Communications in Mathematics and Applications
Vol. 13, No. 5, pp. 1363–1372, 2022
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications
DOI: 10.26713/cma.v13i5.2246

http://www.rgnpublications.com

Special Issue
Mathematics & Its Relevance to Science and Engineering
***Editors*: N. Kishan and K. Phaneendra**

Research Article

A Fixed Point Theorem in Multiplicative Metric Space Based on Common EA-Like Property

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Received: May 2, 2022 **Accepted:** August 14, 2022

Abstract. In the present paper, we prove the existence of a common fixed point theorem with the new structure of contractive condition in multiplicative metric space for four self maps. In this result, we use weaker compatible conditions like weakly compatible mappings and common EA-like properties. Furthermore, it is observed that non-compatible self-maps of multiplicative metric space satisfy the EA properties and weakly compatible mappings. Eventually, weakly compatible mappings and EA properties are independent of each other. Further, the outcome of this theorem will extend and generalize the existing results of multiplicative metric space. At the beginning of this paper, we discuss some basic definitions and examples which are useful for our main theorem. At the end of this paper, some examples are discussed to validate our result.

Keywords. Multiplicative metric space, Weakly compatible mappings, Common EA-like property

Mathematics Subject Classification (2020). 54H25

1. Introduction

In 2008, the concept of *Multiplicative Metric Spaces* (MMS) was introduced by Bashirov *et al.* [4], and Abbas and Jungck [2] invented more generalized commuting maps called compatible mappings, which were more general than commuting and weakly commuting maps. Aamri and Moutawakil [1] developed the concept of EA property and generated some results in MMS. Further, non-compatible self-maps of MMS satisfying EA properties and weakly compatible mappings (weakly compatible mappings) and EA properties are independent. In [14], Wadhwa *et al*. established the notation of common EA-like property and generate some CFP theorems. In this paper, Srinivas and Mallaiah [11], and Srinivas *et al*. [12] developed a new contractive condition using the concept of common EA-like property and *Weakly Compatible Mappings* (WCM) in MMS to generate a *Unique Common Fixed Point* (UCFP) theorem. Some examples are presented to support our outcome.

2. Preliminaries

Definition 2.1 ([2]). Let X be a non-empty set and $\delta : X \times X \to \mathbb{R}^+$ holding in the conditions below:

(2.1.1) $\delta(\alpha,\eta) \geq 1$, and $\delta(\alpha,\eta) = 1 \iff \alpha = \eta$,

(2.1.2) $\delta(\alpha,\eta) = \delta(\eta,\alpha)$,

(2.1.3) $\delta(\alpha,\eta) \leq \delta(\alpha,\gamma)\delta(\gamma,\eta)$, $\forall\ \alpha,\eta,\gamma \in X$.

Then (X,δ) is called *Multiplicative Metric Space* (MMS).

Definition 2.2. Let (X,δ) be a MMS, then sequence $\{\alpha_\iota\}$ is said that

(2.2.1) *Cauchy sequence* $\iff \delta(\alpha_\iota,\alpha_l) \to 1$, for all $\iota, l \to \infty$;

(2.2.2) *convergent* $\iff \exists\ \alpha \in X$ such that $\delta(\alpha_\iota,\alpha) \to 1$ as $\iota \to \infty$;

(2.2.3) is *complete* if every multiplicative Cauchy sequence in it is multiplicative convergent.

Definition 2.3 ([2]). We define mappings G and I of a MMS as

(2.3.1) *Compatible:* if $\lim_{\iota\to\infty} \delta(GI\alpha_\iota, IG\alpha_\iota) = 1$, whenever $\{\alpha_\iota\}$ is a sequence in X such that $G\alpha_\iota = I\alpha_\iota = \phi$, as $\iota \to \infty$ for some $\phi \in X$.

(2.3.2) *Weak-compatible:* $GI\phi = IG\phi$ whenever $G\phi = I\phi$.

Definition 2.4 ([1]). We define self maps G and I of a MMS are said to hold

(2.4.1) *EA-property*, if $\exists$ a sequence$\{\alpha_\iota\} \in X$ such that

$$\lim_{\iota\to\infty} G\alpha_\iota = \lim_{\iota\to\infty} I\alpha_\iota = \phi, \quad \text{for some} \quad \phi \in X.$$

(2.4.2) *EA-like property*, if $\exists$ a sequence $\{\alpha_\iota\} \in X$ such that

$$\lim_{\iota\to\infty} G\alpha_\iota = \lim_{\iota\to\infty} I\alpha_\iota = \phi, \quad \text{for some} \quad \phi \in G(X) \text{ or } \phi \in I(X).$$

Example 2.1. Let $X = [0,2]$ and $\delta : X \times X \to [1,\infty)$ defined as $\delta(\alpha,\eta) = e^{|\alpha-\eta|}$ where $\alpha,\eta \in X$, then (X,δ) is MMS.

Define mappings $G\alpha = \alpha^2 - 3\alpha + 2$, $I\alpha = 2\alpha^2 - 5\alpha + 2$ $\forall\ \alpha$ in X.

Take sequence $\{\alpha_\iota\}$ as $\alpha_\iota = 2 - \frac{1}{\iota}$ and $\iota \in \mathbb{N}$.

Then

$$\lim_{\iota\to\infty} G\alpha_\iota = \lim_{\iota\to\infty}\left\{\left(2-\frac{1}{\iota}\right)^2 - 3\left(2-\frac{1}{\iota}\right)+2\right\} = 0$$

and

$$\lim_{\iota\to\infty} I\alpha_\iota = \lim_{\iota\to\infty}\left\{2\left(2-\frac{1}{\iota}\right)^2 - 5\left(2-\frac{1}{\iota}\right)+2\right\} = 0,$$

$$\lim_{\iota\to\infty} G\alpha_\iota = \lim_{\iota\to\infty} I\alpha_\iota = 0 \in X$$

which implies the pair (G,I) satisfies EA-property.

Also, $I(2) = G(2)$ and $I(0) = G(0) \implies 2$ and 0 are coincidence points.

Further $IG(2) = GI(2)$ and $IG(0) = GI(0)$.

Hence G and I are weakly compatible mappings.

Definition 2.5 ([14]). G, I, H and J are four self maps of a MMS (X,δ), then the pairs (H,J) and (G,I) satisfy

(2.5.1) *common EA-property*, if $\exists$ sequences $\{\alpha_\iota\}$ and $\{\eta_\iota\} \in X$ such that

$$\lim_{\iota\to\infty} G\alpha_\iota = \lim_{\iota\to\infty} I\alpha_\iota = \lim_{\iota\to\infty} H\eta_\iota = \lim_{\iota\to\infty} J\eta_\iota = \phi \text{ for some } \phi \in X.$$

(2.5.2) *common EA-like property*, if $\exists$ sequences $\{\alpha_\iota\}$ and $\{\eta_\iota\} \in X$ such that

$$\lim_{\iota\to\infty} G\alpha_\iota = \lim_{\iota\to\infty} I\alpha_\iota = \lim_{\iota\to\infty} H\eta_\iota = \lim_{\iota\to\infty} J\eta_\iota = \phi$$

for some $\phi \in G(X)\cap I(X)$ or $\phi \in H(X)\cap J(X)$.

Now, let us discuss an example on common EA-like property.

Example 2.2. Let $X = \left[0,\frac{\pi}{2}\right]$ with $\delta(\alpha,\eta) = e^{|\alpha-\eta|}$ then (X,δ) is MMS.

Define mappings $G(\alpha) = \sin(\alpha)$, $I(\alpha) = \frac{1}{\cos(\alpha)+\sin(\alpha)}$, $H(\alpha) = \frac{\cos(\alpha)}{\sin(\alpha)+\cos(\alpha)}$ and $J(\alpha) = \sqrt{1-\cos(\alpha)}$, $G(X) = I(X) = H(X) = J(X) = [0,1]$.

Take two sequences $\{\alpha_\iota\} = \frac{\pi}{2}-\frac{1}{\iota}$ and $\{\eta_\iota\} = \frac{1}{\iota}$ where $\iota \in \mathbb{N}$.

Then

$$\lim_{\iota\to\infty} G(\alpha_\iota) = \lim_{\iota\to\infty} \sin\left(\frac{\pi}{2}-\frac{1}{\iota}\right) = \lim_{\iota\to\infty}\cos\left(\frac{1}{\iota}\right) = 1 \in G(X)$$

and

$$\lim_{\iota\to\infty} I(\alpha_\iota) = \lim_{\iota\to\infty} I\left(\frac{\pi}{2}-\frac{1}{\iota}\right) = \lim_{\iota\to\infty}\frac{1}{\cos\left(\frac{\pi}{2}-\frac{1}{\iota}\right)+\sin\left(\frac{\pi}{2}-\frac{1}{\iota}\right)}$$

$$= \lim_{\iota\to\infty}\frac{1}{\sin\left(\frac{1}{\iota}\right)+\cos\left(\frac{1}{\iota}\right)} = 1 \in I(X).$$

Further

$$\lim_{\iota\to\infty} H(\eta_\iota) = \lim_{\iota\to\infty}\frac{\cos\left(\frac{1}{\iota}\right)}{\sin\left(\frac{1}{\iota}\right)+\cos\left(\frac{1}{\iota}\right)} = 1 \in H(X),$$

also

$$\lim_{\iota\to\infty} J(\eta_\iota) = \lim_{\iota\to\infty}\sqrt{1-\cos\left(\frac{\pi}{2}-\frac{1}{\iota}\right)} = \lim_{\iota\to\infty}\sqrt{1-\sin\left(\frac{1}{\iota}\right)} = 1 \in J(X).$$

This gives

$$\lim_{\iota\to\infty} G\alpha_\iota = \lim_{\iota\to\infty} I\alpha_\iota = \lim_{\iota\to\infty} H\eta_\iota = \lim_{\iota\to\infty} J\eta_\iota = 1,$$

where $1 \in G(X)\cap I(X)$ or $1 \in H(X)\cap J(X)$.
Hence G,H,I and J satisfy common EA-like property.

Now we prove a theorem satisfying common EA-like property on MMS.

3. Main Theorem

Theorem 3.1. *Suppose in a complete MMS (X,δ), there are four mappings G,H,I and J holding the conditions:*

(3.1.1) *$H(X)\subseteq I(X)$ and $G(X)\subseteq J(X)$*

(3.1.2)

$$\delta(G\alpha,H\eta) \le \left\{\max\left[\frac{\delta(G\alpha,I\alpha)\delta(H\eta,J\eta)}{1+\delta(I\alpha,J\eta)}, \frac{\delta(G\alpha,J\eta)\delta(I\alpha,H\eta)}{1+\delta(J\eta,I\alpha)}, \frac{\delta(G\alpha,J\eta)\delta(H\eta,J\eta)}{1+\delta(J\eta,I\alpha)}, \frac{\delta(G\alpha,I\alpha)\delta(H\eta,I\alpha)}{1+\delta(I\alpha,J\eta)}\right]\right\}^\lambda,$$

where $\alpha,\eta\in X$ and $\lambda\in[0,\frac{1}{2}]$,

(3.1.3) *the pairs (H,J) and (G,I) are satisfying common E.A-like property,*

(3.1.4) *the pairs (H,J) and (G,I) are weakly compatible mappings.*

Then the above mappings will have a unique common fixed point.

Proof. We Begin with using (3.1.1), then there is a point $\alpha_0\in X$, such that $\exists\ G\alpha_0 = J\alpha_1 = \eta_0$.
For this point α_1 then there $\exists\ \alpha_2\in X$ such that $H\alpha_1 = I\alpha_2 = \eta_1$.
Continuing this process, it is possible to construct a sequence $\{\eta_\iota\}\in X$ so that

$$\eta_{2\iota} = G\alpha_{2\iota} = J\alpha_{2\iota+1} \text{ and } \eta_{2\iota+1} = H\alpha_{2\iota+1} = I\alpha_{2\iota+2} \text{ for } \iota\ge 0. \tag{3.1}$$

Now we show that $\{\eta_\iota\}$ is a cauchy sequence in MMS.

$\boldsymbol{\delta(\eta_{2\iota},\eta_{2\iota+1})}$

$$\delta(G\alpha_{2\iota},H\alpha_{2\iota+1}) \le \left\{\max\left[\frac{\delta(G\alpha_{2\iota},I\alpha_{2\iota})\delta(H\alpha_{2\iota+1},J\alpha_{2\iota+1})}{1+\delta(I\alpha_{2\iota},J\alpha_{2\iota+1})}, \frac{\delta(G\alpha_{2\iota},J\alpha_{2\iota+1})\delta(I\alpha_{2\iota},H\alpha_{2\iota+1})}{1+\delta(I\alpha_{2\iota},J\alpha_{2\iota+1})}, \frac{\delta(G\alpha_{2\iota},J\alpha_{2\iota+1})\delta(H\alpha_{2\iota+1},J\alpha_{2\iota+1})}{1+\delta(I\alpha_{2\iota},J\alpha_{2\iota+1})}, \frac{\delta(G\alpha_{2\iota},I\alpha_{2\iota})\delta(H\alpha_{2\iota+1},I\alpha_{2\iota})}{1+\delta(I\alpha_{2\iota},J\alpha_{2\iota+1})}\right]\right\}^\lambda$$

this gives

$$\delta(\eta_{2\iota},\eta_{2\iota+1}) \le \left\{\max\left[\frac{\delta(\eta_{2\iota},\eta_{2\iota-1})\delta(\eta_{2\iota+1},\eta_{2\iota})}{1+\delta(\eta_{2\iota-1},\eta_{2\iota)}}, \frac{\delta(\eta_{2\iota},\eta_{2\iota})\delta(\alpha_{2\iota-1},\eta_{2\iota+1})}{1+\delta(\eta_{2\iota-1},\eta_{2\iota-1})}, \frac{\delta(\eta_{2\iota},\eta_{2\iota})\delta(\eta_{2\iota+1},\eta_{2\iota})}{1+\delta(\eta_{2\iota},\eta_{2\iota-1})}, \frac{\delta(\eta_{2\iota},\eta_{2\iota-1})\delta(\eta_{2\iota+1},\eta_{2\iota-1})}{1+\delta(\eta_{2\iota},\eta_{2\iota-1})}\right]\right\}^\lambda$$

which implies

$$\delta(\eta_{2\iota},\eta_{2\iota+1}) \le \{\max[\delta(\eta_{2\iota-1},\eta_{2\iota-1}),\delta(\eta_{2\iota-1},\eta_{2\iota+1}),\delta(\eta_{2\iota+1},\eta_{2\iota}),\delta(\eta_{2\iota},\eta_{2\iota+1})]\}^\lambda$$

on simplification

$$\delta(\eta_{2\iota},\eta_{2\iota+1}) \le \{\delta(\eta_{2\iota-1},\eta_{2\iota+1})\}^{\lambda},$$

$$\delta(\eta_{2\iota},\eta_{2\iota+1}) \le \{\delta(\eta_{2\iota-1},\eta_{2\iota}),\delta(\eta_{2\iota},\eta_{2\iota+1})\}^{\lambda},$$

$$\delta^{(1-\lambda)}(\eta_{2\iota},\eta_{2\iota+1}) \le \delta^{\lambda}(\eta_{2\iota-1},\eta_{2\iota}),$$

$$\delta(\eta_{2\iota},\eta_{2\iota+1}) \le \delta^{\left(\frac{1-\lambda}{\lambda}\right)}(\eta_{2\iota-1},\eta_{2\iota}),$$

$$\delta(\eta_{2\iota},\eta_{2\iota+1}) \le \delta^{h}(\eta_{2\iota-1},\eta_{2\iota}), \quad \text{where } h = \left(\frac{\lambda}{1-\lambda}\right) \in (0,1).$$

Now it gives

$$\delta(\eta_{\iota},\eta_{\iota+1}) \le \delta^{h}(\eta_{\iota-1},\eta_{\iota}) \le \delta^{h^2}(\eta_{\iota-2},\eta_{\iota-1}) \le \cdots \le \delta^{h^{\iota}}(\eta_0,\eta_1).$$

By using the multiplicative triangle inequality $\iota \le l$ we get

$$\delta(\eta_{\iota},\eta_{l}) \le \delta^{h^{\iota}}(\eta_0,\eta_1) \le \delta^{h^{\iota}+1}(\eta_0,\eta_1) \le \cdots \le \delta^{\iota^{l}-1}(\eta_0,\eta_1),$$

$$\delta(\eta_{\iota},\eta_{l}) \le \delta^{\frac{h^{\iota}}{1-h}}(\eta_0,\eta_1).$$

$\{\eta_{\iota}\}$ is a Cauchy sequence in MMS.
Now X being complete in MMS $\exists\ \phi \in X$ such that $\lim_{\iota\to\infty} \eta_{\iota} \to \phi$.
Therefore, the sub sequences $\{G\alpha_{2\iota}\}$, $\{I\alpha_{2\iota}\}$, $\{J\alpha_{2\iota+1}\}$ and $\{H\alpha_{2\iota+1}\}$ of $\{\eta_{\iota}\}$ also converges to the same point $\phi \in X$.
Since on using (3.1.3) the pairs (H,J) and (G,I) are satisfying common EA-like property $\exists$ sequences $\{\alpha_{\iota}\}$ and $\{\eta_{\iota}\} \in X$ such that

$$\lim_{\iota\to\infty} G\alpha_{\iota} = \lim_{\iota\to\infty} I\alpha_{\iota} = \lim_{\iota\to\infty} H\eta_{\iota} = \lim_{\iota\to\infty} J\eta_{\iota} = \phi, \tag{3.2}$$

where $\phi \in I(X) \cap J(X)$ or $\phi \in G(X) \cap H(X)$.
Suppose $\phi \in I(X) \cap J(X)$ we have

$$\lim_{\iota\to\infty} G\alpha_{\iota} = \phi \in I(X) \text{ then } \phi = I(v) \quad \text{for some } v \in X.$$

We claim that $Gv = Iv$.
Now let us consider that

$$\lim_{\iota\to\infty} G\alpha_{\iota} = \phi \quad \text{for all } v \in I(X) \text{ such that } Iv = \phi.$$

Put $\alpha = v$ and $\beta = \beta_{\iota}$ in (3.1.2) we get

$$\delta(Gv,H\eta_{\iota}) \le \left\{\max\left[\frac{\delta(Gv,Iv)\delta(H\eta_{\iota},J\eta_{\iota})}{1+\delta(Iv,J\eta_{\iota})}, \frac{\delta(Gv,J\eta_{\iota})\delta(Iv,H\eta_{\iota})}{1+\delta(Iv,J\eta_{\iota})}, \frac{\delta(Gv,J\eta_{\iota})\delta(H\eta_{\iota},J\eta_{\iota})}{1+\delta(Iv,J\eta_{\iota})}, \frac{\delta(Gv,Iv)\delta(H\eta_{\iota},Iv)}{1+\delta(Iv,J\eta_{\iota})}\right]\right\}^{\lambda}$$

this implies

$$\delta(Gu,\phi) \le \left\{\max\left[\frac{\delta(Gu,\phi)\delta(\phi,\phi)}{1+\delta(\phi,\phi)}, \frac{\delta(Gu,\phi)\delta(\phi,\phi)}{1+\delta(\phi,\phi)}, \frac{\delta(Gu,\phi)\delta(\phi,\phi)}{1+\delta(\phi,\phi)}, \frac{\delta(Gu,\phi)\delta(\phi,\phi)}{1+\delta(\phi,\phi)}\right]\right\}^{\lambda}$$

which gives

$$\delta(Gv,\phi) \le \{\max[\delta(Gv,\phi),\delta(Gv,\phi),\delta(Gv,\phi),\delta(Gv,\phi)]\}^{\lambda},$$

$$\delta(Gv,\phi)\le[\delta(Gv,\phi)]^{\lambda} \text{ implies } Gv=\phi$$

which gives

$$Gv=Iv=\phi. \tag{3.3}$$

Since the self maps G and I are weakly compatible mappings then $Gv=Iv$ and this gives

$$\begin{aligned} & GIv=IGv \\ \Longrightarrow\quad & G\phi=I\phi. \end{aligned} \tag{3.4}$$

Now let us assume that

$$\lim_{\iota\to\infty} H\eta_\iota=\phi \text{ for some } \phi\in X \text{ and } \mu\in J(X) \text{ such that } J\mu=\phi.$$

Put $\alpha=\alpha_\iota$ and $\eta=\mu$ in (3.1.2) we get

$$\delta(G\alpha_\iota,H\mu)\le\left\{\max\left[\frac{\delta(G\alpha_\iota,I\alpha_\iota)\delta(H\mu,J\mu)}{1+\delta(I\alpha_\iota,Jv)},\frac{\delta(G\alpha_\iota,J\mu)\delta(I\alpha_\iota,H\mu)}{1+\delta(I\alpha_\iota,J\mu)},\frac{\delta(G\alpha_\iota,J\mu)\delta(H\mu,J\mu)}{1+\delta(I\alpha_\iota,J\mu)},\frac{\delta(G\alpha_\iota,I\alpha_\iota)\delta(H\mu,I\alpha_\iota)}{1+\delta(I\alpha_\iota,J\mu)}\right]\right\}^{\lambda}$$

this implies

$$\delta(\phi,H\mu)\le\left\{\max\left[\frac{\delta(\phi,\phi)\delta(H\mu,\phi)}{1+\delta(\phi,\phi)},\frac{\delta(\phi,\phi)\delta(\phi,H\mu)}{1+\delta(\phi,\phi)},\frac{\delta(\phi,\phi)\delta(H\mu,\phi)}{1+\delta(\phi,\phi)},\frac{\delta(\phi,\phi)\delta(H\mu,\phi)}{1+\delta(\phi,\phi)}\right]\right\}^{\lambda}$$

this gives

$$\begin{aligned}\delta(\phi,H\mu)&\le\{\max[\delta(H\mu,\phi),\delta(H\mu,\phi),\delta(H\mu,\phi),\delta(H\mu,\phi)]\}^{\lambda}\\ &\le[\delta(H\mu,\phi)]^{\lambda} \quad \text{which implies } H\mu=\phi.\end{aligned}$$

$$H\mu=J\mu=\phi. \tag{3.5}$$

Since the pair (H,J) is weakly compatible mappings, $H\mu=J\mu$ since $H(J\mu)=J(H\mu)$, this gives

$$H\phi=J\phi. \tag{3.6}$$

Put $\alpha=\phi$ and $\eta=v$ in (3.1.2) then we get

$$\delta(G\phi,Hv)\le\left\{\max\left[\frac{\delta(G\phi,I\phi)\delta(Hv,Jv)}{1+\delta(I\phi,Jv)},\frac{\delta(G\phi,Jv)\delta(I\phi,Hv)}{1+\delta(Jv,I\phi)},\frac{\delta(G\phi,Jv)\delta(Hv,Jv)}{1+\delta(Jv,I\phi)},\frac{\delta(G\phi,I\phi)\delta(Hv,I\phi)}{1+\delta(I\phi,Jv)}\right]\right\}^{\lambda}$$

this implies

$$\delta(G\phi,\phi)\le\left\{\max\left[\frac{\delta(G\phi,G\phi)\delta(\phi,\phi)}{1+\delta(G\phi,\phi)},\frac{\delta(G\phi,\phi)\delta(G\phi,\phi)}{1+\delta(\phi,G\phi)},\frac{\delta(G\phi,\phi)\delta(\phi,\phi)}{1+\delta(\phi,G\phi)},\frac{\delta(G\phi,G\phi)\delta(\phi,G\phi)}{1+\delta(G\phi,\phi)}\right]\right\}^{\lambda}$$

and this gives

$$\delta(G\phi,\phi)\le\{\max[\delta(G\phi,G\phi),\delta(\phi,\phi),\delta(\phi,\phi),\delta(G\phi,G\phi)]\}^{\lambda},$$

$$\delta(G\phi,\phi)\le 1, \quad \text{this implies } G\phi=\phi.$$

$$G\phi=I\phi=\phi. \tag{3.7}$$

Put $\alpha=\mu$ and $\eta=\phi$ in (3.1.2) we get

$$\delta(G\mu,H\phi)\le\left\{\max\left[\frac{\delta(G\mu,I\mu)\delta(H\phi,J\phi)}{1+\delta(I\mu,J\phi)},\frac{\delta(G\mu,J\phi)\delta(I\mu,H\phi)}{1+\delta(J\phi,I\mu)},\frac{\delta(G\mu,J\phi)\delta(H\phi,J\phi)}{1+\delta(J\phi,I\mu)},\frac{\delta(G\mu,I\mu)\delta(H\phi,I\mu)}{1+\delta(I\mu,J\phi)}\right]\right\}^{\lambda}$$

which gives

$$\delta(G\mu,H\phi)\le\left\{\max\left[\frac{\delta(G\mu,I\mu)\delta(H\phi,J\phi)}{1+\delta(I\mu,J\phi)},\frac{\delta(G\mu,J\phi)\delta(I\mu,H\phi)}{1+\delta(J\phi,I\mu)},\frac{\delta(G\mu,J\phi)\delta(H\phi,J\phi)}{1+\delta(J\phi,I\mu)},\frac{\delta(G\mu,I\mu)\delta(H\phi,I\mu)}{1+\delta(I\mu,J\phi)}\right]\right\}^{\lambda}$$

consequently

$$\delta(\phi,H\phi)\le\left\{\max\left[\frac{\delta(\phi,\phi)\delta(H\phi,H\phi)}{1+\delta(\phi,H\phi)},\frac{\delta(\phi,H\phi)\delta(\phi,H\phi)}{1+\delta(H\phi,\phi)},\frac{\delta(\phi,H\phi)\delta(H\phi,H\phi)}{1+\delta(H\phi,\phi)},\frac{\delta(\phi,\phi)\delta(H\phi,\phi)}{1+\delta(H\phi,\phi)}\right]\right\}^{\lambda}$$

which gives

$$\delta(\phi,H\phi)\le\left\{\max\frac{1}{1+\delta(\phi,H\phi)},\delta(\phi,H\phi),\delta(H\phi,H\phi),\delta(H\phi,\phi)\right\}^{\lambda},$$

$$\delta(\phi,H\phi)\le\{\delta(H\phi,\phi)\}^{\lambda}\ \text{this implies}\ \delta(H\phi,\phi)\le 1,$$

which gives $H\phi=\phi$.

$$H\phi=J\phi=\phi. \tag{3.8}$$

By using (3.7) and (3.8) we have

$$G\phi=H\phi=J\phi=I\phi=\phi. \tag{3.9}$$

Therefore ϕ is a common fixed point of G,I,J and H.

Uniqueness: Let ϕ and ρ be two common fixed points of G,H,J and I.
If possible $\phi\ne\rho$, put $\alpha=\phi$ and $\eta=\rho$ in (3.1.2) we get

$$\delta(\phi,\rho)=\delta(G\phi,H\rho)\le\left\{\max\left[\frac{\delta(G\phi,I\phi)\delta(H\rho,J\rho)}{1+\delta(I\phi,J\rho)},\frac{\delta(G\phi,J\rho)\delta(I\phi,H\rho)}{1+\delta(J\rho,I\phi)},\frac{\delta(G\phi,J\rho)\delta(H\rho,J\rho)}{1+\delta(J\rho,I\phi)},\frac{\delta(G\phi,I\phi)\delta(H\rho,I\phi)}{1+\delta(I\phi,J\rho)}\right]\right\}^{\lambda}$$

this implies

$$\delta(\phi,\rho)\le\left\{\max\left[\frac{\delta(\phi,\phi)\delta(\rho,\rho)}{1+\delta(\phi,\rho)},\frac{\delta(\phi,\rho)\delta(\phi,\rho)}{1+\delta(\rho,\phi)},\frac{\delta(\phi,\rho)\delta(\rho,\rho)}{1+\delta(\rho,\phi)},\frac{\delta(\phi,\phi)\delta(\rho,\phi)}{1+\delta(\phi,\rho)}\right]\right\}^{\lambda}$$

which implies

$$\delta(\phi,\rho)\le\{\max[1,\delta^{2}(\phi,\rho),\delta(\phi,\rho),\delta(\phi,\phi)]\}^{\lambda},$$

$$\delta(\phi,\rho)\le\{\delta(\phi,\rho)\}^{\lambda},\quad \text{which gives}\ \ \delta(\phi,\rho)\le 1$$

and this implies $\phi=\rho$.
Therefore ϕ is the unique common fixed point of G, H, I and J. □

Now we present an example to validate our theorem.

Example 3.1. Let $X = [0,\pi]$ be define in MMS with $\delta(\alpha,\eta) = e^{|\alpha-\eta|}$ $\forall$ $\alpha,\eta \in X$.
Consider four self maps G, I, J and H as shown below:

$$H(\alpha) = G(\alpha) = \begin{cases} \pi\sin(\alpha) & \text{if } 0 \le \alpha \le \frac{\pi}{2}; \\ \pi\cos^2(\alpha) & \text{if } \frac{\pi}{2} < \alpha \le \pi, \end{cases}$$

$$J(\alpha) = I(\alpha) = \begin{cases} 2\alpha & \text{if } 0 \le \alpha \le \frac{\pi}{2}; \\ \frac{-\pi}{\sin(\alpha)+\cos(\alpha)} & \text{if } \frac{\pi}{2} < \alpha \le \pi. \end{cases}$$

Then $G(X) = H(X) = [0,\pi] \cup (0,\pi]$ while $I(X) = J(X) = \left[0,\frac{\pi}{2}\right] \cup (-\pi,\pi]$

$$\Longrightarrow H(X) \subseteq J(X) \text{ and } G(X) \subseteq I(X).$$

Consider sequences $\{\alpha_\iota\}$ as $\alpha_\iota = \frac{\pi}{2} - \frac{1}{\iota}$ and $\{\eta_\iota\}$ as $\eta_\iota = \pi - \frac{1}{\iota}$ as $\iota > 0$.

Now

$$\lim_{\iota\to\infty} G(\alpha_\iota) = \lim_{\iota\to\infty} G\left(\frac{\pi}{2} - \frac{1}{\iota}\right) = \lim_{\iota\to\infty} \pi\sin\left(\frac{\pi}{2} - \frac{1}{\iota}\right) = \lim_{\iota\to\infty} \pi\cos(\frac{1}{\iota}) = \pi,$$

and

$$\lim_{\iota\to\infty} I(\alpha_\iota) = \lim_{\iota\to\infty} I\left(\frac{\pi}{2} - \frac{1}{\iota}\right) = \lim_{\iota\to\infty} 2\left(\frac{\pi}{2} - \frac{1}{\iota}\right) = \lim_{\iota\to\infty} \pi - \frac{2}{\iota} = \pi,$$

$$\lim_{\iota\to\infty} G\alpha_\iota = \lim_{\iota\to\infty} I\alpha_\iota = \pi.$$

Also

$$\lim_{\iota\to\infty} H(\eta_\iota) = \lim_{\iota\to\infty} H(\pi - \frac{1}{\iota}) = \lim_{\iota\to\infty} \pi\cos^2\left(\pi - \frac{1}{\iota}\right) = \lim_{\iota\to\infty} \pi\cos^2\left(\frac{1}{\iota}\right) = \pi$$

and

$$\lim_{\iota\to\infty} J(\eta_\iota) = \lim_{\iota\to\infty} J\left(\pi - \frac{1}{\iota}\right) = \lim_{\iota\to\infty} \frac{-\pi}{\sin\left(\pi - \frac{1}{\iota}\right) + \cos\left(\pi - \frac{1}{\iota}\right)} = \pi,$$

$$\lim_{\iota\to\infty} H\eta_\iota = \lim_{\iota\to\infty} J\eta_\iota = \pi$$

which implies

$$\lim_{\iota\to\infty} G\alpha_\iota = \lim_{\iota\to\infty} I\alpha_\iota = \lim_{\iota\to\infty} H\eta_\iota = \lim_{\iota\to\infty} J\eta_\iota = \pi$$

where $\pi \in G(X) \cap I(X)$ or $\pi \in H(X) \cap J(X)$.
Therefore the self maps G and I satisfy the common EA-like property
so that the condition (3.1.3) is satisfied.
Moreover $G(0) = I(0)$, $G(\pi) = I(\pi)$ and $G\left(\frac{\pi}{2}\right) = I\left(\frac{\pi}{2}\right)$ therefore 0, π and $\frac{\pi}{2}$ are coincidence points.
Also

$$GI\left(\frac{\pi}{2}\right) = IG\left(\frac{\pi}{2}\right),\ GI(\pi) = IG(\pi) \text{ and } IG(0) = GI(0).$$

Hence the pair (G,I) satisfies weakly compatible mappings.
Now we prove the contractive condition in various cases.
Case I: If α and η are in $\left[0,\frac{\pi}{2}\right]$, then we have $\delta(G\alpha,H\eta) = e^{|G\alpha-H\eta|}$.
Put $\alpha = 0$, $\eta = \frac{\pi}{2}$, in (3.1.2) implies

$G(\alpha)=0$, $H(\eta)=\pi$, $I(\alpha)=0$ and $J(\eta)=\pi$.

$$\delta(0,\pi)\le\left\{\max\left[\frac{\delta(0,0),\delta(\pi,\pi)}{1+\delta(0,\pi)},\frac{\delta(0,\pi),\delta(\pi,0)}{1+\delta(0,\pi)},\frac{\delta(0,\pi),\delta(\pi,\pi)}{1+\delta(0,\pi)},\frac{\delta(0,0),\delta(\pi,0)}{1+\delta(0,\pi)}\right]\right\}^{\lambda}$$

this implies

$$e^{\pi}\le\{\max[e^{0},e^{2\pi},e^{\pi},e^{\pi}]\}^{\lambda} \text{ which gives } e^{\pi}=e^{2\pi\lambda}.$$

Therefore $\lambda=0.5$, where $\lambda\in\left[0,\frac{1}{2}\right]$.

Case II: If $\alpha,\eta\in\left[\frac{\pi}{2}\pi\right]$, then we have $\delta(G\alpha,H\eta)=e^{|G\alpha-H\eta|}$.
Put $\alpha=3\frac{\pi}{2}$, $\eta=\pi$, in (3.1.2) implies
$G(\alpha)=0$, $H(\eta)=0$, $I(\alpha)=\pi$ and $J(\eta)=\pi$.

$$\delta(0,0)\le\left\{\max\left[\frac{\delta(0,\pi),\delta(0,\pi)}{1+\delta(\pi,\pi)},\frac{\delta(0,\pi),\delta(\pi,0)}{1+\delta(\pi,\pi)},\frac{\delta(0,\pi),\delta(0,\pi)}{1+\delta(\pi,\pi)},\frac{\delta(0,\pi),\delta(\pi,0)}{1+\delta(\pi,\pi)}\right]\right\}^{\lambda}$$

this implies

$$e^{0}\le\{\max[e^{2\pi},e^{2\pi},e^{2\pi},e^{2\pi}]\}^{\lambda} \text{ which gives } e^{\pi}=e^{2\pi\lambda}.$$

Therefore $\lambda=0.5$,where $\lambda\in\left[0,\frac{1}{2}\right]$.

Case III: If $\alpha\in\left[0,\frac{\pi}{2}\right]$, $\eta\in\left(\frac{\pi}{2},\pi\right)$, then we have $\delta(G\alpha,H\eta)=e^{|G\alpha-H\eta|}$.
Put $\alpha=\frac{\pi}{4}$, $\eta=\frac{3\pi}{4}$, in (3.1.2) implies
$G(\alpha)=\frac{\pi}{\sqrt{2}}$, $H(\eta)=\frac{\pi}{2}$, $I(\alpha)=\frac{\pi}{2}$ and $J(\eta)=\frac{\pi}{\sqrt{2}}$

$$\delta\left(\frac{\pi}{\sqrt{2}},\frac{\pi}{2}\right)\le\left\{\max\left[\frac{\delta\left(\frac{\pi}{\sqrt{2}},\frac{\pi}{2}\right)\delta\left(\frac{\pi}{\sqrt{2}},\frac{\pi}{2}\right)}{1+\delta\left(\frac{\pi}{\sqrt{2}},\frac{\pi}{2}\right)},\frac{\delta\left(\frac{\pi}{\sqrt{2}},\frac{\pi}{\sqrt{2}}\right)\delta\left(\frac{\pi}{2},\frac{\pi}{2}\right)}{1+\delta\left(\frac{\pi}{\sqrt{2}},\frac{\pi}{2}\right)},\frac{\delta\left(\frac{\pi}{\sqrt{2}},\frac{\pi}{\sqrt{2}}\right)\delta\left(\frac{\pi}{\sqrt{2}},\frac{\pi}{2}\right)}{1+\delta\left(\frac{\pi}{\sqrt{2}},\frac{\pi}{2}\right)},\frac{\delta\left(\frac{\pi}{\sqrt{2}},\frac{\pi}{2}\right)\delta\left(\frac{\pi}{2},\frac{\pi}{2}\right)}{1+\delta\left(\frac{\pi}{\sqrt{2}},\frac{\pi}{2}\right)}\right]\right\}^{\lambda}$$

this implies

$$e^{\frac{(\sqrt{2}-1)\pi}{2}}\le\left\{e^{2\frac{\sqrt{2}-1)\pi}{2}}\right\}^{\lambda} \quad \text{which gives} \quad e^{\pi}=e^{2\pi\lambda}.$$

Therefore $\lambda=0.5$, where $\lambda\in\left[0,\frac{1}{2}\right]$.
Hence the condition (3.1.2) holds.
We observe that $\frac{\pi}{2}$ is the unique common fixed point for the four self mappings.

4. Conclusion

In this paper, we established a common fixed point theorem in MMS by using the concept of weakly compatible mappings and common EA-like property. Further, we also substantiated our result with an appropriate example.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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