Communications in Mathematics and Applications
Vol. 14, No. 5, pp. 1767–1797, 2023
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications
DOI: 10.26713/cma.v14i5.2268

http://www.rgnpublications.com

Research Article

Fuzzy Inventory Model for Deteriorating Items With Low Carbon Emission Cost Under Preservation Technology and Trade Credit

S. Hemalatha*[1,2] and K. Annadurai[3]

[1] *Mother Teresa Women's University, Kodaikanal 624101, Tamil Nadu, India*
[2] *Department of Mathematics, SSM Institute of Engineering and Technology (Anna University), Dindigul 624002, Tamil Nadu, India*
[3] *Department of Mathematics, M.V. Muthiah Government Arts College for Women (Mother Teresa Women's University), Dindigul 624001, Tamil Nadu, India*
***Corresponding author:** shemalatha1974@gmail.com

Received: April 19, 2023 **Accepted:** September 29, 2023

Abstract. Organizations are eager on rethinking and optimizing their current stock techniques in order to achieve profitability. When managing a supply chain, the phenomenon of deterioration becomes a major consideration. It is able to control the expenditure incurred on preservation technology for deteriorating things up to a specified limit. Nowadays, the world's consciousness is to reduce carbon footprints to mitigate global warming. Consumers have become cognizant of surroundings protection and like low carbon evolved products. We explore a mathematical model for the retailer under the conditions of permissible delay in payments; thereby trade credit is implemented to draw customers. In this study, a crisp model is evolved to reduce the total cost. However, parameters are obscure. To model this impreciseness, a fuzzy model is taken into consideration by taking the parameters as triangular fuzzy numbers. Total cost function is defuzzified through Signed-distance method and is proven to be convex. Comparison of crisp and fuzzy models via special cases is carried out. Moreover, sensitivity analysis and graphical representation are given. Finally, the model obtains the minimal supply chain cost with decision variables as confirmed through the numerical study.

Keywords. Trade credit, Preservation technology investment, Carbon emission, Triangular fuzzy number, Signed distance method

Mathematics Subject Classification (2020). 94D05, 35Q93

1. Introduction

In today's market, product demand is dynamic, affected by a variety of factors such as price, stock-on-hand, time, and so on. Choudhury *et al.* [5] were the first suggested in the field of stock-dependent demand. They investigated an inventory model for deteriorating items with stock-dependent demand, time-varying holding costs, and shortages. Preservation technology is a critical component in minimizing the deteriorating effect. Distinct corporate enterprises/organizations are bound to use preservation technology in their inventory control systems in this aspect. With the preservation technology, they established an inventory model with constant demand. Later, many researchers, e.g., Bardhan *et al.* [3], Mala and Priyan [13], and Sahu *et al.* [18] addressed with the preservation technology under various environment.

Rivalry in business has been increasing rapidly in a globalized and liberalized world, and the entire business community is attempting to take proactive actions in dealing with such severe competition by experimenting with various tools and tactics for survival and growth. One of the techniques used is the trade credit policy. In a trade credit policy, the supplier agrees to provide the retailer a defined period of time to pay his account. During this time, the retailer may earn interest on his sales proceeds and amass revenue. This concept has also been formally introduced in our model. Various researchers have studied the inventory model under trade credit policy extensively in recent years. Some stupendous findings of trade credit policy had been offered below.

The impact of credit-linked demand on the retailer's optimal replenishment policy was explored by Jaggi *et al.* [10]. Deteriorating goods are not considered in their paper. Jaggi *et al.* [10] model was extended by Annadurai and Uthayakumar [1] along with the deteriorating products. Annadurai and Uthayakumar [2] developed a two-echelon inventory model for deteriorating items with credit-period-dependent demand and shortages under two-level trade credit financing and determined the retailer's optimal replenishment policy when both the supplier and the retailer offer the credit period to stimulate customer demand in another paper. The advantages related with efforts to minimize setup costs can be easily interpreted from the Japanese experience of employing just in time (JIT) production. Later, Shah *et al.* [17], Singh *et al.* [19], and Shen *et al.* [20] addressed trade credit under various assumptions.

Global warming is an exceptional hazard to our earth. Nowadays, world's attention is at the depletion of carbon emissions. Dye and Yang [8] expected carbon outflows come from ordering and holding stock. They take into consideration sustainability at the backdrop of joint trade credit, where demand pertains to the credit period. Later, Daryanto and Wee [6], Patel *et al.* [7], Karthick and Uthayakumar [11], Malleeswaran and Uthayakumar [14], Mishra and Talati [15], Yu *et al.* [21], and Tao and Xu [22] addressed carbon emissions under various assumptions.

All parameters are expected to be known precisely in the above analysis. However, in the actual world, parameters are inherently imprecise, and one must deal with numerical approximations that are close to genuine quantities. This epistemic uncertainty and its transmission can be modelled using fuzzy numbers. In a fuzzy environment, Bjork [4] examined an EOQ model. Later, Hemalatha and Annadurai [9], Khanna [12], and Shah *et al.* [16] addressed fuzzy inventory model under various assumptions.

In Table 1, we have summarized our contribution. In contrast to the study models and conclusions stated above. Then, in order to make our model distinctive and more adaptable to business needs, we incorporate a few key aspects. The model is developed by adopting the following points which emphasize the model's uniqueness.

(1) The deteriorating effect of products is addressed, and a preservation technology method to control the rate of deterioration is considered.

(2) The concept of trade credit is presented, as well as its implications for overall demand.

(3) Stock-dependent demand is considered by maintaining holding cost constant.

(4) The goal is to find out the optimal investment in preservation technology and cycle time under carbon emissions regulations (Dye and Yang [8]).

(5) Developing EOQ models with uncertainty expressed as fuzzy numbers has shown to be quite profitable. The fuzzy model is discussed in this study. The parameters are treated as triangular fuzzy numbers. The total cost function is defuzzified and proven to be convex using the Signed distance method.

(6) A comparison of crisp and fuzzy models is made using special cases.

The current research focuses on the creation of an inventory model for deteriorating products that considers preservation technology investment. The demand for the product is determined by the is defined by the level of stock on hand, and the holding cost is expected to rise over time. The impact of carbon emissions and the permissible delay is also considered. The goal of this article is to find the optimal cycle length and preservation technology investment for the lowest total cost. The model is examined under unique circumstances, revealing that reduced carbon emission costs under trade credit have a beneficial effect on customer preference. It helps to enhance sales while lowering the overall cost and investment in preservation technology. The goal of this paper is to scrutinize at a fuzzy inventory model for deteriorating items in a green supply chain with low carbon economic manufacturing quantities, as well as trade credit and preservation measures. Following an introduction, the remainder of the article is organized as follows: The second section is devoted to preliminary definitions. The model's notations and assumptions are shown in Section 3. In Section 4, a mathematical model for the crisp model is developed. Numerical example is provided to illustrate the crisp model in Section 5. In Section 6, a mathematical model for a fuzzy model is developed. In Section 7, sensitivity analysis and managerial insights are provided to validate the concept. The special cases for crisp and fuzzy models are discussed in Section 8. A comparison research is offered in Section 9. After that, there is a conclusion and a plan for the future.

2. Preliminaries

The following definitions of fuzzy sets are relevant to the method used in the proposed model.

Definition 2.1. A fuzzy set $\widetilde{B}$ on the given universal set X is a set of ordered pairs on the real line R, $\widetilde{B} = \{(x, \mu_{\widetilde{B}}(x)) : x \in X\}$ called as membership function. The membership function is also called as degree of compatibility or a degree of truth of X in $\widetilde{B}$ which is defined as $\mu_{\widetilde{B}} : X \to [0,1]$.

Definition 2.2 (α-cut of a Fuzzy Set)**.** An α-cut of a fuzzy set B is a crisp set B_α that contains all the elements of the universal set X and have a membership grade in B which is greater than or equal to the specified value α. That is $B_\alpha = \{x \in X/\mu_B(x) \geq \alpha\}$.

Definition 2.3 (Triangular Fuzzy Numbers)**.** Let $\widetilde{B} = (p_1, p_2, p_3)$, $p_1 < p_2 < p_3$, be a triangular fuzzy number (Figure 1) with membership function:

$$\mu_{\widetilde{B}}(x) = \begin{cases} \dfrac{x - p_1}{p_2 - p_1}, & p_1 \leq x \leq p_2, \\ \dfrac{p_3 - x}{p_3 - p_2}, & p_2 \leq x \leq p_3, \\ 0, & \text{otherwise.} \end{cases}$$

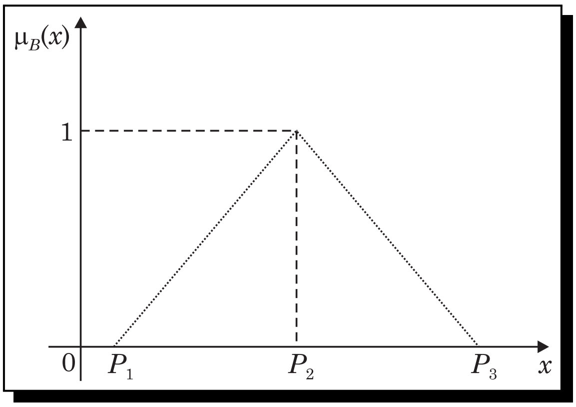


Figure 1. Triangular fuzzy number

Here, p_1, p_2 and p_3 are the lower boundary, mode and upper boundary of the fuzzy number. The left and right cuts of the triangular fuzzy number $\widetilde{B} = (p_1, p_2, p_3)$ is given by $\widetilde{B}_L(\alpha) = p_1 + \alpha(p_2 - p_1)$ and $\widetilde{B}_R(\alpha) = p_3 + \alpha(p_3 - p_2)$.

Definition 2.4 (Signed distance method, Björk [4])**.** For any $b \in R$, $b > 0$, then the signed distance between b and 0 is $d_0(b,0) = b$. If $b < 0$, then the distance between b and 0 is $-d_0(b,0) = -b$. Therefore, signed distance between b and 0 is $d_0(b,0) = b$. For the fuzzy set $\widetilde{B} \in R^+$, $0 \leq \alpha \leq 1$, then we obtain $\widetilde{B} = \bigcup\limits_{0 \leq \alpha \leq 1} \widetilde{B}_\alpha = \bigcup\limits_{0 \leq \alpha \leq 1} [L_\alpha, R_\alpha]$. The signed distance of $[L_\alpha, R_\alpha]$ measured from 0 is $d_0([L_\alpha, R_\alpha], \widetilde{0}) = \frac{\widetilde{B}_L(\alpha) + \widetilde{B}_R(\alpha)}{2}$.

For the triangular fuzzy number $\widetilde{B} \in R^-$, the distance from $\widetilde{B}$ to 0 is written as:

$$\begin{aligned} d(\widetilde{B}, \widetilde{0}) &= \int_0^1 d_0(\widetilde{B}_\alpha, \widetilde{0}) d\alpha \\ &= \int_0^1 d([L_\alpha, R_\alpha], \widetilde{0}) d\alpha \\ &= \frac{1}{2} \int_0^1 (\widetilde{B}_L(\alpha) + \widetilde{B}_R(\alpha)) d\alpha \\ &= \frac{1}{2} \int_0^1 (p_1 + \alpha(p_2 - p_1) + p_3 + \alpha(p_3 - p_2)) d\alpha \\ &= \frac{1}{4}(p_1 + 2p_2 + p_3). \end{aligned}$$

3. Notations and Assumptions

Khanna *et al*. [12] model's notations and assumptions are used. For the sake of completeness and ease of reference, we have delineated these notations below.

3.1 Notations

The following variables and parameters are used to create the suggested model.

Decision Variables

T	Length of cycle
τ	Investment in preservation technology per unit time

Constant Parameters

A_{ce}	Fixed carbon emission per order
c_{ce}	Carbon emission per unit per order
C_{oc}	Cost of ordering (per order)
C_{dc}	Unit cost due to deterioration
C_{hc}	Holding cost per unit time t ($C_{hc}(t) = h_{hc} + r_{hc}t$)
C_{pc}	Purchasing cost per unit
h_{ce}	Carbon emission for inventory per unit time
M	Period of credit presented by the supplier to the retailer
I_c	Interest charged per \$ for unsold item per year by the supplier
I_e	Interest earned per \$ per year
$I_{eh}(t)$	Level of inventory at time t, $0 \le t \le T$
Q	Size of order
S_{sp}	Sales price per unit, where $S_{sp} > C_{pc}$
$TC_p(T,\tau)$	Total cost of the system
y_0	Deterioration rate without investment in preservation technology
$y(\tau)$	Deterioration rate with investment in preservation technology

3.2 Assumptions

(1) The rate of demand is directly proportional to the stock level. i.e.,

$$D(I_{eh}(t)) = a + bI_{eh}(t), \quad a > 0,\ 0 < b < 1.$$

(2) The time horizon is infinite with negligible lead time.

(3) No Shortages.

(4) Preservation technology investment reduces the rate of deterioration gradually. The following function is used to represent this is, $y(\tau) = y_0 e^{-u\tau}$, which satisfies the conditions: $\frac{\partial TC_p}{\partial \tau} < 0$, $\frac{\partial^2 TC_p}{\partial \tau^2} > 0$ and $y(0) = y_0$, where u is sensitivity parameter of investment $0 < u < 1$.

(5) The holding cost is considered to be dependent on time as $C_{hc}(t) = h_{hc} + r_{hc}t$, $0 < r < 1$.

(6) The vendor allows the buyer a delay in payment.

Table 1. Literature summary

Authors	Stock-dependent demand	Deterioration	Preservation Technology	Time varying holding cost	Carbon concerned cost	Trade credit	Fuzzy model
Choudhury *et al.* [5]	Yes	Yes	No	Yes	No	No	No
Singh *et al.* [19]	Yes	Yes	Yes	No	No	No	No
Sahu *et al.* [18]	No	Yes	Yes	No	No	Yes	No
Mishra and Talati [15]	Yes	Yes	Yes	No	No	No	No
Bardhan *et al.* [3]	Yes	Yes	Yes	No	No	No	No
Daryanto and Wee [6]	No	Yes	No	No	Yes	No	No
Shen *et al.* [20]	No	Yes	Yes	No	yes	No	No
Khanna *et al.* [12]	Yes	Yes	Yes	Yes	No	No	No
Yu *et al.* [21]	No	Yes	Yes	No	Yes	No	No
Shah *et al.* [17]	Yes	Yes	Yes	No	No	No	No
Our model (crisp model)	Yes	Yes	Yes	Yes	Yes	Yes	Yes

4. Mathematical Model

A mathematical model is developed in this section to predict the cycle time and the optimal investment in preservation technologies. Deteriorating items are likewise regarded with low carbon economic production amount under preservation technology and trade credit in our inventory system. The following equation governs the inventory level (Khanna *et al.* [12]) can be expressed as follows:

$$\frac{dI(t)}{dt} + y(\tau)I(t) = -(a + bI(t)), \quad 0 \le t \le T. \tag{1}$$

The solution of equation (1) using the boundary condition $I(T) = 0$, is given by

$$I(t) = a\Big[(T-t) + \frac{(T-t)^2(y(\tau)+b)}{2}\Big], \tag{2}$$

and the initial inventory level is:

$$Q = I(0) = a\Big[T + \frac{T^2(y(\tau)+b)}{2}\Big]. \tag{3}$$

The total cost of the system (Khanna *et al.* [12]) is calculated by adding the following costs:

Ordering cost: $OC = C_{oc}$,

Preservation technology investment: $PT = \tau T$,

Holding cost: $HC = \int_0^T (h_{hc} + r_{hc}t)I(t)dt$,

Deterioration cost: $DC = C_{dc}\Big(Q - \int_0^T D(I(t))dt\Big) = C_{dc}\Big[\frac{T^2 y(\tau)(3a - abT) - ab^2T^3}{6}\Big]$, Purchasing cost: $PC = C_{pc}Q = aC_{pc}\left[T + \frac{T^2(y(\tau)+b)}{2}\right]$, and

Total carbon emissions TEC in a finite time horizon T (Dye and Yang [8]) is:

$$TEC = A_{ce} + c_{ce}Q + h_{ce}\int_0^T I(t)dt = A_{ce} + ac_{ce}\left(T + \frac{T^2(y(\tau)+b)}{2}\right) + ah_{ce}\left(\frac{T^2}{2} + \frac{T^3(y(\tau)+b)}{3}\right).$$

Case 1: $M \le T$

$$\begin{aligned} IE_1 &= S_{sp}I_e\int_0^M D(I(t))dt \\ &= S_{sp}I_e\int_0^M a + ba\left((T-t) + \frac{(T-t)^2(y(\tau)+b)}{2}\right)dt \\ &= S_{sp}I_e\left(aM + \frac{abM(2T-M)}{2} + \frac{ab(y(\tau)+b)(3T^2M + M^3 - 3M^2T)}{6}\right), \end{aligned}$$

and the interest charged is for all stuff is given as:

$$\begin{aligned} IC_1 &= C_{pc}I_c\int_M^T I(t)dt \\ &= \frac{aC_{pc}I_c(T-M)^2}{6T}[3 + (y(\tau)+b)(T-M)]. \end{aligned}$$

Thus, the total cost of the system is given by

$$\begin{aligned} TC_p(T,\tau) &= \frac{OC + HC + DC + PT + PC + TCE + IC_1 - IE_1}{T} \\ &= \frac{C_{oc}}{T} + \frac{1}{T}\left[\frac{ah_{hc}T^2}{2} + \frac{ar_{hc}T^3}{6} + \frac{a(y(\tau)+b)}{2}\left(\frac{h_{hc}T^3}{3} + \frac{r_{hc}T^4}{12}\right)\right] \\ &\quad + \frac{aC_{pc}}{T}\left[T + \frac{T^2(y(\tau)+b)}{2}\right] + \frac{C_{dc}}{T}\left[\frac{T^2y(\tau)(3a - abT) - ab^2T^3}{6}\right] + \frac{\tau T}{T} \\ &\quad + \frac{1}{T}\left[A_{ce} + ac_{ce}\left(T + \frac{T^2(y(\tau)+b)}{2}\right) + ah_{ce}\left(\frac{T^2}{2} + \frac{T^3(y(\tau)+b)}{3}\right)\right] \\ &\quad + \frac{aC_{pc}I_c(T-M)^2}{6T}[3 + (y(\tau)+b)(T-M)] \\ &\quad - \frac{S_{sp}I_e}{T}\left[aM + \frac{abM(2T-M)}{2} + \frac{ab(y(\tau)+b)(3T^2M + M^3 - 3M^2T)}{6}\right]. \end{aligned} \tag{4}$$

The intention is to reduce the total cost by jointly optimizing the cycle time T and the investment in preservation technology τ. To establish optimality, taking the necessary conditions

$$\frac{\partial TC_p}{\partial T} = 0 \quad \text{and} \quad \frac{\partial TC_p}{\partial \tau} = 0. \tag{5}$$

We get

$$\begin{aligned} \frac{\partial TC_p}{\partial T} &= \frac{-C_{oc}}{T^2} + \frac{ah_{hc}}{2} + \frac{ar_{hc}T}{3} + \frac{ah_{hc}T}{3}(y_0e^{-u\tau} + b) + \frac{ar_{hc}T^2}{8}(y_0e^{-u\tau} + b) \\ &\quad + \frac{C_{dc}}{6}(3ay_0e^{-u\tau} - 2abTy_0e^{-u\tau} - 2ab^2T) \\ &\quad + \frac{aC_{pc}}{2}(y_0e^{-u\tau} + b) + \frac{aC_{pc}I_c}{6T^2}[3(T^2 - M^2) + (y_0e^{-u\tau} + b)(2T^3 - 3MT^2 + M^3)] \\ &\quad - S_{sp}I_e\left[\frac{-aM}{T^2} + \frac{abM^2}{2T^2} + \frac{ab}{6T^2}(y_0e^{-u\tau} + b)(3MT^2 - M^3)\right] - \frac{A_{ce}}{T^2} + \frac{ac_{ce}}{2}(y_0e^{-u\tau} + b) \end{aligned}$$

$$+\frac{ah_{ce}}{6}[3+4T(y_0e^{-u\tau}+b)]=0, \tag{6}$$

$$\begin{aligned}\frac{\partial TC_p}{\partial \tau} &= 1-\frac{auy_0e^{-u\tau}h_{hc}T^2}{6}-\frac{auy_0e^{-u\tau}r_{hc}T^3}{24}-\frac{C_{dc}uy_0e^{-u\tau}T(3a-abT)}{6}\\ &\quad -\frac{ah_{ce}T^2uy_0e^{-u\tau}}{3}-\frac{a(C_{pc}+c_{ce})Tuy_0e^{-u\tau}}{2}-\frac{aC_{pc}I_cuy_0e^{-u\tau}(T-M)^3}{6T}\\ &\quad +\frac{S_{sp}I_eabMuy_0e^{-u\tau}(3T^2+M^2-3MT)}{6T}=0.\end{aligned} \tag{7}$$

Now, solving equations (6) and (7) concurrently, we get the optimal values of T and τ as T^* and τ^*. After substituting these values in equation (4), we get total cost of the system. The optimal order quantity is found by using equation (3).

Similarly, taking for sufficient condition, it is easy to verify that $\frac{\partial^2 TC_p}{\partial T^2}>0$, $\frac{\partial^2 TC_p}{\partial \tau^2}>0$ and

$$\left(\left(\frac{\partial^2 TC_p}{\partial T^2}\cdot\frac{\partial^2 TC_p}{\partial \tau^2}\right)-\left(\frac{\partial^2 TC_p}{\partial T\partial\tau}\cdot\frac{\partial^2 TC_p}{\partial \tau\partial T}\right)\right)>0. \tag{8}$$

All the second order derivatives are calculated in Appendix. Since all the second order derivatives are extremely non-linear in nature, the optimality is established graphically (Figure 2).

Case 2: $M\geq T$

The retailer trades all stock prior than the credit period M, hence the interest IC_2 charged is zero. Retailer generates income from initiation of the period T and settle the dues at $M\geq T$. Here, the retailer's interest earned is

$$IE_2=S_{sp}I_e\left[\int_0^T(a+bI(t))dt+\left(M-T\right)\int_0^T(a+bI(t))dt\right],$$

i.e.,

$$IE_2=S_{sp}I_e(1+M-T)\left[aT+ab\left(\frac{T^2}{2}+\frac{T^3(y(\tau)+b)}{3}\right)\right].$$

Thus, the total cost of the system is given by

$$TC=\frac{OC+HC+DC+PT+PC+TCE+IC_2-IE_2}{T}.$$

Therefore,

$$\begin{aligned}TC_p(T,\tau) &= \frac{C_{oc}}{T}+\frac{1}{T}\left[\frac{ah_{hc}T^2}{2}+\frac{ar_{hc}T^3}{6}+\frac{a(y(\tau)+b)}{2}\left(\frac{h_{hc}T^3}{3}+\frac{r_{hc}T^4}{12}\right)\right]\\ &\quad +\frac{C_{dc}}{T}\left[\frac{T^2y(\tau)(3a-abT)-ab^2T^3}{6}\right]+\frac{\tau T}{T}+\frac{aC_{pc}}{T}\left[T+\frac{T^2(y(\tau)+b)}{2}\right]\\ &\quad +\frac{1}{T}\left[A_{ce}+ac_{ce}\left(T+\frac{T^2(y(\tau)+b)}{2}\right)+ah_{ce}\left(\frac{T^2}{2}+\frac{T^3(y(\tau)+b)}{3}\right)\right]\\ &\quad -\frac{S_{sp}I_e}{T}(1+M-T)\left[aT+ab\left(\frac{T^2}{2}+\frac{T^3(y(\tau)+b)}{3}\right)\right].\end{aligned} \tag{9}$$

Now $\frac{\partial TC_p}{\partial T}=0$ and $\frac{\partial TC_p}{\partial \tau}=0$ yield

$$\frac{\partial TC}{\partial T}=\frac{-C_{oc}}{T^2}+\frac{ah_{hc}}{2}+\frac{ar_{hc}T}{3}+\frac{ah_{hc}T}{3}(y_0e^{-u\tau}+b)+\frac{ar_{hc}T^2}{8}(y_0e^{-u\tau}+b)$$

$$
\begin{aligned}
&+\frac{C_{dc}}{6}(3ay_0e^{-u\tau}-2abTy_0e^{-u\tau}-2ab^2T)+\frac{aC_{pc}}{2}(y_0e^{-u\tau}+b)-\frac{A_{ce}}{T^2}+\frac{ac_{ce}}{2}(y_0e^{-u\tau}+b)\\
&+\frac{ah_{ce}}{6}[3+4T(y_0e^{-u\tau}+b)]-\frac{S_{sp}I_eab(1+M)(3+4T(y_0e^{-u\tau}+b))}{6}\\
&+S_{sp}I_ea[1+b(T+T^2(y_0e^{-u\tau}+b))]=0
\end{aligned}
\tag{10}
$$

and

$$
\begin{aligned}
\frac{\partial TC}{\partial \tau}&=1-\frac{auy_0e^{-u\tau}h_{hc}T^2}{6}-\frac{auy_0e^{-u\tau}r_{hc}T^3}{24}-\frac{C_{dc}auy_0e^{-u\tau}T(3-bT)}{6}-\frac{h_{ce}auy_0e^{-u\tau}T^2}{3}\\
&\quad-\frac{(C_{pc}+c_{ce})auy_0e^{-u\tau}T}{2}+\frac{S_{sp}I_eabuy_0e^{-u\tau}T^2(1+M)}{3}-\frac{S_{sp}I_eabuy_0e^{-u\tau}T^3}{3}=0.
\end{aligned}
\tag{11}
$$

Solving equations (10) and (11) simultaneously, we get the optimal values of T and τ as T^* and τ^*. After substituting these values in equation (9), we get total cost of the system. The optimal order quantity is found by using equation (3). All the second order derivatives are calculated in Appendix. Since all the second order derivatives are extremely non-linear in nature, the optimality is established graphically (Figure 3).

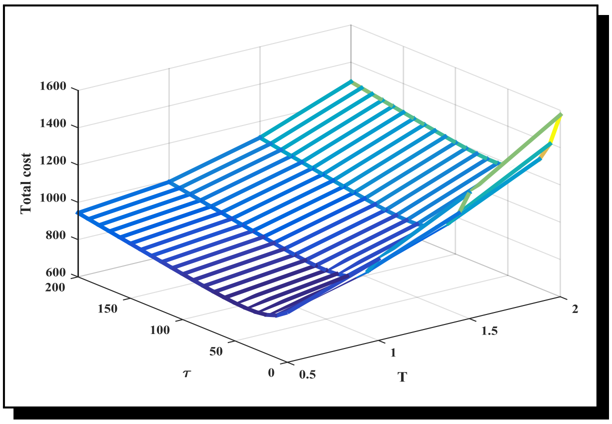


Figure 2. Graphical representation of the total total cost versus the investment in preservation technology and cycle time when $M \le T$

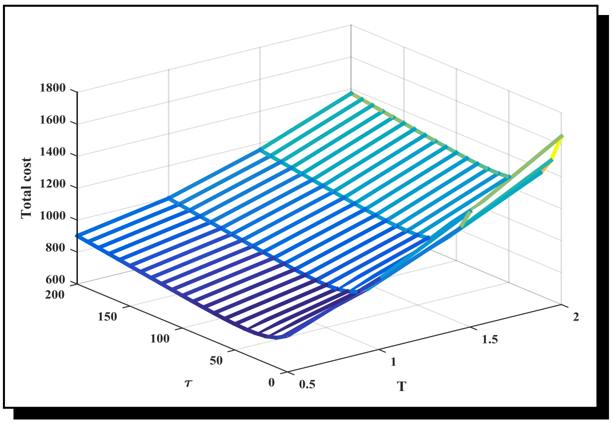


Figure 3. Graphical representation of the cost versus the investment in preservation technology and cycle time when $M \ge T$

5. Numerical Example

A relevant example is presented in this section to explain the model. We explore an inventory system using the same data as in Khanna *et al.* [12], and Yu *et al.* [21]. C_{oc} = 40/order, C_{dc} = 50/year, $u = 0.05$, $y_0 = 0.09$, $h_{hc} = 0.7$ per unit per year, $r_{hc} = 5$, $a = 100$, $b = 0.15$, $C_{pc} = 6$ per unit, $S_{sp} = \$15$, $A_{ce} = 0.02$, $c_{ce} = 0.1$, $h_{ce} = 0.1$, $I_e = 15\%$ per \$ per year, $I_c = 3\%$ per \$ per year and $M = \frac{90}{365} = 0.25$ per year. The total cost of this model for the *Case* 1 is $TC_p(T,\tau) = 780.05$, the cycle time is $T = 0.44$ year and investment in preservation technology $\tau = 34.17$. For *Case* 2, $I_e = 5\%$ per \$ per year, $I_c = 12\%$ per \$ per year and $M = \frac{45}{365} = 0.12$ per year. The total cost is $TC_p(T,\tau) = 767.32$, the cycle time is $T = 0.57$ year and investment in preservation technology $\tau = 38.96$.

6. Fuzzy Model

The fuzzy inventory models, as well as the fuzzification and defuzzification methods, are described in this section. Fuzzification is a process that involves converting crisp parameters to fuzzy parameters. In the event of improbability due to imprecision, ambiguity, or vagueness, the membership function developed in the introductory section for triangular fuzzy numbers can be used to represent fuzzy variables.

6.1 Inventory Model in Fuzzy Nature: Triangular Fuzzy Model

Case 1: $M \le T$

Here, we consider the ordering cost, holding cost, holding cost component, demand parameters, selling price cost, purchasing cost, interest earned and credit period as uncertain. They are represented as triangular fuzzy numbers as follows:

$$\left.\begin{aligned}
&\widetilde{C}_{oc} = (C_{oc} - \Delta_1, C_{oc}, C_{oc} + \Delta_2), && 0 < \Delta_1 < C_{oc}, \Delta_2 > 0, \\
&\widetilde{C}_{dc} = (C_{dc} - \Delta_3, C_{dc}, C_{dc} + \Delta_4), && 0 < \Delta_3 < C_{dc}, \Delta_4 > 0, \\
&\widetilde{a} = (a - \Delta_5, a, a + \Delta_6), && 0 < \Delta_5 < a, \Delta_6 > 0, \\
&\widetilde{b} = (b - \Delta_7, b, b + \Delta_8), && 0 < \Delta_7 < b, \Delta_8 > 0, \\
&\widetilde{h}_{hc} = (h_{hc} - \Delta_9, h_{hc}, h_{hc} + \Delta_{10}), && 0 < \Delta_9 < h_{hc}, \Delta_{10} > 0, \\
&\widetilde{r}_{hc} = (r_{hc} - \Delta_{11}, r_{hc}, r_{hc} + \Delta_{12}), && 0 < \Delta_{11} < r_{hc}, \Delta_{12} > 0, \\
&\widetilde{C}_{pc} = (C_{pc} - \Delta_{13}, C_{pc}, C_{pc} + \Delta_{14}), && 0 < \Delta_{13} < C_{pc}, \Delta_{14} > 0, \\
&\widetilde{S}_{sp} = (S_{sp} - \Delta_{15}, S_{sp}, S_{sp} + \Delta_{16}), && 0 < \Delta_{15} < S_{sp}, \Delta_{16} > 0, \\
&\widetilde{I}_e = (I_e - \Delta_{17}, I_e, I_e + \Delta_{18}), && 0 < \Delta_{17} < I_e, \Delta_{18} > 0, \\
&\widetilde{M} = (M - \Delta_{19}, M, M + \Delta_{20}), && 0 < \Delta_{19} < M, \Delta_{20} > 0.
\end{aligned}\right\} \tag{12}$$

Accordingly, when the parameters C_{oc}, C_{dc}, h_{hc}, r_{hc}, a, b, C_{pc}, S_{sp}, I_e and M, in equation (4) are fuzzified to be $\widetilde{C}_{oc}$, $\widetilde{C}_{dc}$, $\widetilde{h}_{hc}$, $\widetilde{r}_{hc}$, $\widetilde{a}$, $\widetilde{b}$, $\widetilde{C}_{pc}$, $\widetilde{S}_{sp}$, $\widetilde{I}_e$ and $\widetilde{M}$, as expressed in equation (12), the expected total cost function in the fuzzy sense is given by

$$FTC_{p1}(T,\tau) = \frac{\widetilde{C}_{oc}}{T} + \frac{1}{T}\Big[\frac{\widetilde{a}\widetilde{h}_{hc}T^2}{2} + \frac{\widetilde{a}\widetilde{r}_{hc}T^3}{6} + \frac{\widetilde{a}(y(\tau)+\widetilde{b})}{2}\Big(\frac{\widetilde{h}_{hc}T^3}{3} + \frac{\widetilde{r}_{hc}T^4}{12}\Big)\Big]$$

$$+\frac{C_{dc}}{T}\left[\frac{T^2 y(\tau)\widetilde{a}(3-\widetilde{b}T)-\widetilde{a}\widetilde{b}^2T^3}{6}\right]+\frac{\tau T}{T}+\frac{\widetilde{a}\widetilde{C}_{pc}}{T}\left[T+\frac{T^2(y(\tau)+\widetilde{b})}{2}\right]$$
$$+\frac{1}{T}\left[A_{ce}+\widetilde{a}c_{ce}\left(T+\frac{T^2(y(\tau)+\widetilde{b})}{2}\right)+\widetilde{a}h_{ce}\left(\frac{T^2}{2}+\frac{T^3(y(\tau)+\widetilde{b})}{3}\right)\right]$$
$$+\frac{\widetilde{a}\widetilde{C}_{pc}I_c(T-\widetilde{M})^2}{6T}[3+(y(\tau)+\widetilde{b})(T-\widetilde{M})]$$
$$-\frac{\widetilde{S}_{sp}\widetilde{I}_e\widetilde{a}}{T}\left[M+\frac{\widetilde{b}\widetilde{M}(2T-\widetilde{M})}{2}+\frac{\widetilde{b}(y(\tau)+\widetilde{b})(3T^2\widetilde{M}+\widetilde{M}^3-3\widetilde{M}^2T)}{6}\right]. \tag{13}$$

6.2 Defuzzification

Defuzzification is a procedure of converting the fuzzified outcomes into measurable values. Since, we cannot obtain the optimal values of the proposed model by means of fuzzified integrated cost equation (12). Thereby, it is significance to transform an ambiguous (fuzzy) number into a crisp value which can be achieved through a defuzzification method. Here, signed distance method is used for defuzzification. The left and right αcuts of the various parameters C_{oc}, C_{dc}, h_{hc}, r_{hc}, a, b, C_{pc}, S_{sp}, I_e and M are given by

$$\left.\begin{array}{ll}
\widetilde{C}_{oc_L}(\alpha)=C_{oc}-\Delta_1+\alpha\Delta_1>0; & \widetilde{C}_{oc_R}(\alpha)=C_{oc}-\Delta_2+\alpha\Delta_2>0,\\
\widetilde{C}_{dc_L}(\alpha)=C_{dc}-\Delta_3+\alpha\Delta_3>0; & \widetilde{C}_{dc_R}(\alpha)=C_{dc}-\Delta_4+\alpha\Delta_4>0,\\
\widetilde{a}_L(\alpha)=a-\Delta_5+\alpha\Delta_5>0; & \widetilde{a}_R(\alpha)=a-\Delta_6+\alpha\Delta_6>0,\\
\widetilde{b}_L(\alpha)=b-\Delta_7+\alpha\Delta_7>0; & \widetilde{b}_R(\alpha)=b-\Delta_8+\alpha\Delta_8>0,\\
\widetilde{h}_{hc_L}(\alpha)=h_{hc}-\Delta_9+\alpha\Delta_9>0; & \widetilde{h}_{hc_R}(\alpha)=h_{hc}-\Delta_{10}+\alpha\Delta_{10}>0,\\
\widetilde{r}_{hc_L}(\alpha)=r_{hc}-\Delta_{11}+\alpha\Delta_{11}>0; & \widetilde{r}_{hc_R}(\alpha)=r_{hc}-\Delta_{12}+\alpha\Delta_{12}>0,\\
\widetilde{C}_{pc_L}(\alpha)=C_{pc}-\Delta_{13}+\alpha\Delta_{13}>0; & \widetilde{C}_{pc_R}(\alpha)=C_{pc}-\Delta_{14}+\alpha\Delta_{14}>0,\\
\widetilde{S}_{sp_L}(\alpha)=S_{sp}-\Delta_{15}+\alpha\Delta_{15}>0; & \widetilde{S}_{sp_R}(\alpha)=S_{sp}-\Delta_{16}+\alpha\Delta_{16}>0,\\
\widetilde{I}_{e_L}(\alpha)=I_e-\Delta_{17}+\alpha\Delta_{17}>0; & \widetilde{I}_{e_R}(\alpha)=I_e-\Delta_{18}+\alpha\Delta_{18}>0,\\
\widetilde{M}_L(\alpha)=M-\Delta_{19}+\alpha\Delta_{19}>0; & \widetilde{M}_R(\alpha)=M-\Delta_{20}+\alpha\Delta_{20}>0.
\end{array}\right\}$$

The left and right αcuts of $FTC_p(T,\tau)$ is given as

$$FTC_{p1_{L(\alpha)}}(T,\tau)=\frac{\widetilde{C}_{oc_L}(\alpha)}{T}+\frac{1}{T}\left[\frac{\widetilde{a}_L(\alpha)\widetilde{h}_{hc_L}(\alpha)T^2}{2}+\frac{\widetilde{a}_L(\alpha)\widetilde{r}_{hc_L}T^3}{6}\right.$$
$$\left.+\frac{\widetilde{a}_L(\alpha)(y(\tau)+\widetilde{b}_L(\alpha))}{2}\left(\frac{\widetilde{h}_{hc_L}(\alpha)T^3}{3}+\frac{\widetilde{r}_{hc_L}T^4}{12}\right)\right]$$
$$+\frac{\widetilde{C}_{dc_L}(\alpha)}{T}\left[\frac{T^2y(\tau)\widetilde{a}_L(\alpha)\left(3-\widetilde{b}_L(\alpha)T\right)-\widetilde{a}_L(\alpha)\widetilde{b}_L(\alpha)^2T^3}{6}\right]$$
$$+\frac{\tau T}{T}+\frac{\widetilde{a}_L(\alpha)\widetilde{C}_{pc_L}(\alpha)}{T}\left[T+\frac{T^2(y(\tau)+\widetilde{b}_L(\alpha))}{2}\right]$$
$$+\frac{1}{T}\left[A_{ce}+\widetilde{a}_L(\alpha)c_{ce}\left(T+\frac{T^2(y(\tau)+\widetilde{b}_L(\alpha))}{2}\right)\right.$$
$$\left.+\widetilde{a}_L(\alpha)h_{ce}\left(\frac{T^2}{2}+\frac{T^3(y(\tau)+\widetilde{b}_L(\alpha))}{3}\right)\right]$$

$$
\begin{aligned}
&+\frac{\widetilde{a}_L(\alpha)\widetilde{C}_{pc_L}(\alpha)I_c(T-\widetilde{M}_L(\alpha))^2}{6T}[3+(y(\tau)+\widetilde{b}_L(\alpha))(T-\widetilde{M}_L(\alpha))]\\
&-\frac{\widetilde{S}_{sp_L}(\alpha)\widetilde{I}_{e_L}(\alpha)\widetilde{a}_L(\alpha)}{T}\Big[\widetilde{M}_L(\alpha)+\frac{\widetilde{b}_L(\alpha)\widetilde{M}_L(\alpha)(2T-\widetilde{M}_L(\alpha))}{2}\\
&+\frac{\widetilde{b}_L(\alpha)(y(\tau)+\widetilde{b}_L(\alpha))(3T^2\widetilde{M}_L(\alpha)+\widetilde{M}_L(\alpha)^3-3\widetilde{M}_L(\alpha)^2T)}{6}\Big],
\end{aligned}
$$

$$
\begin{aligned}
FTC_{p_{1R(\alpha)}}(T,\tau)=&\frac{\widetilde{C}_{oc_R}(\alpha)}{T}+\frac{1}{T}\Big[\frac{\widetilde{a}_R(\alpha)\widetilde{h}_{hc_R}(\alpha)T^2}{2}+\frac{\widetilde{a}_R(\alpha)\widetilde{r}_{hc_R}(\alpha)T^3}{6}\\
&+\frac{\widetilde{a}_R(\alpha)(y(\tau)+\widetilde{b}_R(\alpha))}{2}\Big(\frac{\widetilde{h}_{hc_R}(\alpha)T^3}{3}+\frac{\widetilde{r}_{hc_R}(\alpha)T^4}{12}\Big)\Big]\\
&+\frac{\widetilde{C}_{dc_R}(\alpha)}{T}\Big[\frac{T^2y(\tau)\widetilde{a}_R(\alpha)(3-\widetilde{b}_R(\alpha)T)-\widetilde{a}_R(\alpha)\widetilde{b}_R(\alpha)^2T^3}{6}\Big]\\
&+\frac{\tau T}{T}+\frac{\widetilde{a}_R(\alpha)\widetilde{C}_{pc_R}(\alpha)}{T}\Big[T+\frac{T^2(y(\tau)+\widetilde{b}_R(\alpha))}{2}\Big]\\
&+\frac{1}{T}\Big[A_{ce}+\widetilde{a}_R(\alpha)c_{ce}\Big(T+\frac{T^2(y(\tau)+\widetilde{b}_R(\alpha))}{2}\Big)\\
&+\widetilde{a}_R(\alpha)h_{ce}\Big(\frac{T^2}{2}+\frac{T^3(y(\tau)+\widetilde{b}_R(\alpha))}{3}\Big)\Big]\\
&+\frac{\widetilde{a}_R(\alpha)\widetilde{C}_{pc_R}(\alpha)I_c(T-\widetilde{M}_R(\alpha))^2}{6T}[3+(y(\tau)+\widetilde{b}_R(\alpha))(T-\widetilde{M}_R(\alpha))]\\
&-\frac{\widetilde{S}_{sp_R}(\alpha)\widetilde{I}_{e_R}(\alpha)\widetilde{a}_R(\alpha)}{T}\Big[\widetilde{M}_R(\alpha)+\frac{\widetilde{b}_R(\alpha)\widetilde{M}_R(\alpha)(2T-\widetilde{M}_R(\alpha))}{2}\\
&+\frac{\widetilde{b}_R(\alpha)(y(\tau)+\widetilde{b}_R(\alpha))(3T^2\widetilde{M}_R(\alpha)+\widetilde{M}_R(\alpha)^3-3\widetilde{M}_R(\alpha)^2T)}{6}\Big]
\end{aligned}
$$

$$
FTC_{Dp1}(T,\tau)=\frac{1}{2}\int_0^1(FTC_{p1}(T,\tau)_{L(\alpha)}+FTC_{p1}(T,\tau)_{R(\alpha)})d\alpha, \tag{14}
$$

$$
\begin{aligned}
FTC_{p1}(T,\tau)=&\frac{C_{oc}+\frac{\Delta_2-\Delta_1}{4}}{T}+\frac{1}{T}\Big[\Big(a+\frac{1}{4}(\Delta_6-\Delta_5)\Big)\Big(\frac{(h_{hc}+\frac{1}{4}(\Delta_{10}-\Delta_9))T^2}{2}\\
&+\frac{(r_{hc}+\frac{1}{4}(\Delta_{12}-\Delta_{11}))T^3}{6}\Big)+\frac{(a+\frac{1}{4}(\Delta_6-\Delta_5))(y(\tau)+(b+\frac{1}{4}(\Delta_8-\Delta_7)))}{2}\\
&\cdot\Big(\frac{(h_{hc}+\frac{1}{4}(\Delta_{10}-\Delta_9))T^3}{3}+\frac{(r_{hc}+\frac{1}{4}(\Delta_{12}-\Delta_{11}))T^4}{12}\Big)\Big]\\
&+\frac{C_{dc}+\frac{1}{4}(\Delta_4-\Delta_3)}{T}\Big[\Big(a+\frac{1}{4}(\Delta_6-\Delta_5)\Big)\\
&\cdot\Big(\frac{T^2y(\tau)(3-(b+\frac{1}{4}(\Delta_8-\Delta_7))T)-(b+\frac{1}{4}(\Delta_8-\Delta_7))^2T^3}{6}\Big)\Big]\\
&+\frac{\tau T}{T}+\frac{(a+\frac{1}{4}(\Delta_6-\Delta_5))(C_{pc}+\frac{1}{4}(\Delta_{14}-\Delta_{13}))}{T}\\
&\cdot\Big[T+\frac{T^2(y(\tau)+(b+\frac{1}{4}(\Delta_8-\Delta_7)))}{2}\Big]+\frac{1}{T}\Big[A_{ce}+c_{ce}\Big(a+\frac{1}{4}(\Delta_6-\Delta_5)\Big)
\end{aligned}
$$

$$
\begin{aligned}
&\cdot\left(T+\frac{T^2(y(\tau)+b+\frac{1}{4}(\Delta_8-\Delta_7))}{2}\right)+\left(a+\frac{1}{4}(\Delta_6-\Delta_5)\right)h_{ce}\\
&\cdot\left(\frac{T^2}{2}+\frac{T^3(y(\tau)+(b+\frac{1}{4}(\Delta_8-\Delta_7)))}{3}\right)\Big]\\
&+\frac{(a+\frac{1}{4}(\Delta_6-\Delta_5))\widetilde{C}_{pc}I_c(T-(M+\frac{1}{4}(\Delta_{20}-\Delta_{19})))^2}{6T}\\
&\cdot\left[3+\left(y(\tau)+\left(b+\frac{1}{4}(\Delta_8-\Delta_7)\right)\right)\left(T-\left(M+\frac{1}{4}(\Delta_{20}-\Delta_{19})\right)\right)\right]\\
&-\frac{(S_{sp}+\frac{1}{4}(\Delta_{16}-\Delta_{15}))(I_e+\frac{1}{4}(\Delta_{18}-\Delta_{17}))(a+\frac{1}{4}(\Delta_6-\Delta_5))}{T}\\
&\cdot\Big[\left(M+\frac{1}{4}(\Delta_{20}-\Delta_{19})\right)\\
&+\frac{(M+\frac{1}{4}(\Delta_{20}-\Delta_{19}))(b+\frac{1}{4}(\Delta_8-\Delta_7))(2T-(M+\frac{1}{4}(\Delta_{20}-\Delta_{19})))}{2}\\
&+\left(\left(y(\tau)+\left(b+\frac{1}{4}(\Delta_8-\Delta_7)\right)\right)\left(b+\frac{1}{4}(\Delta_8-\Delta_7)\right)\right)\\
&\cdot\frac{(3T^2(M+\frac{1}{4}(\Delta_{20}-\Delta_{19}))+(M+\frac{1}{4}(\Delta_{20}-\Delta_{19}))^3-3(M+\frac{1}{4}(\Delta_{20}-\Delta_{19}))^2T)}{6}\Big].
\end{aligned}
\tag{15}
$$

Hence, the fuzzified cost function equation (13) narrated with triangular fuzzy number is transformed into the crisp function by utilizing signed distance formula by equation (14), then the defuzzified cost function is given in equation (15). The objective is to minimize the total cost by jointly optimizing the cycle time T and the investment in preservation technology τ. To establish optimality, taking the necessary conditions $\frac{\partial FTC_{p1}(T,\tau)}{\partial T}=0$ and $\frac{\partial FTC_{p1}(T,\tau)}{\partial \tau}=0$. Accordingly, by taking first order partial derivative of the equation (15) with respect to T and τ and equating to zero, the optimal values of T and τ are obtained as $T^*$and τ^*. After substituting these values in equation (15), we get total cost of the system. The optimal order quantity is found by using equation (3). Similarly taking for sufficient conditions, it is easy to verify that $\frac{\partial^2 FTC_{p1}(T,\tau)}{\partial T^2}>0$, $\frac{\partial^2 FTC_{p1}(T,\tau)}{\partial \tau^2}>0$ and

$$\left(\left(\frac{\partial^2 FTC_{p1}(T,\tau)}{\partial T^2}\cdot\frac{\partial^2 FTC_{p1}(T,\tau)}{\partial \tau^2}\right)-\left(\frac{\partial^2 FTC_{p1}(T,\tau)}{\partial T\partial\tau}\cdot\frac{\partial^2 FTC_{p1}(T,\tau)}{\partial \tau\partial T}\right)\right)>0.$$

Since all the second order derivatives are extremely non-linear in nature, therefore the optimality is established graphically (Figure 4).

Case 2: $M\ge T$

Here, we consider the ordering cost, holding cost, holding cost component, demand parameters, selling price cost and purchasing cost as uncertain and is stated as in *Case* 1. Interest earned and credit period are represented as triangular fuzzy numbers as follows:

$$\left.\begin{aligned}
&\widetilde{I}_e=(I_e-\Delta_{21},I_e,I_e+\Delta_{22}),\quad 0<\Delta_{21}<I_e,\Delta_{22}>0,\\
&\widetilde{M}=(M-\Delta_{23},M,M+\Delta_{24}),\quad 0<\Delta_{23}<M,\Delta_{24}>0.
\end{aligned}\right\}\tag{16}$$

Accordingly, when the parameters C_{oc}, C_{dc}, h_{hc}, r_{hc}, a, b, C_{pc}, S_{sp}, I_e and M, in equation (9) are fuzzified to be $\widetilde{C}_{oc}$, $\widetilde{C}_{dc}$, $\widetilde{h}_{hc}$, $\widetilde{r}_{hc}$, $\widetilde{a}$, $\widetilde{b}$, $\widetilde{C}_{pc}$, $\widetilde{S}_{sp}$, $\widetilde{I}_e$ and $\widetilde{M}$, as expressed in equations (12) and (16), the expected total cost function in the fuzzy sense is given by

$$\begin{aligned}FTC_{p2}(T,\tau)=&\frac{\widetilde{C}_{oc}}{T}++\frac{1}{T}\Big[\frac{\widetilde{a}\widetilde{h}_{hc}T^2}{2}+\frac{\widetilde{a}\widetilde{r}_{hc}T^3}{6}+\frac{\widetilde{a}(y(\tau)+\widetilde{b})}{2}\Big(\frac{\widetilde{h}_{hc}T^3}{3}+\frac{\widetilde{r}_{hc}T^4}{12}\Big)\Big]\\&+\frac{C_{dc}}{T}\Big[\frac{T^2y(\tau)\widetilde{a}(3-\widetilde{b}T)-\widetilde{a}\widetilde{b}^2T^3}{6}\Big]+\frac{\tau T}{T}+\frac{\widetilde{a}\widetilde{C}_{pc}}{T}\Big[T+\frac{T^2(y(\tau)+\widetilde{b})}{2}\Big]\\&+\frac{1}{T}\Big[A_{ce}+\widetilde{a}c_{ce}\Big(T+\frac{T^2(y(\tau)+\widetilde{b})}{2}\Big)+\widetilde{a}h_{ce}\Big(\frac{T^2}{2}+\frac{T^3(y(\tau)+\widetilde{b})}{3}\Big)\Big]\\&-\frac{\widetilde{S}_{sp}\widetilde{I}_e}{T}(1+\widetilde{M}-T)\Big[\widetilde{a}T+\widetilde{a}\widetilde{b}\Big(\frac{T^2}{2}+\frac{T^3(y(\tau)+\widetilde{b})}{3}\Big)\Big].\end{aligned}\tag{17}$$

The left and right αcuts of the various parameters C_{oc}, C_{dc}, h_{hc}, r_{hc}, a, b, C_{pc}, and S_{sp} are considered as stated in *Case* 1. I_e and M are as follows:

$$\left.\begin{aligned}&\widetilde{I}_{e_L}(\alpha)=I_e-\Delta_{21}+\alpha\Delta_{21}>0;\quad \widetilde{I}_{e_R}(\alpha)=I_e-\Delta_{22}+\alpha\Delta_{22}>0,\\&\widetilde{M}_L(\alpha)=M-\Delta_{23}+\alpha\Delta_{23}>0;\quad \widetilde{M}_R(\alpha)=M-\Delta_{24}+\alpha\Delta_{24}>0,\end{aligned}\right\}$$

$$\begin{aligned}FTC_{p2_{L(\alpha)}}(T,\tau)=&\frac{\widetilde{C}_{oc_L}(\alpha)}{T}+\frac{1}{T}\Big[\frac{\widetilde{a}_L(\alpha)\widetilde{h}_{hc_L}(\alpha)T^2}{2}+\frac{\widetilde{a}_L(\alpha)\widetilde{r}_{hc_L}T^3}{6}\\&+\frac{\widetilde{a}_L(\alpha)(y(\tau)+\widetilde{b}_L(\alpha))}{2}\Big(\frac{\widetilde{h}_{hc_L}(\alpha)T^3}{3}+\frac{\widetilde{r}_{hc_L}T^4}{12}\Big)\Big]\\&+\frac{\widetilde{C}_{dc_L}(\alpha)}{T}\Big[\frac{T^2y(\tau)\widetilde{a}_L(\alpha)(3-\widetilde{b}_L(\alpha)T)-\widetilde{a}_L(\alpha)\widetilde{b}_L(\alpha)^2T^3}{6}\Big]\\&+\frac{\tau T}{T}+\frac{\widetilde{a}_L(\alpha)\widetilde{C}_{pc_L}(\alpha)}{T}\Big[T+\frac{T^2(y(\tau)+\widetilde{b}_L(\alpha))}{2}\Big]\\&+\frac{1}{T}\Big[A_{ce}+\widetilde{a}_L(\alpha)c_{ce}\Big(T+\frac{T^2(y(\tau)+\widetilde{b}_L(\alpha))}{2}\Big)\\&\cdot+\widetilde{a}_L(\alpha)h_{ce}\Big(\frac{T^2}{2}+\frac{T^3(y(\tau)+\widetilde{b}_L(\alpha))}{3}\Big)\Big]\\&-\frac{\widetilde{S}_{sp_L}(\alpha)\widetilde{I}_{e_L}(\alpha)}{T}(1+\widetilde{M}_L(\alpha)-T)\\&\cdot\Big[\widetilde{a}_L(\alpha)T+\widetilde{a}_L(\alpha)\widetilde{b}_L(\alpha)\Big(\frac{T^2}{2}+\frac{T^3(y(\tau)+\widetilde{b}_L(\alpha))}{3}\Big)\Big]\end{aligned}$$

$$\begin{aligned}FTC_{p2_{R(\alpha)}}(T,\tau)=&\frac{\widetilde{C}_{oc_R}(\alpha)}{T}+\frac{1}{T}\Big[\frac{\widetilde{a}_R(\alpha)\widetilde{h}_{hc_R}(\alpha)T^2}{2}+\frac{\widetilde{a}_R(\alpha)\widetilde{r}_{hc_R}(\alpha)T^3}{6}\\&+\frac{\widetilde{a}_R(\alpha)(y(\tau)+\widetilde{b}_R(\alpha))}{2}\Big(\frac{\widetilde{h}_{hc_R}(\alpha)T^3}{3}+\frac{\widetilde{r}_{hc_R}(\alpha)T^4}{12}\Big)\Big]\\&+\frac{\widetilde{C}_{dc_R}(\alpha)}{T}\Big[\frac{T^2y(\tau)\widetilde{a}_R(\alpha)(3-\widetilde{b}_R(\alpha)T)-\widetilde{a}_R(\alpha)\widetilde{b}_R(\alpha)^2T^3}{6}\Big]\\&+\frac{\tau T}{T}+\frac{\widetilde{a}_R(\alpha)\widetilde{C}_{pc_R}(\alpha)}{T}\Big[T+\frac{T^2\Big(y(\tau)+\widetilde{b}_R(\alpha)\Big)}{2}\Big]\\&+\frac{1}{T}\Big[A_{ce}+\widetilde{a}_R(\alpha)c_{ce}\Big(T+\frac{T^2(y(\tau)+\widetilde{b}_R(\alpha))}{2}\Big)\end{aligned}$$

$$
\begin{aligned}
&+\widetilde{a}_R(\alpha)h_{ce}\Big(\frac{T^2}{2}+\frac{T^3(y(\tau)+\widetilde{b}_R(\alpha))}{3}\Big)\Big]\\
&-\frac{\widetilde{S}_{sp_R}(\alpha)\widetilde{I}_{e_R}(\alpha)}{T}(1+\widetilde{M}_R(\alpha)-T)\\
&\cdot\Big[\widetilde{a}_R(\alpha)T+\widetilde{a}_R(\alpha)\widetilde{b}_R(\alpha)\Big(\frac{T^2}{2}+\frac{T^3(y(\tau)+\widetilde{b}_R(\alpha))}{3}\Big)\Big],
\end{aligned}
$$

$$
FTC_{Dp2}(T,\tau)=\frac{1}{2}\int_0^1(FTC_{p2}(T,\tau)_{L(\alpha)}+FTC_2(T,\tau)_{R(\alpha)})d\alpha. \tag{18}
$$

Hence, the fuzzified cost function equation (17) narrated with triangular fuzzy number is transformed into the crisp function by utilizing signed distance formula by equation (18), then the defuzzified cost function is given in equation (19). The objective is to minimize the total cost by jointly optimizing the cycle time T and the investment in preservation technology τ. To establish optimality, the necessary conditions $\frac{\partial FTC_{p2}(T,\tau)}{\partial T}=0$ and $\frac{\partial FTC_{p2}(T,\tau)}{\partial \tau}=0$. Accordingly, by taking first order partial derivative of the equation (18) with respect to T and τ and equating to zero, the optimal values of T and τ are obtained as T^* and τ^*. After substituting these values in equation (19), we get total cost of the system. The optimal order quantity is found by using equation (3). Similarly, taking for sufficient conditions, it is easy to verify that $\frac{\partial^2 FTC_{p2}(T,\tau)}{\partial T^2}>0$, $\frac{\partial^2 FTC_{p2}(T,\tau)}{\partial \tau^2}>0$ and

$$
\Big(\Big(\frac{\partial^2 FTC_{p2}(T,\tau)}{\partial T^2}\cdot\frac{\partial^2 FTC_{p2}(T,\tau)}{\partial \tau^2}\Big)-\Big(\frac{\partial^2 FTC_{p2}(T,\tau)}{\partial T\partial\tau}\cdot\frac{\partial^2 FTC_{p2}(T,\tau)}{\partial \tau\partial T}\Big)\Big)>0.
$$

Since all the second order derivatives are extremely non-linear in nature, therefore the optimality is established graphically (Figure 5). Moreover, we summarize the input parameters as fuzzy triangular values and defuzzified values in Table 2.

$$
\begin{aligned}
FTC_{p1}(T,\tau)=&\frac{C_{oc}+\frac{\Delta_2-\Delta_1}{4}}{T}+\frac{1}{T}\Big[\Big(a+\frac{1}{4}(\Delta_6-\Delta_5)\Big)\\
&\cdot\Big(\frac{(h_{hc}+\frac{1}{4}(\Delta_{10}-\Delta_9))T^2}{2}+\frac{(r_{hc}+\frac{1}{4}(\Delta_{12}-\Delta_{11}))T^3}{6}\Big)\\
&+\frac{(a+\frac{1}{4}(\Delta_6-\Delta_5))(y(\tau)+(b+\frac{1}{4}(\Delta_8-\Delta_7)))}{2}\\
&\cdot\Big(\frac{(h_{hc}+\frac{1}{4}(\Delta_{10}-\Delta_9))T^3}{3}+\frac{(r_{hc}+\frac{1}{4}(\Delta_{12}-\Delta_{11}))T^4}{12}\Big)\Big]\\
&+\frac{C_{dc}+\frac{1}{4}(\Delta_4-\Delta_3)}{T}\Big[\Big(a+\frac{1}{4}(\Delta_6-\Delta_5)\Big)\\
&\cdot\Big(\frac{T^2y(\tau)(3-(b+\frac{1}{4}(\Delta_8-\Delta_7))T)-(b+\frac{1}{4}(\Delta_8-\Delta_7))^2T^3}{6}\Big)\Big]\\
&+\frac{\tau T}{T}+\frac{(a+\frac{1}{4}(\Delta_6-\Delta_5))(C_{pc}+\frac{1}{4}(\Delta_{14}-\Delta_{13}))}{T}\\
&\cdot\Big[T+\frac{T^2(y(\tau)+(b+\frac{1}{4}(\Delta_8-\Delta_7)))}{2}\Big]+\frac{1}{T}\Big[A_{ce}+c_{ce}\Big(a+\frac{1}{4}(\Delta_6-\Delta_5)\Big)\\
&\cdot\Big(T+\frac{T^2(y(\tau)+b+\frac{1}{4}(\Delta_8-\Delta_7))}{2}\Big)+\Big(a+\frac{1}{4}(\Delta_6-\Delta_5)\Big)h_{ce}
\end{aligned}
$$

$$\cdot\left(\frac{T^2}{2}+\frac{T^3(y(\tau)+(b+\frac{1}{4}(\Delta_8-\Delta_7)))}{3}\right)\Big]$$
$$-\frac{(S_{sp}+\frac{1}{4}(\Delta_{16}-\Delta_{15}))(I_e+\frac{1}{4}(\Delta_{22}-\Delta_{21}))}{T}$$
$$\cdot\left(1+\left(M+\frac{1}{4}(\Delta_{24}-\Delta_{23})\right)-T\right)\left[\left(a+\frac{1}{4}(\Delta_6-\Delta_5)\right)T+\left(a+\frac{1}{4}(\Delta_6-\Delta_5)\right)\right.$$
$$\left.\cdot\left(b+\frac{1}{4}(\Delta_8-\Delta_7)\right)\left(\frac{T^2}{2}+\frac{T^3(y(\tau)+(b+\frac{1}{4}(\Delta_8-\Delta_7)))}{3}\right)\right]. \tag{19}$$

Table 2. Input parameters as fuzzy triangular values

Input parameters		Input parameters as fuzzy triangular values	Defuzzified values
For both *Case* 1 and *Case* 2	C_{oc}	(30, 40, 50)	37.5
	C_{dc}	(40, 50, 60)	45
	a	(90, 100, 110)	82.5
	b	(0.14, 0.15, 0.16)	0.12
	h_{hc}	(0.6, 0.7, 0.8)	0.6
	r_{hc}	(4, 5, 6)	4.5
	C_{pc}	(5, 6, 7)	5.25
	S_{sp}	(13, 15, 17)	12.75
Case 1	I_e	(0.14, 0.15, 0.16)	0.12
	M	(0.22, 0.25, 0.27)	0.21
Case 2	I_e	(0.04, 0.05, 0.06)	0.045
	M	(0.11, 0.12, 0.13)	0.10

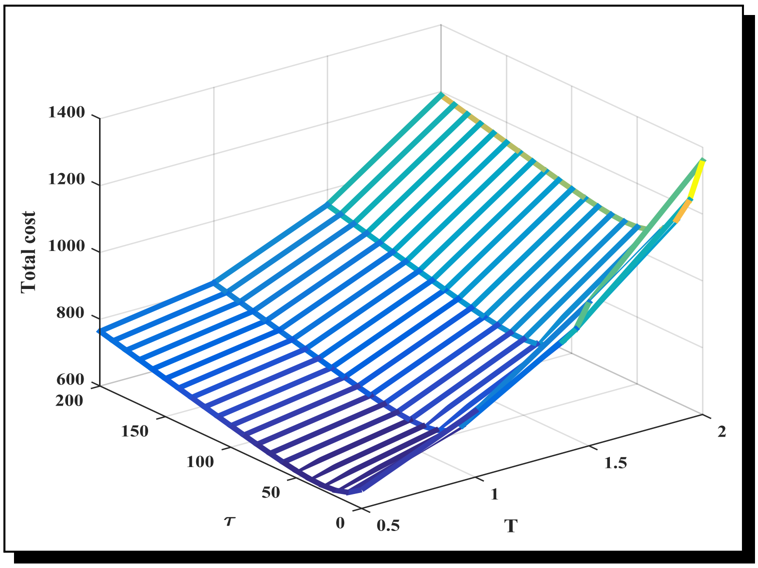


Figure 4. Graphical representation for optimality when $M \leq T$

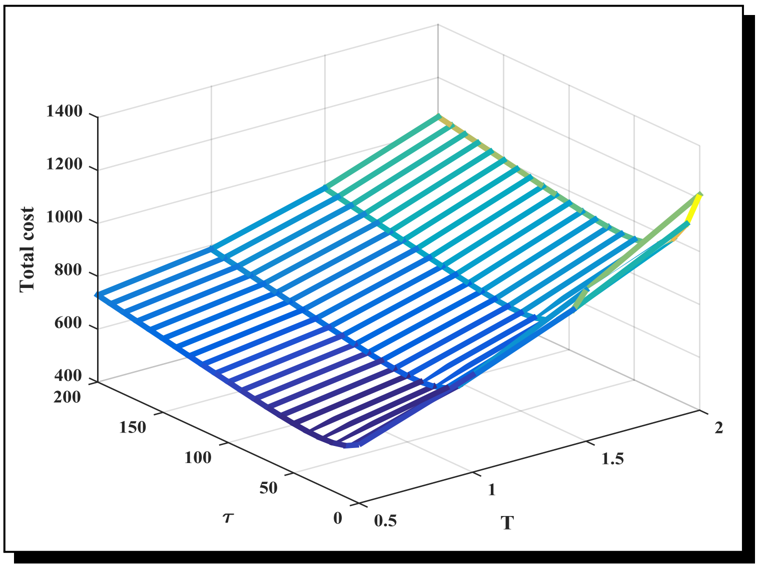


Figure 5. Graphical representation for optimality when $M \geq T$

7. Sensitivity Analysis

In this section, we scrutinize the influence of the parameters in the system variables. It is critical in an inventory system for a retailer to know the behavior of the system parameters which influenced upon the total cost function. When the relevant parameters are increased or decreased, the retailer should know when the minimal expense is reached. To illustrate the applicability of the model, we study sensitivity analysis with the variation of different parameters.

In this part, we inspect the effects of variations in the system variables h_{hc}, r_{hc}, u, a, b, C_{oc}, C_{dc}, C_{pc}, S_{sp}, I_e and M on the optimal ordering quantity Q, the cycle time is T and investment in preservation technology τ with minimum total expected cost. The optimal values of Q, T, τ and $TC_p(T,\tau)$ are derived, when one of the parameters changes (increases or decreases) by 25% and all other parameters remain unchanged. The results of sensitivity analysis are presented for both the cases in Table 3 and Table 4 and Figures 6–27. On the basis of the results of Table 3 and Table 4 and Figures 6–27, we see that fuzzy model provides best optimal solution as compared to crisp model.

(1) It's interesting to note that increasing the value of the holding costs components h_{hc} and r_{hc} has a positive effect. This will lead to a decrease in Q, T and τ but increase of $TC_p(T,\tau)$.

(2) The optimal solution for several values of D, increase in demand parameter a results increase in Q, and τ. This result has implication on the holding cost, ordering cost as well as delivery cost. Therefore, an increase in a will lead to an increase of $TC_p(T,\tau)$ and decrease in T.

(3) From Table 4, the values of Q, T, τ and $TC_p(T,\tau)$ increase with increase in the values of parameter b, whereas, from Table 3, increase in the values of parameter b will result in decrease of Q, T and τ but increase of $TC_p(T,\tau)$.

Table 3. Effects of parameters on optimal solution for *Case* 1: $M \leq T$

Parameter	%	Crisp Parameter values	Fuzzy Parameter values	Crisp Optimal values				Fuzzy Optimal values			
				T	τ	Q	$TC_p(T,\tau)$	T	τ	Q	$TC_p(T,\tau)$
h_{hc}	-50	0.35	0.34	0.46	34.99	47.75	770.46	0.51	30.78	43.57	589.01
	-25	0.525	0.47	0.45	34.57	46.68	775.30	0.50	30.45	42.69	592.10
	0	0.70	0.60	0.44	34.17	45.61	780.05	0.49	30.13	41.81	595.14
	+25	0.875	0.73	0.44	33.78	45.61	784.00	0.48	29.82	40.93	598.14
	+50	1.05	0.83	0.42	33.41	43.47	789.31	0.48	29.59	40.94	600.16
r_{hc}	-50	2.50	2.25	0.49	36.15	50.98	767.13	0.55	32.27	47.10	584.72
	-25	3.75	3.56	0.47	35.07	48.83	773.48	0.51	30.93	43.57	591.19
	0	5	4.5	0.44	34.17	45.61	780.05	0.49	30.13	41.81	595.14
	+25	6.25	5.63	0.43	33.4	44.54	784.79	0.47	29.31	40.06	599.43
	+50	7.5	6.75	0.41	32.71	42.41	789.94	0.45	28.59	38.31	603.41
u	-50	0.025	0.023	0.40	36.60	41.49	808.33	0.42	25.59	35.89	620.90
	-25	0.0375	0.031	0.43	35.01	44.59	791.04	0.45	30.83	38.42	611.34
	0	0.05	0.05	0.44	34.17	45.61	780.05	0.49	30.13	41.81	595.14
	+25	0.0625	0.055	0.45	31.22	46.65	772.02	0.49	29.32	41.79	592.31
	+50	0.075	0.065	0.46	28.62	47.70	765.83	0.50	27.63	42.64	587.21
a	-50	50	45.00	0.56	24.97	29.38	439.51	0.60	21.95	28.22	365.44
	-25	75	63.75	0.49	30.40	38.28	611.69	0.53	26.69	35.07	481.94
	0	100	82.50	0.44	34.17	45.61	780.05	0.49	30.13	41.81	595.14
	+25	125	112.5	0.41	37.06	52.97	945.44	0.44	34.22	50.98	772.80
	+50	150	127.5	0.38	39.39	58.76	1109.9	0.42	35.85	55.07	860.54
b	-50	0.075	0.06	0.46	34.89	46.91	771.66	0.50	30.76	42.07	590.12
	-25	0.1125	0.09	0.45	34.49	46.30	776.09	0.49	30.41	41.51	592.94
	0	0.15	0.12	0.44	34.17	45.61	780.05	0.49	30.13	41.81	595.14
	+25	0.1875	0.14	0.44	33.95	45.97	782.81	0.49	29.98	42.01	596.46
	+50	0.225	0.17	0.44	33.82	46.34	785.11	0.48	29.81	41.41	598.55
C_{oc}	-50	20	18.75	0.32	27.61	32.88	736.11	0.35	23.46	29.62	555.10
	-25	30	26.25	0.39	31.54	40.28	759.60	0.41	26.81	34.82	572.87
	0	40	37.50	0.44	34.17	45.61	780.05	0.49	30.13	41.81	595.14
	+25	50	41.25	0.49	36.13	50.98	797.74	0.51	30.99	43.57	601.96
	+50	60	52.50	0.53	37.68	55.3	814.26	0.57	33.09	48.86	620.71
C_{dc}	-50	25	22.50	0.43	22.10	44.66	770.90	0.48	18.13	41.09	584.79
	-25	37.5	29.88	0.44	28.98	45.66	775.95	0.48	22.87	41.01	589.11
	0	50	45.00	0.44	34.17	45.61	780.05	0.49	30.13	41.81	595.14
	+25	62.5	56.25	0.45	38.36	46.65	782.39	0.49	34.27	41.77	598.61
	+50	75	67.50	0.45	41.89	46.63	784.77	0.50	37.74	48.86	601.85
C_{pc}	-50	3	2.63	0.47	34.09	48.84	466.37	0.51	29.83	43.58	370.61
	-25	4.5	3.75	0.46	34.13	47.76	622.94	0.50	29.96	42.70	466.67
	0	6	5.25	0.44	34.17	45.61	780.05	0.49	30.13	41.81	595.14
	+25	7.5	6.00	0.43	34.22	44.54	936.27	0.48	30.22	40.93	659.50
	+50	9	7.50	0.42	34.27	43.47	1092.4	0.47	30.38	40.05	787.78
S_{sp}	-50	7.5	6.00	0.45	34.42	46.68	792.63	0.49	30.33	41.81	602.34
	-25	11.25	8.63	0.45	34.29	46.68	785.98	0.49	30.25	41.81	599.54
	0	15	12.75	0.44	34.17	45.61	780.05	0.49	30.13	41.81	595.14
	+25	18.75	14.63	0.44	34.05	45.61	773.57	0.49	30.08	41.81	593.13
	+50	22.5	17.25	0.44	33.94	45.61	767.08	0.49	30.01	41.81	590.34
I_e	-50	0.075	0.06	0.45	34.42	46.68	792.63	0.49	30.32	41.81	601.94
	-25	0.1125	0.09	0.45	34.42	46.68	785.98	0.49	30.23	41.81	598.54
	0	0.15	0.12	0.44	34.17	45.61	780.05	0.49	30.13	41.81	595.14
	+25	0.1875	0.17	0.44	34.05	45.61	773.57	0.49	29.99	41.81	589.47
	+50	0.225	0.18	0.44	33.94	45.61	767.08	0.48	29.96	40.93	588.85
M	-50	0.125	0.10	0.45	34.42	46.68	792.77	0.49	30.33	41.81	602.46
	-25	0.1875	0.14	0.45	34.29	46.68	786.01	0.49	30.25	41.81	599.78
	0	0.25	0.21	0.44	34.17	45.61	780.05	0.49	30.13	41.81	595.14
	+25	0.3125	0.23	0.44	34.06	45.61	773.64	0.49	30.10	41.81	593.83
	+50	0.375	0.29	0.44	33.95	45.61	767.31	0.49	30.00	41.81	589.92

Table 4. Effects of parameters on optimal solution for *Case* 2: $M \geq T$

Parameter	%	Crisp Parameter values	Fuzzy Parameter values	Crisp Optimal values				Fuzzy Optimal values			
				T	τ	Q	$TC_p(T,\tau)$	T	τ	Q	$TC_p(T,\tau)$
h_{hc}	-50	0.350	0.34	0.61	40.26	64.01	762.75	0.62	34.78	53.30	581.89
	-25	0.525	0.47	0.59	39.60	61.83	765.12	0.61	34.32	52.42	584.32
	0	0.70	0.60	0.57	38.96	59.65	767.32	0.59	33.87	50.64	585.70
	+25	0.875	0.73	0.55	38.36	57.47	769.36	0.58	33.43	49.97	587.97
	+50	1.05	0.83	0.53	37.78	55.3	771.26	0.57	33.11	48.86	589.45
r_{hc}	-50	2.50	2.25	0.73	43.90	77.26	774.34	0.73	38.12	63.16	585.65
	-25	3.75	3.56	0.63	40.91	66.21	768.80	0.64	35.28	55.09	585.53
	0	5	4.5	0.57	38.96	59.65	767.32	0.59	33.87	50.64	585.70
	+25	6.25	5.63	0.53	37.51	55.3	767.70	0.55	32.53	47.09	587.07
	+50	7.5	6.75	0.50	36.35	52.06	768.79	0.52	31.45	44.45	588.76
u	-50	0.025	0.023	0.50	45.30	52.24	786.92	0.50	32.68	42.93	604.77
	-25	0.0375	0.031	0.54	43.22	56.45	774.69	0.54	36.48	46.34	598.05
	0	0.05	0.05	0.57	38.96	59.65	767.32	0.59	33.87	50.64	585.70
	+25	0.0625	0.055	0.58	35.12	60.69	760.55	0.60	32.74	51.50	583.48
	+50	0.075	0.065	0.59	31.90	61.75	755.95	0.61	30.56	52.36	579.14
a	-50	50	45.0	0.70	29.22	37.09	441.68	0.69	33.87	27.62	363.78
	-25	75	63.75	0.60	34.95	47.24	605.01	0.64	30.23	42.63	477.40
	0	100	82.5	0.57	38.96	59.65	767.32	0.59	33.87	50.64	585.70
	+25	125	112.50	0.53	42.05	69.08	923.33	0.54	38.21	62.94	755.02
	+50	150	127.50	0.50	44.56	77.99	1077.1	0.52	39.95	68.58	838.09
b	-50	0.075	0.06	0.44	33.82	44.89	746.50	0.61	34.62	51.49	582.31
	-25	0.1125	0.09	0.57	39.32	59.03	763.76	0.60	34.19	51.08	584.22
	0	0.15	0.12	0.57	38.96	59.65	767.32	0.59	33.87	50.64	585.70
	+25	0.1875	0.14	0.57	38.78	60.26	770.11	0.59	33.71	50.92	587.10
	+50	0.225	0.17	0.57	38.79	60.87	772.13	0.59	33.56	51.36	588.86
C_{oc}	-50	20	18.75	0.41	32.73	42.41	703.05	0.43	27.48	36.56	534.03
	-25	30	26.25	0.5	36.47	52.06	738.16	0.50	30.69	42.69	556.69
	0	40	37.50	0.57	38.96	59.65	767.32	0.59	33.87	50.64	585.70
	+25	50	41.25	0.62	40.82	65.11	791.63	0.62	34.68	53.30	594.77
	+50	60	52.50	0.67	42.29	70.61	814.64	0.68	36.69	58.66	617.66
C_{dc}	-50	25	22.50	0.55	26.59	57.63	755.57	0.58	21.72	49.94	574.56
	-25	37.5	29.88	0.56	33.61	58.61	762.31	0.58	26.50	49.85	578.71
	0	50	45.0	0.57	38.96	59.65	767.32	0.59	33.87	50.64	585.70
	+25	62.5	56.25	0.58	43.33	60.7	771.28	0.60	38.09	51.48	589.87
	+50	75	67.5	0.59	47.06	61.76	774.50	0.60	41.65	51.45	592.40
C_{pc}	-50	3	2.63	0.62	39.45	65.12	460.43	0.62	33.86	53.32	363.45
	-25	4.5	3.75	0.59	39.20	61.83	613.26	0.61	33.86	52.42	458.78
	0	6	5.25	0.57	38.96	59.65	767.32	0.59	33.87	50.64	585.70
	+25	7.5	6.00	0.55	38.75	57.46	921.13	0.58	33.87	49.74	649.11
	+50	9	7.50	0.53	38.57	55.29	1074.70	0.57	33.88	48.85	776.76
S_{sp}	-50	7.5	6.00	0.50	36.65	52.06	782.33	0.54	31.99	46.21	596.06
	-25	11.25	8.63	0.53	37.75	55.30	773.25	0.56	32.69	47.98	591.62
	0	15	12.75	0.57	38.96	59.65	767.32	0.59	33.87	50.64	585.70
	+25	18.75	14.63	0.60	40.30	62.92	765.87	0.61	34.44	52.41	584.13
	+50	22.5	17.25	0.65	41.78	68.40	762.41	0.64	35.28	55.09	582.94
I_e	-50	0.025	0.02	0.50	36.65	52.06	782.33	0.54	31.91	46.21	596.78
	-25	0.0375	0.036	0.53	37.75	55.30	773.25	0.57	33.13	48.86	589.10
	0	0.05	0.045	0.57	38.96	59.65	767.32	0.59	33.87	50.64	585.70
	+25	0.0625	0.055	0.61	40.30	64.01	763.74	0.62	34.74	53.30	583.48
	+50	0.075	0.065	0.65	41.78	68.40	762.41	0.65	35.68	55.98	582.24
M	-50	0.06	0.05	0.57	38.99	59.64	772.63	0.59	33.88	50.64	588.16
	-25	0.09	0.08	0.57	38.98	59.65	769.68	0.59	33.87	50.64	586.68
	0	0.12	0.10	0.57	38.96	59.65	767.32	0.59	33.87	50.64	585.70
	+25	0.15	0.12	0.57	38.95	59.65	764.97	0.59	33.86	50.64	584.72
	+50	0.18	0.14	0.57	38.94	59.65	762.62	0.59	33.86	50.64	583.74

(4) From Table 3 and Table 4, it is observed that increase in the effectiveness parameter u will result in decrease of $TC_p(T,\tau)$ and τ but increase of T and Q,.

(5) It is foreseeable that if the buyer's ordering cost C_{oc} rises, $TC_p(T,\tau)$and Q, will increases. This is because, for high values of ordering cost, departing from the optimal solution has a substantial effect on T, and τ respectively. As a result, an increase in C_{oc} will result in an increase in T, and τ in both circumstances.

(6) From Table 3 and Table 4, increase in the values of the parameter C_{dc} will result in increase of Q, T, τ and $TC_p(T,\tau)$.

(7) In Table 3, with an increase in purchasing cost C_{pc}, $TC_p(T,\tau)$increases. However, Q, T and τ decreases. From Table 4, it is seen that increase in the values of C_{pc} will result in increase of $TC_p(T,\tau)$and τ but decrease of T and Q.

(8) Table 3 ensure that, with an increase in the values of the parameter S_{sp} will result in decrease of Q, T, τ and $TC_p(T,\tau)$. From Table 4, we observe that increase in selling price S_{sp} increases Q, T and τ. But decrease in $TC_p(T,\tau)$.

(9) From Table 3, it is interesting to observe that increase in the values of the parameter I_e will result in decrease of Q, T, τ and $TC_p(T,\tau)$. From Table 4, we observe that increase in interest earned I_e increases Q, T and τ. But decrease in $TC_p(T,\tau)$.

(10) When the credit period M increases, Q, T, τ and $TC_p(T,\tau)$ decreases from Table 3. But from Table 4, it shows that increase in M, results decrease in $TC_p(T,\tau)$ and with slight fluctuation in Q, T and τ.

8. Special Cases

The proposed model has been explored under various special conditions. Following assumption are made based on the relative results summarized in Table 5 and Table 6.

Special case 1. In this case, increase in total cost, as the system avert the investment in preservation technology ($\tau = 0$) for both crisp and fuzzy models. Since, there is a lack of controlling the rate of deterioration. Finally, increases total cost as there is an increase in deterioration units. We observe that it would be better to place a small order more frequently. Also, there is a positive influence in the total cost made by the investment in preservation technology.

Special case 2. For $b = 0$, $\tau = 0$ the rate of demand is constant. Deterioration rate cannot be controlled as the system does not consider the investment in preservation technology for both crisp and fuzzy models. Further, total cost of the system increases. Hence, there is a dual impact of constant demand and constant deterioration rate.

Special case 3. Throughout the cycle, there is a constant holding cost and constant deterioration parameter for $\tau = 0$, $r_{hc} = 0$. Here, total cost is minimized as the holding cost is not increasing with time. But there is no investment in preservation technology for both crisp and fuzzy models which results in loss due to deterioration. Eventually increases the total cost.

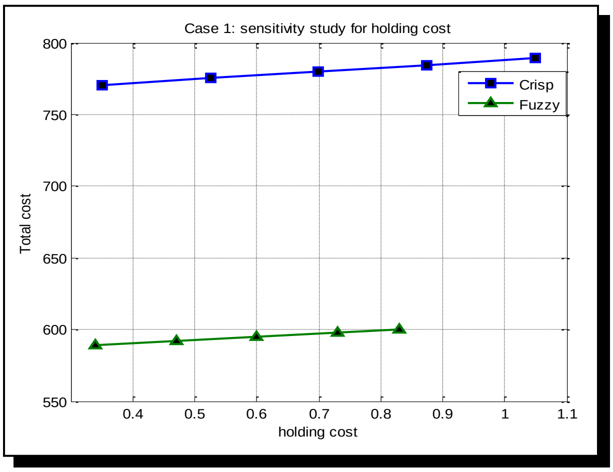


Figure 6. Effect of holding cost h_{hc}

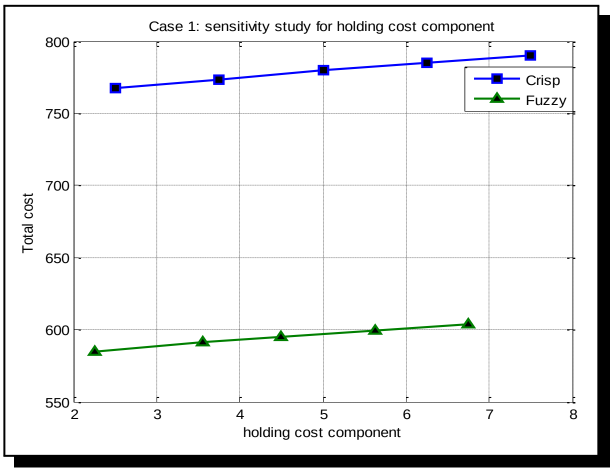


Figure 7. Effect of holding cost component r_{hc}

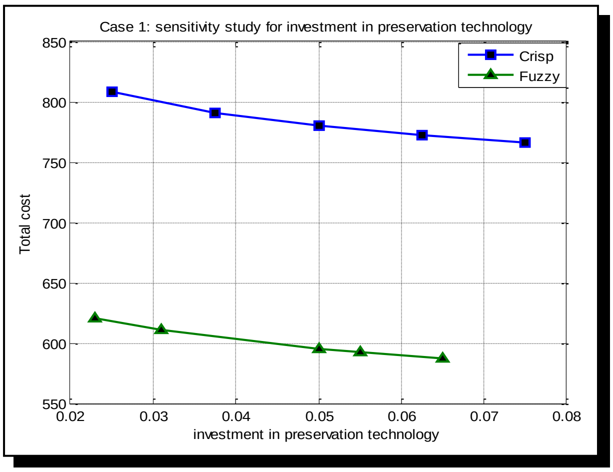


Figure 8. Effect of investment in preservation technology u

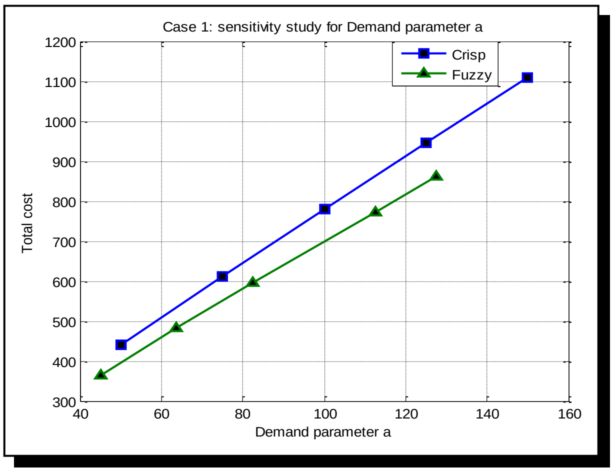


Figure 9. Effect of demand parameter a

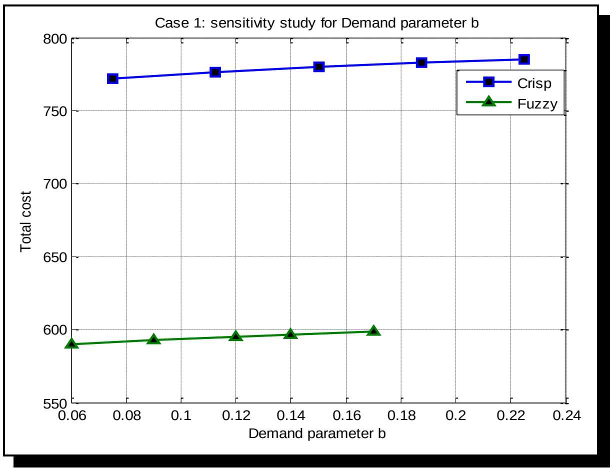


Figure 10. Effect of demand parameter b

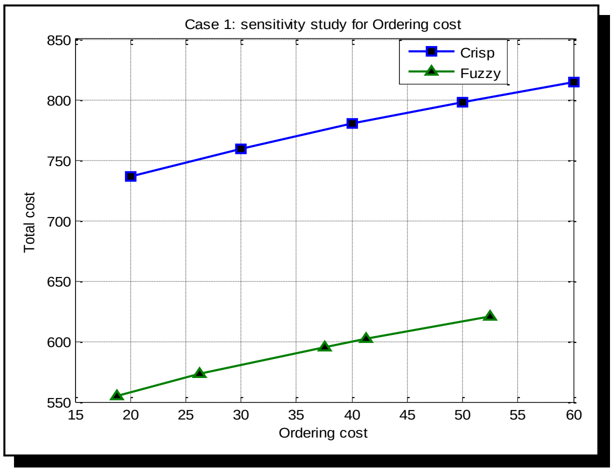


Figure 11. Effect of ordering cost C_{oc}

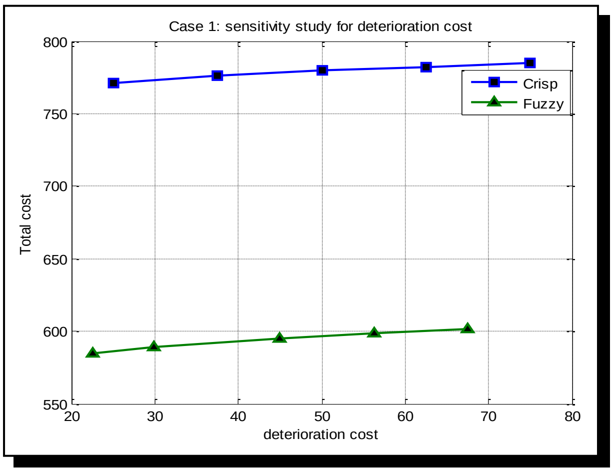


Figure 12. Effect of deterioration cost C_{dc}

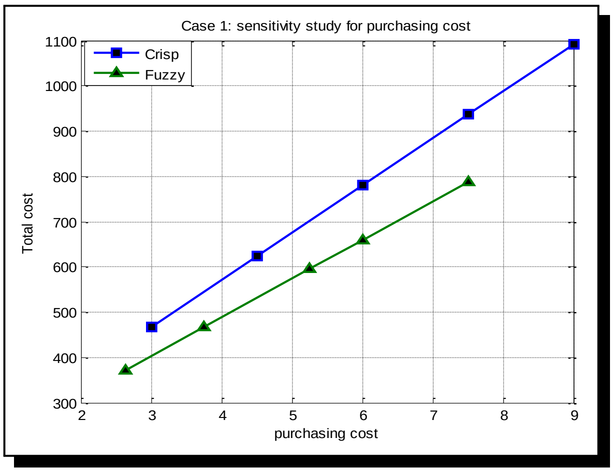


Figure 13. Effect of purchasing cost C_{pc}

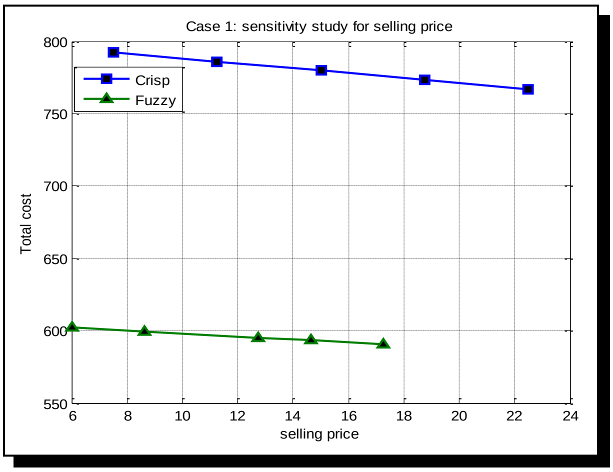


Figure 14. Effect of selling price S_{sp}

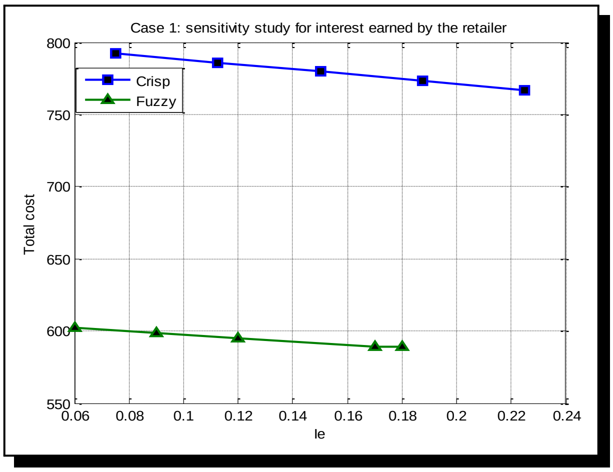


Figure 15. Effect of I_e

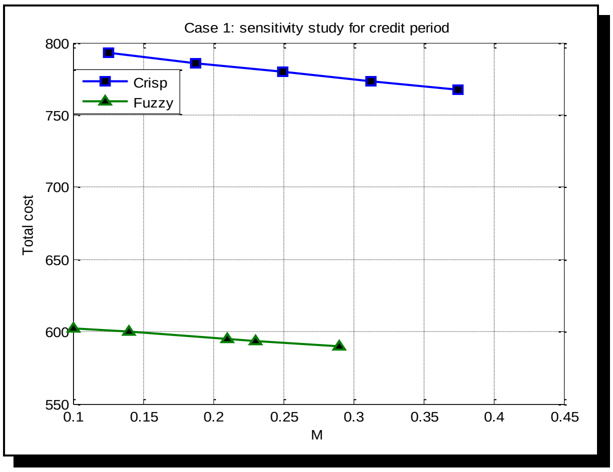


Figure 16. Effect of credit period M

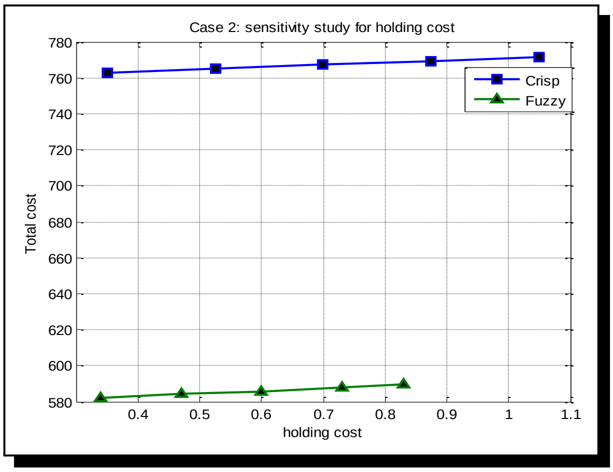


Figure 17. Effect of holding cost h_{hc}

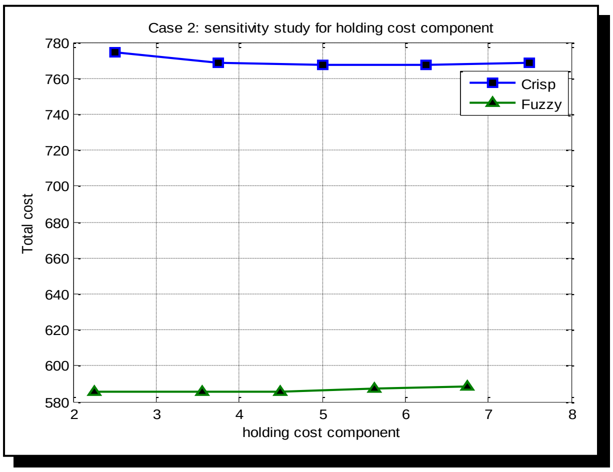


Figure 18. Effect of holding cost component r_{hc}

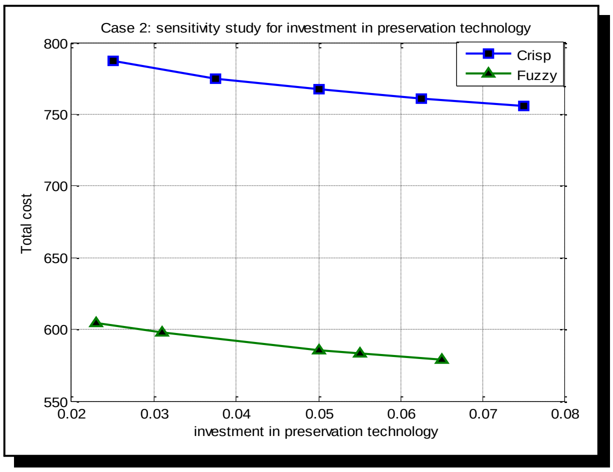


Figure 19. Effect of investment in preservation technology u

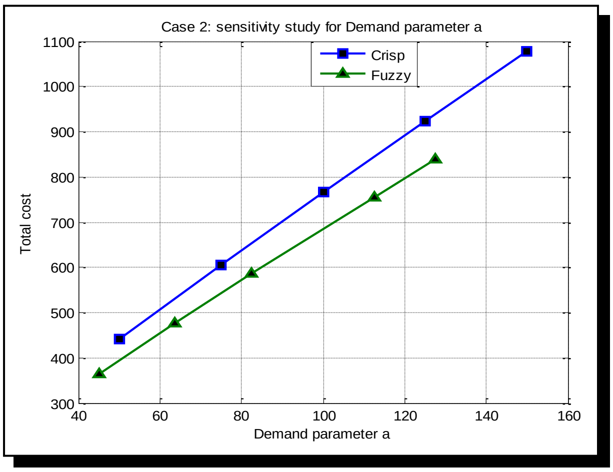


Figure 20. Effect of demand parameter a

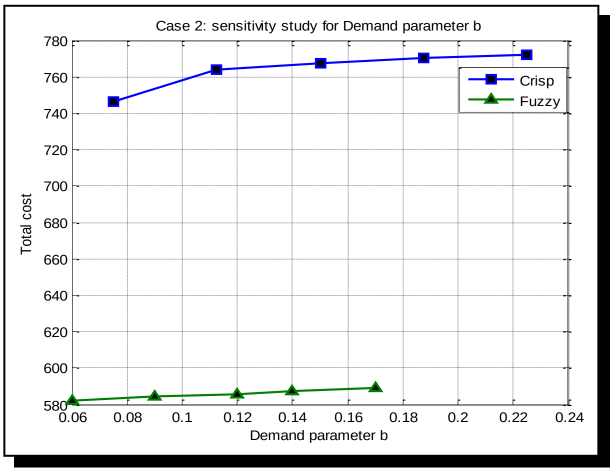


Figure 21. Effect of demand parameter b

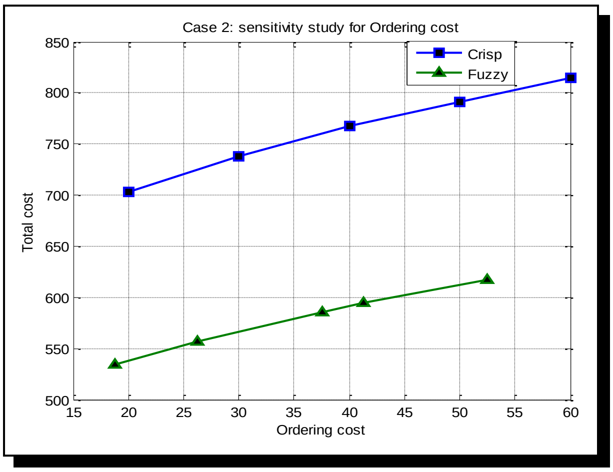


Figure 22. Effect of ordering cost C_{oc}

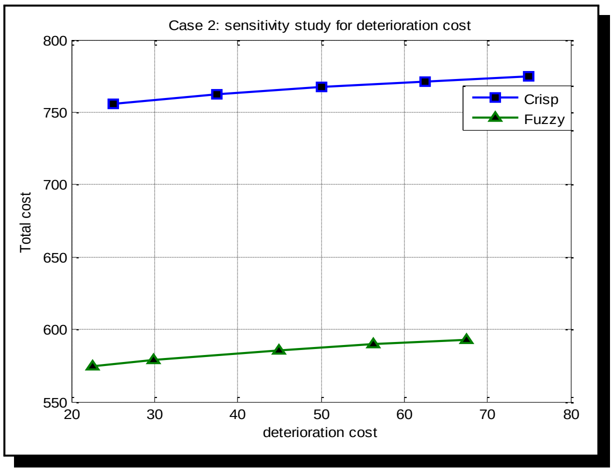


Figure 23. Effect of deterioration cost C_{dc}

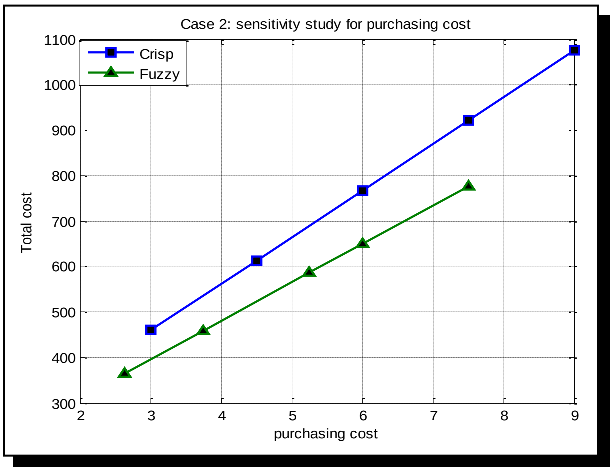


Figure 24. Effect of purchasing cost C_{pc}

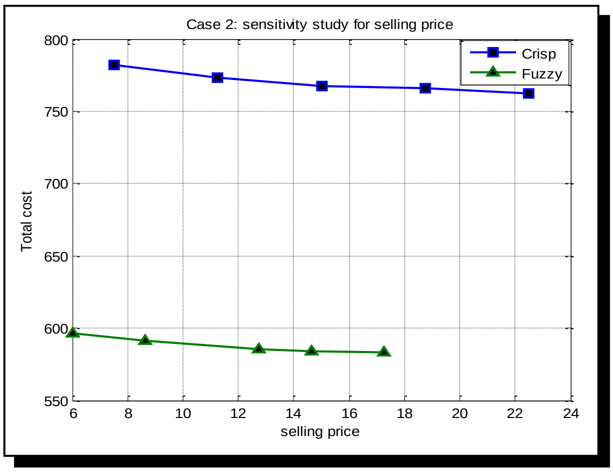


Figure 25. Effect of selling price S_{sp}

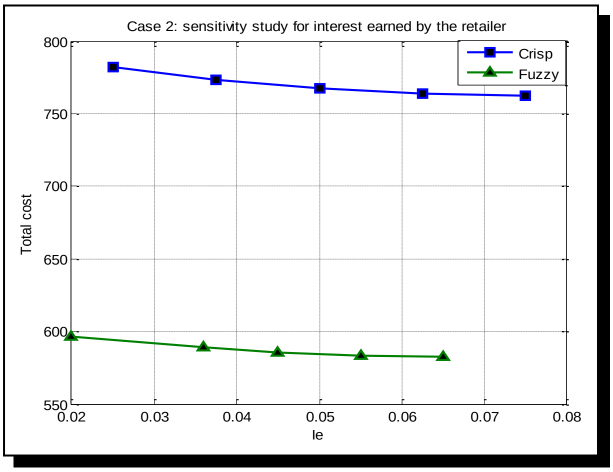


Figure 26. Effect of I_e

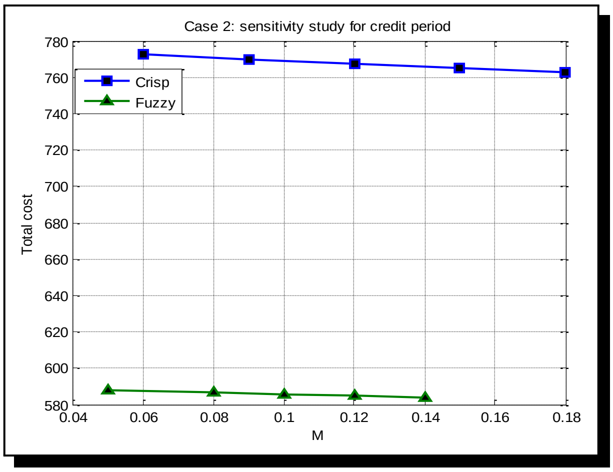


Figure 27. Effect of credit period M

Special case 4. This case represents EOQ model for deterioration under the condition$\tau = 0$, $r_{hc} = 0$, $b = 0$. Thus, it shows that stock-dependent demand, time-varying holding cost as well as investment in preservation technology for both crisp and fuzzy models which results a positive impact in the inventory system (for case 1).

Special case 5. In this case ($\tau = A_{ce} = c_{ce} = h_{ce} = 0$), an inventory model for deterioration under trade credit is represented. As there is no preservation technology and reduction of carbon emission, the system does not control the deterioration rate. This result increase in total cost. Hence, there is a positive impact of preservation technology and reduction of carbon emission in this article.

Special case 6. ($\tau = 0, I_e = 0, I_c = 0$) Even though, there is reduction of carbon emission, the system does not control the deterioration rate. Since, there is no investment in preservation technology. Finally, increase in total cost. Therefore, by incorporating investment in preservation technology and trade credit, savings may occur, by reducing the total cost.

Table 5. Special cases for case $M \leq T$

Special cases for *Case* 1: $M \leq T$		T^*	τ^*	TC^*	Q^*
1	Crisp $\tau = 0$ Triangular fuzzy model	0.32 0.36	0 0	819.46 629.48	32.46 30.82
2	Crisp $b = 0$, $\tau = 0$ Triangular fuzzy model	0.34 0.37	0 0	817.93 621.56	34.52 31.03
3	Crisp $\tau = 0$, $r_{hc} = 0$ Triangular fuzzy model	0.35 0.40	0 0	819.14 619.07	36.47 34.39
4	Crisp $\tau = 0$, $r_{hc} = 0$, $b = 0$. Triangular fuzzy model	0.36 0.41	0 0	806.41 610.77	36.58 34.45
5	Crisp $\tau = A_{ce} = c_{ce} = h_{ce} = 0$ Triangular fuzzy model	0.33 0.37	0 0	817.93 618.95	34.31 31.71
6	Crisp $\tau = 0$,$I_e = 0$,$I_c = 0$Triangular Fuzzy model	0.33 0.37	0 0	837.06 639.24	34.31 31.71

Table 6. Special Cases for case $M \geq T$

Special cases for case $M \geq T$		T^*	τ^*	TC^*	Q^*
1	Crisp $\tau = 0$ Triangular fuzzy model	0.36 0.40	0 0	791.90 604.65	37.56 33.90
2	Crisp $b = 0$, $\tau = 0$ Triangular fuzzy model	0.38 0.41	0 0	783.28 597.96	38.65 34.45
3	Crisp $\tau = 0$, $r_{hc} = 0$ Triangular fuzzy model	0.41 0.45	0 0	774.49 598.04	41.76 38.88
4	Crisp $\tau = A_{ce} = c_{ce} = h_{ce} = 0$ Triangular fuzzy model	0.36 0.40	0 0	779.51 594.26	37.56 34.39
5	Crisp $\tau = 0$ $I_e = 0$, $I_c = 0$ Triangular fuzzy model	0.33 0.37	0 0	849.26 639.24	34.31 31.71

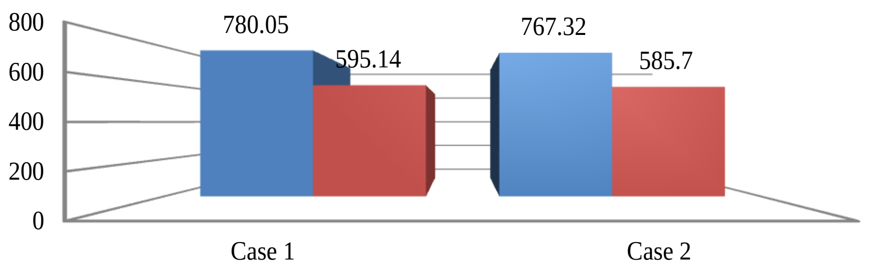
Table 7. Summary of optimal solution

	Crisp model		Triangular fuzzy model		% savings	
	Case 1	*Case* 2	*Case* 1	*Case* 2	*Case* 1	*Case* 2
T^*	0.44	0.57	0.49	0.59	—	—
τ^*	34.17	38.96	30.13	33.87	11.82	13.06
TC^*	780.05	767.32	595.14	585.70	23.70	23.67
Q^*	45.61	59.65	41.81	50.64	8.33	15.10

9. Comparative Study

From the comparative study of Table 5 and Table 6, we observe that triangular fuzzy number gives the optimum solution in this paper. The fuzzy model with triangular fuzzy numbers generates a better result than the crisp model with the total cost with 23.70% savings under *Case* 1 and 23.67% savings under *Case* 2. In this paper, it is shown that the knowledge of the crisp model is gradually improved to a fuzzy model with triangular fuzzy and fine-tuned our model into more specific knowledge with minimum total cost. The main reason for this situation is the low carbon emission cost under trade credit which helps to increase the sales and a positive impact on customer preference.

The triangular fuzzy model finds lower values of $\tau = 30.13$ and total cost $TC = 595.14$ (better) at each performance criterion than the crisp model $\tau = 34.17$ and total cost $TC = 780.05$ under full credit period (*Case* 1) and for *Case* 2, $\tau = 33.87$ and total cost $TC = 585.70$ in the triangular fuzzy model but $\tau = 38.96$ and total cost $TC = 767.32$ in the crisp model, indicating that total cost is higher than the fuzzy model with triangular fuzzy numbers (Figure 28). In each case, we conclude that fuzzy model gives a better result than the crisp model. Hence, fuzzy model gives the advantages of the application of fuzzy in real-world environment on Supply Chain management.



	Case 1	Case 2
crisp	780.05	767.32
Fuzzy	595.14	585.7

Figure 28. Comparison of crisp and fuzzy model

10. Conclusion

We investigated the inventory model in two different methods in this research. The crisp model is developed by taking into consideration that the parameters are precisely known. However, these parameters are inherently imprecise in nature. The parameters of a fuzzy model are represented as triangular fuzzy integers. Total cost function is defuzzified and proven to be convex using the signed distance method. A comparison of crisp and fuzzy models is made using special cases. The concept of triangular fuzzy numbers is highlighted in this study, and the signed distance method for defuzzification is found to be the most cost-effective. Further, model parameters were subjected to sensitivity analysis, which yielded management insights.

The model developed here may further be outstretched for more criteria of permissible delay, seasonal and expiry products, inflation and multi-items.

Appendix A

Crisp Model

Case 1. $M \le T$:

$$\begin{aligned}\frac{\partial TC_p}{\partial T} &= \frac{-C_{oc}}{T^2} + \frac{ah_{hc}}{2} + \frac{ar_{hc}T}{3} + \frac{ah_{hc}T}{3}(y_0e^{-u\tau}+b) + \frac{ar_{hc}T^2}{8}(y_0e^{-u\tau}+b) \\ &\quad + \frac{C_{dc}}{6}(3ay_0e^{-u\tau} - 2abTy_0e^{-u\tau} - 2ab^2T) + \frac{aC_{pc}}{2}(y_0e^{-u\tau}+b) \\ &\quad + \frac{aC_{pc}I_c}{6T^2}[3(T^2-M^2) + (y_0e^{-u\tau}+b)(2T^3 - 3MT^2 + M^3)] \\ &\quad - S_{sp}I_e\left[\frac{-aM}{T^2} + \frac{abM^2}{2T^2} + \frac{ab}{6T^2}(y_0e^{-u\tau}+b)(3MT^2 - M^3)\right] \\ &\quad - \frac{A_{ce}}{T^2} + \frac{ac_{ce}}{2}(y_0e^{-u\tau}+b) + \frac{ah_{ce}}{6}[3 + 4T(y_0e^{-u\tau}+b)],\end{aligned} \tag{A.1}$$

$$\begin{aligned}\frac{\partial TC_p}{\partial \tau} &= 1 - \frac{auy_0e^{-u\tau}h_{hc}T^2}{6} - \frac{auy_0e^{-u\tau}r_{hc}T^3}{24} - \frac{C_{dc}uy_0e^{-u\tau}T(3a-abT)}{6} \\ &\quad - \frac{ah_{ce}T^2uy_0e^{-u\tau}}{3} - \frac{a(C_{pc}+c_{ce})Tuy_0e^{-u\tau}}{2} - \frac{aC_{pc}I_cuy_0e^{-u\tau}(T-M)^3}{6T} \\ &\quad + \frac{S_{sp}I_eabMuy_0e^{-u\tau}(3T^2+M^2-3MT)}{6T},\end{aligned} \tag{A.2}$$

$$\begin{aligned}\frac{\partial^2 TC_p}{\partial T^2} &= \frac{2C_{oc}}{T^3} + \frac{ar_{hc}}{3} + \frac{ah_{hc}}{3}(y_0e^{-u\tau}+b) + \frac{ar_{hc}T}{4}(y_0e^{-u\tau}+b) \\ &\quad - \frac{C_{dc}}{3}(aby_0e^{-u\tau}+ab^2) + \frac{2ah_{ce}(y_0e^{-u\tau}+b)}{3} + \frac{2A_{ce}}{T^3} \\ &\quad + \frac{aC_{pc}I_cM^2}{T^3} + \frac{aC_{pc}I_c(y_0e^{-u\tau}+b)(T^3-M^3)}{3T^3} \\ &\quad - \frac{S_{sp}I_eaM(2-bM)}{T^3} - \frac{S_{sp}I_eabM^3(y_0e^{-u\tau}+b)}{3T^3},\end{aligned} \tag{A.3}$$

$$\begin{aligned}\frac{\partial^2 TC_p}{\partial \tau^2} &= \frac{au^2h_{hc}T^2y_0e^{-u\tau}}{6} + \frac{au^2r_{hc}T^3y_0e^{-u\tau}}{24} + \frac{C_{dc}Tu^2y_0e^{-u\tau}(3a-abT)}{6} \\ &\quad + \frac{ah_{ce}T^2u^2y_0e^{-u\tau}}{3} + \frac{a(C_{pc}+c_{ce})Tu^2y_0e^{-u\tau}}{2} + \frac{aC_{pc}I_cu^2y_0e^{-u\tau}(T-M)^3}{6T} \\ &\quad - \frac{S_{sp}I_eabMu^2y_0e^{-u\tau}(3T^2+M^2-3MT)}{6T},\end{aligned} \tag{A.4}$$

$$\frac{\partial^2 TC_p}{\partial T\partial\tau} = \frac{\partial^2 TC_p}{\partial\tau\partial T}$$

$$= -\frac{auy_0e^{-u\tau}h_{hc}T}{3} - \frac{auy_0e^{-u\tau}r_{hc}T^2}{8} - \frac{C_{dc}uy_0e^{-u\tau}(3a-2abT)}{6} - \frac{2ah_{ce}Tuy_0e^{-u\tau}}{3} - \frac{auy_0e^{-u\tau}(C_{pc}+c_{ce})}{2} - \frac{aC_{pc}I_cuy_0e^{-u\tau}(2T^3-3MT^2+M^3)}{6T^2} + \frac{S_{sp}I_eMabuy_0e^{-u\tau}(3T^2-M^2)}{6T^2}. \tag{A.5}$$

Case 2. $M \geq T$:

$$\frac{\partial TC}{\partial T} = \frac{-C_{oc}}{T^2} + \frac{ah_{hc}}{2} + \frac{ar_{hc}T}{3} + \frac{ah_{hc}T}{3}(y_0e^{-u\tau}+b) + \frac{ar_{hc}T^2}{8}(y_0e^{-u\tau}+b) + \frac{C_{dc}}{6}(3ay_0e^{-u\tau}-2abTy_0e^{-u\tau}-2ab^2T) + \frac{aC_{pc}}{2}(y_0e^{-u\tau}+b) - \frac{A_{ce}}{T^2} + \frac{ac_{ce}}{2}(y_0e^{-u\tau}+b) + \frac{ah_{ce}}{6}[3+4T(y_0e^{-u\tau}+b)] - \frac{S_{sp}I_eab(1+M)(3+4T(y_0e^{-u\tau}+b))}{6} + S_{sp}I_ea[1+b(T+T^2(y_0e^{-u\tau}+b))], \tag{A.6}$$

$$\frac{\partial TC}{\partial\tau} = 1 - \frac{auy_0e^{-u\tau}h_{hc}T^2}{6} - \frac{auy_0e^{-u\tau}r_{hc}T^3}{24} - \frac{C_{dc}auy_0e^{-u\tau}T(3-bT)}{6} - \frac{h_{ce}auy_0e^{-u\tau}T^2}{3} - \frac{(C_{pc}+c_{ce})auy_0e^{-u\tau}T}{2} + \frac{S_{sp}I_eabuy_0e^{-u\tau}T^2(1+M)}{3} - \frac{S_{sp}I_eabuy_0e^{-u\tau}T^3}{3}, \tag{A.7}$$

$$\frac{\partial^2 TC}{\partial T^2} = \frac{2C_{oc}}{T^3} + \frac{ar_{hc}}{3} + \frac{ah_{hc}}{3}(y_0e^{-u\tau}+b) + \frac{ar_{hc}T}{4}(y_0e^{-u\tau}+b) - \frac{C_{dc}}{3}(aby_0e^{-u\tau}+ab^2) + \frac{2A_{ce}}{T^3} + \frac{2ah_{ce}(y_0e^{-u\tau}+b)}{3} - \frac{2S_{sp}I_eab(y_0e^{-u\tau}+b)(1+M)}{3} + S_{sp}I_eab(1+2T(y_0e^{-u\tau}+b)), \tag{A.8}$$

$$\frac{\partial^2 TC}{\partial\tau^2} = \frac{au^2h_{hc}T^2y_0e^{-u\tau}}{6} + \frac{au^2r_{hc}T^3y_0e^{-u\tau}}{24} + \frac{C_{dc}Tau^2y_0e^{-u\tau}(3-bT)}{6} + \frac{ah_{ce}T^2u^2y_0e^{-u\tau}}{3} + \frac{a(C_{pc}+c_{ce})Tu^2y_0e^{-u\tau}}{2} - \frac{S_{sp}I_eabu^2y_0e^{-u\tau}T^2(1+M)}{3} + \frac{S_{sp}I_eabu^2y_0e^{-u\tau}T^3}{3}, \tag{A.9}$$

$$\frac{\partial^2 TC_p}{\partial T\partial\tau} = \frac{\partial^2 TC_p}{\partial\tau\partial T}$$

$$
\begin{aligned}
&= -\frac{auy_0e^{-u\tau}h_{hc}T}{3} - \frac{auy_0e^{-u\tau}r_{hc}T^2}{8} - \frac{C_{dc}auy_0e^{-u\tau}(3-2bT)}{6} \\
&\quad - \frac{2ah_{ce}uy_0e^{-u\tau}T}{3} - \frac{auy_0e^{-u\tau}(C_{pc}+c_{ce})}{2} + \frac{2S_{sp}I_eabuy_0e^{-u\tau}T(1+M)}{3} \\
&\quad - S_{sp}I_eabuy_0e^{-u\tau}T^2. \qquad \text{(A.10)}
\end{aligned}
$$

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] K. Annadurai and R. Uthayakumar, Analysis of partial trade credit financing in a supply chain by EOQ-based model for decaying items with shortages, *The International Journal of Advanced Manufacturing Technology* **61** (2012), 1139 – 1159, DOI: 10.1007/s00170-011-3765-9.

[2] K. Annadurai and R. Uthayakumar, Two-echelon inventory model for deteriorating items with credit period dependent demand including shortages under trade credit, *Optimization Letters* **7** (2013), 1227 – 1249, DOI: 10.1007/s11590-012-0499-z.

[3] S. Bardhan, H. Pal and B. C. Giri, Optimal replenishment policy and preservation technology investment for a non-instantaneous deteriorating item with stock-dependent demand, *Operational Research* **19**(2) (2019), 347 – 368, DOI: 10.1007/s12351-017-0302-0.

[4] K.-M. Björk, An analytical solution to a fuzzy economic order quantity problem, *International Journal of Approximate Reasoning* **50**(3) (2009), 485 – 493, DOI: 10.1016/j.ijar.2008.10.001.

[5] K. D. Choudhury, B. Karmakar, M. Das and T. K. Datta, An inventory model for deteriorating items with stock-dependent demand, time-varying holding cost and shortages, *Opsearch* **52**(1) (2015), 55 – 74, DOI: 10.1007/s12597-013-0166-x.

[6] Y. Daryanto and H. M. Wee, Low carbon economic production quantity model for imperfect quality deteriorating item, *International Journal of Industrial Engineering and Engineering Management* **1**(1) (2019), 1 – 8, DOI: 10.24002/ijieem.v1i1.2291.

[7] D. G. Patel, D. B. Shah and N. H. Shah, Optimal carbon footprints for chemical industries incorporating trade credit under trended demand, *Revista Investigacion Operacional* **41**(6) (2020), 793 – 803, URL: https://rev-inv-ope.pantheonsorbonne.fr/sites/default/files/inline-files/41620-01_0.pdf.

[8] C.-Y. Dye and C.-T. Yang, Sustainable trade credit and replenishment decisions with credit-linked demand under carbon emission constraints, *European Journal of Operational Research* **244**(1) (2015), 187 – 200, DOI: 10.1016/j.ejor.2015.01.026.

[9] S. Hemalatha and K. Annadurai, An integrated production-distribution inventory system for deteriorating products in fuzzy environment, *Malaya Journal of Matematik* **8**(4) (2020), 1527 – 1538, DOI: 10.26637/MJM0804/0033.

[10] C. K. Jaggi, S. K. Goyal and S. K. Goel, Retailer's optimal replenishment decisions with credit-linked demand under permissible delay in payments, *European Journal of Operational Research* **190**(1) (2008), 130 – 135, DOI: 10.1016/j.ejor.2007.05.042.

[11] B. Karthick and R. Uthayakumar, Optimizing an imperfect production model with varying setup cost, price discount, and lead time under fuzzy demand, *Process Integration and Optimization for Sustainability* **5** (2021), 13 – 29, DOI: 10.1007/s41660-020-00133-8.

[12] A. Khanna, Priyamvada and C. K. Jaggi, Optimizing preservation strategies for deteriorating items with time-varying holding cost and stock-dependent demand, *Yugoslav Journal of Operations Research* **30**(2) (2020), 237 – 250, DOI: 10.2298/YJOR190215003K.

[13] P. Mala and S. Priyan, A sustainable approach for smart supply chain with quality humiliation and carbon emission cost, *Asian Journal of Fuzzy and Applied Mathematics* **9**(1) (2021), 1 – 8, DOI: 10.24203/ajfam.v9i1.6513.

[14] B. Malleeswaran and R. Uthayakumar, An integrated vendor–buyer supply chain model for backorder price discount and price-dependent demand using service level constraints and carbon emission cost, *International Journal of Systems Science: Operations & Logistics* **9**(1) (2020), 111 – 120, DOI: 10.1080/23302674.2020.1833258.

[15] P. Mishra and I. Talati, Quantity discount for integrated supply chain model with back order and controllable deterioration rate, *Yugoslav Journal of Operations Research* **28**(3) (2018), 355 – 369, DOI: 10.2298/YJOR171014012M.

[16] N. H. Shah, E. Patel and K. Rabari, Optimal ordering policies under conditional trade-credit for retailer, *Revista Investigacion Operacional* **41**(7) (2020), 970 – 978, URL: https://rev-inv-ope.pantheonsorbonne.fr/sites/default/files/inline-files/41720-06.pdf.

[17] N. H. Shah, K. Rabari and E. Patel, Inventory and preservation investment for deteriorating system with stock-dependent demand and partial backlogged shortages, *Yugoslav Journal of Operations Research* **31**(2) (2021), 181 – 192, DOI: 10.2298/YJOR200217038S.

[18] S. Sahu, G. C. Panda and A. K. Das, A fully backlogged deteriorating inventory model with price dependent demand using preservation technology investment and trade credit policy, *International Journal of Engineering Research & Technology* **6**(6) (2017), 851 – 858.

[19] S. R. Singh, D. Khurana and S. Tayal, An economic order quantity model for deteriorating products having stock dependent demand with trade credit period and preservation technology, *Uncertain Supply Chain Management* **4**(1) (2016), 29 – 42, DOI: 10.5267/j.uscm.2015.8.001.

[20] Y. J. Shen, K. F. Shen and C. T. Yang, A production inventory model for deteriorating items with collaborative preservation technology investment under carbon tax, *Sustainability* **11** (2019), 5027, DOI: 10.3390/su11185027.

[21] C. Yu, Z. Qu, T. W. Archibald and Z. Luan, An inventory model of a deteriorating product considering carbon emissions, *Computers & Industrial Engineering* **148** (2020), 106694, DOI: 10.1016/j.cie.2020.106694.

[22] Z. Tao and J. Xu, Carbon-regulated EOQ models with consumers' low-carbon awareness, *Sustainability* **11** (2019), 1004, DOI: 10.3390/su11041004.