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Research Article

Influence of Hall Current and Chemical Reaction on MHD Unsteady Mass and Heat Transfer Flow Under Rotation

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Abstract. A literal explanation on the dilemma about a transient MHD heat and mass transfer flow under Hall current and chemical reaction past a uniformly accelerated porous plate influenced by magnetic intensity with rotation has been presented. Having assumed a magnetic field with uniform intensity, it is applied perpendicular to the plate oriented to the fluid region. The governing equations of flow are solved analytically by adopting Laplace transform technique in closed form. The profiles of concentration field and Sherwood number concerning the plate are graphically verified with respect to different values of the parameters involving the problem. The consequent results have been explained physically. It is observed that consumption of chemical species leads to fall in species concentration.

Keywords. Magnetohydrodynamics (MHD), Chemical reaction, Hall current, Concentration, Sherwood number

Mathematics Subject Classification (2020). 76W05

1. Introduction

Varieties of naturally occurring phenomena and engineering issues find link to analysis to MHD (*Magnetohydrodynamics*). Characteristics of MHD within interaction of conducting and magnetic filled are studied in geophysics. MHD is devoted to study of fluid motion characterised by electric conduction under the effects of magnetic field. Model studies concerning MHD forced and free convection involving mass and heat transfer issues are applied by several authors because of their applications in several fields of scientific and technological studies and research. Few of those authors include Singh and Singh [8], and Elbashbeshy [4] etc. Georgantopoulos *et al*. [5] analyzed free convection of two-dimensional unsteady nature in association with transfer of mass passing an infinite erect porous plate. Several researchers have examined reaction-induced impact resulting from flows involving transfer of mass and heat, of those Apelblat [2] and Anderson *et al.* [1] are considered important. Chambré and Young [3] studied chemical reaction of primary order in nearest of plate oriented horizontally. Muthucumaraswamy [6] conferred effects of heat and mass transfer considering homogeneous 1st order reaction on endlessly moving surface of isothermal condition with uniformly occurring suction. Muthucumaraswamy and Meenakshisundaram [7] analyzed analytically the influence of chemical reaction on erect oscillatory plate. Sinha and Sarma [9] analyzed consequence of radiation and Hall current on an unsteady MHD mass and heat transfer flow in existence of rotation.

Because of the importance in finding out issues of chemical reaction with MHD, the present authors have planned to investigate the consequences of chemical species on unsteady MHD flow passing through a uniformly accelerated plate with pores. This work is an extension of the work of Sinha and Sarma [9] to consider the effect of chemical reaction.

2. Mathematical Formulation

To idealize the present fluid flow problem, Figure 1 fluid structure is taking into account. The 3D co-ordinate system is introduced with x-axis going uphill vertically along a plate, y-axis being at perpendicular to plate and z-axis is along its width. the u' and v' are velocities along the x and y axes, respectively. The plate and the fluid are set in fixed rotation having uniform angular velocity Ω around z-axis.

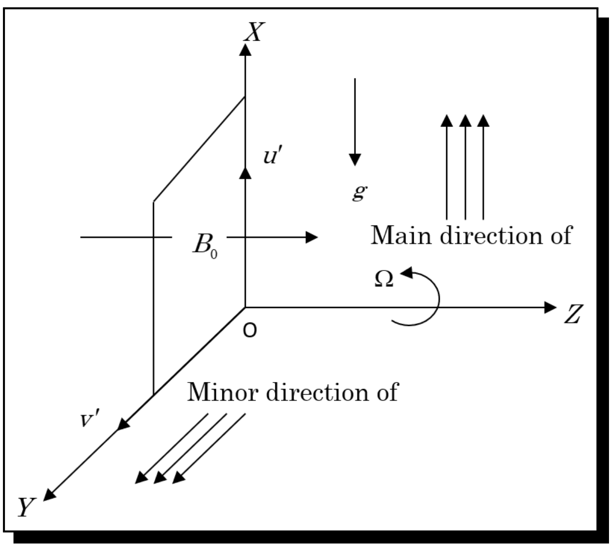

Figure 1. Structure of flow

The governing Navier-Stoke's equations describing the flow model are given by:

1. *Momentum Equation*:

$$\frac{\partial u'}{\partial t'} - w'_0 \frac{\partial u'}{\partial z'} - 2\Omega' v' = \upsilon \frac{\partial^2 u'}{\partial z'^2} + \frac{\sigma B_0^2 (m v' - u')}{\rho(1+m^2)}, \tag{2.1}$$

$$\frac{\partial v'}{\partial t'} - w'_0 \frac{\partial v'}{\partial z'} + 2\Omega' u' = \upsilon \frac{\partial^2 v'}{\partial z'^2} - \frac{\sigma B_0^2 (m u' + v')}{\rho(1+m^2)} \tag{2.2}$$

2. *Energy Equation*:

$$\rho C_p \frac{\partial T'}{\partial t'} = K_T \frac{\partial^2 T'}{\partial z'^2} - \frac{\partial q_r}{\partial z'} \tag{2.3}$$

3. *Concentration Equation*:

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial z'^2} - K'_r (C' - C_\infty) \tag{2.4}$$

The limiting situations are:

$$\left.\begin{array}{ll} u'=0,\ v'=0,\ T'=T'_\infty,\ C'=C'_\infty & \text{for } t'\le 0,\ \forall\ z', \\ u'=a't',\ v'=0,\ T'=T'_w,\ C'=C'_w & \text{for } z'=0, \\ u'=0,\ v'=0,\ T'=T'_\infty,\ C'=C'_\infty & \text{for } z'\to\infty \end{array}\right\}. \tag{2.5}$$

The radiative heat flux term of an optically thin gray gas is written as:

$$\frac{\partial q_r}{\partial z'} = -4K\sigma'(T'^4_\infty - T'^4). \tag{2.6}$$

By escalating T'^4 as Taylor series with $T'_\infty$and omitting terms of higher order, it is obtained as:

$$T'^4 \cong 4T'^3_\infty - 3T'^4_\infty. \tag{2.7}$$

By virtue of equation (2.6) and (2.7), equation (2.3) becomes

$$\rho C_p \frac{\partial T'}{\partial t'} = K_T \frac{\partial^2 T'}{\partial z'^2} + 16K\sigma' T'^3_\infty (T'_\infty - T'). \tag{2.8}$$

To make the flow model normalized, the dimensionless parameters are introduced:

$$\left.\begin{array}{l} z=\frac{w'_0 z'}{\upsilon},\ t=\frac{w'^2_0 t'}{\upsilon},\ u=\frac{u'}{u'_0},\ v=\frac{v'}{u'_0},\ M=\frac{\sigma B_0^2 \upsilon}{\rho w'^2_0}, \\ \Omega=\frac{2\Omega'\upsilon}{w'_0},\ a=\frac{a'\upsilon}{w_0^3},\ \theta=\frac{T'-T'_\infty}{T'_\omega-T'_\infty},\ R=\frac{16K\upsilon^2\sigma' T'^3_\infty}{K_T w_0^2}, \\ Pr=\frac{\upsilon}{\alpha},\ \phi=\frac{C'-C'_\infty}{C'_\omega-C'_\infty},\ Sc=\frac{\upsilon}{D},\ Pr=\frac{K_T}{\upsilon\rho C_p},\ Kr=\frac{Kr'\upsilon}{w'^2_0} \end{array}\right\}. \tag{2.9}$$

The following dimensional equations are derived from equation (2.1)-(2.4) by using the identities given in the equations (2.6)–(2.9).

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial z} - v\Omega = \frac{\partial^2 u}{\partial z^2} + M \cdot \frac{mv - u}{1+m^2}, \tag{2.10}$$

$$\frac{\partial v}{\partial t} - \frac{\partial v}{\partial z} + u\Omega = \frac{\partial^2 v}{\partial z^2} - M \cdot \frac{mu + v}{1+m^2}, \tag{2.11}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial z^2} - \frac{1}{Pr} R\theta, \tag{2.12}$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial z^2} - Kr\,\phi. \tag{2.13}$$

Using $q = u + iv$ and $A = i\Omega + \frac{M(1+m_i)}{1+m^2}$, equation (2.10) and (2.11) implies:

$$\frac{\partial q}{\partial t} = \frac{\partial q}{\partial z} + \frac{\partial^2 q}{\partial z^2} - qA\,. \tag{2.14}$$

The consequent marginal circumstances are:

$$\left.\begin{array}{ll} t \le 0: & q = 0,\ \theta = 0,\ \phi = 0 \text{ for all values of } z \\ t > 0: & q = at,\ \theta = 1,\ \phi = 1 \text{ at } z = 0, \\ & \text{and } q \to 0,\ \theta \to 0,\ \phi \to 0 \text{ as } z \to \infty \end{array}\right\} \tag{2.15}$$

3. Solution of Problem

Adopting Laplace transform technique in equations (2.14), (2.12), and (2.13), the equations are found as follows:

$$\frac{d^2\bar{q}}{dz^2} + \frac{d\bar{q}}{dz} - (A + S)\bar{q} = 0, \tag{3.1}$$

$$\frac{d^2\bar{\theta}}{dz^2} - Pr(s + C)\bar{\theta} = 0, \quad \text{where}\, C = \frac{R}{Pr}, \tag{3.2}$$

$$\frac{d^2\bar{\phi}}{dz^2} - Sc(s + Kr)\bar{\phi} = 0\,, \tag{3.3}$$

With boundary conditions

$$\left.\begin{array}{ll} \bar{q} = \frac{a}{s^2},\ \bar{\theta} = \frac{1}{s},\ \bar{\phi} = \frac{1}{s} & \text{at } z = 0, \\ \bar{q} = 0,\ \bar{\theta} = 0,\ \bar{\phi} = 0, & \text{at } z \to \infty \end{array}\right\}. \tag{3.4}$$

The solutions of the equations (3.1)-(3.3) with respect to the boundary conditions (3.4) are:

$$\theta = \lambda_1\,, \tag{3.5}$$

$$\phi = \lambda_2\,, \tag{3.6}$$

$$q = \lambda_3\,. \tag{3.7}$$

The expressions for Skin friction co-efficient (τ), Nusselt number (Nu), and Sherwood number (Sh) are as follows:

$$\tau = \left(\frac{\partial q}{\partial z}\right)_{z=0} = \omega_1, \quad Nu = -\left(\frac{\partial \theta}{\partial z}\right)_{z=0} = \omega_2 \text{ and } Sh = -\left(\frac{\partial \phi}{\partial z}\right)_{z=0} = \omega_3\,.$$

4. Results and Discussion

Necessary computations for representative concentration field and the mass transfer rate in terms of Sherwood number are made to acquire the physical imminent into the problem. In this regard, it has been carried out by taking some randomly selected particular values of chemical reaction parameter Kr and Schmdit number Sc involved in the problem. The mathematical fallouts estimated during solution of problem are analyzed through Figures 2-5. Figures 2 and 3 depict the influence of chemical reaction parameter Kr and Schmdit number Sc on species concentration. Figure 2 observes that the stage of the concentration level of fluid falls due to high consumption of chemical species satisfying the physical reality. The rising behavior of the concentration level due to the action of high mass diffusivity is experienced in Figure 3. Figure 4 represents deviation of mass transfer rate (Sh) from the plate to the fluid. This figure explains that Sherwood number *Sh* mounted up due to effect of *Sc*, i.e., the mass flux *Sh* gets accelerated due to impact of mass diffusivity *Sc*. The consequence of chemical reaction parameter *Kr* on Sherwood number has been presented in Figure 5. Here, the rate of change of mass is steadily amplified for escalating the parameter of chemical reaction *Kr*. This fact implies that high consumption of chemical species enhances the mass flux phenomena.

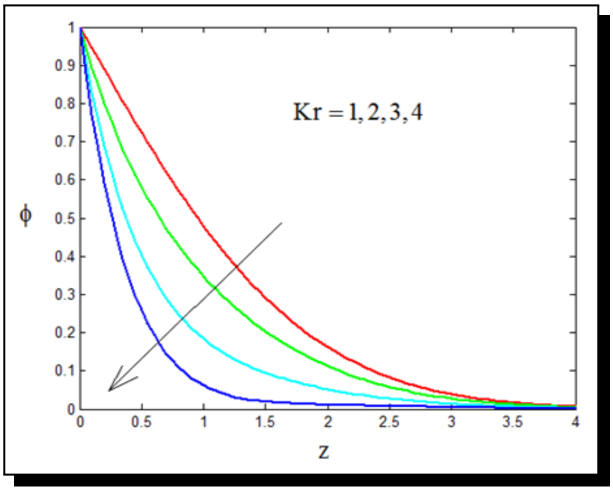

Figure 2. Concentration against z with $Sc = 0.60$, $t = 1$

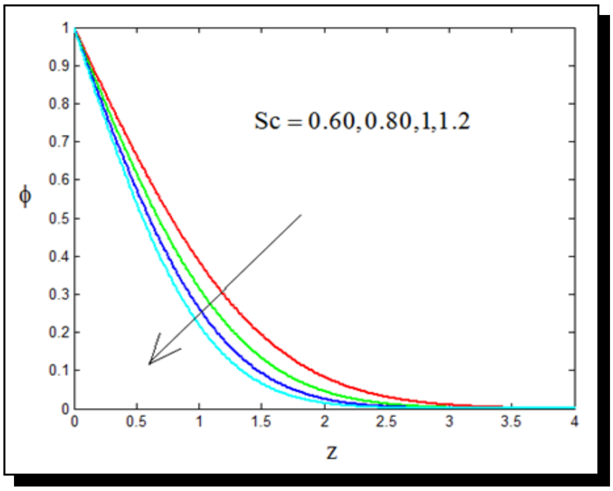

Figure 3. Concentration against z with $Kr = 1$, $t = 1$

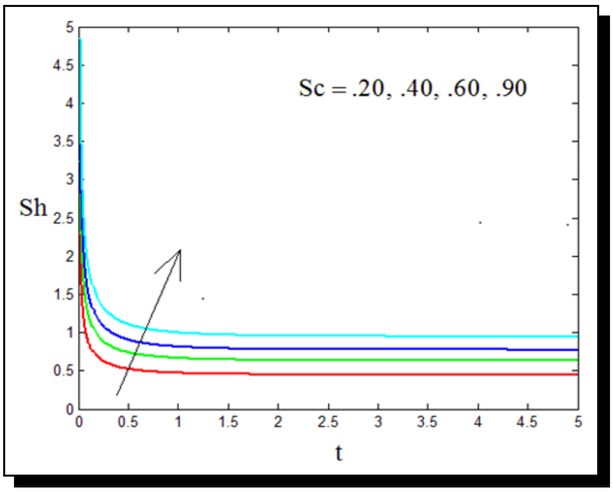

Figure 4. Sh against t for $Kr = 1$

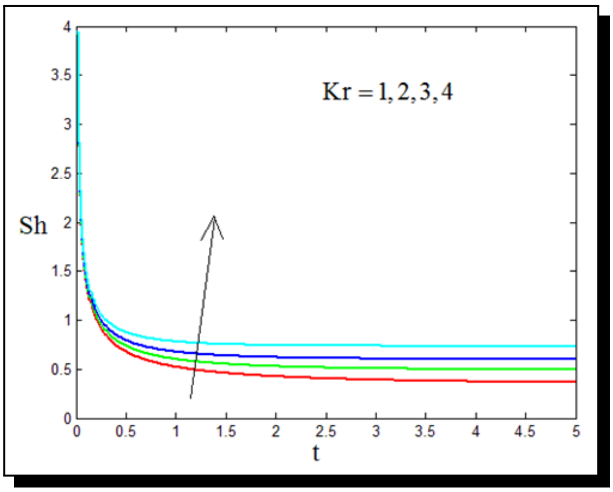

Figure 5. Sh against t for $Sc = 0.60$

5. Comparison of Outcomes

For comparing outcomes of present research, the results of Sinha and Sarma [9] are used. Comparing Figures 6 and 7 of this article with Figures 7 and 8 of Sinha and Sarma [9], it is observed same kind of behaviour due to the implementation of Schmdit number *Sc*, i.e., there is a significant effect of Schmdit number on both fluid concentration profile and rate of mass transfer. The concentration profile and changing rate of mass are almost similar making an admirable pact with the findings investigated by Sinha and Sarma [9] and the present authors.

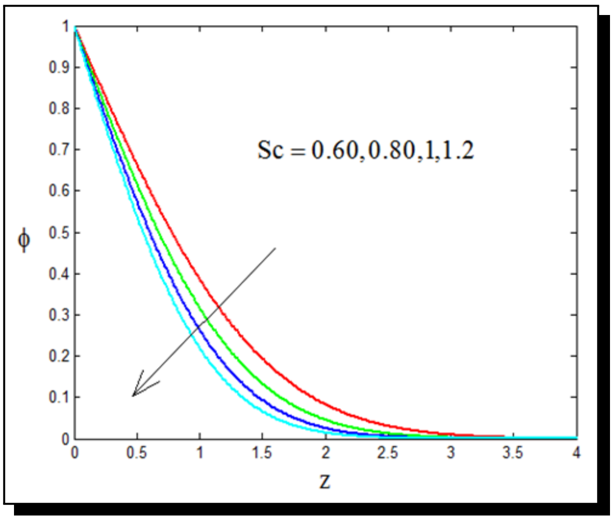

Figure 6. Concentration versus z for $Kr = 0$, $t = 1$

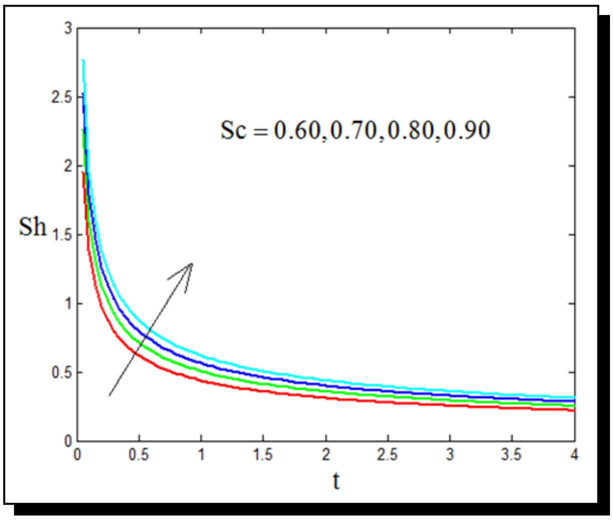

Figure 7. Sh versus t for $Kr = 0$

6. Conclusion

Investigation on the current problem leads to the following opinions:

(1) The low mass diffusivity and consumption of chemical species reduces the concentration level of fluid steadily.

(2) The mass flux occurring from plate to fluid and high consumption (chemical reaction) have made considerable increase in mass transfer rate.

Appendix

$$\lambda_1 = \alpha(Pr,C,z,t) = \frac{1}{2}\left[e^{\sqrt{Pr}\sqrt{C}z} erfc\left(\frac{\sqrt{Pr}z}{2\sqrt{t}} + \sqrt{Ct}\right) + e^{-\sqrt{Pr}\sqrt{C}z} erfc\left(\frac{\sqrt{Pr}z}{2\sqrt{t}} - \sqrt{Ct}\right)\right],$$

$$\lambda_2 = f(Sc,Kr,z,t) = \frac{1}{2}\left[e^{\sqrt{Sc}\sqrt{Kr}z} erfc\left(\frac{\sqrt{Sc}z}{2\sqrt{t}} + \sqrt{Kr\,t}\right) + e^{-\sqrt{Sc}\sqrt{Kr}z} erfc\left(\frac{\sqrt{Sc}z}{2\sqrt{t}} - \sqrt{Kr\,t}\right)\right],$$

$$\lambda_3 = \gamma(b,z,t) = \frac{1}{2}ae^{-\frac{1}{2}z}\left[\left(t + \frac{z}{2\sqrt{b}}\right)e^{\sqrt{b}z} erfc\left(\frac{z}{2\sqrt{t}} + \sqrt{bt}\right) + \left(t - \frac{z}{2\sqrt{b}}\right)e^{-\sqrt{b}z} erfc\left(\frac{z}{2\sqrt{t}} - \sqrt{bt}\right)\right],$$

$$\omega_1 = f(b,t) = a\left[-\sqrt{\frac{t}{\pi}}e^{-bt} - \left(\sqrt{b}t + \frac{1}{2\sqrt{b}}\right)erf(\sqrt{bt}) - \frac{t}{2}\right],$$

$$\omega_2 = g(Pr,C,t) = \sqrt{\frac{Pr}{\pi t}}e^{-Ct} + \sqrt{Pr}\sqrt{C}\,erf(\sqrt{Ct}),$$

$$\omega_3 = h(Sc,Kr,t) = \sqrt{\frac{Sc}{\pi t}}e^{-Kr\,t} + \sqrt{Sc}\sqrt{Kr}erf(\sqrt{Kr\,t}),$$

$$A = i\Omega + \frac{M(1+mi)}{1+m^2},\ b = \frac{1+4A}{4},\ C = \frac{R}{Pr},$$

$$\xi = \frac{1+\sqrt{1+4(A+s)}}{2} = \frac{1}{2} + \sqrt{b+s},$$

$$\mu = \nu\rho,\ Pr = \frac{\nu}{\alpha},\ \frac{1}{Pr} = \frac{K_T}{\nu\rho Cp},\ Sc = \frac{\nu}{D},\ q = u + iv\,.$$

Nomenclature

B_0 : Strength of applied magnetic field
u' : Components of velocity along X-axis
v' : Components of velocity along Y-axis
Ω' : Uniform angular velocity
g : Acceleration due to gravity
w'_0 : Constant Suction velocity
v : Kinematic viscosity
σ : Electrical conductivity
ρ : Fluid density
C_p : Specific heat at constant pressure
T' : Dimensional temperature
T'_w : Dimensional temperature at the plate
T'_∞: Dimensional temperature in the free stream
t' : Dimensional time
K_T : Thermal conductivity
C' : Dimensional concentration
C'_w : Dimensional concentration at the wall
C'_∞: Dimensional concentration in the free stream
D : Molecular mass diffusivity
K'_r : Dimensional chemical reaction parameter
q_r : Radiative heat flux
a : Absorption co-efficient
K : Non-dimensional porosity number
M : Magnetic parameter
R : Radiation parameter
Pr : Prandtl number
Sc : Schmidt number
m : Hall parameter
μ : Co-efficient of viscosity
θ : Non-dimensional temperature
ϕ : Non-dimensional concentration

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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