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Research Article

A Method to Evaluate the Evidence Dependability on the Stimulus of Neutrosophic Set

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Abstract. In evidence theory, basic probability assignment plays an important role. The basic probability assignment is usually provided by experts. The evaluation of evidence dependability is till open issue, when preliminary data is unavailable. In this paper, we propose a new method to evaluate evidence dependability on the stimulus of neutrosophic set. The dependability of evidence was evaluated based on the truth degree between *Basic Probability Assignments* (BPAs). First, basic probability assignments were revamp to neutrosophic set. By the similarity degree between the neutrosophic set, we can obtain the truth degree between the Basic Probability Assignments. Then dependability of evidence can be computed based on its rapport with supporting degree. Based on the new evidence dependability, we formulated a new method for combining evidence sources with different dependability degrades. Finally, the validity of the proposed method is exemplified by the real life example.

Keywords. Evidence theory, Neutrosophic set, Evidence dependability, Basic probability assignment

Mathematics Subject Classification (2020). 03E72, 05D40

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1. Introduction

Dempster-Shafer (D-S) theory is a mathematical theory of evidence. The D-S theory of evidence [4, 15], one of the most popular uncertainty theories used in many areas. It is also known as the theory of probable or evidential reasoning. It is a powerful theoretical tool which can be pertained for the representation of incomplete knowledge, belief updating. The Dempster-Shafer model of representation and processing of uncertainty has led to a huge number of practical applications in a wide range of domain such as information fusion expert systems, decision making and risk assignment. *Basic Probability Assignment* (BPA) plays a vital role in evidence theory. All other measures can be defined in the terms of basic probability assignment. In actual practice the basic probability assignment is usually provided by experts subjectivity. Moreover, the evidence theory, many other theories are expanded to deal with uncertainty, such as fuzzy sets, intuitionistic fuzzy sets and so on.

The neutrosophy is a new branch of philosophy. F. Smarandache [6] introduced the notion of neutrosophic sets to handle with incomplete, inconsistent and indeterminate information. Neutrosophic set is a useful mathematical tool which is the generalization of the classic sets, conventional fuzzy set and intuitionistic fuzzy set. In neutrosophic logic, each proposition has a truth degree (T), an indeterminancy degree (I) and a falsity degree (F), where T, I, F are standard or non standard subsets of $]^{-}0, 1^{+}[$.

Multi sensor data fusion has been applied in many fields, like pattern recognition, target identification [3, 7, 12, 13], and decision making [10, 22]. When fusing information from multiple sensors, the information provided by different sensors may be uncertain, imprecise or even contradictory with each other. Many theories including the probability theory, fuzzy theory and evidence theory have been applied in data fusion [2, 8, 14, 17, 18]. The evaluation of evidence is important for the combination of basic probability assignments. Some methods to evaluate the reliability of evidences sources, but most of these methods are developed to evaluate evidence reliability when prior knowledge is available. The method proposed by Elouedi *et al*. [5] assessed the reliability of an evidence sources in the model of transferable belief, which is developed from evidence theory. In [19], Wu *et al*. proposed a new method to evaluate evidence reliability on the basis of intuitionistic fuzzy sets by using similarity measures of basic probability assignments.

In this paper, we proposed a new method for evaluating the evidence dependability on the stimulus of neutrosophic sets. By using of neutrosophic set, we have evaluated the evidence dependability. Numerical examples were used to validate the performance of the proposed method.

This paper is organized as follows: In Section 2 gives a brief recall of Dempster-Shafer theory and neutrosophic set. In Section 3 we proposed a new method to evaluate the evidence dependability on the stimulus neutrosophic set and numerical example are used to exemplify the validity of the proposed method. Finally, some concluding remarks are provided in Section 4.

2. Preliminaries

2.1 Basic Concepts

The evidence theory, initiated by Dempster and developed by Shafer, was modeled based on the frame of discernment denoted by Θ, which is a finite set with mutually exclusive elements. The power set of Θ, denoted by 2^{Θ}, contains all the possible unions of the sets in Θ including Θ itself. Singleton sets in a frame of discernment Θ are called atomic sets because they do not contain nonempty subsets. The following terminologies are central in the Dempster-Shafer theory [4].

Let $\Theta = \{\theta_1, \theta_2, \theta_3, \dots, \theta_n\}$ be the frame of discernment. A basic probability assignment is a function $m : 2^{\Theta} \to [0,1]$, satisfying the two following conditions:

$$m(\emptyset) = 0, \tag{2.1}$$

$$\sum_{A \subseteq \Theta} m(A) = 1, \tag{2.2}$$

where $\emptyset$ is the empty set, and A denotes the subset of Θ. A basic probability assignment is also called as a belief structure. For $A \subseteq \Theta$, the value assigned by the basic probability assignment on A is the basic probability mass of A, expressed by $m(A)$.

For $A \subseteq \Theta$, if $m(A) > 0$, A is the focal element of m. The set of all focal elements is expressed by $\{A \mid A \subseteq \Theta, m(A) > 0\}$. If the focal elements of a basic probability assignment 'm' are all atomic sets with only one element, the basic probability assignment is called *Bayesian Belief Structure* (BBS). The basic probability assignment with the following form:

$$m(A) = 1, \ \text{for all } A \subseteq \Theta \quad \text{and} \quad m(B) = 0, \ \text{for all } B \subseteq \Theta, B \neq A,$$

is called as a categorical belief structure. The basic probability assignment with $m(\Theta) = 1$ and $m(A) = 0$, for all $A \neq \Theta$, is called as a vacuous basic probability assignment.

Given a basic probability assignment m defined on Θ, its belief function and plausibility function can be, respectively, defined as:

$$Bel(A) = \sum_{B \subseteq A} m(B), \tag{2.3}$$

$$Pl(A) = \sum_{B \cap A} m(B) = 1 - \sum_{B \cap A} m(B), \tag{2.4}$$

$Bel(A)$ quantifies all basic probability masses exactly assigned to A and its subsets. $Pl(A)$ measures all possible basic probability masses that could be assigned to A and its subsets. In such sense, $Bel(A)$ and $Pl(A)$ can be regarded as the lower bound and upper bound of the probability to which A is supported. So, the belief degree of A can be considered as an interval number $BI(A) = [Bel(A), Pl(A)]$.

One of the four basic evidential belief functions is uncertainty. The uncertainty function [9] is defined by

$$u(A) = Bel(A) - Pl(A). \tag{2.5}$$

The pignistic transformation [16] is defined to transform a belief structure m to the so-called pignistic probability function, which is helpful for decision making. For a basic probability assignment m defined on $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$, the pignistic transformation is expressed by

$$Bet\,P(A) = \sum_{B \subseteq \Theta} \frac{|A \cap B|}{|B|} \frac{m(B)}{(1 - m(\emptyset))}, \quad \text{for all } A \subseteq \Theta, \tag{2.6}$$

where $|A|$ is the number of elements in set A, which is also called as the cardinality of set A. Particularly, given $m(\emptyset) = 0$ and $\Theta \in \Theta$, we have

$$BetP(\{\Theta\}) = \sum_{\Theta \in B} \frac{m(B)}{|B|}, \quad \theta = \theta_1, \theta_2, \dots, \theta_n,\ B \subset \Theta. \tag{2.7}$$

2.2 Dempster's Combination Rule

Given two basic probability assignments m_1 and m_2 defined on Θ, the basic probability assignment that results from their combination, denoted as $m_1 \oplus m_2$, or m_{12} for short, can be obtained by Dempster's combination rule [4], shown as:

$$m_{12}(A) = \begin{cases} \dfrac{\sum\limits_{B \cap C = A} m_1(B) m_2(C)}{1 - \sum\limits_{B \cap C = \emptyset} m_1(B) m_2(C)}, & \text{for all } A \subseteq \Theta, A \neq \emptyset, \\ 0, & A = \emptyset. \end{cases} \tag{2.8}$$

For more than two basic probability assignments to be combined, the combination results of all basic probability assignments can be obtained as:

$$m(A) = \begin{cases} \dfrac{\sum\limits_{\cap A_i = A} \prod_{i=1}^{n} m_i(A_i)}{1 - \sum\limits_{\cap A_i = \emptyset} m_1(B) m_2(C)}, & \text{for all } A \subseteq \Theta, A = \emptyset, \\ 0, & A = \emptyset. \end{cases} \tag{2.9}$$

Here, 'n' is the number of evidence pieces in the process of combination, i denotes the ith piece of evidence, and $m_i(A_i)$ is the basic probability assignment of hypothesis A_i supported by basic probability assignment i. The amount of conflict among 'n' mutually independent pieces of evidence is equal to the mass of the empty set after the conjunctive combination and before the normalization step. It represents contradictory evidence. It is calculated as:

$$k = \sum_{\cap A_i = \emptyset} \prod_{i=1}^{n} m_i(A_i). \tag{2.10}$$

The case of $k = 0$ indicates that there is no conflict among basic probability assignments, while $k = 1$ indicates that all basic probability assignments are in complete conflict. Dempster's rule has many good properties, such as commutativity and associativity. So, it has been widely applied in many areas. However, when the basic probability assignment to be combined are completely contradictory, that is, $k = 1$, the combination rule cannot be performed. When they are in high conflict, that is, $k \to 1$, we may get counter-intuitive combination results, which do not coincide with the actual situation. This can be demonstrated by the following example [21].

When the evidence source is not dependable, and its reliability degree is assigned as λ with $\lambda \in [0,1]$, we can use the discounting operation introduced by Shafer [15] to modify the original basic probability assignment. Based on Shafer's discounting operation, the basic probability assignment m^λ obtained by discounting is expressed as:

$$\begin{cases} m^\lambda(A) = \lambda m(A), \quad A \subset \Theta, \\ m^\lambda(\Theta) = 1 - \lambda + \lambda m(\Theta). \end{cases} \tag{2.11}$$

We note that if the evidence source is totally dependable, i.e., $\lambda = 1$, then the basic probability assignment m^λ is identical to the original basic probability assignment m. If the evidence source is completely undependable, i.e., $\lambda = 0$, we can get $m^\lambda(\Theta) = 1$, which means that the discounted basic probability assignment is a vacuous one providing no information.

2.3 Neutrosophic Set

Let X be a space of points, with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth membership function T_A, indeterminancy membership function I_A and falsity membership function $F_A \cdot T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non standard subsets of $]0^{-1}, 1^{+}[$.

That is, $T_A : X \to]0^{-1}, 1^{+}[$,

$I_A : X \to]0^{-1}, 1^{+}[$,

$F_A : X \to]0^{-1}, 1^{+}[$.

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, thus $0^{-1} \le \sup T_A(x) + \sup I_A(x) + \sup F(A)(x) \le 3^{+}$.

3. Evaluating the Evidence Dependability

3.1 The Relation between Basic Probability Assignment and Neutrosophic Set

Let m be basic probability assignment. If m is regarded as an neutrosophic set A defined in $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$, $Bel(\theta)$ is the degree of truth membership, $1 - u(\theta)$ is the degree of indeterminancy membership, $1 - Pl(\theta)$ is the degree of falsity membership. Based on these analysis, the basic probability assignment m, defined on the discernment frame $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$, can be expressed as an neutrsophic set A defined on $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$. The neutrosophic set A is written as:

$$\begin{aligned} A &= \{\langle \theta, T_A(\theta), I_A(\theta), F_A(\theta)\rangle \mid \theta \in \Theta\} \\ &= \{\langle \theta_1, Bel(\theta_1), 1 - u(\theta_1), 1 - Pl(\theta_1)\rangle, \langle \theta_2, Bel(\theta_2), 1 - u(\theta_2), 1 - Pl(\theta_2)\rangle, \ldots, \\ &\quad \langle \theta_n, Bel(\theta_n), 1 - u(\theta_n), 1 - Pl(\theta_n)\rangle\}. \end{aligned} \tag{3.1}$$

The relation between basic probability assignment and neutrosophic set has its physical interpretation from the viewpoint of target identification. Let the discernment frame be $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$, i.e., all possible classes of the target are contained in the set $\Theta = \{\theta_1, \theta_2, \theta_3\}$. The output of the sensor expressed by a basic probability assignment m indicates that the target is identified as an neutrosophic set A with

$$A = \{\langle\theta_1, Bel(\theta_1), 1-u(\theta_1), 1-Pl(\theta_1)\rangle, \langle\theta_2, Bel(\theta_2), 1-u(\theta_1), 1-Pl(\theta_2)\rangle,$$
$$\langle\theta_3, Bel(\theta_3), 1-u(\theta_3), 1-Pl(\theta_3)\rangle\}.$$

Specially, if a sensor identifies the target as a singleton subset of Θ, taking $\{\theta_1\}$ as an example, the basic probability assignment can be written as:

$$m(\{\theta_1\}) = 1, \quad m(\{\theta_2\}) = 0, \quad m(\{\theta_3\}) = 0.$$

Then the corresponding neutrosophic set is $A = \{\langle\theta_1, 1, 0, 0\rangle, \langle\theta_2, 0, 0, 1\rangle, \langle\theta_3, 0, 0, 1\rangle\}$, which is same as the set $\{\theta_1\}$.

3.2 Supporting Degree of Basic Probability Assignments

Supporting degree of basic probability assignment has been introduced to develop the modified combination rules [11, 20]. Generally, the supporting degree is calculated on the basis of the similarity or distance measures between basic probability assignments. If we use Sup to express the supporting degree, we have $\mathrm{Sup}(m_1, m_2) = \mathrm{Sup}(m_1, m_2)$. Taking Sim and Dis as the similarity and distance measures between basic probability assignments, respectively, we can get the following relations:

$$\mathrm{Sup}(m_1, m_2) \propto \mathrm{Sim}(m_1, m_2), \quad \mathrm{Sup}(m_1, m_2) \propto 1 - \mathrm{Dis}(m_1, m_2).$$

In other words, the higher similarity degree between the two basic probability assignments indicates the higher supporting degree between them. The lower distance between the two basic probability assignments also indicates higher supporting degree between the basic probability assignments. For clarity, the supporting degree between basic probability assignments can be usually considered as consistent to the similarity degree between basic probability assignments.

The relation between basic probability assignment and neutrosophic set allow us to calculate the supporting degree of basic probability assignments in the framework of neutrosophic set. Thus, the supporting degree $\mathrm{Sup}(m_1, m_2)$ can be obtained by calculating the supporting degree between neutrosophic set A_1 and A_2, where A_1 and A_2 are neutrosophic sets derived from m_1 and m_2, respectively. So, we have:

$$\mathrm{Sup}(m_1, m_2) = \mathrm{Sup}(A_1, A_2) = \mathrm{Sim}(A_1, A_2). \tag{3.2}$$

In recent years, a lot of similarity measures of neutrosophic sets have been proposed [1]. This provides us much convenience in calculating the supporting degree of basic probability assignments. In the following, similarity measure between A and B is defined as following:

Let $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}$ and $B = \{\langle x, T_B(x), I_B(x), F_B(x)\rangle \mid x \in X\}$ be two neutrosophic set defined in $X = \{x_1, x_2, \dots, x_n\}$. The similarity degree between A and B are calculated by:

$$S(A,B) = \frac{\sum_1^n (T_A(x_i)\cdot T_B(x_i) + I_A(x_i)\cdot I_B(x_i) + F_A(x_i)\cdot F_B(x_i)}{\max\left(\sum_1^n ((T_A(x_i))^2 + (I_A(x_i))^2 + (F_A(x_i))^2, \sum_1^n ((T_B(x_i))^2 + (I_B(x_i))^2 + (F_B(x_i))^2)\right)}. \tag{3.3}$$

It has been proved that the similarity measure $S(A,B)$ satisfies all axiomatic properties of neutrosophic similarity measure [1].

Based on the above analysis, we can obtain the supporting degree between two basic probability assignments m_1 and m_2, by the following steps:

Step 1: From equations (2.3), (2.4) and (2.5), we can get the values of the belief function, uncertainty function and plausibility function of all singleton subsets, corresponding to the basic probability assignments m_1 and m_2.

Step 2: From equation (3.1), we can get the two neutrosophic sets A_1 and A_2 according to m_1 and m_2.

Step 3: Following equation (3.3), we can calculate the similarity degrees $S(A_1,A_2)$.
Finally, we can get the degree to which m_1 supports m_2 is $\text{Sup}(m_1,m_2) = S(A_1,A_2)$, the degree of m_2 supporting m_1

$$\text{Sup}(m_2,m_1) = \text{Sup}(m_1,m_2) = S(A_1,A_2).$$

Based on the axiomatic properties of $S(A,B)$, we have

$$m_1 = m_2 \;\Rightarrow\; \text{Sup}(m_1,m_2) = \text{Sup}(m_2,m_1) = 1.$$

3.3 Evidence Reliability

Suppose that there are N basic probability assignments expressed as $m_1, m_2, \dots, m_N$. Based on the supporting degree between any two basic probability assignments, we can construct the *Supporting Degree Matrix* (SDM) as:

$$SDM = \left\{ \begin{bmatrix} \text{Sup}(m_1,m_2) & \text{Sup}(m_1,m_2) & \dots & \text{Sup}(m_1,m_2) \\ \text{Sup}(m_1,m_2) & \text{Sup}(m_1,m_2) & \dots & \text{Sup}(m_1,m_2) \\ \vdots & \vdots & \ddots & \vdots \\ \text{Sup}(m_1,m_2) & \text{Sup}(m_1,m_2) & \dots & \text{Sup}(m_1,m_2) \end{bmatrix} \right\}. \tag{3.4}$$

Now, the elements in the ith row represent the degree to which m_i is supported by other basic probability assignments. So, the total supporting degree of m_i can be calculates as:

$$\text{Total}_{\text{Sup}}(m_i) = \sum_{\substack{j=1 \\ j\neq 1}}^{N} \text{Sup}(m_i,m_j). \tag{3.5}$$

Generally, the larger support degree of a basic probability assignment indicates that this basic probability assignment is more reliable. Otherwise, the basic probability assignment is less reliable. So the reliability of each basic probability assignment can be calculated by its total support degree. In application, the reliability should be normalized. If we consider the relative reliability of all basic probability assignments [19], they can be normalized to the dependability of m_i as:

$$D'(m_i) = \frac{\text{Total}_{\text{Sup}}(m_i)}{\sum_{j=1}^{N} \text{Total}_{\text{Sup}}(m_j)}. \tag{3.6}$$

If the reliability of the most reliable basic probability assignment is set, the absolute dynamic reliability of m_i can be obtained as:

$$D(m_i) = \frac{\mathrm{Total}_{\mathrm{Sup}}(m_i)}{\max\limits_{j=1,2,\dots,N} \mathrm{Total}_{\mathrm{Sup}}(m_j)}. \tag{3.7}$$

3.4 A New Method to Combine the discounted Basic Probability Assignments

Once the dependability of all basic probability assignments are obtained, we can use evidence dependability to modify the original basic probability assignments by the discounting operation. Then, we can combine the discounted basic probability assignments using Dempster's combination rule. So, we can propose a new method for evidence combination. Suppose that there are 'N' basic probability assignments $m_1, m_2, \dots, m_N$ to be combined, they can be combined as the following steps:

Step 1: *Calculate the supporting degree of each basic probability assignment*. From equations (2.3), (2.4) and (2.5), we can get the value of the belief function, the uncertainty function and the plausibility function, for all singleton subsets with respect to m_i, $i = 1,2,\dots,N$. From equation (3.1), we can get Neutrosophic sets corresponding to all basic probability assignments. Following equation (3.3), we can calculate the similarity degrees $S(A_i, A_j)$, $i = 1,2,\dots,N$. Finally, we get the supporting degree between m_i and m_j, shown as:

$$\mathrm{Sup}(m_i, m_j) = \mathrm{Sup}(m_j, m_i) = S(A_i, A_j) = S(A_j, A_i).$$

Step 2: *Calculate the dependability of each basic probability assignment*. From the supporting degree between every two basic probability assignments, the support degree matrix can be constructed as equation (3.4). Then the dependability of each basic probability assignment can be obtained based on equation (3.6).

Step 3: *Modify the original basic probability assignments*. Using the evidence dependability and evidence discounting operation shown in equation (2.11), we can modify the original basic probability assignments $m_1, m_2, \dots, m_N$. The discounted basic probability assignments are denoted by $m_1^D, m_2^D, \dots, m_N^D$.

Step 4: *Evidence combination by Dempster's combination rule*. By Dempster's combination rule, the discounted basic probability assignments $m_1^D, m_2^D, \dots, m_N^D$ can be combined to an integrated basic probability assignment.

Example 3.1. In a target identification system based on multiple sensors, three sensors X_1, X_2, X_3 are employed to recognize the identification of a target. Three possible types of the target are denoted by θ_1, θ_2 and θ_3. So the frame of discernment can be expressed as $\Theta = \{\theta_1, \theta_2, \theta_3\}$. The outputs of three sensors are expressed by three basic probability assignments. They are listed as the following:

$$m_1(\{\theta_1\}) = 0.6,\quad m_1(\{\theta_2\}) = 0.2,\quad m_1(\{\theta_3\}) = 0.2,$$

$$m_2(\{\theta_1\}) = 0.3,\quad m_2(\{\theta_2\}) = 0.6,\quad m_2(\{\theta_3\}) = 0.1,$$

$$m_3(\{\theta_1\}) = 0.3, \quad m_3(\{\theta_2\}) = 0.3, \quad m_1(\{\theta_3\}) = 0.4.$$

Three neutrosophic set in $\Theta = \{\theta_1, \theta_2, \theta_3\}$ can be generated from these basic probability assignments. They are expressed as:

$$A_1 = \{\langle \theta_1, 0.6, 0.8, 0.4\rangle, \langle \theta_2, 0.2, 0.4, 0.8\rangle, \langle \theta_3, 0.2, 0.4, 0.8\rangle\},$$

$$A_2 = \{\langle \theta_1, 0.3, 0.6, 0.7\rangle, \langle \theta_2, 0.6, 0.8, 0.4\rangle, \langle \theta_3, 0.1, 0.2, 0.9\rangle\},$$

$$A_3 = \{\langle \theta_1, 0.3, 0.6, 0.7\rangle, \langle \theta_2, 0.3, 0.6, 0.7\rangle, \langle \theta_3, 0.4, 0.8, 0.6\rangle\}.$$

The *Supporting Degree Matrix* (SDM) for the three basic probability assignments is:

$$SDM = \begin{pmatrix} 1 & 0.8514 & 0.8818 \\ 0.8514 & 1 & 0.8618 \\ 0.8818 & 0.8618 & 1 \end{pmatrix}.$$

Based on equation (3.5), the total supporting degree of the basic probability assignment can be calculated:

$$\mathrm{Total}_{\mathrm{Sup}}(m_1) = 0.8514 + 0.8818 = 1.7333,$$

$$\mathrm{Total}_{\mathrm{Sup}}(m_2) = 0.8514 + 0.8618 = 1.7132,$$

$$\mathrm{Total}_{\mathrm{Sup}}(m_3) = 0.8818 + 0.8618 = 1.7436.$$

Finally, the absolute dependability of each basic probability assignment can be yielded according to equation (3.7):

$$D(m_1) = \frac{1.7333}{1.7436} = 0.99,$$

$$D(m_2) = \frac{1.7132}{1.7436} = 0.98,$$

$$D(m_3) = \frac{1.7436}{1.7436} = 1.$$

Based on the dependability factor, we can modify three original basic probability assignments by the discounting operation. We can get the discounted basic probability assignments as:

$$m_1^{(D)}(\{\theta_1\}) = 0.594, \quad m_1^{(D)}(\{\theta_2\}) = 0.198, \quad m_1^{(D)}(\{\theta_3\}) = 0.198,$$

$$m_2^{(D)}(\{\theta_1\}) = 0.294, \quad m_2^{(D)}(\{\theta_2\}) = 0.588, \quad m_2^{(D)}(\{\theta_3\}) = 0.098,$$

$$m_3^{(D)}(\{\theta_1\}) = 0.3, \quad m_3^{(D)}(\{\theta_2\}) = 0.3, \quad m_3^{(D)}(\{\theta_3\}) = 0.4.$$

Combining these discounted basic probability assignments by using Dempster's rule, we get the final result:

$$m(\{\theta_1\}) = 0.5357, \quad m(\{\theta_2\}) = 0.4413, \quad m(\{\theta_3\}) = 0.1947.$$

Based on the final fusion result, a comprehensive recognition on the target can be obtained. As shown in the result, the unknown target is identified as Θ_1, according to the outputs of three sensors. This example demonstrates that the proposed method provides an alternative way to combine uncertain evidence sources with different dependability, when a prior knowledge is not available.

4. Conclusion

The estimation of basic probability assignment plays a very important role in the application of DS theory in complex uncertain problems. In this paper, the supporting degree based on the similarity measure between basic probability assignment has been computed and then the dependability of evidence can be evaluated by using the total supporting degree of each basic probability assignment. A numerical example is used to illustrate the efficiency of the proposed method.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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