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Research Article

On Super Heronian Mean Labeling of Some Subdivision Graphs

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Abstract. Let $f : V(G) \to \{1,2,\dots,p+q\}$ be an injective function, where $p = |V(G)|$ and $q = |E(G)|$. For a vertex labeling f the induced edge labeling $f^*(e = uv)$ is defined by,

$$f^*(e) = \left\lfloor \frac{f(u) + \sqrt{f(u)f(v)} + f(v)}{3} \right\rfloor \text{ or } \left\lceil \frac{f(u) + \sqrt{f(u)f(v)} + f(v)}{3} \right\rceil.$$

Then f is called a super Heronian mean labeling if $\{f(V(G))\} \cup \{f(e) : e \in E(G)\} = \{1,2,3,\dots,p+q\}$. A graph which admits super Heronian mean labeling is called super Heronian mean graph.

Keywords. Super Heronian mean graph, Subdivision graph, Snake graphs

Mathematics Subject Classification (2020). 54C05, 54C08, 54C10

1. Introduction

Graphs considered in this paper are connected, finite and undirected, i.e., with no loops and parallel edges and where the vertices and edges of a graph G is denoted by $V(G)$ and $E(G)$, respectively. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions (Sandhya *et al*. [5]). All of the detailed survey of graph labeling are refered to Gallian [2].

The super Heronian mean labeling is a type of labeling was introduced by Sandhya *et al*. [3–6] and proved that P_n, C_n, L_n, TL_n, $M(P_n)$, $T(P_n)$, T_n, Q_n, $A(T_n)$, $A(Q_n)$, $D(Q_n)$ and other corona graphs are super Heronian mean graph. So far, there were already 25 results published regarding this topic. In this paper, we proved that the subdivision of the graphs: Triangular snake T_n, quadrilateral snake Q_n, double triangular snake $D(T_n)$ and double quadrilateral snake $D(Q_n)$ are super Heronian mean graphs for all $n \geq 3$.

2. Proof of the Theorem

Theorem 2.1. *The subdivision of any triangular snake graph T_n, $n \geq 3$ is a super Heronian mean graph.*

Proof. Let $S(T_n)$ be a subdivision of T_n. Let u_i, x_i and y_i, $1 \leq i \leq n-1$ be the vertices which subdivide the edges v_iv_{i+1}, v_iz_i and $v_{i+1}z_i$, respectively. Note that $|V(S(T_n))| = 5n-4$ and $|E(S(T_n))| = 6n-6$ (refer to Figure 1).

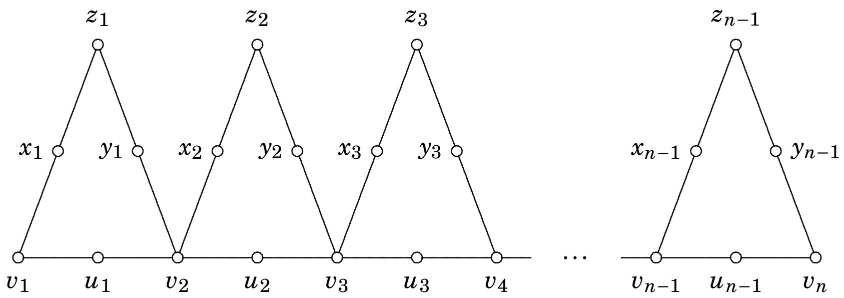

Figure 1. The subdivision of the triangular snake graph T_n

Define a function $f : V(S(T_n)) \to \{1, 2, \dots, 11n-10\}$ by:

For $1 \leq i \leq n$,

$$f(v_i) = 11i - 10$$

and for $1 \leq i \leq n-1$,

$$f(u_i) = 11i - 5; \quad f(x_i) = 11i - 7;$$

$$f(y_i) = 11i - 1; \quad f(z_i) = 11i - 4.$$

And the edges are labeled with:

For $1 \leq i \leq n-1$,

$$f(v_iu_i) = 11i - 8; \quad f(u_iv_{i+1}) = 11i - 3;$$

$$f(v_ix_i) = 11i - 9; \quad f(x_iz_i) = 11i - 6;$$

$$f(y_iv_{i+1}) = 11i; \quad f(y_iz_i) = 11i - 2.$$

Therefore, $S(T_n)$ is a super Heronian mean graph, for all $n \geq 3$. □

Example 2.1. The graph in Figure 2 shows the subdivision of the triangular snake T_5 and its super Heronian mean labeling.

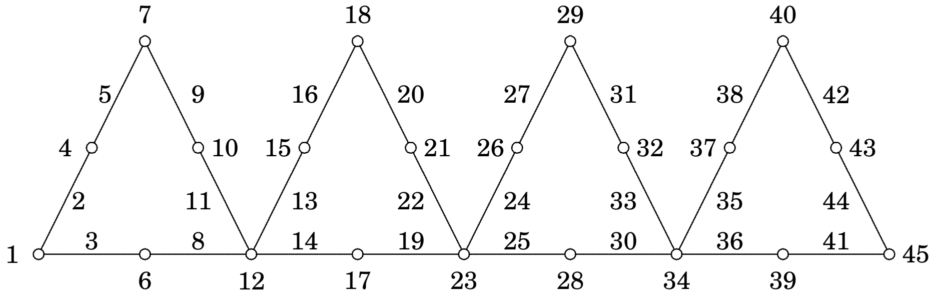

Figure 2. The subdivision of the triangular snake T_5 and its super Heronian mean labeling

Theorem 2.2. *The subdivision of any quadrilateral snake graph Q_n, $n \geq 3$ is a super Heronian mean graph.*

Proof. Let $S(Q_n)$ be a subdivision of Q_n. Let u_i, x_i, y_i, and b_i, $1 \leq i \leq n-1$ be the vertices which subdivide the edges $v_iv_{i+1}, v_ia_i, v_{i+1}c_i$ and a_ic_i, respectively. Note that $|V(S(T_n))| = 7n-6$ and $|E(S(T_n))| = 8n-8$ (refer to Figure 3).

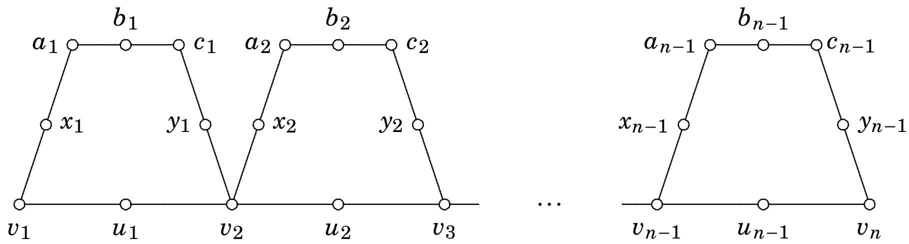

Figure 3. The subdivision of the quadrilateral snake graph Q_n

Define a function $f : V(S(Q_n)) \to \{1, 2, \ldots, 15n-14\}$ by:

For $1 \leq i \leq n$,

$$f(v_i) = 15i - 14$$

and for $1 \leq i \leq n-1$,

$$f(u_i) = 15i - 7; \quad f(a_i) = 15i - 9;$$
$$f(x_i) = 15i - 12; \quad f(b_i) = 15i - 6;$$
$$f(y_i) = 15i - 1; \quad f(c_i) = 15i - 3.$$

And the edges are labeled with:

For $1 \leq i \leq n-1$,

$$f(v_iu_i) = 15i - 11; \quad f(u_iv_{i+1}) = 15i - 4;$$
$$f(v_ix_i) = 15i - 13; \quad f(x_ia_i) = 15i - 10;$$
$$f(a_ib_i) = 15i - 8; \quad f(b_ic_i) = 15i - 5;$$
$$f(c_iy_i) = 15i - 2; \quad f(y_iv_{i+1}) = 15i.$$

Therefore, $S(Q_n)$ is a super Heronian mean graph, for all $n \geq 3$. □

Example 2.2. The graph in Figure 4 shows the subdivision of the quadrilateral snake Q_4 and its super Heronian mean labeling.

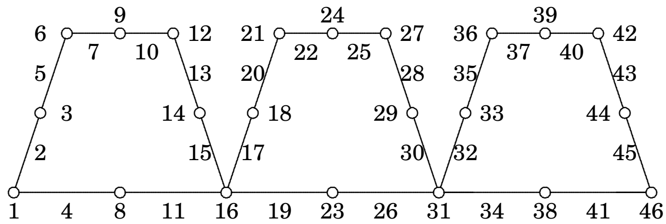

Figure 4. The subdivision of the quadrilateral snake Q_4 and its super Heronian mean labeling

Theorem 2.3. *The subdivision of any double triangular snake $D(T_n)$, $n \geq 2$ is a super Heronian mean graph.*

Proof. Let $S(D(T_n))$ be a subdivision of $D(T_n)$. Let $u_i, x_i, y_i, \overline{x}_i$ and $\overline{y}_i, 1 \leq i \leq n-1$ be the vertices which subdivide the edges $v_i v_{i+1}, v_i z_i, v_{i+1} z_i, v_i \overline{z}_i$ and $\overline{z}_i v_{i+1}$, respectively. Note that $|V(S(T_n))| = 8n-7$ and $|E(S(T_n))| = 10n-10$ (refer to Figure 5).

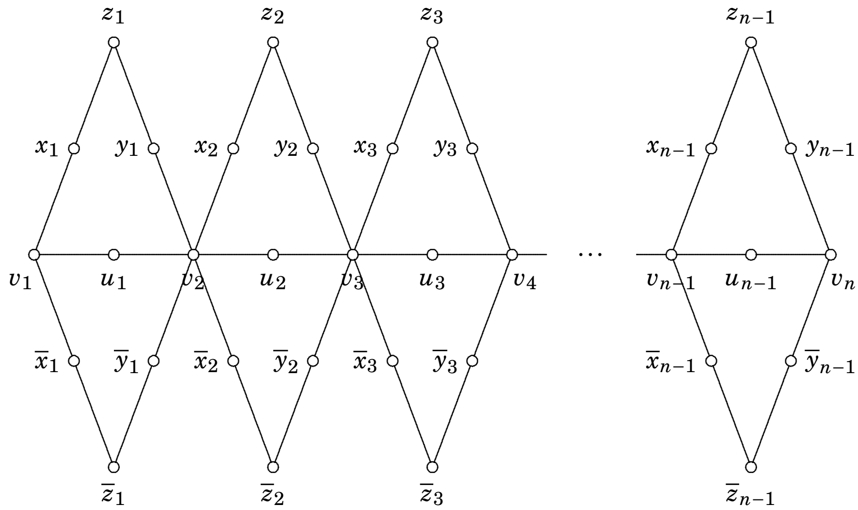

Figure 5. The subdivision of double triangular snake graph $D(T_n)$

Define a function $f : V(D(T_n)) \to \{1, 2, \ldots, 18n-17\}$ by:

For $1 \leq i \leq n$,

$$f(v_i) = 18i - 17$$

and for $1 \leq i \leq n-1$,

$$f(u_i) = 18i - 8; \quad f(\overline{x}_i) = 18i - 12;$$
$$f(x_i) = 18i - 14; \quad f(\overline{y}_i) = 18i - 2;$$
$$f(y_i) = 18i - 4; \quad f(\overline{z}_i) = 18i - 7;$$
$$f(z_i) = 18i - 9.$$

And the edges are labeled with:

For $1 \le i \le n-1$,

$f(v_i u_i) = 18i - 13$; $f(u_i v_{i+1}) = 18i - 3$;

$f(v_i x_i) = 18i - 16$; $f(x_i z_i) = 18i - 11$;

$f(y_i v_{i+1}) = 18i - 1$; $f(y_i z_i) = 18i - 6$;

$f(v_i \overline{x}_i) = 18i - 15$; $f(\overline{y}_i v_{i+1}) = 18i$;

$f(\overline{x}_i \overline{z}_i) = 18i - 10$; $f(\overline{y}_i \overline{z}_i) = 18i - 5$.

Therefore, $S(D(T_n)$ is a super Heronian mean graph, for all $n \ge 2$. □

Example 2.3. The graph in Figure 6 shows the subdivision of the double triangular snake T_5 and its super Heronian mean labeling.

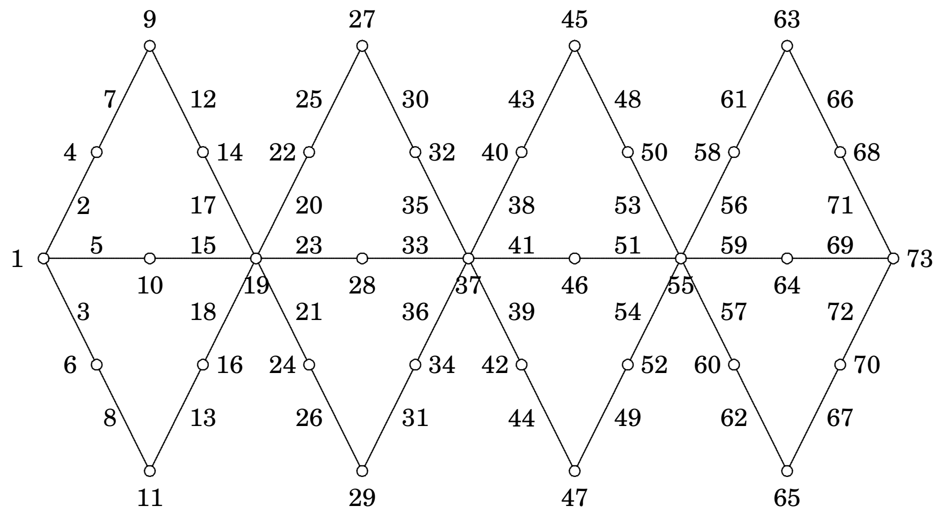

Figure 6. The subdivision of the double triangular snake T_5 and its super Heronian mean labeling

Theorem 2.4. *The subdivision of any double quadrilateral snake $D(Q_n)$, $n \ge 2$ is a super Heronian mean graph.*

Proof. Let $S(D(Q_n))$ be a subdivision of $D(Q_n)$. Let $u_i, x_i, y_i, b_i, \overline{x}_i, \overline{y}$, and $\overline{b}_i$, $1 \le i \le n-1$ be the vertices which subdivide the edges $v_i v_{i+1}, v_i a_i, v_{i+1} c_i, a_i c_i, v_i \overline{a}_i, v_{i+1} \overline{c}_i$ and $\overline{a}_i \overline{c}_i$, respectively. Note that $|V(S(T_n))| = 12n - 11$ and $|E(S(T_n))| = 14n - 14$ (refer to Figure 7).

Define a function $f : V(D(Q_n)) \to \{1, 2, \dots, 26n - 25\}$ by:

For $1 \le i \le n$,

$f(v_i) = 26i - 25$

and for $1 \le i \le n-1$,

$f(u_i) = 26i - 12$; $f(\overline{x}_i) = 26i - 21$;

$f(x_i) = 26i - 22$; $f(\overline{y}_i) = 26i - 2$;

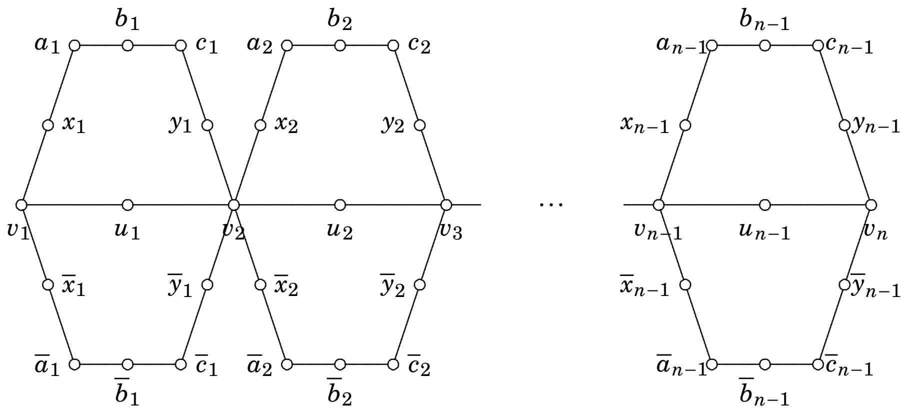

Figure 7. The subdivision of the double quadrilateral snake graph $S(D(Q_n))$

$f(y_i) = 26i-3$; $f(\overline{a}_i) = 26i-16$;
$f(a_i) = 26i-17$; $f(\overline{b}_i) = 26i-11$;
$f(b_i) = 26i-13$; $f(\overline{c}_i) = 26i-7$;
$f(c_i) = 26i-8$.

And the edges are labeled with:

For $1 \le i \le n-1$,

$f(v_iu_i) = 26i-19$; $f(u_iv_{i+1}) = 26i-5$;
$f(v_ix_i) = 26i-24$; $f(x_ia_i) = 26i-20$;
$f(y_iv_{i+1}) = 26i-1$; $f(y_ic_i) = 26i-6$;
$f(a_ib_i) = 26i-15$; $f(b_ic_i) = 26i-10$;
$f(v_i\overline{x}_i) = 26i-23$; $f(\overline{x}_i\overline{a}_i) = 26i-18$;
$f(\overline{y}_iv_{i+1}) = 26i$; $f(\overline{y}_i\overline{c}_i) = 26i-4$;
$f(\overline{a}_i\overline{b}_i) = 26i-14$; $f(\overline{b}_i\overline{c}_i) = 26i-9$.

Therefore, $S(D(Q_n)$ is a super Heronian mean graph, for all $n \ge 2$. □

Example 2.4. The graph in Figure 8 shows the subdivision of the double quadrilateral snake $D(Q_4)$ and its super Heronian mean labeling.

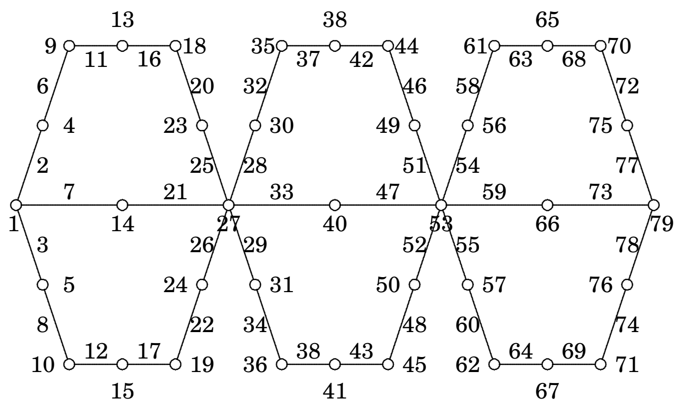

Figure 8. The subdivision of the double quadrilateral snake $D(Q_4)$ and its super Heronian mean labeling

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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