



# On Super Heronian Mean Labeling of Some Subdivision Graphs

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**Abstract.** Let  $f : V(G) \rightarrow \{1, 2, \dots, p + q\}$  be an injective function, where  $p = |V(G)|$  and  $q = |E(G)|$ . For a vertex labeling  $f$  the induced edge labeling  $f^*(e = uv)$  is defined by,

$$f^*(e) = \left\lfloor \frac{f(u) + \sqrt{f(u)f(v)} + f(v)}{3} \right\rfloor \text{ or } \left\lceil \frac{f(u) + \sqrt{f(u)f(v)} + f(v)}{3} \right\rceil.$$

Then  $f$  is called a super Heronian mean labeling if  $\{f(V(G))\} \cup \{f(e) : e \in E(G)\} = \{1, 2, 3, \dots, p + q\}$ . A graph which admits super Heronian mean labeling is called super Heronian mean graph.

**Keywords.** Super Heronian mean graph, Subdivision graph, Snake graphs

**Mathematics Subject Classification (2020).** 54C05, 54C08, 54C10

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## 1. Introduction

Graphs considered in this paper are connected, finite and undirected, i.e., with no loops and parallel edges and where the vertices and edges of a graph  $G$  is denoted by  $V(G)$  and  $E(G)$ , respectively. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions (Sandhya *et al.* [5]). All of the detailed survey of graph labeling are referred to Gallian [2].

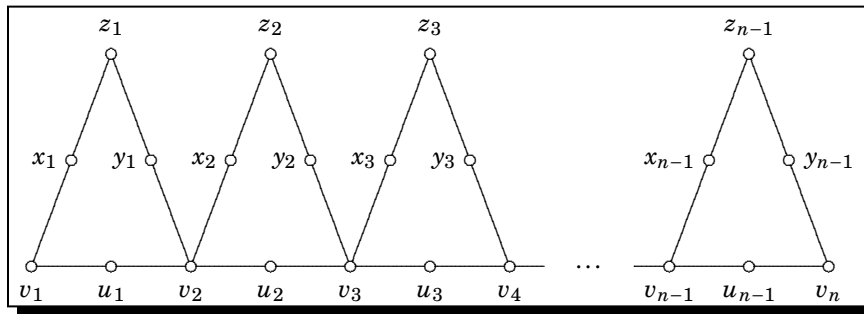
The super Heronian mean labeling is a type of labeling was introduced by Sandhya *et al.* [3–6] and proved that  $P_n, C_n, L_n, TL_n, M(P_n), T(P_n), T_n, Q_n, A(T_n), A(Q_n), D(Q_n)$  and

other corona graphs are super Heronian mean graph. So far, there were already 25 results published regarding this topic. In this paper, we proved that the subdivision of the graphs: Triangular snake  $T_n$ , quadrilateral snake  $Q_n$ , double triangular snake  $D(T_n)$  and double quadrilateral snake  $D(Q_n)$  are super Heronian mean graphs for all  $n \geq 3$ .

## 2. Proof of the Theorem

**Theorem 2.1.** *The subdivision of any triangular snake graph  $T_n$ ,  $n \geq 3$  is a super Heronian mean graph.*

*Proof.* Let  $S(T_n)$  be a subdivision of  $T_n$ . Let  $u_i, x_i$  and  $y_i$ ,  $1 \leq i \leq n - 1$  be the vertices which subdivide the edges  $v_i v_{i+1}$ ,  $v_i z_i$  and  $v_{i+1} z_i$ , respectively. Note that  $|V(S(T_n))| = 5n - 4$  and  $|E(S(T_n))| = 6n - 6$  (refer to Figure 1).



**Figure 1.** The subdivision of the triangular snake graph  $T_n$

Define a function  $f : V(S(T_n)) \rightarrow \{1, 2, \dots, 11n - 10\}$  by:

For  $1 \leq i \leq n$ ,

$$f(v_i) = 11i - 10$$

and for  $1 \leq i \leq n - 1$ ,

$$f(u_i) = 11i - 5; \quad f(x_i) = 11i - 7;$$

$$f(y_i) = 11i - 1; \quad f(z_i) = 11i - 4.$$

And the edges are labeled with:

For  $1 \leq i \leq n - 1$ ,

$$f(v_i u_i) = 11i - 8; \quad f(u_i v_{i+1}) = 11i - 3;$$

$$f(v_i x_i) = 11i - 9; \quad f(x_i z_i) = 11i - 6;$$

$$f(y_i v_{i+1}) = 11i; \quad f(y_i z_i) = 11i - 2.$$

Therefore,  $S(T_n)$  is a super Heronian mean graph, for all  $n \geq 3$ . □

**Example 2.1.** The graph in Figure 2 shows the subdivision of the triangular snake  $T_5$  and its super Heronian mean labeling.

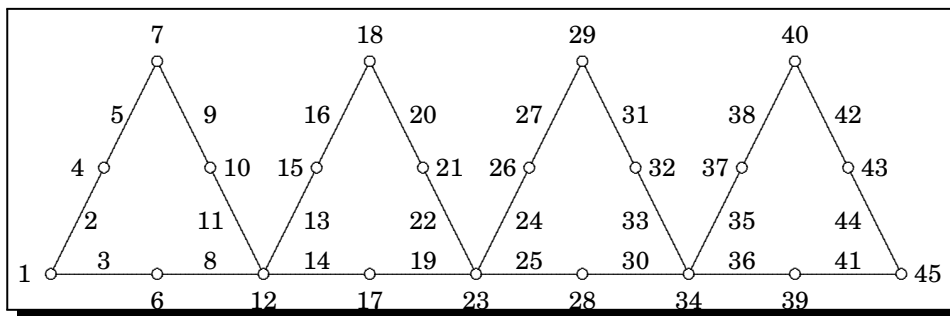


Figure 2. The subdivision of the triangular snake  $T_5$  and its super Heronian mean labeling

**Theorem 2.2.** *The subdivision of any quadrilateral snake graph  $Q_n$ ,  $n \geq 3$  is a super Heronian mean graph.*

*Proof.* Let  $S(Q_n)$  be a subdivision of  $Q_n$ . Let  $u_i, x_i, y_i$ , and  $b_i$ ,  $1 \leq i \leq n - 1$  be the vertices which subdivide the edges  $v_i v_{i+1}, v_i a_i, v_{i+1} c_i$  and  $a_i c_i$ , respectively. Note that  $|V(S(T_n))| = 7n - 6$  and  $|E(S(T_n))| = 8n - 8$  (refer to Figure 3).

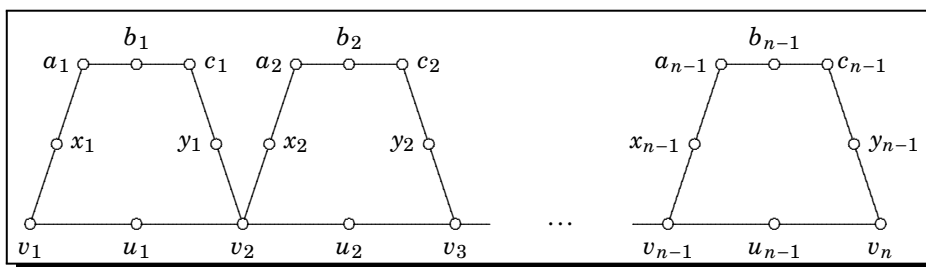


Figure 3. The Subdivision of the Quadrilateral Snake Graph  $Q_n$

Define a function  $f : V(S(Q_n)) \rightarrow \{1, 2, \dots, 15n - 14\}$  by:

For  $1 \leq i \leq n$ ,

$$f(v_i) = 15i - 14$$

and for  $1 \leq i \leq n - 1$ ,

$$f(u_i) = 15i - 7; \quad f(a_i) = 15i - 9;$$

$$f(x_i) = 15i - 12; \quad f(b_i) = 15i - 6;$$

$$f(y_i) = 15i - 1; \quad f(c_i) = 15i - 3.$$

And the edges are labeled with:

For  $1 \leq i \leq n - 1$ ,

$$f(v_i u_i) = 15i - 11; \quad f(u_i v_{i+1}) = 15i - 4;$$

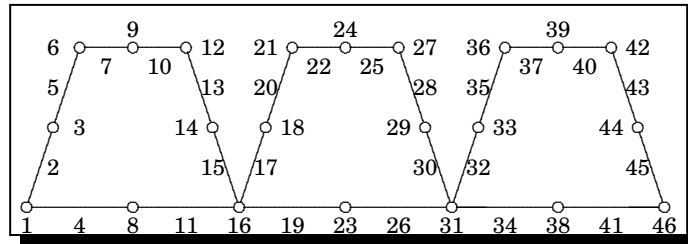
$$f(v_i x_i) = 15i - 13; \quad f(x_i a_i) = 15i - 10;$$

$$f(a_i b_i) = 15i - 8; \quad f(b_i c_i) = 15i - 5;$$

$$f(c_i y_i) = 15i - 2; \quad f(y_i v_{i+1}) = 15i.$$

Therefore,  $S(Q_n)$  is a super Heronian mean graph, for all  $n \geq 3$ . □

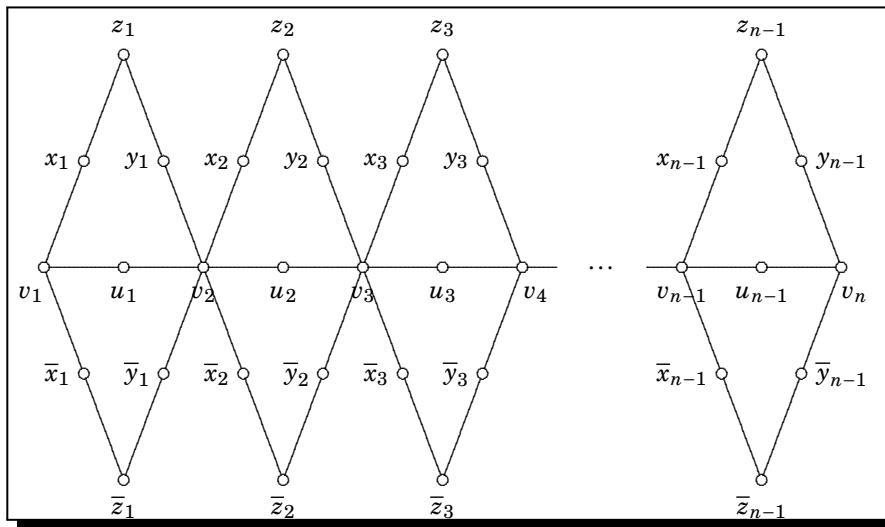
**Example 2.2.** The graph in Figure 4 shows the subdivision of the quadrilateral snake  $Q_4$  and its super Heronian mean labeling.



**Figure 4.** The subdivision of the quadrilateral snake  $Q_4$  and its super Heronian mean labeling

**Theorem 2.3.** The subdivision of any double triangular snake  $D(T_n)$ ,  $n \geq 2$  is a super Heronian mean graph.

*Proof.* Let  $S(D(T_n))$  be a subdivision of  $D(T_n)$ . Let  $u_i, x_i, y_i, \bar{x}_i$  and  $\bar{y}_i, 1 \leq i \leq n - 1$  be the vertices which subdivide the edges  $v_i v_{i+1}, v_i z_i, v_{i+1} z_i, v_i \bar{z}_i$  and  $\bar{z}_i v_{i+1}$ , respectively. Note that  $|V(S(D(T_n)))| = 8n - 7$  and  $|E(S(D(T_n)))| = 10n - 10$  (refer to Figure 5).



**Figure 5.** The subdivision of double triangular snake graph  $D(T_n)$

Define a function  $f : V(D(T_n)) \rightarrow \{1, 2, \dots, 18n - 17\}$  by:

For  $1 \leq i \leq n$ ,

$$f(v_i) = 18i - 17$$

and for  $1 \leq i \leq n - 1$ ,

$$f(u_i) = 18i - 8; \quad f(\bar{x}_i) = 18i - 12;$$

$$f(x_i) = 18i - 14; \quad f(\bar{y}_i) = 18i - 2;$$

$$f(y_i) = 18i - 4; \quad f(\bar{z}_i) = 18i - 7;$$

$$f(z_i) = 18i - 9.$$

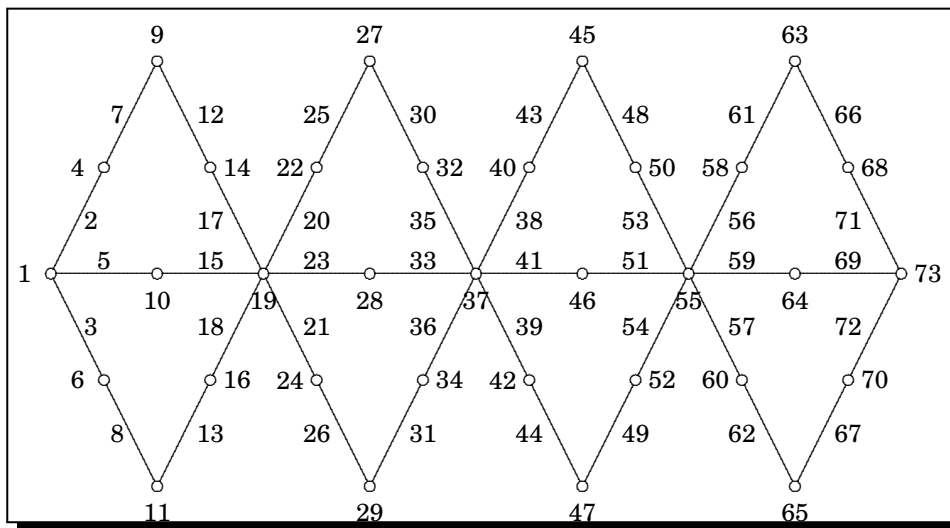
And the edges are labeled with:

For  $1 \leq i \leq n - 1$ ,

$$\begin{aligned}
 f(v_i u_i) &= 18i - 13; & f(u_i v_{i+1}) &= 18i - 3; \\
 f(v_i x_i) &= 18i - 16; & f(x_i z_i) &= 18i - 11; \\
 f(y_i v_{i+1}) &= 18i - 1; & f(y_i z_i) &= 18i - 6; \\
 f(v_i \bar{x}_i) &= 18i - 15; & f(\bar{y}_i v_{i+1}) &= 18i; \\
 f(\bar{x}_i \bar{z}_i) &= 18i - 10; & f(\bar{y}_i \bar{z}_i) &= 18i - 5.
 \end{aligned}$$

Therefore,  $S(D(T_n))$  is a super Heronian mean graph, for all  $n \geq 2$ . □

**Example 2.3.** The graph in Figure 6 shows the subdivision of the double triangular snake  $T_5$  and its super Heronian mean labeling.



**Figure 6.** The subdivision of the double triangular snake  $T_5$  and its super Heronian mean labeling

**Theorem 2.4.** The subdivision of any double quadrilateral snake  $D(Q_n)$ ,  $n \geq 2$  is a super Heronian mean graph.

*Proof.* Let  $S(D(Q_n))$  be a subdivision of  $D(Q_n)$ . Let  $u_i, x_i, y_i, b_i, \bar{x}_i, \bar{y}$ , and  $\bar{b}_i$ ,  $1 \leq i \leq n - 1$  be the vertices which subdivide the edges  $v_i v_{i+1}, v_i a_i, v_{i+1} c_i, a_i c_i, v_i \bar{a}_i, v_{i+1} \bar{c}_i$  and  $\bar{a}_i \bar{c}_i$ , respectively. Note that  $|V(S(T_n))| = 12n - 11$  and  $|E(S(T_n))| = 14n - 14$  (refer to Figure 7).

Define a function  $f : V(D(Q_n)) \rightarrow \{1, 2, \dots, 26n - 25\}$  by:

For  $1 \leq i \leq n$ ,

$$f(v_i) = 26i - 25$$

and for  $1 \leq i \leq n - 1$ ,

$$f(u_i) = 26i - 12; \quad f(\bar{x}_i) = 26i - 21;$$

$$f(x_i) = 26i - 22; \quad f(\bar{y}_i) = 26i - 2;$$

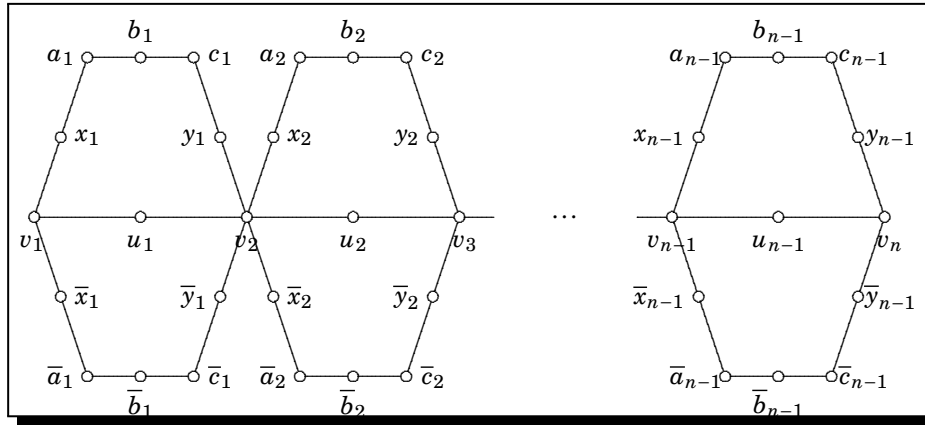


Figure 7. The subdivision of the double quadrilateral snake graph  $S(D(Q_n))$

$$\begin{aligned}
 f(y_i) &= 26i - 3; & f(\bar{a}_i) &= 26i - 16; \\
 f(a_i) &= 26i - 17; & f(\bar{b}_i) &= 26i - 11; \\
 f(b_i) &= 26i - 13; & f(\bar{c}_i) &= 26i - 7; \\
 f(c_i) &= 26i - 8.
 \end{aligned}$$

And the edges are labeled with:

For  $1 \leq i \leq n - 1$ ,

$$\begin{aligned}
 f(v_i u_i) &= 26i - 19; & f(u_i v_{i+1}) &= 26i - 5; \\
 f(v_i x_i) &= 26i - 24; & f(x_i a_i) &= 26i - 20; \\
 f(y_i v_{i+1}) &= 26i - 1; & f(y_i c_i) &= 26i - 6; \\
 f(a_i b_i) &= 26i - 15; & f(b_i c_i) &= 26i - 10; \\
 f(v_i \bar{x}_i) &= 26i - 23; & f(\bar{x}_i \bar{a}_i) &= 26i - 18; \\
 f(\bar{y}_i v_{i+1}) &= 26i; & f(\bar{y}_i \bar{c}_i) &= 26i - 4; \\
 f(\bar{a}_i \bar{b}_i) &= 26i - 14; & f(\bar{b}_i \bar{c}_i) &= 26i - 9.
 \end{aligned}$$

Therefore,  $S(D(Q_n))$  is a super Heronian mean graph, for all  $n \geq 2$ . □

**Example 2.4.** The graph in Figure 8 shows the subdivision of the double quadrilateral snake  $D(Q_4)$  and its super Heronian mean labeling.

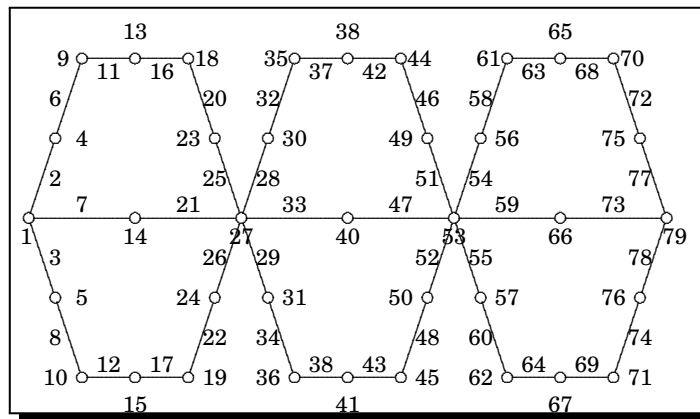


Figure 8. The subdivision of the double quadrilateral snake  $D(Q_4)$  and its super Heronian mean labeling

### Competing Interests

The authors declare that they have no competing interests.

### Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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