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Research Article

Topological Indices for Inverse Graphs Associated With Finite Cyclic Group

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Abstract. Topological indices are mathematical descriptors for molecular structures. These descriptors are used to describe physico-chemical properties such as solubility, molecular shape and molecular weight. In this paper, we present distance-based topological indices such as Wiener index and hyper-Wiener index by using Hosoya polynomial for inverse graphs associated with finite cyclic group. Also, we have found eccentricity based topological index of inverse graphs associated with finite cyclic group.

Keywords. Hosoya polynomial, Wiener index, Hyper-Wiener index, Inverse graph

Mathematics Subject Classification (2020). 05C10, 05C25, 05C75

1. Introduction

A study on algebraic structures is a very interesting and astonishing thing in mathematics. The graphs constructed from algebraic structures are used to prove the accurate results in algebra very easily. That's why the algebraic graph theory has been an active research area in long time. Recently, Alfuraidan and Zakariya [1] introduced the inverse graphs associated with finite groups. More over, the authors established the results on inverse graph on finite group in [2, 3, 6, 11, 12]

The topological index gives the numerical values to the graphs. The topological index of chemical component is useful to analyse the various chemical properties of the material. There is an close relation between graph and chemical structure. This accommodates as the underlying motivation for finding the topological index of graph from algebraic structures.

The Hosoya polynomial of a graph was introduced by Haruo Hosoya in [9] (1988). It paves way towards determining almost all distance based topological indices which are used to predict physico-chemical properties and bio activities of compounds. In this paper, we present the Hosoya polynomial for inverse graphs associated with finite cyclic group. We also derive distance based topological indices such as Wiener index, hyper-Wiener index by using Hosoya polynomial for inverse graphs associated with finite cyclic group. For undefined terms, we refer [4] and [13].

2. Preliminaries

Now, let us recall basic definitions and notations about graphs. A graph $G=(V,E)$, we mean a simple graph with non-empty vertex set V and edge set E. The number of elements in V is called order of G and the number of elements in E is called the size of G. A graph G is said to be complete if any pair of distinct vertices are adjacent in G. We denote the complete graph of order n by K_n. A graph G is connected if there exists a path between every pair of distinct vertices in G. The degree of the vertex $v\in V$, denoted by $d(v)$, is the number of edges in G which are incident with v. The distance between any two vertices u and v is length of the shortest path between u and v. It is denoted by $d(u,v)$. The maximum distance of a vertex v among its distances with all the vertices of G is called the eccentricity of v, denoted by $ecc(v)$. For any vertex v of G, the total distance of v is defined as $D(v)=\sum\limits_{u\in V(G)} d(v,u)$. The diameter of a connected graph is supremum of shortest distances between vertices in G and is denoted by $\text{diam}(G)$. The distance number $D(v|G)$ of the vertex v in G is defined as sum of the distance between all other vertices in G ($D(v|G)=\sum\limits_{u\in V(G)} d(v,u)$). We define $d(G,t)$ to be the number of unordered pairs of vertices in G that are at exactly distance t. The girth of G is defined as length of the shortest cycle in G and is denoted by $\text{gr}(G)$. We take $\text{gr}(G)=\infty$ if G contains no cycles. For undefined terms in graph theory, we refer [4]. In this paper we use the notion that $(\Gamma,\star)$ for finite group (Simply G) and G for simple undirected graph.

Alfuraidan and Zakariya [1] introduced the inverse graphs associated with finite groups as follows, let $(\Gamma,\star)$ be a finite group and S be the set of all non-self-invertible elements in Γ. Then the inverse graph of G is $G(\Gamma_S)$ (Simply Γ_S) with vertex set Γ and two distinct vertices u and v are adjacent if and only if either $u\star v\in S$ or $v\star u\in S$.

Let us consider the following observations which will help to prove the results:

Let Γ be cyclic group with order n and $u\in V(\Gamma_S)$, then

$$deg(u)=\begin{cases}\left.\begin{cases}n-1, & \text{if } u\notin S\\ n-2, & \text{if } u\in S\end{cases}\right\}, & \text{if } n \text{ is odd,}\\ \left.\begin{cases}n-2, & \text{if } u\notin S \text{ or } u^2\notin S\\ n-3, & \text{otherwise}\end{cases}\right\}, & \text{if } n \text{ is even.}\end{cases}$$

Now consider the following topological indices for a simple graph G is given by

- The *Hosoya polynomial* [8]

$$H(G,x):=\sum_{\{u,v\}\in V(G)} x^{d(u,v)}.$$

- The *Wiener index*
$$W(G) := \frac{1}{2} \sum_{u,v \in V(G)} d(u,v).$$
Note, that the first derivative of the Hosoya polynomial at $x = 1$ is equal to the Wiener index, i.e.,
$$W(G) := \frac{\partial H(G,x)}{\partial x}\Big|_{x=1}.$$
- The *first Zagreb index*
$$M_1(G) := \sum_{v \in V(G)} (d(v))^2.$$
- The *second Zagreb index*
$$M_2(G) := \sum_{(u,v) \in E(G)} d(u)d(v).$$
- The *Hyper-Wiener index* [5]
$$WW(G) := \frac{1}{2} W(G) + \frac{1}{2} \sum_{\{u,v\} \subseteq V(G)} d(u,v)^2 = \frac{1}{2} \sum_{k=1}^{\delta(G)} k(k+1)d(G,k).$$
- The *Eccentric-connectivity index* [10]
$$\xi(G) = \sum_{v \in V(G)} d(v)ecc(v).$$
- The *Total eccentricity index*
$$\zeta(G) = \sum_{v \in V(G)} ecc(v).$$
- The *New version (eccentricity based) of Zagreb index* [7]
$$M_1^*(G) = \sum_{uv \in E(G)} (ecc(u) + ecc(v)),$$
$$M_1^{**}(G) = \sum_{v \in V(G)} (ecc(v))^2,$$
$$M_2^*(G) = \sum_{uv \in E(G)} ecc(u)ecc(v).$$
- *Average eccentricity index*
$$aveg(G) = \frac{1}{n} \sum_{v \in V(G)} ecc(v).$$
- *Eccentric distance sum index*
$$\xi^{DS}(G) = \sum_{v \in V(G)} ecc(v)D(v|G),$$
where $D(v|G) = \sum_{u \in V(G)} d(v,u)$.
- The *ABC-index* [14]
$$ABC(\Gamma(\mathbb{V}_{\mathfrak{B}})) = \sum_{uv \in E} \sqrt{\frac{d(u)+d(v)-2}{d(u)d(v)}}.$$

Theorem 2.1 ([1, Theorem 3.4]]). *The diameter of a connected inverse graph is two.*

Theorem 2.2 ([1, Theorem 3.6]). *Let Γ be a finite abelian group with exactly two non-self invertible elements. Then, the associated inverse graph $G(\Gamma_s)$ is complete bipartite.*

Observation 2.3. *Let $(\Gamma,\star)$ be a finite cyclic group order n, then $\Gamma \cong \mathbb{Z}_n$, where $\mathbb{Z}_n$ is addition modulo group of order n.*

- *If n is odd then $(\Gamma,\star)$ has one self invertible element that is identity of G.*
- *If n is even then $(\Gamma,\star)$ has two self invertible element are identity and $\frac{n}{2}$.*

Observation 2.4. *Let $(\Gamma,\star)$ be a finite cyclic group order n, then $\Gamma \cong \mathbb{Z}_n$, where $\mathbb{Z}_n$ is addition modulo group of order n.*

- *If n is odd then,* $D(u \mid G(\Gamma_S) = \begin{cases} n-1, & \text{if } u \notin S, \\ n, & \text{otherwise}. \end{cases}$
- *If n is even then,* $D(u \mid G(\Gamma_S) = \begin{cases} n, & \text{if } u \notin S \text{ or } u^2 \notin S, \\ n+1, & \text{otherwise}. \end{cases}$

In this paper, we first study the topological indices of inverse graph for cyclic group and then for abelian group.

3. Topological Index of Inverse Graph of Cyclic Group

In this section, we found some results related to topological index of inverse graph associated with cyclic group.

Theorem 3.1. *Let Γ be cyclic group of order $n \geq 3$ then, Hosoya polynomial*

$$H(G(\Gamma_S),x) = \begin{cases} \frac{(n-1)^2}{2}x + \frac{n-1}{2}x^2, & \text{if } n \text{ is odd}, \\ \frac{(n-1)(n-2)}{2}x + (n-1)x^2, & \text{if } 4 \mid n, \\ \frac{n^2-2n+4}{2}x + (n-2)x^2, & \text{otherwise}. \end{cases}$$

Proof. Let as assume that Γ be cyclic group of order n. By Theorem 3.4 $diam(\Gamma_S) = 2$ so any pair of vertices has distance 1 or 2.

Case 1. If n is odd then S contains only the identity element. Any non-zero vertex v is not adjacent to v^{-1} in Γ_S. Hence the number of vertices having distance 2 is $\frac{n-1}{2}$. Also, the number of adjacent pair of vertices in $G(\Gamma_S)$ is $\frac{n(n-1)}{2} - \frac{n-1}{2} = \frac{(n-1)^2}{2} = \frac{(n-1)^2}{2}$. Hence,

$$H(G(\Gamma_S),x) = \frac{(n-1)^2}{2}x + \frac{n-1}{2}x^2.$$

Case 2. If n is even then S contains identity element and $\frac{n}{2}$. Consider the following subcases:

Subcase 1. If $4 \nmid n$. Any non-zero vertex v is not adjacent to v^{-1} and $|\frac{n}{2} - v|$ in Γ_S. Hence the number of vertices having distance 2 is $n-1$. Also, the number of adjacent pair of vertices in $G(\Gamma_S)$ is $\frac{(n-2)(n-1)}{2}$. Hence,

$$H(G(\Gamma_S),x) = \frac{(n-1)(n-2)}{2}x + (n-1)x^2.$$

Subcase 2. If $4 \mid n$. Any non-zero vertex v with $v^2 \notin S$ is not adjacent to v^{-1} and $|\frac{n}{2} - v|$ in Γ_S. If non-zero vertex v with $v^2 \in S$ is not adjacent to v^{-1}. Hence the number of vertices having distance 2 is $n-2$. Also, the number of adjacent pair of vertices in $G(\Gamma_S)$ is $\frac{n(n-1)}{2} - (n-2)$. Hence,

$$\begin{aligned} H(G(\Gamma_S), x) &= \frac{n(n-1)}{2} - (n-2)x + (n-2)x^2 \\ &= \frac{n^2 - 2n + 4}{2} x + (n-2)x^2. \end{aligned}$$

This, holds the proof. □

First derivative of the above theorem (Hosoya polynomial at $x = 1$) gives the following theorem:

Theorem 3.2. *Let Γ be cyclic group of order $n \geq 3$ then, wiener index*

$$W(G(\Gamma_S)) = \begin{cases} \frac{n^2-1}{2}, & \text{if } n \text{ is odd}, \\ \frac{(n-1)(n+2)}{2}, & \text{if } 4 \mid n, \\ \frac{n^2+2n-4}{2}, & \text{otherwise}. \end{cases}$$

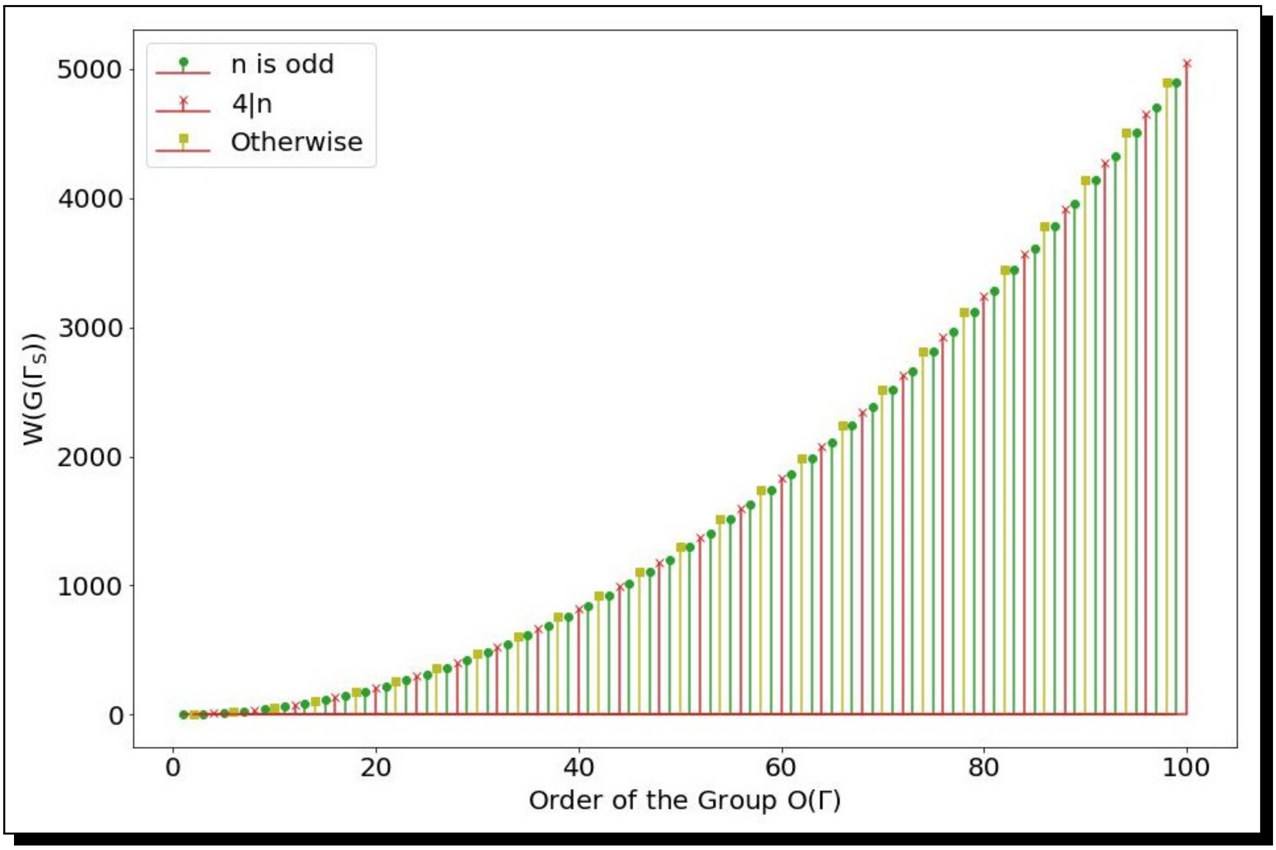

Figure 1. Wiener index of $G(\Gamma_S)$

Theorem 3.3. *Let Γ be cyclic group of order $n \geq 3$ then, first Zagreb index*

$$M_1(G(\Gamma_S),x) = \begin{cases} n^3-4n^2-3, & \text{if } n \text{ is odd},\\ n^3-6n^2+17n-20, & \text{if } 4 \mid n,\\ n^3-6n^2+13n-10, & \text{otherwise}. \end{cases}$$

Proof. Let Γ be cyclic group of order n and $u \in V(\Gamma_S)$. We know that

$$deg(u) = \begin{cases} \left\{ \begin{cases} n-1, & \text{if } u \notin S \\ n-2, & \text{if } u \in S \end{cases} \right\}, & \text{if } n \text{ is odd},\\ \left\{ \begin{cases} n-2, & \text{if } u \notin S \text{ or } u^2 \notin S \\ n-3, & \text{otherwise} \end{cases} \right\}, & \text{if } n \text{ is even}. \end{cases}$$

Case 1. If n is odd then S contains only the identity element.

$$M_1(G(\Gamma_S)) = (n-1)^2+(n-1)(n-2)^2 = (n-1)(n^2-3n+3) = n^3-4n^2-3.$$

Case 2. If n is even then S contains identity element and $\frac{n}{2}$. Consider the following subcases:

Subcase 1. If $4 \mid n$, then there exist two elements $u = \frac{n}{4}$ and u^{-1} satisfies the condition that u^2 and $(u^{-1})^2$ are in S. Hence

$$M_1(G(\Gamma_S)) = 4(n-2)^2+(n-4)(n-3)^2 = 4(n^2-4n+4)+(n-4)(n^2-6n+9) = n^3-6n^2+17n-20.$$

Subcase 2. If $4 \mid n$, then only two elements in S and no such element $u^2 \in S$ exist. Hence

$$M_1(G(\Gamma_S)) = 2(n-2)^2+(n-2)(n-3)^2 = (n-2)(n^2-4n+5) = n^3-6n^2+13n-10. \qquad \square$$

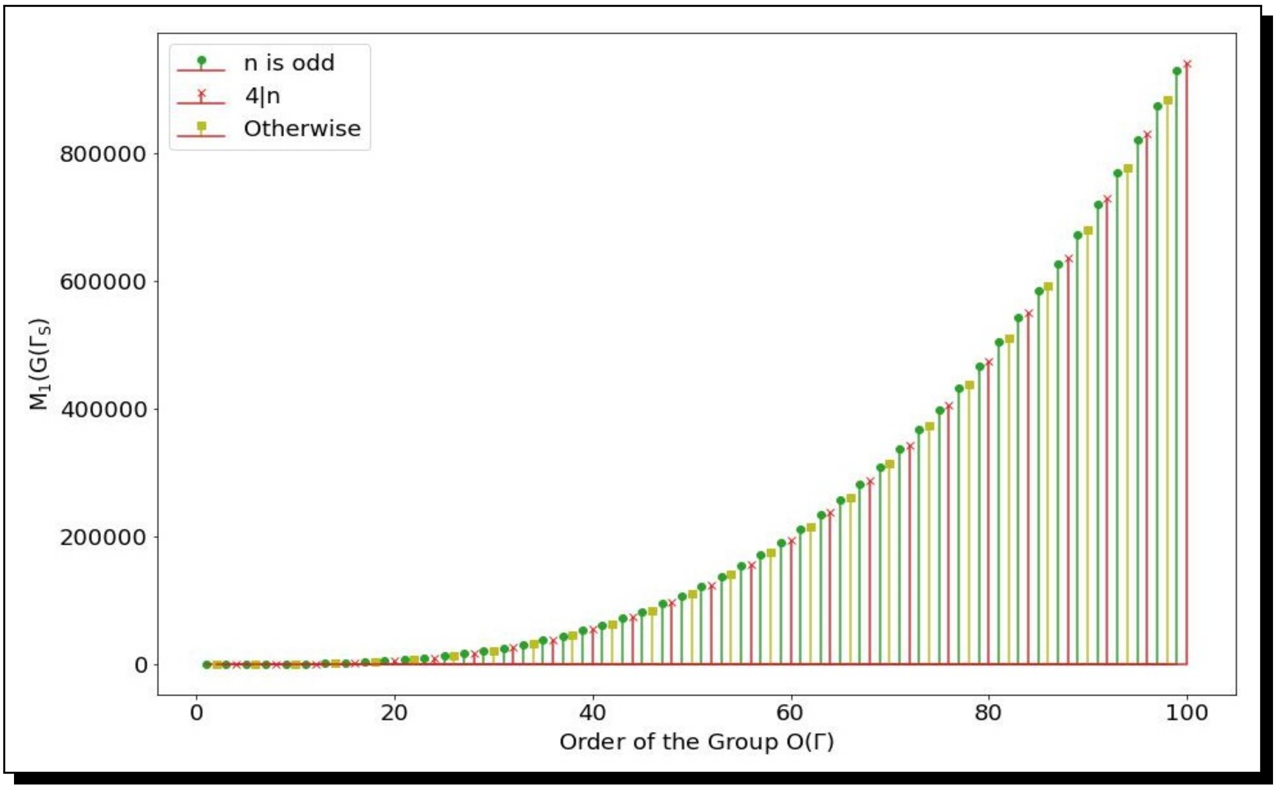

Figure 2. First Zagreb index of $G(\Gamma_S)$

Theorem 3.4. *Let Γ be cyclic group of order $n \geq 3$ then, Second Zagreb index*

$$M_2(G(\Gamma_S)) = \begin{cases} \frac{(n-1)(n-2)(n^2-3n+4)}{2}, & \textit{if } n \textit{ is odd,} \\ 4(n-2)^2 + \frac{(n-3)^2(n-4)(n-7)}{2}, & \textit{if } 4 \mid n, \\ \frac{(n-2)(n-3)(n^2-4n+7)}{2}, & \textit{otherwise.} \end{cases}$$

Proof. Let Γ be cyclic group of order n and S be the set of non self invertible elements. Then consider the partition of the edge set $G(\Gamma_S)$,

- $E_1 = \{(u,v) \in E(\Gamma_S) | (u \notin S (or) u^2 \notin S \text{ and } v \in S \text{ with } v^2 \in S) \text{ or } (v \notin S (or) v^2 \notin S \text{ and } u \in S \text{ with } u^2 \in S)\}$.
- $E_2 = \{(u,v) \in E(\Gamma_S) | u, u^2 \in S \ and \ v, V^2 \in S\}$.
- $E_3 = \{(u,v) \in E(\Gamma_S) | (u \notin S \text{ and } v \in S \text{ with } v^2 \notin S) \text{ or } (v \notin S \text{ and } u \in S \text{ with } u^2 \notin S)\}$.

Now,

$$|E_1| = \begin{cases} n-1, & \text{if } n \text{ is odd,} \\ 4(n-4), & \text{if } 4 \mid n, \\ 2(n-2), & \text{otherwise.} \end{cases}$$

$$|E_1| = \begin{cases} \frac{(n-2)(n-3)}{2}, & \text{if } n \text{ is odd,} \\ \frac{(n-4)(n-7)}{2}, & \text{if } 4 \mid n, \\ \frac{(n-2)(n-5)}{2}, & \text{otherwise.} \end{cases}$$

$$|E_1| = \begin{cases} 0, & \text{if } n \text{ is odd,} \\ 4, & \text{if } 4 \mid n, \\ 0, & \text{otherwise.} \end{cases}$$

One can see that, $E_1 \cup E_2 \cup E_3 = E(\Gamma_S)$. By the degree of the vertex in Γ_S gives the following cases:

Case i (if n is odd): If $(u,v) \in E_1$ then degrees of end points are $n-1$ and $n-2$. If $(u,v) \in E_2$ then degrees of both end is $n-2$. For this case no such element in E_3. Hence,

$$\begin{aligned} M_2(G(\Gamma_S)) &= (n-1)(n-1)(n-2) + \frac{(n-1)(n-3)}{2}(n-2)^2 + 0 \\ &= \frac{(n-1)(n-2)(n^2-3n+4)}{2}. \end{aligned}$$

Case ii (if $4|n$): If $(u,v) \in E_1$ then degrees of end points are $n-2$ and $n-3$. If $(u,v) \in E_2$ then degrees of both end is $n-3$. If $(u,v) \in E_3$ then degrees of both end is $n-2$. Hence,

$$\begin{aligned} M_2(G(\Gamma_S)) &= 4(n-4)(n-2)(n-3) + \frac{(n-4)(n-7)}{2}(n-3)^2 + 4(n-2)^2 \\ &= 4(n-2)^2 + \frac{(n-3)^2(n-4)(n-7)}{2}. \end{aligned}$$

Case iii (if $4 \nmid n$): If $(u,v) \in E_1$ then degrees of end points are $n-2$ and $n-3$. If $(u,v) \in E_2$ then degrees of both end is $n-3$. For this case no such element in E_3. Hence,

$$\begin{aligned} M_2(G(\Gamma_S)) &= 2(n-2)(n-2)(n-3) + \frac{(n-2)(n-5)}{2}(n-3)^2 + 0 \\ &= \frac{(n-2)(n-3)(n^2-4n+7)}{2}. \end{aligned}$$

All the above cases proves this theorem. □

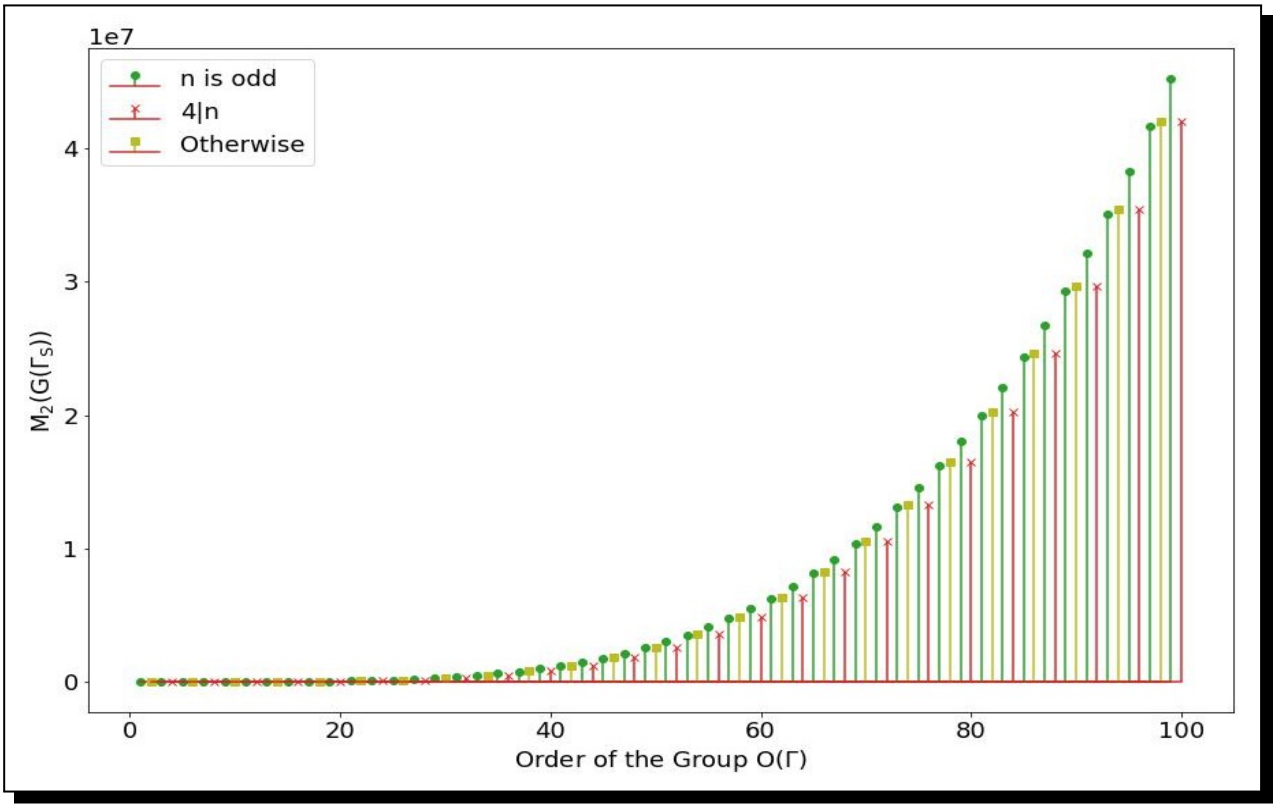

Figure 3. Second Zagreb index of $G(\Gamma_S)$

Consider the following observations it will helpful to prove the remaining theorems.

- $d(\Gamma_S,1) = \begin{cases} \frac{(n-1)^2}{2}, & \text{if } n \text{ is odd}, \\ \frac{n^2-3n+4}{2}, & \text{if } 4 \mid n, \\ \frac{(n-1)(n-2)}{2}, & \text{otherwise}. \end{cases}$
- $d(\Gamma_S,2) = \begin{cases} \frac{(n-1)}{2}, & \text{if } n \text{ is odd}, \\ n-1, & \text{if } 4 \mid n, \\ n-2, & \text{otherwise}. \end{cases}$

Theorem 3.5. *Let Γ be cyclic group of order $n \geq 3$ then, Hyper-Wiener index*

$$WW(G(\Gamma_S)) = \begin{cases} \frac{(n-1)(n+2)}{2}, & \text{if } n \text{ is odd,} \\ \frac{n^2+3n-8}{2}, & \text{if } 4 \mid n, \\ \frac{7n-8}{2}(n-2), & \text{otherwise.} \end{cases}$$

Proof. Let Γ be cyclic group of order n and S be the set of non self invertible elements. We know that $W(G(\Gamma_S);x) = \sum_{k=1}^{\delta(G)} d(G,k)x^k$ here k is 0 or 1. Since diameter of $G(\Gamma_S)$ is 2.

$$W(G(\Gamma_S);x) = 2d(G(\Gamma_S),1) + 6d(G(\Gamma)_S,2).$$

Now, by the above observation the proof is trivial. □

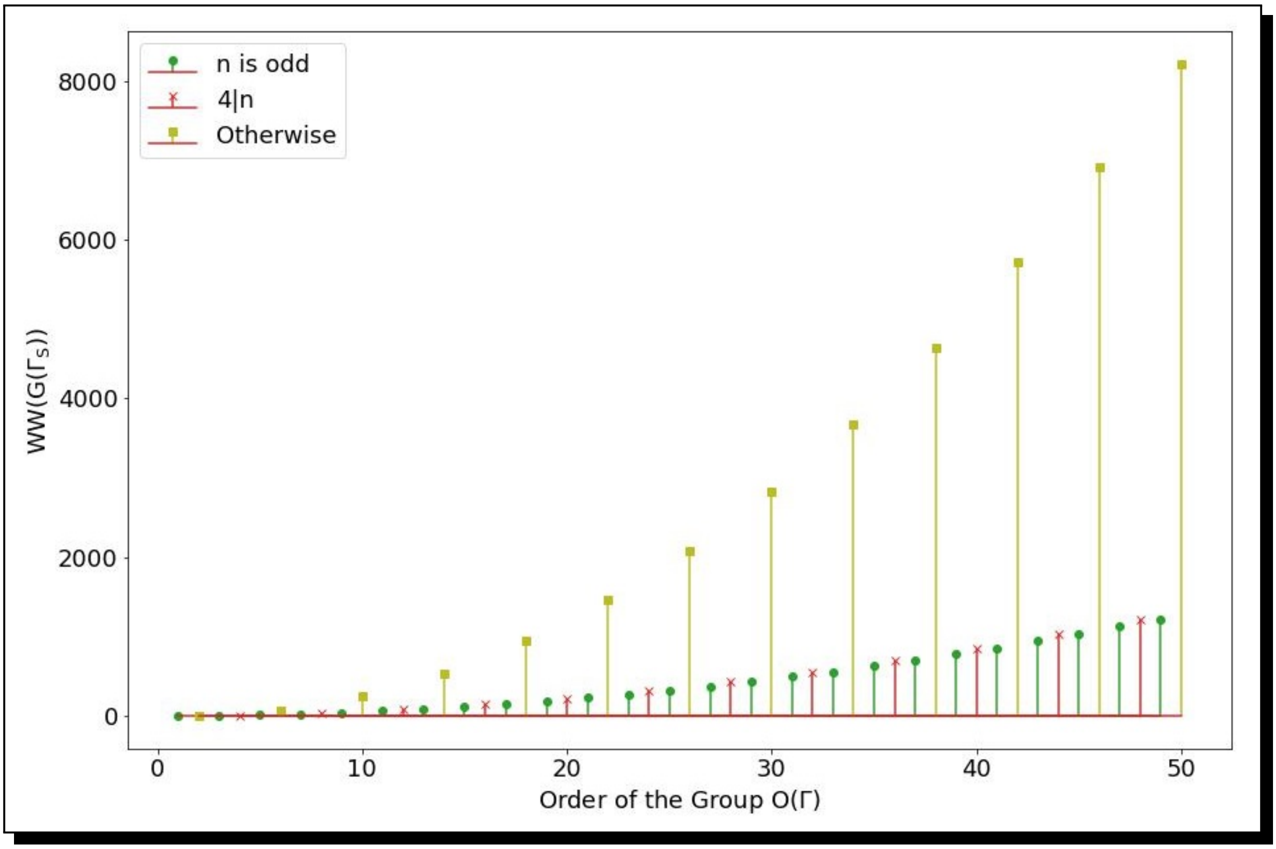

Figure 4. Hyper-Wiener index of $G(\Gamma_S)$

Theorem 3.6. *Let Γ be cyclic group of order $n \geq 3$ then, Eccentric-connectivity index $\xi(G(\Gamma_S)) = \sum_{v \in V} d(v)ecc(v)$*

$$\xi(G(\Gamma_S)) = \begin{cases} (n-1)(2n-3), & \text{if } n \text{ is odd,} \\ 2(n^2-3n+4), & \text{if } 4 \mid n, \\ 2(n-1)(n-2), & \text{otherwise.} \end{cases}$$

Proof. Let Γ be cyclic group of order n and S be the set of non self invertible elements. Now consider the following cases with the consideration of degree of the vertex in $G(\Gamma_S)$, *Case* i (if n is odd): For this case S contain the single element.

$$\begin{aligned}\xi(G(\Gamma_S)) &= (n-1)+(n-1)(n-2)\\ &= (n-1)(2n-3).\end{aligned}$$

Case ii (if n is even and $4 \mid n$): For this case S contains four elements, there are Identity, $\frac{n}{2}$, $\frac{n}{4}$ and $(\frac{n}{4})^{-1}$. The mention above elements have degree $n-2$ in $G(\Gamma_S)$ and all other elements have degree $n-3$. Therefor,

$$\begin{aligned}\xi(G(\Gamma_S)) &= 4(n-2)x2+(n-4)(n-3)x2\\ &= 2(n^2-3n+4).\end{aligned}$$

Case iii (if n is even and $4 \nmid n$): For this case S contains two elements, there are Identity and $\frac{n}{2}$. The mention above elements have degree $n-2$ in $G(\Gamma_S)$ and all other elements have degree $n-3$. Therefor,

$$\begin{aligned}\xi(G(\Gamma_S)) &= 2(n-2)x2+(n-2)(n-3)x2\\ &= 2(n-1)(n-2).\end{aligned}$$

□

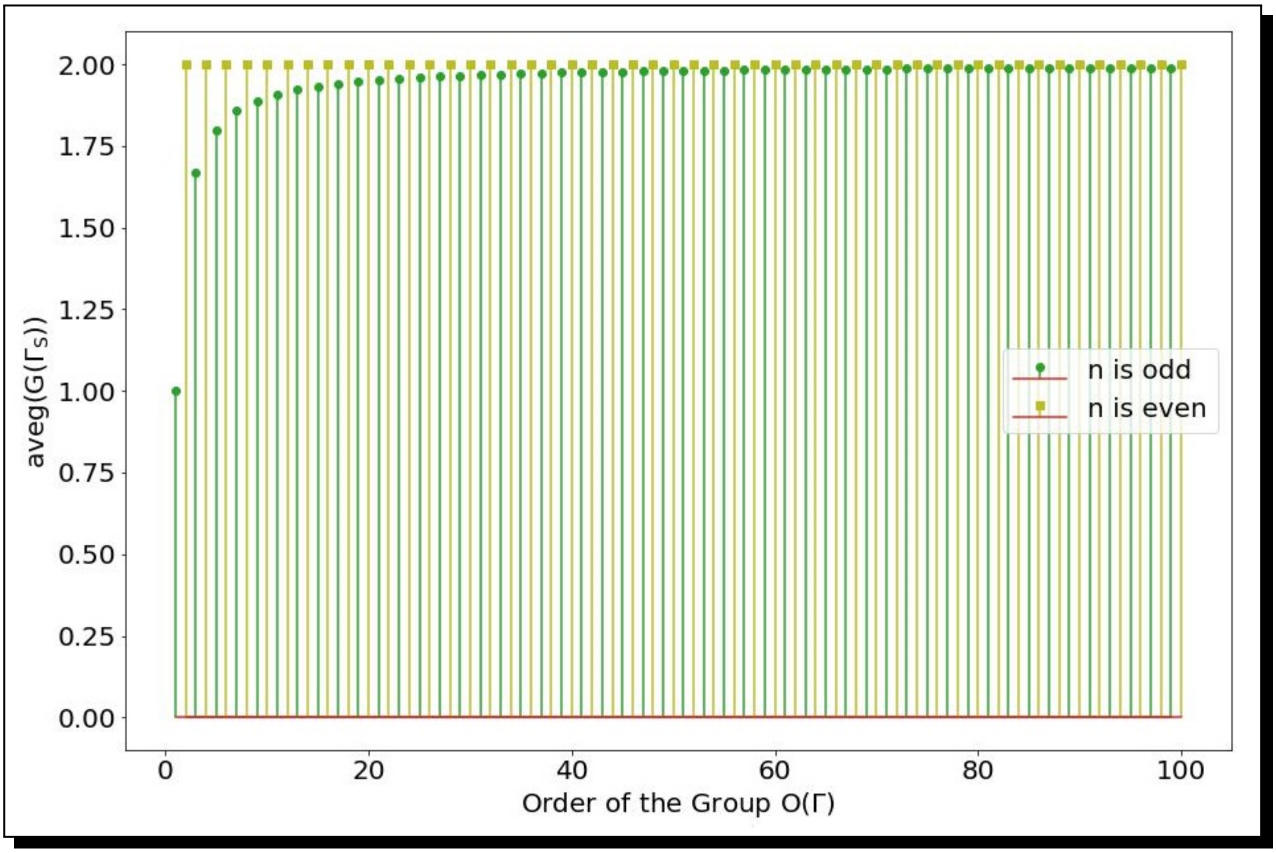

Figure 5. Average eccentricity index of $G(\Gamma_S)$

One can seen that the following theorems is true since, $ecc(0)$ is 1 if n is odd otherwise all the vertices in $G(\Gamma_S)$ has eccentricity 2.

Theorem 3.7. *Let* Γ *be cyclic group of order* $n \geq 3$ *then, Total eccentricity index*

$$\zeta(G(\Gamma_S)) = \begin{cases} 2n-1, & \text{if } n \text{ is odd},\\ 2n, & \text{if } n \text{ is even}. \end{cases}$$

Theorem 3.8. *Let* Γ *be cyclic group of order* $n \geq 3$ *then, Average eccentricity index*

$$aveg(G(\Gamma_S)) = \begin{cases} 2-\frac{1}{n}, & \text{if } n \text{ is odd},\\ 2, & \text{if } n \text{ is even}. \end{cases}$$

Now, consider the edge partition of $G(\Gamma_S)$ define in 3.4 gives the following theorem:

Theorem 3.9. *Let* Γ *be cyclic group of order* $n \geq 3$ *then, ABC index*

$$ABC(G(\Gamma_S)) = \begin{cases} (n-1)\sqrt{\frac{2n-5}{(n-1)(n-3)}}, & \text{if } n \text{ is odd},\\ 4\Big((n-1)\sqrt{\frac{2n-5}{(n-1)(n-3)}}\sqrt{\frac{2(n-3)}{(n-2)^2}}\Big)+\frac{(n-4)(n-7)}{2}\sqrt{\frac{2(n-4)}{(n-3)^2}}, & \text{if } 4 \mid n,\\ 2(n-2)\Big(\sqrt{\frac{2n-5}{(n-1)(n-3)}}+\frac{(n-5)}{4}\sqrt{\frac{2(n-4)}{(n-3)^2}}\Big), & \text{otherwise}. \end{cases}$$

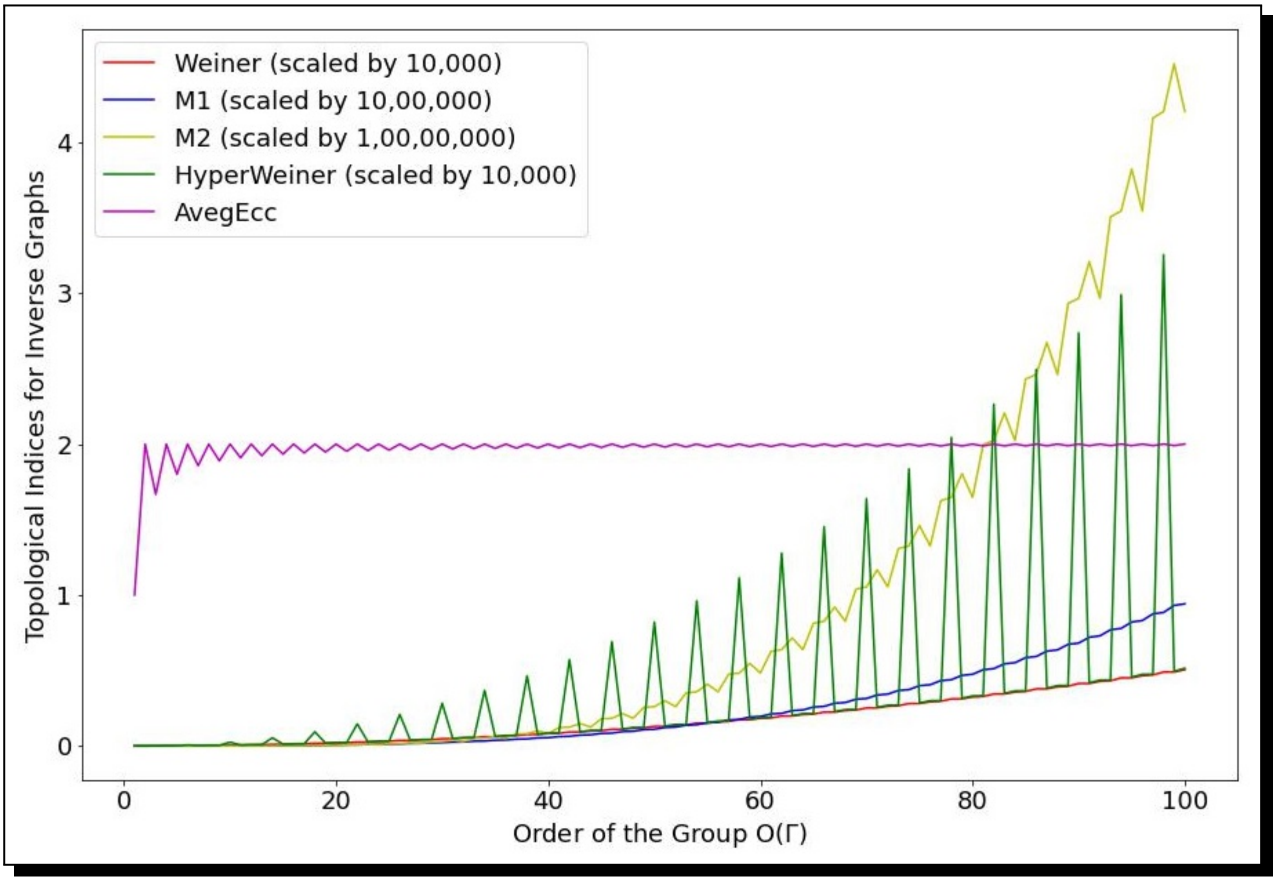

Figure 6. Topological index of $G(\Gamma_S)$

4. Conclusion

In this article we found the different kind of Topological index of inverse graph of finite cyclic group. We can give the comparison of various Topological index in following figure. Future study of this perspective is finding the degree based various Topological indices which is an interesting thing.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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