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Research Article

FRW Viscous Cosmology for Varying *G*, *c* and Λ, with Inhomogeneous Equation of State

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Abstract. In this paper, the Freidmann equations described by inhomogeneous *equation of state* (EOS) of the form $p = (\gamma - 1)\rho c^2 + \Lambda(t)$, for a bulk viscosity media, are considered. The time dependent parameter Λ is considered as a linear combination of the terms as $\Lambda(t) = \Lambda_0 + \Lambda_1 \frac{\dot{R}}{R}$ and the bulk viscosity as $\zeta = \zeta_0 H^{-1} \rho$, where R is a scale factor and Λ_0, Λ_1 and ζ_0, are all constants. The resultant equation to the scale factor is solved to get the exact solution by choosing some special parameters, in the framework of variable speed of light (VSL) theory.

Keywords. VSL theory, Viscous fluid, Dark energy

Mathematics Subject Classification (2020). 83A05, 83C05,83C15, 83C56

1. Introduction

The evolution of the universe in the framework of VSL theory has been proposed by [1] in which it is postulated that light traveled faster in an early universe and a phase transition at a time t_c led to the change of speed of light c^- to c^+. This idea of the varying speed of light seriously has a physical effect that might have happened in the very early universe. The varying speed of light theory [1, 3, 6, 18] had been introduced and discussed as an alternative to the inflationary model of the universe. It was discussed by Moffat [14], Albrecht and Magueijo [1], and Barrow [3], that in the very early universe, a finite period of time during which the speed of light falls at an appropriate rate, can lead to a solution of the flatness, horizon, which can also provide a solution for the cosmological constant problem. Albercht and Magueijo [1], and Barrow [3] discussed the models in the framework of VSL theory, considering G and Λ varying with respect to time, in the conservation equations. Barrow [3] solved the Einstein's field equations for FRW spacetime in VSL theory by considering, $c(t) = c_0 R^n$ and $G(t) = G_0 R^n$.

The cosmological observations have introduced the evidences that our universe is undergoing a late time cosmic acceleration [2, 17]. In the last two decades, a lot of research work on the extended gravity [11], modified equation of state or a new field with dark energy, have been carried out.

The bulk viscosity in cosmology, in various aspects, is studied by some authors [7, 8, 12, 16, 19, 20]. The presence of dissipative processes are thought in any realistic theory of the evolution of the universe. The Friedmann equations with a more general EOS as well as bulk viscosity, were solved by Meng *et al*. [13] and discussed the acceleration expansion of the universe evolution and future singularity in general theory of relativity. In [10] Khadekar and Ghogre have solved the Friedmann equation with inhomogeneous EOS by considering the bulk viscosity as $\zeta = \zeta_0 + \zeta_1 \frac{\dot{R}}{R} + \zeta_2 \frac{\ddot{R}}{R}$ and obtained the exact solution to the scale factor and discussed the acceleration expansion of the universe evolution for a special choice of parameters.

Inspired by all above mentioned work, in this paper, we have extended our previous work Khadekar and Ghogre [10], to solve the Friedmann equations with inhomogeneous EOS by assuming bulk viscosity ζ and cosmological constant Λ as time dependent and discussed the acceleration expansion of the universe evolution and the future singularities in the framework of VSL theory. For this, we considered a non perfect fluid with bulk viscosity as $\zeta = \zeta_0 H^{-1}\rho$, and cosmological constant Λ as $\Lambda(t) = \Lambda_0 + \Lambda_1 \frac{\dot{R}}{R}$, where R is a scale factor and Λ_0, Λ_1 and ζ_0, are all constants.

The paper is organized as follows: The model is described by using inhomogeneous EOS in Section 2. The field equations are solved by following the procedure given by Barrow [3] with the assumption that $G/c^2 = \textit{constant}$ and c is proportional to Hubble parameter: $c \propto H^{1/2}$ and obtained the exact solution. In Section 3, we considered some special cases of the solution for $\gamma \neq 0$ and $\gamma = 0$.

2. The Model and the Field Equations

We consider the FRW metric of the form

$$ds^2 = -c^2 dt^2 + R^2(t)(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2). \tag{2.1}$$

The field equations are in the form

$$R_{ij} - \frac{1}{2} g_{ij} R = \frac{-8\pi G(t)}{c^4(t)} T_{ij}. \tag{2.2}$$

With bulk viscosity in the FRW cosmology, the stress energy momentum tensor can be written as

$$T_{ij} = (\rho\, c^2 + \bar{p}) u_i u_j + \bar{p} g_{ij}, \tag{2.3}$$

where $\bar{p}$ is an equivalent pressure which is given by

$$\bar{p} = p - \zeta\theta. \tag{2.4}$$

Here ζ is the bulk viscosity, θ is the expansion factor defined by $\theta = 3\dot{R}/R$ with u_i as the four velocity, p is pressure and ρ is density.
We assume that the bulk viscosity ζ as

$$\zeta = \zeta_0 H^{-1}\rho, \tag{2.5}$$

where ζ_0 is a constant.
The Einstein field equations (2.2) for the FRW model (2.1) are expressed as:

$$2\dot{H} + 3H^2 = \frac{-8\pi G\bar{p}}{c^2}, \tag{2.6}$$

$$H^2 = \frac{8\pi G\rho}{3}, \tag{2.7}$$

where dot ($\dot{}$) stands for differentiation with respect to time.
The conservation equation is given by

$$\dot{\rho} + 3\left(\rho + \frac{\bar{p}}{c^2}\right)H = -\rho\left(\frac{\dot{G}}{G} - \frac{4\dot{c}}{c}\right). \tag{2.8}$$

The conservation equation for energy $T^{o}_{\nu;0} = 0$ yields

$$\dot{\rho} + 3\left(\rho + \frac{\bar{p}}{c^2}\right)H = 0, \tag{2.9}$$

$$\frac{\dot{G}}{G} - \frac{4\dot{c}}{c} = 0. \tag{2.10}$$

The inhomogeneous EOS is considered of the form

$$p = (\gamma - 1)\rho c^2 + \Lambda(t), \tag{2.11}$$

where $\Lambda(t)$, the time dependent parameter is assumed as a linear combination of a constant term and the term proportional to the Hubble parameter $H = \dot{R}/R$ in the following form as in [10]

$$\Lambda(t) = \Lambda_0 + \Lambda_1\frac{\dot{R}}{R}, \tag{2.12}$$

where Λ_0, Λ_1 are constants.
Using p, $\Lambda(t)$ and ζ from eqs. (2.11), (2.12) and (2.5) respectively, in the eq. (2.4), we get

$$\bar{p} = (\gamma - 1)\rho c^2 + \Lambda_0 + \Lambda_1\frac{\dot{R}}{R} - 3\zeta_0\rho. \tag{2.13}$$

From Einstein Field equations (2.6) and (2.7), we get

$$\frac{\ddot{R}}{R} = \frac{-4\pi G}{3}\left(\rho + \frac{3\bar{p}}{c^2}\right). \tag{2.14}$$

Substituting the value of $\bar{p}$ from eq. (2.13) in the above equation, we get

$$\frac{\ddot{R}}{R} = \frac{-4\pi G}{3}\left[\left(3\gamma - 2 - \frac{9\zeta_0}{c^2}\right)\rho + \frac{3}{c^2}\left(\Lambda_0 + \Lambda_1\frac{\dot{R}}{R}\right)\right]. \tag{2.15}$$

Substituting for ρ from eq. (2.7), we get

$$\frac{\ddot{R}}{R} = -\frac{1}{2}\left(3\gamma - 2 - \frac{9\zeta_0}{c^2}\right)\frac{\dot{R}^2}{R^2} - \frac{4\pi G\Lambda_1}{c^2}\frac{\dot{R}}{R} - \frac{4\pi G\Lambda_0}{c^2}. \tag{2.16}$$

For solving eq. (2.16), we use the procedure given by Barrow [3] with the assumption that $G/c^2 = constant$. In spite of the fact that both constants G and c vary, Belinchón [5] obtained the integration condition as the relationship $G/c^2 = constant$. Also, Belinchón and Chakrabarty [4] used such a relationship as an assumption.

We consider

$$\frac{G}{c^2} = G_0 = constant. \tag{2.17}$$

Barrow [3] assumed that the rate of variation of c is proportional to the expansion rate of the universe, i.e.,

$$c^2 = c_0 H, \tag{2.18}$$

where c_0 is constant.

Using eq. (2.17) in eq. (2.16) we get

$$\frac{\ddot{R}}{R} = -\frac{1}{2}\left(3\gamma - 2 - \frac{9\zeta_0}{c^2}\right)\frac{\dot{R}^2}{R^2} - 4\pi G_0\Lambda_1\frac{\dot{R}}{R} - 4\pi G_0\Lambda_0. \tag{2.19}$$

Using eq. (2.18) and $H = \frac{\dot{R}}{R}$ and then simplifying, we get

$$\frac{\ddot{R}}{R} = -\frac{1}{2}(3\gamma - 2)\frac{\dot{R}^2}{R^2} + \left(\frac{9\zeta_0}{2c_0} - 4\pi G_0\Lambda_1\right)\frac{\dot{R}}{R} - 4\pi G_0\Lambda_0. \tag{2.20}$$

We write eq. (2.20) as

$$\frac{\ddot{R}}{R} = -\frac{(3\gamma - 2)}{2}\frac{\dot{R}^2}{R^2} + \frac{1}{T_1}\frac{\dot{R}}{R} + \frac{1}{T_2^2}, \tag{2.21}$$

where

$$\frac{9\zeta_0 - 8\pi c_0 G_0\Lambda_1}{2c_0} = \frac{1}{T_1}, \tag{2.22}$$

$$4\pi G_0\Lambda_0 = \frac{1}{T_2^2} \tag{2.23}$$

and noting that the dimensions of the two terms $\frac{9\zeta_0 - 8\pi c_0 G_0\Lambda_1}{2c_0}$ and $4\pi G_0\Lambda_0$ are $[time]^{-1}$ and $[time]^{-2}$, respectively. Eq. (2.20) having constants γ, ζ_0, Λ_0 and Λ_1 is condensed to eq. (2.21) having three parameters γ, T_1, and T_2. It is observed that when $T_1 \gg t$, (cosmic time scale), then $\frac{9\zeta_0}{8\pi c_0 G_0} = \Lambda_1$, and if $T_2 \gg t$, the effect of Λ_0 can be neglected.

To solve the differential eq. (2.21), we write $\frac{\dot{R}}{R} = x$, and then simplifying, we get

$$\frac{dx}{dt} = -\frac{3\gamma}{2}x^2 + \frac{1}{T_1}x + \frac{1}{T_2^2}. \tag{2.24}$$

After solving, eq. (2.24) yields

$$T\log\left[\frac{3\gamma x - \frac{1}{T_1} - \frac{1}{T}}{3\gamma x - \frac{1}{T_1} + \frac{1}{T}}\right] = -t + \log\alpha_1, \tag{2.25}$$

where

$$T = \frac{T_1}{\sqrt{1+6\gamma(T_1/T_2)^2}}, \tag{2.26}$$

and α_1 is constant of integration.

It can be observed from eq. (2.26) that when $T_2 \to \infty$, $T = T_1$ and when $T_1 \to \infty$, $T = T_2/\sqrt{6\gamma}$.

For finding the value of α_1, we use $3x = \theta$, in eq. (2.25). Hence, we get

$$\log\alpha_1 = e^{t/T}\left[\frac{\theta\gamma - \frac{1}{T_1} - \frac{1}{T}}{\theta\gamma - \frac{1}{T_1} + \frac{1}{T}}\right]. \tag{2.27}$$

Using the initial condition as $\theta(t_0) = \theta_0$, if $\gamma \neq 0$, in eq. (2.27), we get

$$\log\alpha_1 = \left[\frac{\theta_0\gamma - \frac{1}{T_1} - \frac{1}{T}}{\theta_0\gamma - \frac{1}{T_1} + \frac{1}{T}}\right] e^{(t_0/T)}. \tag{2.28}$$

Using this value of integrating constant α_1, in the eq. (2.25), we get

$$\left[\frac{3\gamma x - \frac{1}{T_1} - \frac{1}{T}}{3\gamma x - \frac{1}{T_1} + \frac{1}{T}}\right] = e^{\left(\frac{-t-t_0}{T}\right)}A_0, \tag{2.29}$$

where $A_0 = \left[\frac{\theta_0\gamma - \frac{1}{T_1} - \frac{1}{T}}{\theta_0\gamma - \frac{1}{T_1} + \frac{1}{T}}\right]$.

Simplifying eq. (2.29), we get

$$3\gamma x = \frac{(T+T_1)e^{\left(\frac{t-t_0}{T}\right)} - (T-T_1)A_0}{TT_1\left(e^{\left(\frac{t-t_0}{T}\right)} - A_0\right)}. \tag{2.30}$$

Replacing x by $\frac{\dot{R}}{R}$ in the above equation, we get

$$\frac{\dot{R}}{R} = \frac{1}{3\gamma}X_1, \tag{2.31}$$

where

$$X_1 = \left[\frac{(1+\gamma\theta_0 T - T/T_1)(1/T + 1/T_1)\exp[(t-t_0)/T] - (1-\gamma\theta_0 T + T/T_1)(1/T - 1/T_1)}{(1+\gamma\theta_0 T - T/T_1)\exp[(t-t_0)/T] + (1-\gamma\theta_0 T + T/T_1)}\right].$$

The above equation can be written in the simple form as,

$$\frac{\dot{R}}{R} = \frac{1}{3\gamma}\left(\frac{1}{T_1} + \frac{1}{T}\right)\left(1 - \frac{(1+d_0)B_0 e^{\frac{-(t-t_0)}{T}}}{1 + B_0 e^{\frac{-(t-t_0)}{T}}}\right), \tag{2.32}$$

where $B_0 = \left(\frac{1-\gamma\theta_0 T + \frac{T}{T_1}}{1+\gamma\theta_0 T - \frac{T}{T_1}}\right)$, $d_0 = \left(\frac{\frac{1}{T} - \frac{1}{T_1}}{\frac{1}{T} + \frac{1}{T_1}}\right)$.

Integrating on both sides of eq. (2.32), the scale factor R can be obtained as

$$R = \alpha_2\left(1 + B_0 e^{-\left(\frac{t-t_0}{T}\right)}\right)^{2/3\bar{\gamma}} e^{\frac{t}{3\gamma}\left(\frac{1}{T} + \frac{1}{T_1}\right)}, \tag{2.33}$$

where α_2 is constant of integration.

The value of integrating constant α_2 can be obtained by using the initial conditions $R(t_0) = R_0$ and $\theta(t_0) = \theta_0$, if $\gamma \neq 0$, and the solution can be obtained as

$$R(t) = R_0(X_2)^{2/3\gamma}, \tag{2.34}$$

where $X_2 = \left[\frac{1}{2}\left(1+\gamma\theta_0 T - \frac{T}{T_1}\right)e^{\left(\frac{t-t_0}{2}\right)\left(\frac{1}{T}+\frac{1}{T_1}\right)} + \frac{1}{2}\left(1-\gamma\theta_0 T + \frac{T}{T_1}\right)e^{-\left(\frac{t-t_0}{2}\right)\left(\frac{1}{T}-\frac{1}{T_1}\right)}\right]$.

3. Some Special Cases

Using the resultant eq. (2.34), we obtain the expressions for $\gamma \neq 0$ and $\gamma = 0$ in some special cases: (i) $T_2 \to \infty$, (ii) $T_1 \to \infty$.

3.1 For $\gamma \neq 0$

In eq. (2.34), applying the condition $T_2 \to \infty$, i.e. $T = T_1$, we get

$$R(t) = R_0\left[1+\frac{1}{2}\gamma\,\theta_0\,T_1\left(\exp\left[\frac{t-t_0}{T_1}\right]-1\right)\right]^{2/3\gamma}. \tag{3.1}$$

Also, using $T_1 \to \infty$ i.e. $T = T_2/\sqrt{6\gamma}$, in eq. (2.34), we get

$$R(t) = R_0\left[\cosh\left(\frac{\sqrt{6\gamma}(t-t_0)}{2T_2}\right) + \frac{\sqrt{\gamma}\theta_0 T_2}{2\sqrt{6}}\sinh\left(\frac{\sqrt{6\gamma}(t-t_0)}{2T_2}\right)\right]^{2/3\bar{\gamma}}. \tag{3.2}$$

For $T \to \infty$, the above two cases become

$$R(t) = R_0\left[1+\frac{1}{2}\gamma\,\theta_0(t-t_0)\right]^{2/3\gamma}, \tag{3.3}$$

which corresponds to the case of $p = (\gamma-1)\rho$ and $\zeta \propto \theta$ obtained earlier by [9] in framework of general theory of relativity.

3.2 For $\gamma = 0$

By substituting $\gamma = 0$ in eq. (2.21), we get

$$\frac{\ddot{R}}{R} = \frac{\dot{R}^2}{R^2} + \frac{1}{T_1}\frac{\dot{R}}{R} + \frac{1}{T_2^2}. \tag{3.4}$$

For solving this equation, we again consider $\frac{\dot{R}}{R} = x$, then the above equation reduces to

$$\frac{dx}{dt} - \frac{x}{T_1} = \frac{1}{T_2^2}. \tag{3.5}$$

Solving the above equation and again replacing x by $\frac{\dot{R}}{R}$, we get

$$\frac{\dot{R}}{R} = \frac{-T_1}{T_2^2} + \alpha_3 e^{(t/T_1)}, \tag{3.6}$$

where α_3 is constant of integration.
Using the initial conditions $R(t_0) = R_0$ and $\theta(t_0) = \theta_0$, we get the solution as

$$R(t) = R_0\exp\left[\left(\frac{\theta_0 T_1}{3} + \frac{T_1^2}{T_2^2}\right)\left(e^{(t-t_0)/T_1}-1\right) - \frac{T_1(t-t_0)}{T_2^2}\right]. \tag{3.7}$$

For $T_2 \to \infty$, the above equation becomes

$$R(t) = R_0\exp\left[\frac{\theta_0 T_1}{3}\left(e^{(t-t_0)/T_1}-1\right)\right], \tag{3.8}$$

and for $T_1 \to \infty$, it becomes

$$R(t) = R_0 \exp\left[\frac{\theta_0}{3}(t-t_0) + \frac{(t-t_0)^2}{2T_2^2}\right]. \tag{3.9}$$

4. Discussion and Remarks

The Friedmann equation with the inhomogeneous equation of state is solved by considering bulk viscosity $\zeta = \zeta_0 H^{-1}\rho$ and cosmological constant Λ, both as time dependent and discussed the accelerated expansion of the universe evolution for the model, in the framework of VSL theory. The dynamical eq. (2.21) for the scale factor is obtained and by using the suitable transformation, it is shown that the dynamical equation of scale factor reduces to the linear differential equation. In this equation, the term $\frac{1}{T_1}\frac{\dot{R}}{R}$ describes the effective viscosity. The scale factor is obtained from this equation as eq. (2.34).

After discussing the special cases $\gamma \neq 0$ and $\gamma = 0$, we have obtained the scale factor when $T_1 \to \infty$ and $T_2 \to \infty$.

We know that, mathematically, if the universe is accelerating then

$$\frac{\ddot{R}}{R} > 0. \tag{4.1}$$

it is observed from eq. (2.19) that the bulk viscosity ζ_0 and the negative cosmological parameters Λ_0, Λ_1, can cause the universe to accelerate. Using eq. (2.19), we discuss a special case, with $\Lambda_0 = 0$.

Now, $\Lambda_0 = 0 \Rightarrow T_2 \to \infty$, then eq. (2.19) reduces as follows:

$$\frac{\ddot{R}}{R} = -\left(\frac{3\gamma-2}{2}\right)\frac{\dot{R}^2}{R^2} - \left[4\pi G_0(\Lambda_1 - \frac{9\zeta_0}{2c_0}\right]\frac{\dot{R}}{R}. \tag{4.2}$$

Hence

$$\frac{\ddot{R}}{R} > 0 \Rightarrow -\left(\frac{3\gamma-2}{2}\right)\frac{\dot{R}^2}{R^2} + \frac{1}{T_1}\frac{\dot{R}}{R} > 0 \tag{4.3}$$

i.e.

$$\frac{\ddot{R}}{R} > 0 \Rightarrow \frac{2}{3} + \frac{2}{3T_1}\frac{R}{\dot{R}} > \gamma \tag{4.4}$$

But when $T_2 \to \infty$, $\frac{R}{\dot{R}} = \frac{3}{\theta_0 \exp\left(\frac{t-t_0}{T}\right)}$ and also $T_1 = T$

$$\Rightarrow \quad \gamma < \frac{2}{3}\exp\left(\frac{t-t_0}{T}\right) + \frac{2}{T\theta_0} \tag{4.5}$$

Hence, using eq. (2.11) we can write

$$\frac{p-\Lambda(t)}{\rho c^2} = \gamma - 1 < \frac{2}{3}\exp\left(\frac{t-t_0}{T}\right) + \frac{2}{T\theta_0} - 1, \tag{4.6}$$

for the accelerating universe.

In eq. (2.21), the term $\frac{1}{T_1}\frac{\dot{R}}{R}$ describes the effective viscosity. In this equation, if the first term is dominant then $\frac{\ddot{R}}{R} = -\frac{(3\gamma-2)}{2}\frac{\dot{R}^2}{R^2} \Rightarrow R\frac{dH}{dR} = -\frac{3\gamma}{2}H$. If the second term is dominant then $\frac{\ddot{R}}{R} = \frac{1}{T_1}\frac{\dot{R}}{R} \Rightarrow R\frac{dH}{dR} = -H + \frac{1}{T_1}$ and the third term $\frac{1}{T_2^2}$ plays the role of effective cosmological constant.

By using inhomogeneous, Hubble parameter-dependent term, the effect of modification of general EOS of dark energy is considered. For realizing such transition in a more natural way, the inhomogeneous equation of state are helpful. By making the dark energy equation more general, it was demonstrated that, this extra freedom in inhomogeneous term, brings a number of possibilities to construct the late time universe by assuming that inhomogeneous terms in EOS are not restricted by energy conservation law [15].

Many interesting dynamical characteristics may be shared by the VSL cosmological model, other than those present in the more conventional models, which will be definitely useful for a more detailed study of the universe, in future.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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