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Research Article

On Comparison of Crisp, Fuzzy, Intuitionistic Fuzzy Unconstrained Optimization Problems Using Newton's Method

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Abstract. This paper is focused on arithmetic operations on fuzzy and intuitionistic fuzzy numbers to solve the fuzzy unconstrained optimization problems with triangular and trapezoidal, fuzzy number coefficients. The optimal solution is obtained by fuzzy Newton's method, and the MATLAB outputs are also provided with illustrative examples. The method proposed in this research work has been compared with the existing Newton's method.

Keywords. Fuzzy set, Fuzzy numbers, Intuitionistic set, Unconstrained optimization

Mathematics Subject Classification (2020). 65K10, 65F10, 90C70

1. Introduction

In this paper, the optimization of linear and nonlinear systems are discussed. The problems involving only the crisp values are inevitable with inexactness and uncertainty when using real-world problems. Zadeh [18] introduced the Fuzzy sets in 1965, it plays an important role and deals with vagueness and dissimilarities. Bellman and Zadeh [3] put forward the approach of making a decision in a fuzzy environment in 1970. Intuitionistic fuzzy set was introduced by Atanassov in 1983 [2]. Chalco-Cano *et al*. [4] deliberated Newton's method for solving optimization problems in 2015. "A comparative solution of fuzzy unconstrained optimization problems with a triangular fuzzy number" was established by Umamaheshwari and Ganesan [16]. Throughout most of the recent decades, many researchers have designed optimization problems with fuzzy valued objective problems ([1, 5–7, 14, 15]). Moreover, the unconstrained problems are solved by differential calculus. Vidhya and Hepzibah [17] introduced "A comparative study on interval arithmetic operations with intuitionistic fuzzy numbers". "A Newton method for nonlinear unconstrained optimization problems with two variables" was proposed by Porchelvi and Sathya [12]. This paper, deals with fuzzy Newton's method with triangular coefficient, trapezoidal coefficient, triangular intuitionistic, and trapezoidal intuitionistic coefficient to solve unconstrained optimization problems. Moreover, a comparison between crisp Newton's method with fuzzy and intuitionistic unconstrained optimization problems have been made. This paper is organised as follows. The second portion covers some preliminary information about this study project. In Section 3, various techniques for solving unconstrained optimization problems in a fuzzy and intuitionistic fuzzy environment are proposed. In Section 4, some illustrative cases are offered to demonstrate the method's resilience. In Section 5, there is a comparative study of the present approaches. Section 6 concludes with some concluding remarks.

2. Preliminaries

This section provides an introduction to fuzzy unconstrained optimization models and stressed the importance to consider the topics like linear and nonlinear optimization problems in fuzzy environment using arithmetic operations and provides certain definitions which are related to this research work.

2.1 Basic Concepts of Fuzzy Sets and Fuzzy Numbers

Definition 2.1 ([9])**.** *A fuzzy set* $\widetilde{A}$ is defined by $\widetilde{A} = \{x, \mu_{\widetilde{A}}(x) : x \in \mu_{\widetilde{A}}(x) \in [0,1]\}$. In the pair $(x, \mu_{\widetilde{A}}(x))$ the first element X belongs to the classical set A, the second element $\mu_{\widetilde{A}}(x)$, belongs to the interval $[0,1]$ called membership function, denoted by $\widetilde{A} = \{\mu_{\widetilde{A}}(x)\backslash x : x \in A, \mu_{\widetilde{A}}(x) \in [0,1]\}$.

Definition 2.2 ([9])**.** Let $\mathfrak{R}$ be the set of real numbers and $\widetilde{A} : \mathfrak{R} \to [0,1]$ be a fuzzy set then we say that $\widetilde{A}$ is a fuzzy number that contains the following properties:

(i) 0 is normal, i.e., there exist $x_0 \in \mathfrak{R}$ such that $\widetilde{A}(x_0) = 1$;

(ii) $\widetilde{A}$ is convex, i.e., $\widetilde{A}(tx + (1-t)y \geq \min\{\widetilde{A}(x), \widetilde{A}(y)\}$, where $x, y \in \mathfrak{R}$ and $t \in [0,1]$.

(iii) $\widetilde{A}(x)$ is upper semi-continuous on $\mathfrak{R}$, i.e., $\left\{\frac{x}{\widetilde{A}(x)} \geq \alpha\right\}$ is a closed subset of $\mathfrak{R}$ for each $\alpha \in [0,1]$.

Definition 2.3 ([9])**.** Let us take a fuzzy number $\widetilde{\mathbb{A}}$ on $\mathfrak{R}$ is said to be a *triangular fuzzy number* (TFN) or linear fuzzy number if its membership function $\widetilde{\mathbb{A}} : \mathfrak{R} \to [0,1]$ has the following characteristics. It is a fuzzy number represents with three points as follows $\widetilde{\mathbb{A}} = (a_1, a_2, a_3)$. This representative is interpreted as membership functions and holds the following conditions:

(i) a_1 to a_2 is increasing function

(ii) a_2 to a_3 is decreasing function.

(iii) $a_1 \le a_2 \le a_3$.

$$\mu_{\widetilde{\mathbb{A}}}(x) := \begin{cases} 0, & \text{for } x < a_1 \\ \frac{x-a_1}{a_2-a_1}, & \text{for } a_1 \le x \le a_2, \\ \frac{a_3-x}{a_3-a_2}, & \text{for } a_2 \le x \le a_3, \\ 0, & \text{otherwise.} \end{cases}$$

The triangular fuzzy number is diagrammatically shown below.

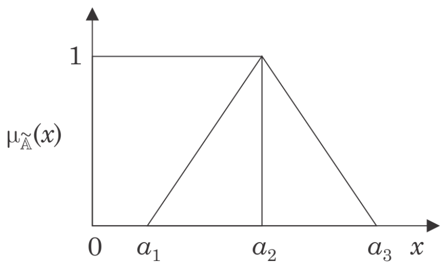

Figure 1. Triangular fuzzy number

Let $F(\mathfrak{R})$ to denote the set of all TFNs. The α level set of $\widetilde{\mathbb{A}}$ is defined as $\widetilde{\mathbb{A}}_\alpha = [(a_2 - a_1)\alpha + a_1, a_3 - (a_3 - a_2)\alpha]$.

Definition 2.4 ([9]). Let us consider fuzzy number $\widetilde{\mathbb{A}}$ on $\mathfrak{R}$ is said to be a trapezoidal fuzzy number (TrFN) or linear fuzzy number if the membership function $\widetilde{\mathbb{A}} : \mathfrak{R} \to [0,1]$ has the following characteristics. It is a fuzzy number represents with four points $\widetilde{\mathbb{A}} = (a_1, a_2, a_3, a_4)$ such that $a_1 \le a_2 \le a_3 \le a_4$ with the membership function defined as

$$\mu_{\widetilde{\mathbb{A}}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & \text{for } a_1 \le x \le a_2, \\ -1, & \text{for } a_2 \le x \le a_3, \\ \frac{a_4-x}{a_4-a_3}, & \text{for } a_2 \le x \le a_3, \\ 0, & \text{otherwise.} \end{cases}$$

The trapezoidal fuzzy number diagrammatically is shown below.

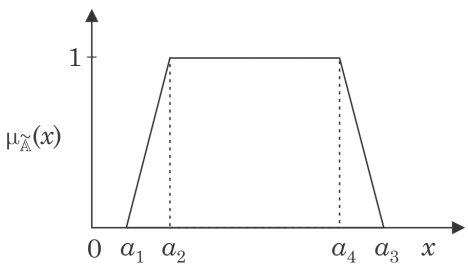

Figure 2. Trapezoidal fuzzy number

2.2 Arithmetic Operations on Fuzzy Numbers

2.2.1 Arithmetic Operations for Triangular Fuzzy Numbers

Let us consider $\widetilde{\mathbb{A}} = (a_1, a_2, a_3)$ and $\widetilde{\mathbb{B}} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers. Then the arithmetic operations are:

Addition [9]. $\widetilde{\mathbb{A}} + \widetilde{\mathbb{B}} = (a_1 + b_1, a_2 + b_2, a_3 + a_3)$.

Subtraction [9]. $\widetilde{\mathbb{A}} - \widetilde{\mathbb{B}} = (a_1 - b_1, a_2 - b_2, a_3 - a_3)$.

Multiplication. $\widetilde{\mathbb{A}} \cdot \widetilde{\mathbb{B}} = (a_1 \cdot R(\widetilde{\mathbb{B}}), a_2 \cdot R(\widetilde{\mathbb{B}}), a_3 \cdot R(\widetilde{\mathbb{B}}))$, where $R(\widetilde{\mathbb{B}}) = (b_1 + 4b_2 + b_3)/6$.

Division. $\widetilde{\mathbb{A}}/\widetilde{\mathbb{B}} = (a_1/R(\widetilde{\mathbb{B}}), a_2/R(\widetilde{\mathbb{B}}), a_3/R(\widetilde{\mathbb{B}}))$, where $R(\widetilde{\mathbb{B}}) = (b_1 + 4b_2 + b_3)/6$.

2.2.2 Arithmetic Operation for Trapezoidal Fuzzy Numbers

Suppose $\widetilde{\mathbb{A}} = (a_1, a_2, a_3, a_4)$ and $\widetilde{\mathbb{B}} = (b_1, b_2, b_3, b_4)$ are two TrFN. Then the arithmetic operations are:

Addition [9]. $\widetilde{\mathbb{A}} + \widetilde{\mathbb{B}} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

Subtraction [9]. $\widetilde{\mathbb{A}} - \widetilde{\mathbb{B}} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$

Multiplication. $\widetilde{\mathbb{A}} \cdot \widetilde{\mathbb{B}} = (a_1 \cdot R(\widetilde{\mathbb{B}}), a_2 \cdot R(\widetilde{\mathbb{B}}), a_3 \cdot R(\widetilde{\mathbb{B}}), a_4 \cdot R(\widetilde{\mathbb{B}}))$, where $R(\widetilde{\mathbb{B}}) = (b_1 + 2b_2 + 2b_3 + b_4)/6$

Division. $\widetilde{\mathbb{A}}/\widetilde{\mathbb{B}} = (a_1/R(\widetilde{\mathbb{B}}), a_2/R(\widetilde{\mathbb{B}}), a_3/R(\widetilde{\mathbb{B}}), a_4/(\widetilde{\mathbb{B}}))$, where $R(\widetilde{\mathbb{B}}) = (b_1 + 2b_2 + 2b_3 + b_4)/6$.

Definition 2.5 ([2]). Let the set $X = \{x_1, x_2, x_3, \dots x_n\}$ an *intuitionistic fuzzy set* (IFS) is defined as $A = (x_i, t_A(x_i), f_A(x_i) : x_i \in X)$ which assigns to each other element x_i a membership degree $t_A(x_i)$ and a non-membership degree $f_A(x_i)$ under the condition $0 \le t_A(x_i) + f_A(x_i) \le 1$, for all $(x_i) \in X$ in [2].

Definition 2.6 ([8]). A *Triangular Intuitionistic Fuzzy Number* (TIFN) $\widetilde{A}^I$ is an intuitionist fuzzy set in R with the following membership function $\mu_{\widetilde{A}}(x)$ and non-membership function $\mathscr{V}_{\widetilde{A}}(x)$

$$\mu_{\widetilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2, \\ \frac{x-a_3}{a_2-a_3}, & a_2 \le x \le a_3, \\ 0, & \text{otherwise}, \end{cases} \qquad \mathscr{V}_{\widetilde{A}^I}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1'}, & a_1' \le x \le a_{12}, \\ \frac{x-a_2}{a_3-a_3'}, & a_2 \le x \le a_3', \\ 1, & \text{otherwise}. \end{cases}$$

The intuitionistic triangular fuzzy number is diagrammatically shown below.

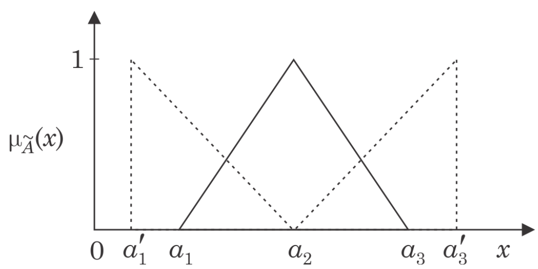

Figure 3. Triangular intuitionistic fuzzy number

Here $a_3' \le a_2 \le a_2 \le a_3 \le a_3$ and $\mu_{\widetilde{A}^I}(x) + V_{\widetilde{A}^I}(x) \le 1$, or $\mu_{\widetilde{A}^I}(x) = \mathscr{V}_{\widetilde{A}^I}(x)$ for all $x \in R$. This TIFN is denoted by $\widetilde{A}^I = (a_1, a_2, a_3; a_1', a_2, a_3') = \{(a_1, a_2, a_3); (a_1', a_2, a_3')\}$.

2.2.3 Arithmetic Operation for Triangular Intuitionistic Fuzzy Numbers

Let $\widetilde{A}^I = \{(a_1, a_2, a_3); (a_1', a_2, a_3')\}$ and $\widetilde{B}^I = \{(b_1, b_2, b_3); (b_1', b_2, b_3')\}$ are two TIFN,

Addition. $\widetilde{A}^I + \widetilde{B}^I = (a_1 + b_1, a_2 + b_2, a_3 + b_3); (a_1' + b_1', a_2 + a_2, a_3' + b_3')$ is also a TIFN.

Subtraction. $\widetilde{A}^I - \widetilde{B}^I = (a_1 - b_3, a_2 - b_2, a_3 - b_1); (a_1' - b_3', a_2 - b_2, a_3' - b_1')$ is also a TIFN.

Multiplication. $\widetilde{A}^I \cdot \widetilde{B}^I = (a_1 \cdot R(\widetilde{B}^I), a_2 \cdot R(\widetilde{B}^I), a_3 \cdot R(\widetilde{K}^I)); (a_1' \cdot R(\widetilde{B}^I), a_2 \cdot R(\widetilde{B}^I), a_3' \cdot R(\widetilde{a}^I))$ is also a TIFN, where $R(\widetilde{K}^I) = \frac{b_1 + 4b_2 + b_3 + b_1' + 4b_2 + b_3'}{12}$.

Division. $\widetilde{A}^I / \widetilde{B}^I = (a_1 / R(\widetilde{B}^I), a_2 / R(\widetilde{B}^I), a_3 / R(\widetilde{K}^I)); (a_1' / R(\widetilde{B}^I), a_2 / R(\widetilde{B}^I), a_3' \cdot R(\widetilde{a}^I))$ is also a TIFN, where $R(\widetilde{K}^I) = \frac{b_1 + 4b_2 + b_3 + b_1' + 4b_2 + b_3'}{12}$.

Definition 2.7 ([11], Trapezoidal Intuitionistic Fuzzy Number [8])**.** Let *trapezoidal intuitionistic fuzzy number* (TrIFN) $\widetilde{A}^I$ is an intuitionistic fuzzy set in R with the following membership function $\mu_{\widetilde{A}^I}(x)$ and non-membership function $\mathscr{V}_{\widetilde{A}^I}(x)$.

$$\mu_{\widetilde{A}^I}(x) = \begin{cases} 0, & x < a_1 \\ \frac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2,\ a_2 \le x \le a_3, \\ \frac{x - a_4}{a_3 - a_4}, & a_3 \le x \le a_4, \\ 0, & \text{otherwise}, \end{cases} \quad \text{and} \quad \begin{cases} 1, & x < a_1' \\ \frac{x - a_1}{a_2' - a_1'}, & a_1' \le x \le a_2', \\ \mathscr{V}_{\widetilde{A}^I}(x) = 0, & a_2' \ge x \ge a_3', \\ \frac{x - a_4'}{a_3' - a_4'}, & a_3 \le x \le a_4', \\ 1, & \text{otherwise}. \end{cases}$$

The trapezoidal intuitionistic fuzzy number is diagrammatically shown below.

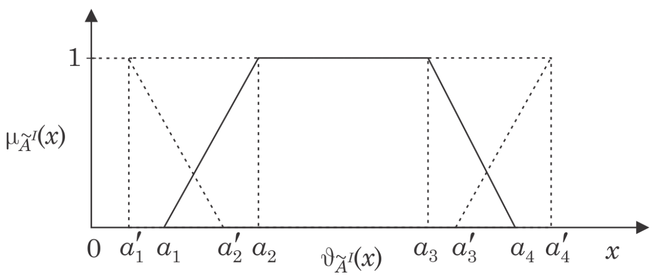

Figure 4. Triangular intuitionistic fuzzy number

2.2.4 New Modified Arithmetic Operations for Trapezoidal Intuitionistic Fuzzy Numbers

The following are the modified operations that can be performed on trapezoidal intuitionistic fuzzy numbers:

Let $\widetilde{A}^I = \{(a_1, a_2, a_3, a_4); (a_1', a_2, a_3', a_4')\}$ and $\widetilde{B}^I = \{(b_1, b_2, b_3, b_4); (b_1', b_2, b_3', b_4')\}$.

Addition. $\widetilde{A}^I + \widetilde{B}^I = \{(a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); (a_1' + b_1', a_2' + b_2', a_3' + b_3', a_4' + b_4')\}$.

Subtraction. $\widetilde{A}^I - \widetilde{B}^I = \{(a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1); (a_1' - b_4', a_2' - b_3', a_3' - b_2', a_4' - b_1')\}$.

Multiplication. $\widetilde{A}^I \cdot \widetilde{B}^I = \{(a_1 \cdot R(\widetilde{B}^I), a_2 \cdot R(\widetilde{B}^I), a_3 \cdot R(\widetilde{B}^I), a_4 \cdot R(\widetilde{B}^I)); (a_1' \cdot R(\widetilde{B}^I), a_2' \cdot R(\widetilde{B}^I), a_3' \cdot$

$R(\widetilde{B}^I), j'_4 \cdot R(\widetilde{B}^I))\}$, where $R(\widetilde{B}^I) = (b_1 + 2b_2 + 2b_3 + b_4 + b'_1 + 2b'_2 + 2b'_3 + b'_4)/12$.

Division. $\widetilde{A}^I/\widetilde{B}^I = \left\{\frac{a_1}{R(\widetilde{B}^I)}, \frac{a_2}{R(\widetilde{B}^I)}, \frac{a_3}{R(\widetilde{B}^I)}, \frac{a_4}{R(\widetilde{B}^I))}; (a'_1/R(\widetilde{B}^I), a'_2/R(\widetilde{B}^I), a'_3/R(\widetilde{B}^I), a'_4/R(\widetilde{B}^I))\right\}$, where $R(\widetilde{K}^I) = (k_1 + 2k_2 + 2k_3 + k_4 + k'_1 + 2k'_2 + 2k'_3 + k'_4)/12$.

3. Proposed Algorithms to Solve Unconstrained Optimization Problems

Newton's Method ([13])**.** Gradient search can be viewed as pursuing the move direction suggested by the first order Taylor's series approximation.

Aligning Δ with gradient $\nabla f(x^{(t)})$ produces the most rapid improvement in this first order approximation to $f(x)$.

To improve on the slow, zigzagging progress characteristic of gradient search requires more information. An obvious possibilities in extending to the second order Taylor approximation.

Newton Step ([13])**.** Until the first order Taylor approximation, which is linear in directional components Δx_j, the quadratic second order version may have a local optimum of the second order approximation, we may fix $\lambda = 1$ and differentiable f_2 with respect to components of Δx with $\lambda = 1$, the scalar notation form of f_2 is

$$f_2(x^{(t)} + \Delta x \triangleq f(x^{(t)}) + \sum_{i=1}^{n}\left(\frac{\partial f}{\partial x_i}\right)\Delta x_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\frac{\partial^2 f}{\partial x_i \partial x_j}\right)\Delta x_i \Delta x_j .$$

The partial derivatives with respect to move components are

$$\frac{\partial f_2}{\partial \Delta x_i} = \left(\frac{\partial f}{\partial x_i}\right) + \sum_{j=i}^{n}\left(\frac{\partial^2 f}{\partial x_i \partial x_j}\right)\Delta x_i, \quad i = 1,2,3,\cdots$$

or in matrix format $\nabla f_2(\Delta x) = \Delta f(x^{(t)}) + H(x^{(t)})\Delta x$.

Either way, setting $\nabla f_2(x) = 0$ to find a stationary point produces the famous *Newton Step*.

Consider $H(x^{(t)})\Delta x = -\nabla f(x^{(t)})$, when Newton step Δx, which move to a stationary point (if there is one) of the second order Taylor series approximation to $f(x)$ at current point $X^{(t)}$ can be obtaining by solving the linear equation system.

3.1 Algorithms

Algorithm 3.1 Newton Method [13]

Step 1: *Initialization*: Choose any starting solution $(x^{(0)})$, pick stopping tolerance $\varepsilon < 0$, and the solution index $t \leftarrow 0$.

Step 2: *Derivatives*: Compute objective function gradient $\nabla f(x^{(t)})$ and Hessian matrix $H(x^{(t)})$ at current point $(x^{(t)})$.

Step 3: *Stationary point*: If $\|\nabla f(x^{(t)})\| < \varepsilon$, stop. Point $(x^{(t)})$ is sufficiently close to a stationary point.

Step 4: *Newton move*: Solve the linear system. $H(x^{(t)})\Delta x = -\nabla f(x^{(t)})$ for Newton move $\Delta(x^{(t+1)})$.

Step 5: *New point*: $(x^{(t+1)}) \leftarrow (x^{(t)}) + \Delta(x^{(t+1)})$.

Step 6: *Increment* $t \leftarrow t+1$, and return to *Step* 1.

Algorithm 3.2 Fuzzy Newton Algorithm

Step 1: *Initialization*: Choose any starting solution $(\tilde{x}^{(0)})$, pick stopping tolerance $\epsilon < 0$, and the solution index $t \leftarrow 0$.

Step 2: *Derivatives*: Compute objective function gradient $\nabla\tilde{f}(\tilde{x}^{(t)})$ and Hessian matrix $\tilde{H}(\tilde{x}^{(t)})$ at current point $(\tilde{x}^{(t)})$.

Step 3: *Stationary point*: If $\|\nabla\tilde{f}(\tilde{x}^{(t)})\| < \epsilon$, stop. Point $(\tilde{x}^{(t)})$ is sufficiently close to a stationary point.

Step 4: *Newton move*: Solve the linear system. $\tilde{H}(\tilde{x}^{(t)})\Delta\tilde{x} = -\nabla\tilde{f}(\tilde{x}^{(t)})$ for Newton move $\Delta(\tilde{x}^{(t+1)})$.

Step 5: *New point*: $(\tilde{x}^{(t+1)}) \leftarrow (\tilde{x}^{(t)}) + \Delta(\tilde{x}^{(t+1)})$.

Step 6: *Increment* $t \leftarrow t+1$, and return to *Step* 1.

Algorithm 3.3 Intuitionistic Fuzzy Newton Algorithm

Step 1: *Initialization*: Choose any starting solution $(\tilde{x}^{I(0)})$, pick stopping tolerance $\epsilon < 0$, and the solution index $t \leftarrow 0$.

Step 2: *Derivatives*: Generate the gradient of both the objective function $\nabla\tilde{f}(\tilde{x}^{(t)})^I$ as well as the Hessian matrix $\tilde{H}^I(\tilde{x}^{I(t)})$ well at current point $(\tilde{x}^{I(t)})$.

Step 3: *Stationary point*: If $\|\nabla\tilde{f}^I(\tilde{x}^{I(t)})\| < \epsilon$, stop. Point $(\tilde{x}^{I(t)})$ is sufficiently close to a stationary point.

Step 4: *Newton move*: The linear system should always be computed. For the Newton move $\Delta(\tilde{x}^{I(t+1)}), \tilde{H}^I(\tilde{x}^{I(t)})\Delta\tilde{x} = -(\Delta\tilde{f}(\tilde{x}^{(t)}))^I$.

Step 5: *New point* $(\tilde{x}^{I(t+1)}) \leftarrow (\tilde{x}^{I(t)}) + \Delta(\tilde{x}^{I(t+1)})$.

Step 6: *Increment* $t \leftarrow t+1$, and return to *Step* 1.

4. Numerical Illustrations

Some numerical examples are provided here to check the robustness of the proposed algorithms.

Illustrative Example 4.1. *Case* (i): Let us consider the unconstrained optimization problem with triangular fuzzy coefficients, $\mathscr{T}(x,y) = (0.5,1,1.5)x^3 - (2,3,4)xy + (0.5,1,1.5)y^3$. Solving this problem by using Algorithm 3.2, the MATLAB outputs are tabulated here.

Iteration	(x_i, y_i)	(x_{i+1}, y_{i+1})
1	$(0.5000000, 1.0000000, 1.5000000)$ $(1.5000000, 2.0000000, 2.5000000)$	$(0.21500000, 1.1428000, 2.0718000)$ $(-0.1901000, 1.2860000, 2.7648000)$
2	$(0.2150000, 1.1428000, 2.0718000)$ $(-0.1901000, 1.2860000, 2.7648000)$	$(0.0601000, 1.0279000, 2.0010000)$ $(-0.5543000, 1.0428000, 0.6427000)$

Case (ii): Let us consider the following unconstrained optimization problems with trapezoidal fuzzy coefficients $\tilde{\mathscr{T}}(x,y) = (0.5,0.75,1,1.75)\tilde{x}^3 - (1.5,2.25,3,5.25)\tilde{x}\tilde{y} + (0.5,0.75,1,1.75)\tilde{y}^3$. Solving this problem by using Algorithm 3.2, the MATLAB outputs are tabulated here.

Iteration	(x_i, y_i)	(x_{i+1}, y_{i+1})
1	(0.5000000, 0.7500000, 1.0000000, 1.7500000)	(0.7500000, 0.8928570, 1.1071430, 0.8214290)
	(1.500000, 1.7500000, 2.0000000, 2.7500000)	(0.2500000, 1.0357140, 1.4642860, 2.3928570)
2	(0.7500000, 0.8928570, 1.1071430, 0.8214290)	(0.5476810, 0.7772468, 1.0204350, 1.7636240)
	(0.2500000, 1.0357140, 1.4642860, 2.3928570)	(−0.1723400, 0.7943750, 1.2832820, 2.272157)

Case (iii): Let us consider the following unconstrained optimization problem with triangular Intuitionistic fuzzy coefficients $\widetilde{\mathscr{T}}(x,y) = (0.5,1,1.5);(0.4,1,1.6)x^3 - (2.5,3,3.5)(2.4,3,3.6)xy + (0.5,1,1.5);(0.4,1,1.6)y^3$. Solving this problem by using Algorithm 3.3, the MATLAB outputs are tabulated here.

Iteration	(x_i, y_i)	(x_{i+1}, y_{i+1})
1	(0.5, 1, 1.5); (0.4, 1, 1.6)	{(0.4285714, 1.1428571, 1.8571428);
	(1.5, 2, 2.5); (1.4, 2, 2.6)	(0.3857142, 1.1428571, 1.9000000)}
		{(1.1904761, 1.2857142, 1.3809523);
		(1.2714285, 1.2857142, 1.3000000)}
2	{(0.4285714, 1.1428571, 1.8571428);	{(0.3360231, 1.0274955, 1.7189679);
	(0.3857142, 1.1428571, 1.9000000)}	(−0.1723400, 0.7943750, 1.2832820, 2.272157)}
	{(1.1904761, 1.2857142, 1.3809523);	{(1.0674437, 1.042438, 1.0174337);
	(1.271428, 1.2857142, 1.300000)}	(1.1714973, 1.0427136, 0.9139298)}

Case (iv): Let us consider the following unconstrained optimization problem with trapezoidal Intuitionistic fuzzy coefficients $\widetilde{\mathscr{T}}(x,y) = \{(0.5,0.75,1,1.75);(0.25,0.50,1,2.25)\}x^3 - \{(1.5,2.25,3,5.25)(0.75,0.50,3,6.75)\}xy + \{(0.5,0.75,1,1.75);(0.25,0.50,1,2.25)\}y^3$. Solving this problem by using Algorithm 3.3, the MATLAB outputs are tabulated here.

Iteration	(x_i, y_i)	(x_{i+1}, y_{i+1})
1	{(0.5, 0.75, 1, 1.75); (0.25, 0.50, 1, 2.25)}	(0.7500000,0.8928571,1.1071428,1.8214285);
		(0.8214285, 0.8928571, 1.0714285, 1.7857142)
	{(1.5, 1.75, 2, 2.75); (1.25, 1.50, 2, 3.25)}	(0.2500000, 1.0357142, 1.46428572.3928571);
		(−0.107142, 1.0357142, 1.64285714, 2.5714285)
2	{(0.750000, 0.8928571, 1.1071428, 1.8214285); (0.8214285, 0.8928571, 1.0714285, 1.7857142)}	{(0.5277777, 0.7658730, 1.0119047, 1.7579365); (0.5992063, 0.7658730, 0.976190, 1.7222222)}
	{(0.2500000, 1.0357142, 1.4642857, 2.3928571); (−0.107142, 1.0357142, 1.6428571, 2.5714285)}	{(−0.1590909, 0.8019480, 1.2889610, 2.2759740); (−0.5162337, 0.8019480, 1.4675324, 2.4545454)}

Illustrative Example 4.2. *Case* (i): Consider the unconstrained optimization with triangular fuzzy coefficients as follows:
$\widetilde{\mathscr{T}}(x,y) = (0.5,1,1.5)x - (0.5,1,1.5)y + (1.5,2,2.5)xy + (1.5,2,2.5)x^2 + (0.5,1,1.5)y^2$. Solving this problem by using Algorithm 3.2, the MATLAB outputs are tabulated here.

Iteration	(x_i, y_i)	(x_{i+1}, y_{i+1})
1	(0, 0, 0)	(−0.5000000, −1.000000, −1.50000)
	(0, 0, 0)	(0.750000, 1.50000, 2.250000)

Case (ii): Consider the unconstrained optimization with trapezoidal fuzzy coefficients shown below.
$\widetilde{\mathscr{T}}(x,y) = (0.5,0.75,1,1.75)x-(0.5,0.75,1,1.75)y+(1.5,1.75,2,2.75)xy+(1.5,1.75,2,2.25)x^2-(0.5,0.75,1,1.75)y^2$. Solving this problem by using Algorithm 3.2, the MATLAB outputs are tabulated here.

Iteration	(x_i,y_i)	(x_{i+1},y_{i+1})
1	(0,0,0)(0,0,0)	(−1.5625,−1,−0.8125,−0.625) (1.125,1.3125,1.5,2.0625)

Case (iii): Consider the unconstrained optimization problem with triangular Intuitionistic fuzzy coefficients that follows:
$\widetilde{\mathscr{T}}(x,y) = (0.4,1,1.6);(0.5,1,1.5)x-(0.4,1,1.6);(0.5,1,1.5)y+(1.5,2,2.5);(1.4,2,2.6)xy+(1.5,2,2.5);(1.4,2,2.6)x^2+(0.4,1,1.6);(0.5,1,1.5)y^2$. Solving this problem by using Algorithm 3.3, the MATLAB outputs are tabulated here.

Iteration	(x_i,y_i)	(x_{i+1},y_{i+1})
1	{(0,0,0)(0,0,0)} {(0,0,0)(0,0,0)}	{(−0.5000000,−1.0000000,−1.5000000); (−0.4500000,−1.0000000,−1.5500000)} {(0.7500000,1.500000,2.250000); (0.700000,1.5000000,2.3000000)}

Case (iv): Consider the unconstrained optimization problem with trapezoidal Intuitionistic fuzzy coefficients that follows:
$\widetilde{\mathscr{T}}(x,y) = (0.5,0.75,1,1.75);(0.25,0.50,1,2.25)x-(0.5,0.75,1,1.75);(0.25,0.50,1,2.25)y+(1.5,1.75,2,2.75);(1.25,1.50,2,3.25)xy+(1.5,1.75,2,2.25);(1.25,1.50,2,3.25)x^2-(0.5,0.75,1,1.75);(0.25,0.50,1,2.25)y^2$. Solving this problem by using Algorithm 3.3, the MATLAB outputs are tabulated here.

Iteration	(x_i,y_i)	(x_{i+1},y_{i+1})
1	{(0,0,0,0);(0,0,0,0);} {(0,0,0,0);(0,0,0,0);}	{(−0.3750000,−0.4375000,−0.500000,−0.6875000); (−1.9375800,−1.004000,0.6202000,−0.4250200)} {(−0.7500000,−0.8750000,−1.000000,−1.3750000); (0.9000350,1.1204400,1.5000590,2.4375950)}

5. Comparative Study of Proposed Method with Existing Method

Function	Number of iterations in existing algorithm [12]	Number of iterations in proposed algorithm	
		Fuzzy Newton method	Intuitionistic fuzzy Newton method
$x^3-3xy+y^3$	8	2	2
$x-y+2xy+2x^2+y^2$	14	1	1

6. Conclusion

A new strategy for solving fuzzy unconstrained optimization issues was proposed in this paper. In addition, triangular and trapezoidal fuzzy number coefficients, as well as triangular and trapezoidal intuitionistic fuzzy number coefficients, are used. For tackling fuzzy unconstrained optimization problems, Newton's approach is employed, and the validity of the proposed method is tested using numerical examples and MATLAB programme outputs. In addition, we conducted a comparison study of crisp, fuzzy, and intuitionistic fuzzy Newton's methods with unconstrained optimization problems and found that our suggested method converges quickly.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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