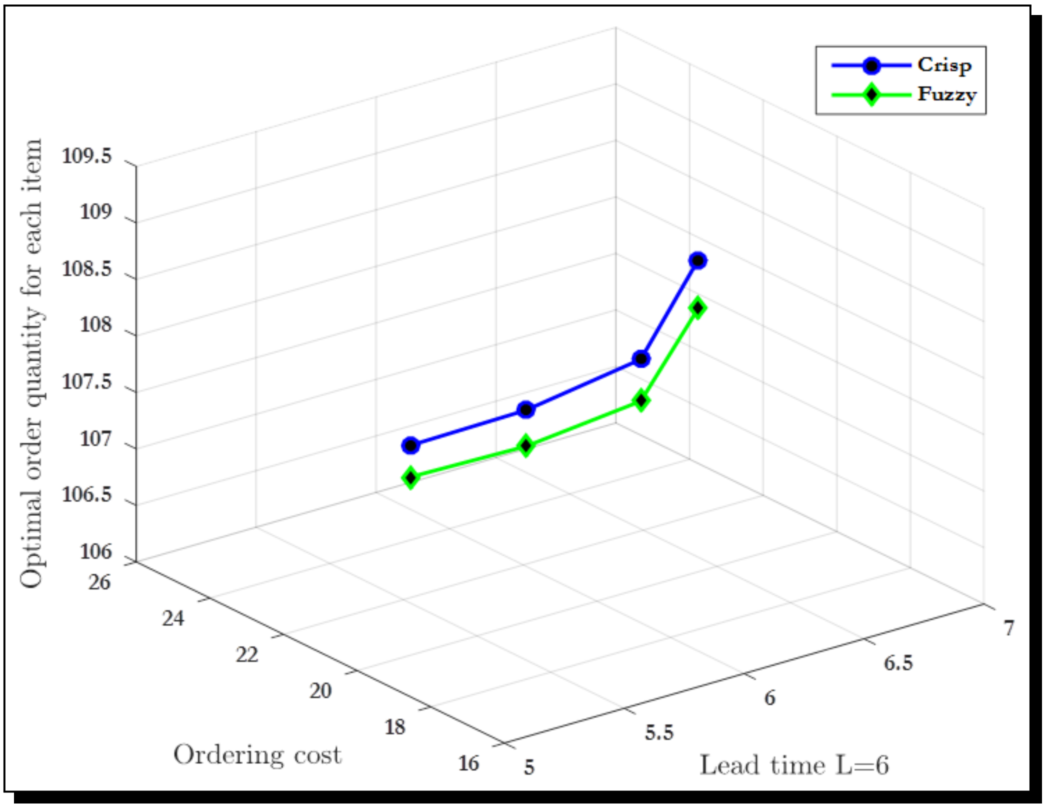


(m) Lead time $L = 3$

Figure *Contd.*

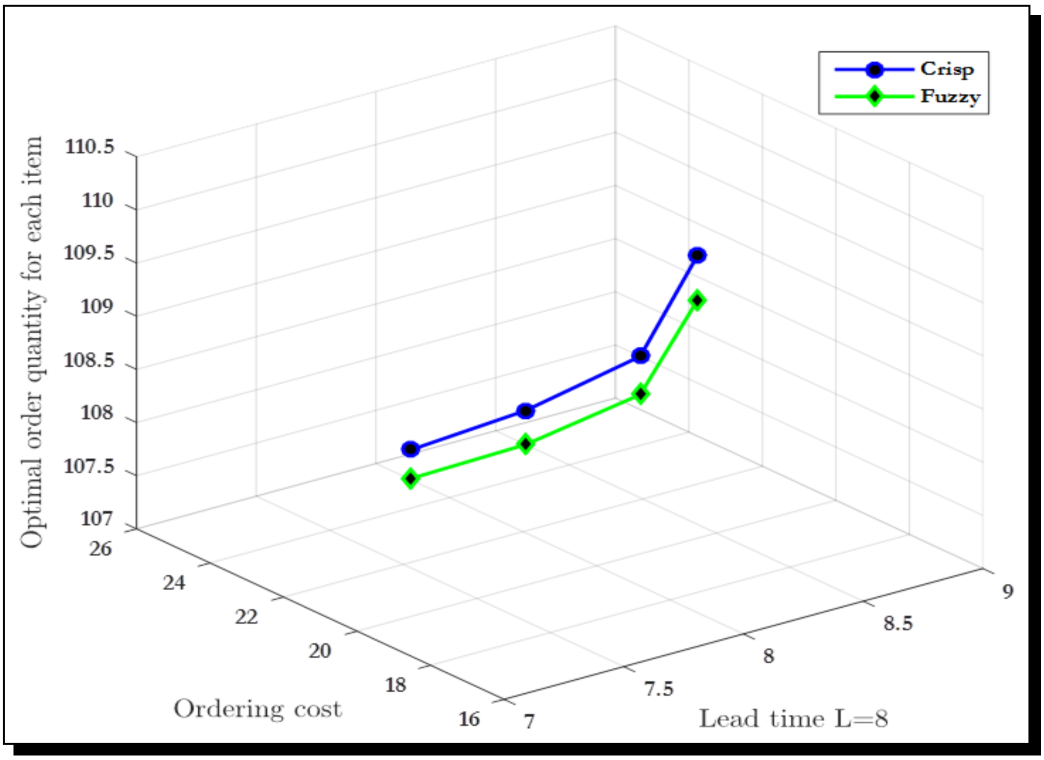


(n) Lead time $L = 4$



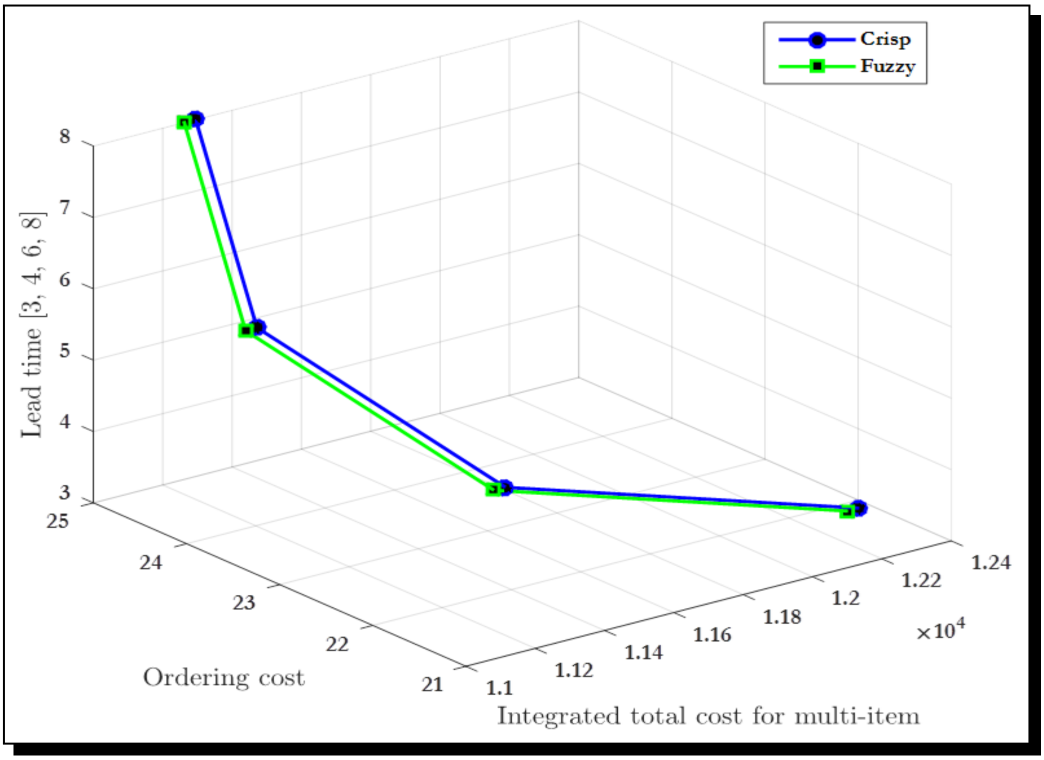
(o) Lead time $L = 6$

Figure *Contd.*



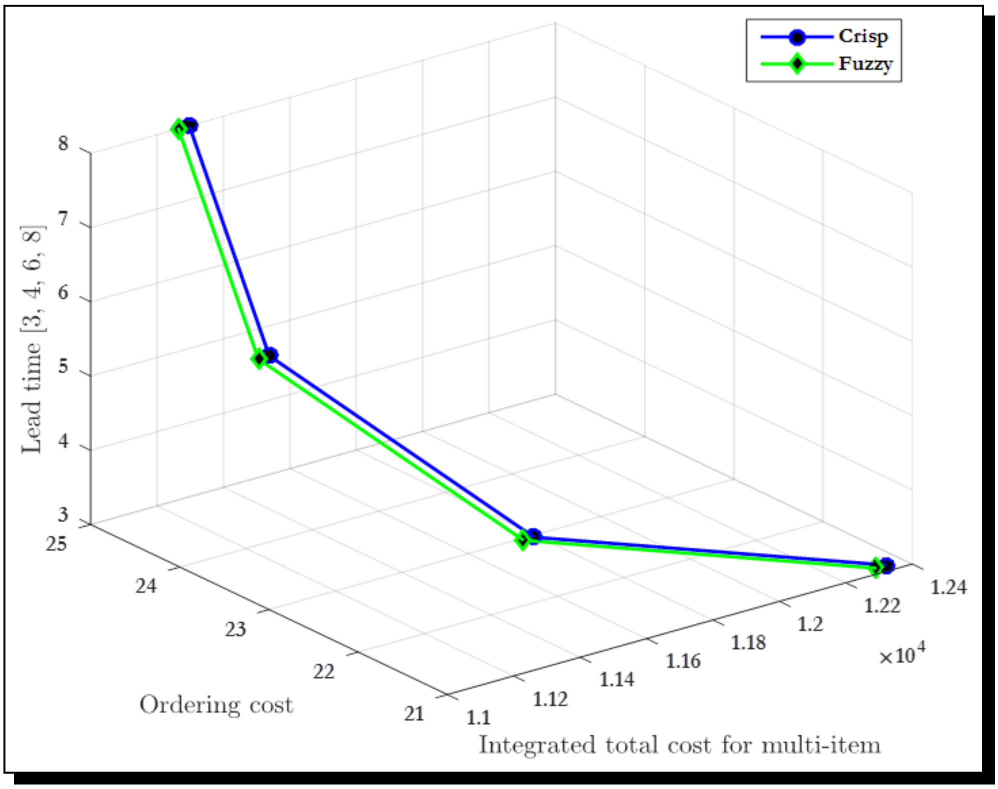
(p) Lead time $L = 8$

Figure 4. (m)-(p): Graphical representation of optimal order quantity for each item versus ordering cost $A_4 = [17.25, 18.81, 21.93, 25.05]$ and lead time

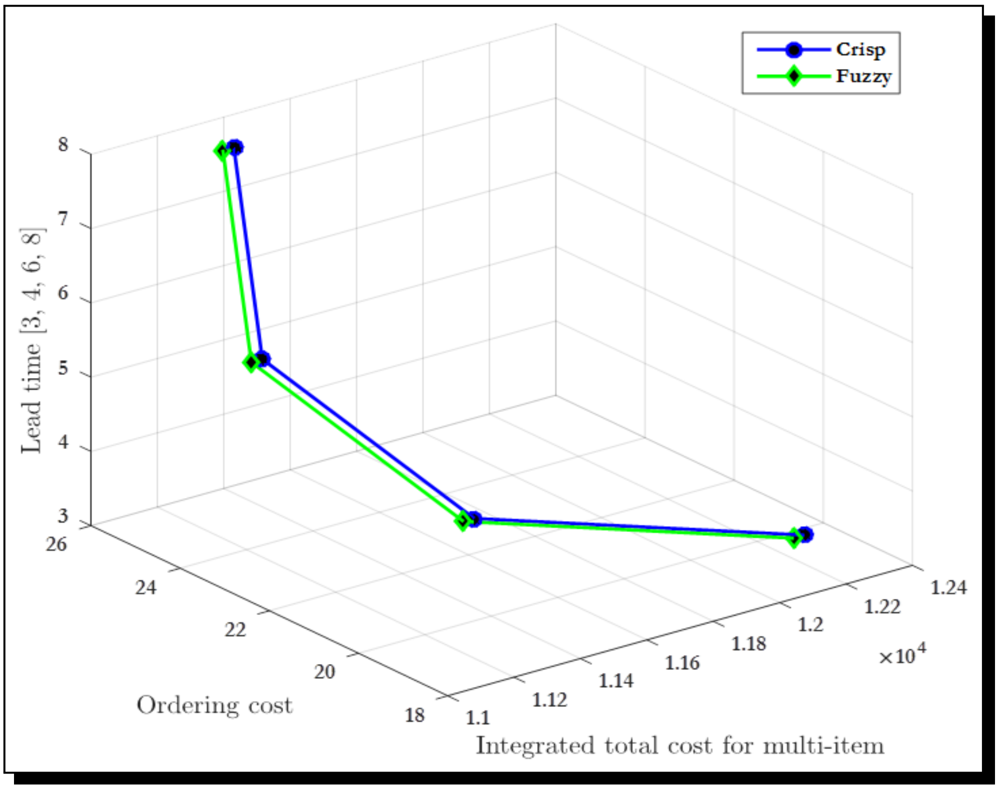


(a) Ordering cost [21.87, 22.5, 23.75, 25]

Figure *Contd.*

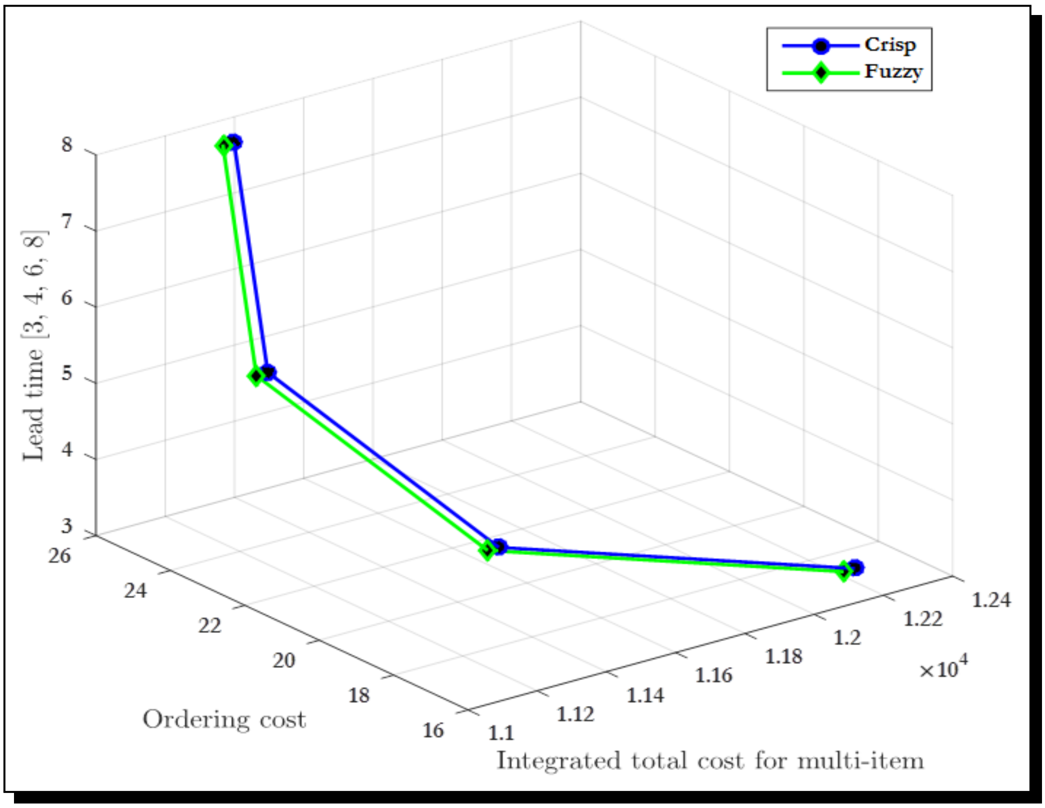


(b) Ordering cost [21.09, 21.87, 23.46, 24.99]



(c) Ordering cost [19.79, 20.83, 22.91, 24.99]

Figure *Contd.*



(d) Ordering cost $[17.25, 18.81, 21.93, 25.03]$

Figure 5. (a)-(d): Graphical representation of minimum integrated total cost for multi-item versus ordering cost and lead time

6. Comparative Study

In Table 1, the arithmetical outcomes of exceeding examples are specified. The optimum standards of crisp multi-item inventory system for all four decision variables are $Q_i^* = (109.46, 108.33, 107.38, 106.57)$ units, $m_i^* = [3,4,5,5]$, $A_4^* = [17.25, 18.81, 21.93, 25.05]$/order, $L_i^* = 6$ weeks and the minimized integrated total cost for multi-item is \$11059.0. The optimal values for fuzzy multi-item inventory system for all four decision variables are $\tilde{Q}_i^* = (109.03, 107.96, 107.06, 106.29)$ units, $m_i^* = [3,4,5,5]$, $A_4^* = [17.25, 18.81, 21.93, 25.05]$/order, $L_i^* = 6$ weeks and the minimized integrated total cost for multi-item is \$11026.0. The relative variations for crisp and fuzzy multi-item model can be grasped in Table 1. The simultaneous variation happening the assessment of L_i, m_i, and A_i, whereas observances of other inputs are stable, takes a major affect continuously in the integrated total cost and optimal order quantity. In Table 2, the percentage variations of integrated total cost and optimal order quantity are presented when lead time varies. Lead time optimal rate reserved at which the integrated total cost for multi-item is smallest amongst entirely four quantities as for four quantities of lead time. Table 3 affords comparison of optimal solution of crisp and fuzzy multi-item inventory model as well for saving percentage of optimal order quantity for each item (0.39%, 0.34%, 0.29%, 0.25%) and minimum Integrated total cost for multi-item 0.30% variations are given for the various data of A_i, and L_i.

Fuzzy multi-item integrated inventory system supports the businesses manage indeterminate inventory price parameters. Indeterminate cost parameters of inventory management models are found to be optimistic and slightly significant. High levels of improbability of the inventory will be difficult to control. Indeterminate cost parameters give (0.39%, 0.34%, 0.29%, 0.25%) and 0.30% variations in optimum order quantity for each item besides minimum integrated total cost for multi-item separately. Indeterminate cost constraints are optimistic as an analyst scientifically changed from nil and ensure slightly significant and straight outcome on inventory. For that reason, administrations are capable to find optimum solution for multi-item in beneficial manner.

Table 2. Summary of crisp and fuzzy optimal solutions

L_i	Ordering cost	Savings (%) optimal order quantity for each item	Savings (%) integrated total cost for multi-item
3	$A_1 = [21.87, 22.5, 23.75, 25]$	(0.26, 0.31, 0.26, 0.26)	0.26
4		(0.55, 0.48, 0.44, 0.39)	0.29
6		(0.39, 0.33, 0.29, 0.26)	0.30
8		(0.39, 0.33, 0.29, 0.25)	0.29
3	$A_2 = [21.09, 21.87, 23.46, 24.99]$	(0.78, 0.71, 0.64, 0.59)	0.26
4		(0.55, 0.49, 0.44, 0.39)	0.28
6		(0.39, 0.34, 0.29, 0.25)	0.30
8		(0.39 ,0.33, 0.29, 0.25)	0.29
3	$A_3 = [19.79, 20.83, 22.91, 24.99]$	(0.78, 0.71, 0.64, 0.59)	0.26
4		(0.55, 0.49, 0.44, 0.39)	0.28
6		(0.39, 0.34, 0.29, 0.25)	0.30
8		(0.39, 0.33, 0.29, 0.25)	0.29
3	$A_4 = [17.25, 18.81, 21.93, 25.05]$	(0.78, 0.71, 0.64, 0.59)	0.26
4		(0.55, 0.49, 0.44, 0.39)	0.28
6		(0.39, 0.34, 0.29, 0.25)	0.30
8		(0.39, 0.33, 0.29, 0.25)	0.28

Table 3. Summary of the comparisons

L_i	m_i	A_i	Comparisons	Optimal order for each item	Integrated total cost for multi-item
3	3	[17.25, 18.81, 21.93, 25.05]	This Crisp multi-item inventory model	(109.46, 108.33, 107.38, 106.57)	11059.0
4	4				
6	5		This fuzzy multi-item inventory model	(109.03, 107.96, 107.06, 106.29)	11026.0
8	5				
			Savings (%)	(0.39, 0.34, 0.29, 0.25)	0.3

7. Conclusion

Integrated system for multi-item and ordering cost depletion contingent on lead time with carbon emission cost is established in fuzzy and crisp situations. In the fuzzy situation, completely interrelated inventory inputs and decision variables are presumed through pentagonal fuzzy quantities. On behalf of defuzzification, the graded mean technique is hired towards estimate minimum integrated total cost for multi-item. The addition of Kuhn-Tucker technique is utilized to obtain the optimal order quantity for each item. An analytical algorithm is trapped into explore the special outcomes for fuzzy inputs on minimum integrated total cost for multi-item, the optimal order quantity for each item based on suggested inventory system. Graphical representations for the numerical examples display for the suggested fuzzy system, unique can find a major quantity of reserves in a multi-item of integrated inventory system. Subsequently comparing together the multi-item of the crisp and fuzzy system, it is perceived that the multi-item of the fuzzy inventory system is better than the multi-item of the crisp inventory system.

Further investigation on this system can arrange using inventory space limitations, setup cost restrictions, ordering constraints, etc. Additionaly, different types of multi-level stream sequence systems can be deliberated in a crisp situation, fuzzy situation, or together.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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