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Research Article

Rayleigh Wave Propagation at Viscous Liquid/Micropolar Micro-stretch Elastic Solid

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Abstract. In this article, the governing equations of a homogeneous, isotropic micropolar micro-stretch elastic solid for xz-plane are considered and solved for surface wave propagation. Two types of frequency equations for Rayleigh waves are derived, in which one is along the free surface of micropolar micro-stretch elastic solid half space and another is at viscous liquid/micropolar micro-stretch solid interface. These are dispersive in nature. In the study of some particular cases, we observed that four types of Rayleigh waves are propagate, out of these, two waves are at free surface of generalized micropolar solid and micro-stretch solid and another two types of waves are at interface of viscous liquid/non-microstretch solid. In these four waves, three Rayleigh waves are dependent on solid density and one of them is non-dispersive in nature. Numerical example is considered for a particular solid and viscous liquid layer and the frequency curves are drawn and discussed with the help of MATLAB programme.

Keywords. Micropolar micro-stretch elasticity, Viscous liquid layer, Rayleigh waves, Frequency equation

Mathematics Subject Classification (2020). 74J15, 74A02, 74A10

1. Introduction

Rayleigh waves are surface type waves which are combination of two types of motions named as longitudinal and transverse motion and these waves are create an elliptic motion. In the year 1885, the Rayleigh waves are predicted by L. Rayleigh [13], so they are named as Rayleigh waves. These waves are very sensitive to surface defects, so they are very useful in the fields like earthquake engineering, geophysics, ocean beds, material sciences and telecommunication industrial etc.

The dispersive relation of Rayleigh waves and Love waves in an elastic solid half-space which is covered by single solid layer derived by Love [11]. Eringen [4] derived the constitutive relations and field equations for micropolar elastic media within the frame work of linear theory by introducing micropolar theory of elasticity. Eringen [3] developed his investigation on the theory of micropolar elasticity by including the axial stretch during the molecules rotation and also he extended his research on this theory by including stretch. The equations of motions, constitutive relations for thermo-micro stretch fluid are given by Eringen [5], also he derived the solution for acoustical waves in bubbly liquids. The microstretch continuum is a model for Bravais lattice with basis on the atomic level and two phase dipolar solids with core on the microscopic level.

The dispersive reflection problem at the interface of solid and viscous liquid by Miles [12]. The propagation of Rayleigh waves in isotropic solids such as isotropic micro stretch thermoelastic solids, micro stretch thermoelastic solids under inviscid fluids are studied by Kumar *et al*. [10], Sharma *et al*. [14] and cited therein. Kumar and Tomar [8] studied the reflection and transmission of elastic waves at viscous liquid. In recent, Srinivas and Somaiah [16] studied the fluid effect on radial vibrations.

In the present paper, we derive the dispersion relation of Rayleigh waves at interface of micropolar micro-stretch elastic solid and viscous liquid layer.

2. Basic Equations

The equations of motion and constitutive relations in a homogeneous, isotropic micropolar microstretch elastic solid in the absence of body forces, body couples and stretch forces are followed by Eringen [2], Sherief *et al*. [15], and Kumar and Kansal [9] as

$$(\lambda+2\mu+K)\nabla(\nabla.\vec{U})-(\mu+K)\nabla\times\nabla\times\vec{U}+K\nabla\times\vec{\phi}+\lambda_0\nabla\psi^*=\rho\ddot{\vec{U}}, \tag{2.1}$$

$$(\alpha+\beta+\gamma)\nabla(\nabla.\vec{\phi})-\gamma\nabla\times(\nabla\times\vec{\phi})+K\nabla\times\vec{U}-2K\vec{\phi}=\rho J\ddot{\vec{\phi}}, \tag{2.2}$$

$$\alpha_0\nabla^2\psi^*-\lambda_1\psi^*-\lambda_0\nabla.\vec{U}=\frac{\rho J_0}{2}\ddot{\psi}^*. \tag{2.3}$$

The constitutive relations are

$$T_{ij}=\lambda U_{r,r}\delta_{ij}+\mu(U_{i,j}+U_{j,i})+K(U_{j,i}-\epsilon_{ijr}\phi_r)+\lambda_0\delta_{ij}\psi^*, \tag{2.4}$$

$$m_{ij}=\alpha\phi_{r,r}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}+b_0\epsilon_{mji}\psi^*_{,m}, \tag{2.5}$$

$$\lambda_i^*=\alpha_0\psi^*_{,i}+b_0\epsilon_{ijm}\phi_{j,m} \tag{2.6}$$

where λ,μ are Lame's elastic constants, α,β,γ,K are micropolar constants, $\lambda_0,\lambda_1,\alpha_0,b_0$ are microstretch elastic constants, ρ is the density of the solid. $\vec{U}=(u,v,w)$ is the macro displacement vector, $\vec{\phi}=(\phi_1,\phi_2,\phi_3)$ is the micro rotation vector, ψ^* is the micro stretch scalar, J is the micro inertia, J_0 is the micro inertia of the micro elements, T_{ij}, m_{ij} are components of stress and couple stress tensor, respectively. λ_i^* is the micro stress tensor, $e_{ij} = \frac{1}{2}(U_{i,j} + U_{j,i})$ are components of infinitesimal strain, e_{kk} is the dilation, δ_{ij} is Kroneckor's delta. Superpose dot is the partial derivative with respect to time t and suffix comma is the partial derivative with respect to the coordinate vector.

3. Problem Formulation

Consider a homogeneous, isotropic micropolar micro-stretch elastic solid M_1 lying under a uniform homogeneous viscid liquid layer medium M_2 of thickness T with cartesian coordinate system $oxyz$ at any point on the plane horizontal surface and z-axis pointing vertically downward in to the half-space.

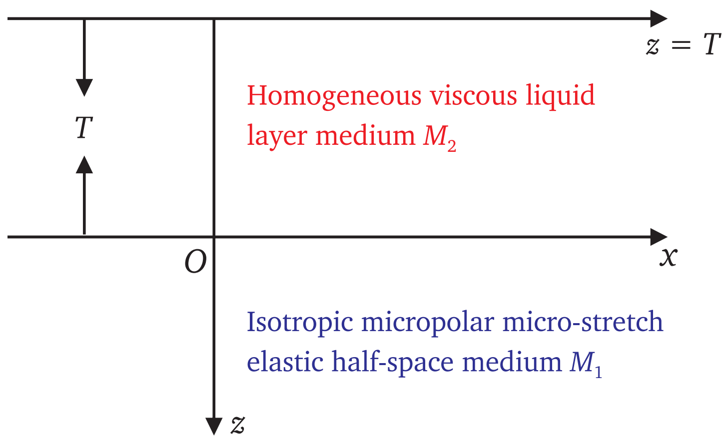

Figure 1. Geometry of the problem

Consider the direction of wave propagation along x-axis and amplitudes decaying in the direction of z-axis, so, all the particles vibrating on a line parallel to y-axis are equally displayed. With these assumptions all the field equations are independent of y-coordinates. For the two dimensional problem, we take macro displacement vector $\vec{U}$ components and micro-rotation vector $\vec{\phi}$ components are

$$\vec{U} = (u, o, w), \quad \vec{\phi} = (0, \phi, 0) \tag{3.1}$$

and stretch component $\psi^*(x,z,t)$ with u, w and ϕ are functions of x, z, t.
Using equation (3.1) in equations (2.1), (2.2) and (2.3), we obtain

$$(\mu + K)\nabla^2 u + (\lambda + \mu)(u_{,xx} + w_{,xz}) - K\phi_{,z} + \lambda_0 \psi^*_{,x} = \rho \ddot{u}, \tag{3.2}$$

$$(\mu + K)\nabla^2 w + (\lambda + \mu)(w_{,zz} + u_{,xz}) + K\phi_{,x} + \lambda_0 \psi^*_{,z} = \rho \ddot{w}, \tag{3.3}$$

$$K(u_{,z} - w_{,x}) - \gamma \nabla^2 \phi - 2K\phi = \rho J \ddot{\phi}, \tag{3.4}$$

$$\alpha_0 \nabla^2 \psi^* - \lambda_1 \psi^* - \lambda_0 (u_{,x} + w_{,z}) = \frac{\rho J_0}{2} \ddot{\psi}, \tag{3.5}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$.

The displacement components u and w can be expressed by Helmholtz potentials χ,ψ as

$$u=\frac{\partial\chi}{\partial x}-\frac{\partial\psi}{\partial z}\,,\quad w=\frac{\partial\chi}{\partial z}+\frac{\partial\psi}{\partial x} \tag{3.6}$$

on using equation (3.6) in equations (3.2) to (3.5), we get

$$(\lambda+2\mu+K)\nabla^2\chi+\lambda_0\psi^*=\rho\frac{\partial^2\chi}{\partial t^2}, \tag{3.7}$$

$$(\mu+K)\nabla^2\psi+K\phi=\rho\frac{\partial^2\psi}{\partial t^2}, \tag{3.8}$$

$$\left(\gamma\nabla^2+2K+\rho J\frac{\partial^2}{\partial t^2}\right)\phi+K\nabla^2\psi=0, \tag{3.9}$$

$$\left(\alpha_0\nabla^2-\lambda_1-\frac{\rho J_0}{2}\frac{\partial^2}{\partial t^2}\right)\psi^*-\lambda_0\nabla^2\chi=0. \tag{3.10}$$

Following Fehler [7], the equations of motion and stresses are given in xz-plane for viscous liquid medium as

$$K^L\nabla^2\phi^L+\frac{4}{3}\eta^L\frac{\partial}{\partial t}\nabla^2\phi^L=\rho\frac{\partial^2\phi^L}{\partial t^2}, \tag{3.11}$$

$$\eta^L\frac{\partial}{\partial t}\nabla^2\psi^L=\rho^L\frac{\partial^2\psi^L}{\partial t^2} \tag{3.12}$$

and

$$T^L_{zx}=\eta^L\frac{\partial}{\partial t}\left[2\frac{\partial^2\phi^L}{\partial x\partial z}+\frac{\partial^2\psi^L}{\partial x^2}-\frac{\partial^2\psi^L}{\partial z^2}\right], \tag{3.13}$$

$$T^L_{zz}=\left[K^L-\frac{2}{3}\eta^L\frac{\partial}{\partial t}\right]\left[\frac{\partial^2\phi^L}{\partial x^2}+\frac{\partial^2\phi^L}{\partial z^2}\right]+2\eta^L\frac{\partial}{\partial t}\left[\frac{\partial^2\psi^L}{\partial x\partial z}-\frac{\partial^2\phi^L}{\partial z^2}\right]. \tag{3.14}$$

The components of displacements are given by

$$u^L=\frac{\partial\phi^L}{\partial x}-\frac{\partial\psi^L}{\partial z}\,,\quad w^L=\frac{\partial\phi^L}{\partial z}-\frac{\partial\psi^L}{\partial x}, \tag{3.15}$$

where K^L is bulk modulus, ρ^L is fluid density, η^L is fluid viscosity and ϕ^L,ψ^L are potentials of longitudinal and transverse waves, respectively.

4. Solution of the Problem

Consider the surface wave solution along the direction of x-axis for the equations (3.7) to (3.10) of the form

$$(\chi,\psi,\psi^*,\phi)(x,z,t)=\left[\bar{\chi}(z),\bar{\psi}(z),\bar{\psi}^*(z),\bar{\phi}(z)\right]e^{i(\eta x-\omega t)}, \tag{4.1}$$

where the wave number η, angular frequency ω are connected by the wave velocity v as $v=\frac{\omega}{\eta}$ and $\bar{\chi},\bar{\psi},\bar{\psi}^*,\bar{\phi}$ are amplitudes of χ,ψ,ψ^*,ϕ, respectively. The Rayleigh surface waves are damped in time with wave velocity v taken as $Re(v)>0$ and $Im(v)\le 0$.

Inserting equation (4.1) in equations (3.7) to (3.10) we obtain the following system of four simultaneous second order differential equations

$$\left[C_1D^2+\rho\omega^2-C_1\eta^2\right]\bar{\chi}+\lambda_0\bar{\psi}^*=0, \tag{4.2}$$

$$\left[C_2D^2+\rho\omega^2-C_2\eta^2\right]\bar{\psi}+K\bar{\phi}=0, \tag{4.3}$$

$$K(D^2-\eta^2)\bar{\psi}+[\gamma D^2+2K-\rho J\omega^2-\gamma\eta^2]\bar{\phi}=0, \tag{4.4}$$

$$-\lambda_0(D^2-\eta^2)\bar{\chi}+\left[\alpha_0 D^2+\frac{\rho J_0}{2}\omega^2-\lambda_1-\alpha_0\eta^2\right]\bar{\psi}^*=0, \tag{4.5}$$

where $C_1=\lambda+2\mu+K$, $C_2=\mu+K$ and $D\equiv\frac{d}{dx}$.

From equation (4.1), we have the surface waves, so the quantity η must be real and positive. According to Suhubi and Eringen [6] theory, we consider the solution of simultaneous equations (4.2) to (4.5) in the form of surface wave solutions for half-space solid as

$$\bar{\chi}(z)=A_1e^{-r_1z}+A_2e^{-r_2z}, \tag{4.6}$$

$$\bar{\psi}(z)=A_3e^{-r_3z}+A_4e^{-r_4z}, \tag{4.7}$$

$$\bar{\psi}^*(z)=A_1\xi_1e^{-r_1z}+A_2\xi_2e^{-r_2z}, \tag{4.8}$$

$$\bar{\phi}(z)=A_3\xi_3e^{-r_3z}+A_4\xi_4e^{-r_4z}, \tag{4.9}$$

where

$$\left.\begin{aligned}
&r_1^2,r_2^2=\frac{1}{2}[-P\pm(P^2-4Q)^{\frac{1}{2}}],\\
&r_3^2,r_4^2=\frac{1}{2}[-P_1\pm(P_1^2-4Q_1)^{\frac{1}{2}}],\\
&P=\frac{1}{2\alpha_0}[\rho J_0\omega^2-2\lambda_1]+\frac{\rho\omega^2\alpha_0+\lambda_0^2}{\alpha_0(\lambda+2\mu+K)}-2\eta^2,\\
&Q=\frac{[\rho J_0\omega^2-2(\lambda_1+\alpha_0\eta^2)][\rho\omega^2-\eta^2(\lambda+2\mu+K)]}{2\alpha_0(\lambda+2\mu+K)}-2\lambda_0\eta^2,\\
&P_1=\frac{2K-\rho J\omega^2}{\gamma}+\frac{\gamma\rho\omega^2-K^2}{\gamma(\mu+K)}-2\eta^2,\\
&Q_1=\frac{[\rho\omega^2-(\mu+K)\eta^2][2K-\rho J\omega^2-\gamma\eta^2]+\eta^2K^2}{\gamma(\mu+K)},\\
&\xi_j=\frac{C_1(\eta^2-r_j^2)-\rho\omega^2}{\lambda_0},\quad j=1,2,\\
&\xi_l=\frac{C_2(\eta^2-r_l^2)-\rho\omega^2}{K},\quad l=3,4.
\end{aligned}\right\}. \tag{4.10}$$

On substituting amplitudes given in equations (4.6) to (4.9) in equation (4.1), we obtain

$$\left.\begin{aligned}
&\chi(x,z,t)=\sum_{j=1}^{2}A_je^{-r_jz}e^{i(\eta x-\omega t)},\\
&\psi(x,z,t)=\sum_{j=3}^{4}A_je^{-r_jz}e^{i(\eta x-\omega t)},\\
&\psi^*(x,z,t)=\sum_{j=1}^{2}A_j\xi_je^{-r_jz}e^{i(\eta x-\omega t)},\\
&\phi(x,z,t)=\sum_{j=3}^{4}A_j\xi_je^{-r_jz}e^{i(\eta x-\omega t)},
\end{aligned}\right\} \tag{4.11}$$

where A_j, $j=1,2,3,4$ are arbitrary constants.

The solutions of equations (3.11) and (3.12) are

$$\phi^L(x,z,\omega,t) = A\exp[i(\eta x-\omega t)-\epsilon_1 z], \tag{4.12}$$

$$\psi^L(x,z,\omega,t) = B\exp[i(\eta x-\omega t)+\epsilon_2 z], \tag{4.13}$$

where A,B are arbitrary constants, x-axis is the direction of propagation and

$$\left.\begin{aligned} \epsilon_1 &= \frac{3\rho^L\omega^2-(4i\omega\eta^L+3K^L)\eta^2}{3K^L+4i\omega\eta^L}, \\ \epsilon_2 &= \eta^2-\frac{i\rho^L\omega}{\eta^L}. \end{aligned}\right\} \tag{4.14}$$

5. Boundary Conditions and Dispersion Equations

With the help of equation (3.1), the constitutive relations (2.4) to (2.6) reduces to

$$T_{zz} = \lambda\frac{\partial u}{\partial x}+(\lambda+2\mu+K)\frac{\partial w}{\partial z}+\lambda_0\psi^*, \tag{5.1}$$

$$T_{zx} = (\mu+K)\frac{\partial u}{\partial z}+\mu\frac{\partial w}{\partial x}-K\phi, \tag{5.2}$$

$$m_{zy} = \gamma\frac{\partial \phi}{\partial z}+b_0\frac{\partial \psi^*}{\partial x}, \tag{5.3}$$

$$\lambda_z^* = \alpha_0\frac{\partial \psi^*}{\partial z}-b_0\frac{\partial \phi}{\partial x} \tag{5.4}$$

with the help of equation (4.11), equation (3.6) reduces to

$$u = i\eta e^{-r_1 z}A_1+i\eta e^{-r_2 z}A_2+r_3e^{-r_3 z}A_3+r_4e^{-r_4 z}A_4, \tag{5.5}$$

$$w = -r_1e^{-r_1 z}A_1-r_2e^{-r_2 z}A_2+i\eta e^{-r_3 z}A_3+i\eta e^{-r_4 z}A_4. \tag{5.6}$$

The boundary conditions at the interface separating M_1 and M_2 at $z=0$ are given as

(i) Continuity of the longitudinal displacement is

$$u = u^L. \tag{5.7}$$

(ii) Continuity of the transversal displacement is

$$w = w^L. \tag{5.8}$$

(iii) Continuity of the normal stress is

$$T_{zz} = T_{zz}^L. \tag{5.9}$$

(iv) Continuity of shear stress is

$$T_{zx} = T_{zx}^L. \tag{5.10}$$

(v) Vanishing of tangential couple stress is

$$m_{zy} = 0. \tag{5.11}$$

(vi) Vanishing of normal microstress is

$$\lambda_z^* = 0. \tag{5.12}$$

Inserting equations (3.13), (3.14) and (5.1) to (5.4) with the help of equations (3.15), (5.5) and (5.6) in equations (5.7) to (5.12) we obtain, the following system of six equations

$$\left.\begin{aligned}
&i\eta A_1+i\eta A_2+r_3A_3+r_4A_4-i\eta A+\epsilon_2 B=0,\\
&-r_1A_1-r_2A_2+i\eta A_3+i\eta A_4+\epsilon_1 A+\epsilon_2 B=0,\\
&[-\lambda\eta^2-i\eta r_1(\lambda+2\mu+K)+\lambda_0\xi_1]A_1+[-\lambda\eta^2-i\eta r_2(\lambda+2\mu+K)+\lambda_0\xi_2]A_2+(i\lambda\eta r_3-\eta^2)A_3\\
&\qquad+(i\lambda\eta r_4-\eta^2)A_4-\left[(\epsilon_1^2-(\eta^L)^2)+\frac{2i\omega\eta^L}{3}+K^L+2i\omega\eta^L\epsilon_1^2\right]A-2\epsilon_2\omega(\eta^L)^2B=0,\\
&-i\eta r_1(2\mu+K)A_1-i\eta r_2(2\mu+K)A_2+[r_3^2(\mu+K)-\mu\eta^2-K\xi_3]A_3\\
&\qquad+[r_4^2(\mu+K)-\mu\eta^2-K\xi_4]A_4-2\omega\epsilon_1(\eta^L)^2A+i(\epsilon_2^2-(\eta^L)^2)\eta^L\omega B=0,\\
&ib_0\xi_1\eta A_1+ib_0\xi_2\eta A_2-\gamma r_3\xi_3A_3-\gamma r_4\xi_4A_4=0,\\
&[\alpha_0r_3+ib_0\eta\xi_3]A_3+[\alpha_0r_4+ib_0\eta\xi_4]A_4=0.
\end{aligned}\right\}\tag{5.13}$$

The system (5.13) has a non-trivial solution if and only if

$$\det(a_{ij})=0,\quad 1\le i,\ j\le 6.\tag{5.14}$$

Solving the determinant (5.14) we get the following dispersion relations

$$2\delta_2\omega^2=-\left(2\delta_3+\delta_4+\frac{4\delta_1}{\delta_2\delta}(\rho^2J+\delta_5)\right)\pm\left\{\left(2\delta_3+\delta_4+\frac{4\delta_1}{\delta_2\delta}(\rho^2J+\delta_5)\right)^2-4\left(\delta_1^2+\delta_3^2+\delta_3\delta_4-\frac{4\delta_1\delta_6}{\delta}\right)\right\}^{\frac{1}{2}}\tag{5.15}$$

and

$$\Delta_1=-\Delta_2,\tag{5.16}$$

where a_{ij} $(1\le i,\ j\le 6)$, δ_l $(1\le l\le 6)$, δ, Δ_1 and Δ_2 are given in Appendix.

Equation (5.15) represents dispersion relation for Rayleigh waves along free surface of Micropolar microstretch elastic solid half-space and equation (5.16) is a dispersion relation between viscous liquid layer and micropolar microstretch solid half-space.

Particular Cases

1. When $\alpha_0\to 0$ in equation (5.15), we get the following frequency equation of Rayleigh waves in free surface of generalized micropolar elastic solid. It is dispersive in nature.

$$\rho\left(\frac{\gamma}{C_2}-J\right)\omega^2=2(\gamma\eta^2-K).\tag{5.17}$$

2. When $\gamma\to 0$ in equation (5.15), we get

$$\rho\omega^2=(2C_2K^2-6K)\quad\text{or}\quad\rho\omega^2=(K-C_2K^2)\tag{5.18}$$

which is a frequency equation of Rayleigh waves in free surface of generalized micropolar elastic solid and it is non-dispersive in nature.

3. By applying the properties of determinants, the equation (5.16) reduces to

$$r_3=r_4=i\eta\quad\text{and}\quad\epsilon_1=-i\eta.\tag{5.19}$$

Under using equation (5.19), we get the following frequency Type-I and Type-II equations at interface of viscous liquid and non-microstretch elastic solid respectively given by

$$2(3\eta^2+1)\omega = 3i\eta^L\left[\frac{\lambda\eta^2+K^2}{(\eta^L)^2}-1\right] \tag{5.20}$$

and

$$\rho\omega = \eta\eta^L\left[-i\eta^L \pm \left\{4i\eta\left(\mu+\frac{K}{2}+C_2\right)-(\eta^L)^2\right\}^{\frac{1}{2}}\right]. \tag{5.21}$$

6. Numerical Application

To study the effect of fluid and stretch on the dispersion curves in a micropolar micro-stretch elastic solid, we adopt the available physical parameters as:
The micropolar elastic solid parameters given below Chiroiu and Munteanu [1]: $\lambda = 7.583$ Gpa; $\mu = 6.334$ Gpa; $K = 14.905$ Mpa; $\rho = 1189\,\text{kg/m}^3$; $J = 6.25\times10^{-7}\,\text{m}^2$; $\gamma = 2.896$ N; $\alpha = 3.688\,\text{Gpam}^2$.
With the unavailability, the micro-stretch parameters taken from Kumar *et al.* [10]: $\alpha_0 = 0.779\times10^{-9}$ N; $b_0 = 0.5\times10^{-9}$ N; $\lambda_0 = 0.5\times10^{10}\,\text{N/m}^2$; $\lambda_1 = 0.5\times10^{10}\,\text{N/m}^2$.
Viscous liquid parameters given below Fehler [7]: $K^L = 0.119\times10^9\,\text{dyne/m}^2$; $\rho^L = 1.01\times10^{-3}\,\text{kg/m}^2$; $\eta^L = 0.0014$ poise.
The non-dimensional wave number η taken as $10\le\eta\le100$.

The variation of frequency with wave number for free surface of stretched solid and micropolar solid are shown in Figure 2 and Figure 3, respectively.

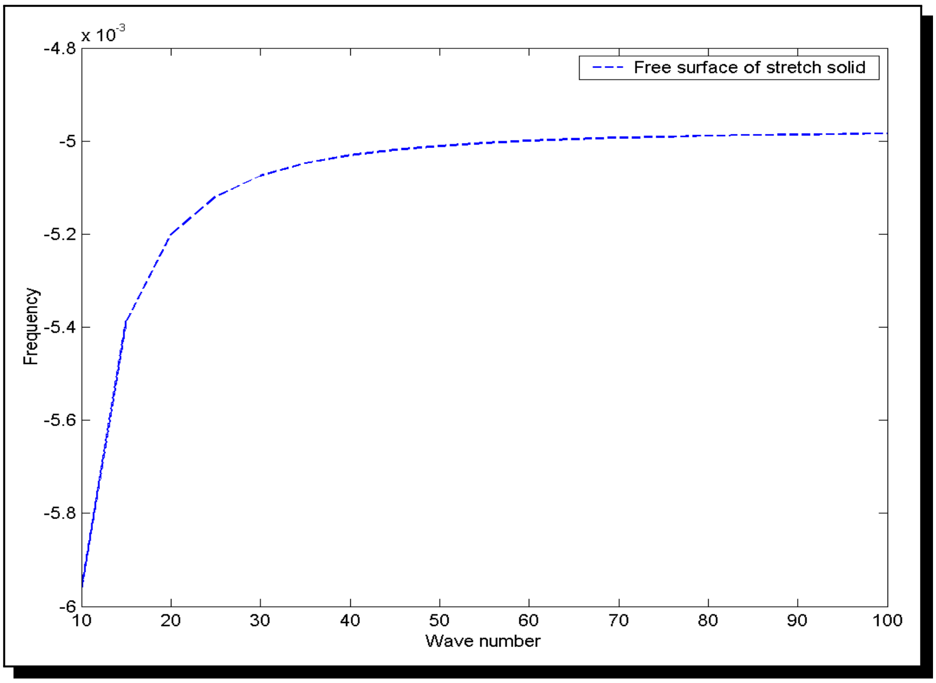

Figure 2. Frequency vs. wave number

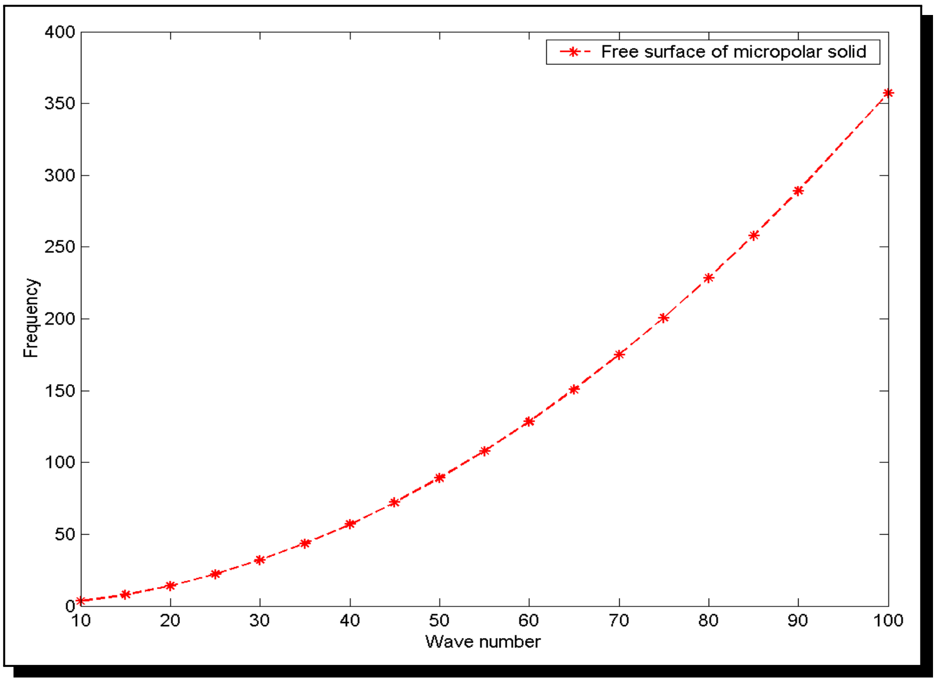

Figure 3. Frequency vs. wave number

Type-I dispersion curve for interface of viscous liquid and non micro-stretch (i.e., micropolar) solid is shown in Figure 4 and this frequency curve decreasing in parabolic path in given range of wave number 0 to 60 and nearly constant in the range of 60 to 100. Type-II dispersion curve pertaining to viscous liquid and non micro-stretch (i.e., micropolar) solid shown in Figure 5 and from this figure we observed that the frequencies are increasing in parabolic path.

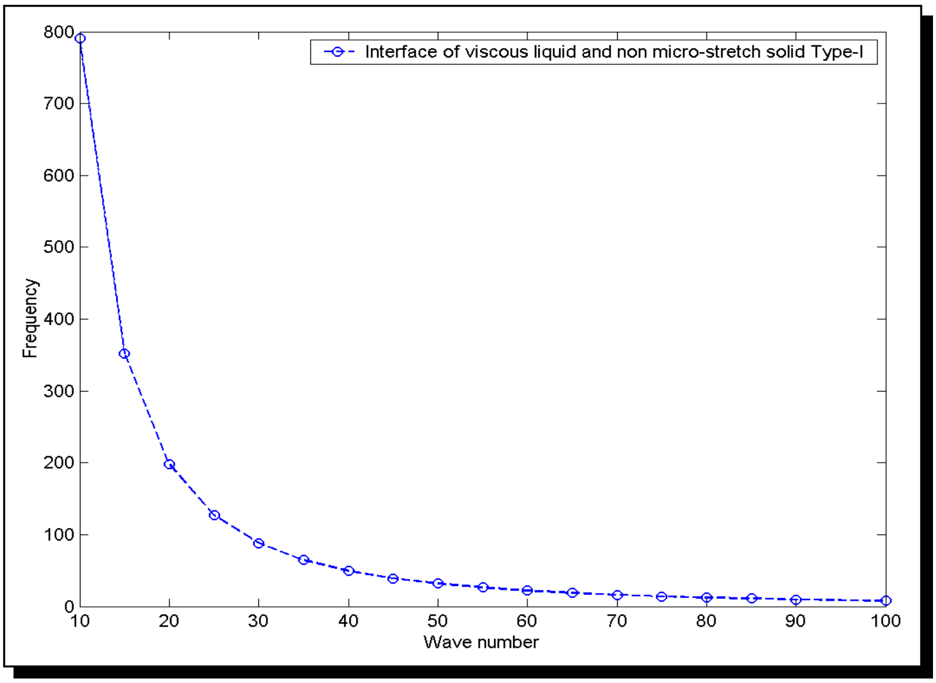

Figure 4. Frequency vs. wave number

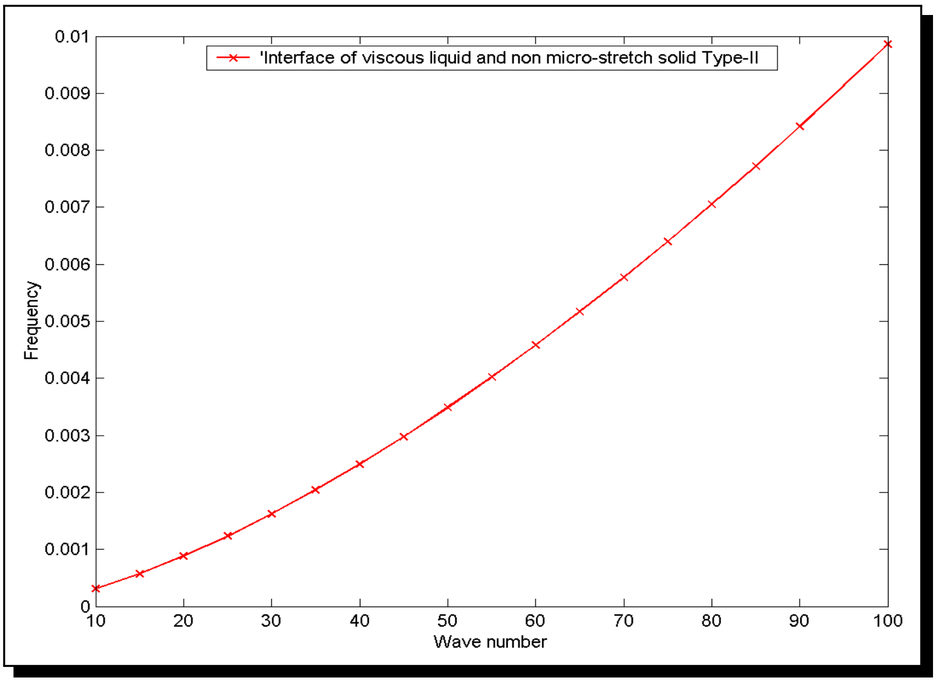

Figure 5. Frequency vs. wave number

The comparative graphs are shown in Figure 6 to Figure 10. From these figures we observed that the Rayleigh surface waves are vanishes at free surface of micropolar micro-stretch solid and interface of viscous liquid/non micro-stretch (i.e., micropolar) solid. Also, the Rayleigh waves are propagate in parabolic path at free surface of non micro-stretch (i.e., micropolar) solid and interface of viscous liquid/non micro-stretch (i.e., micropolar) solid.

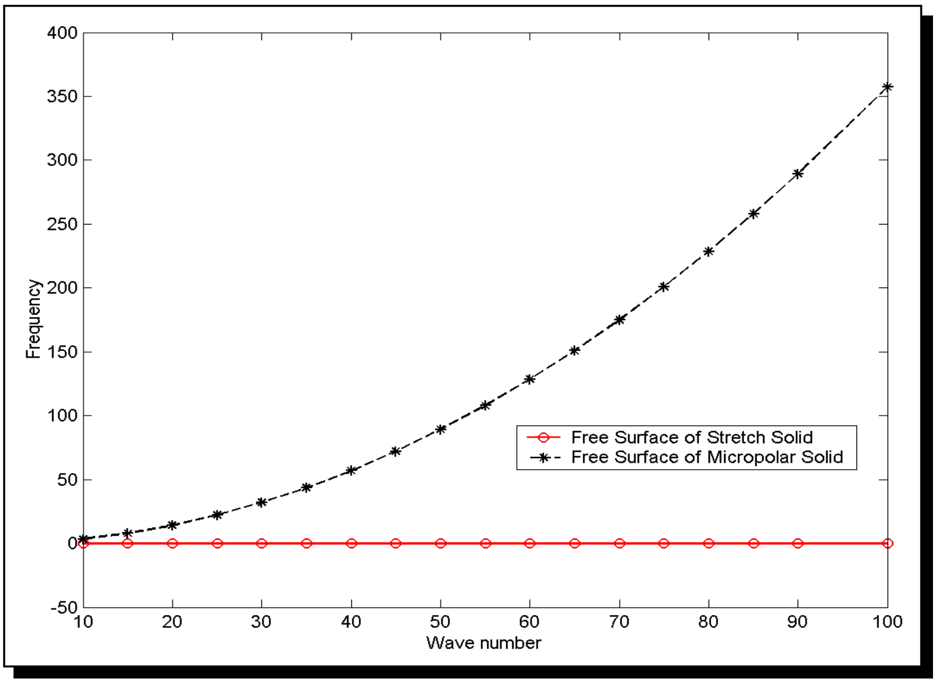

Figure 6. Frequency vs. wave number

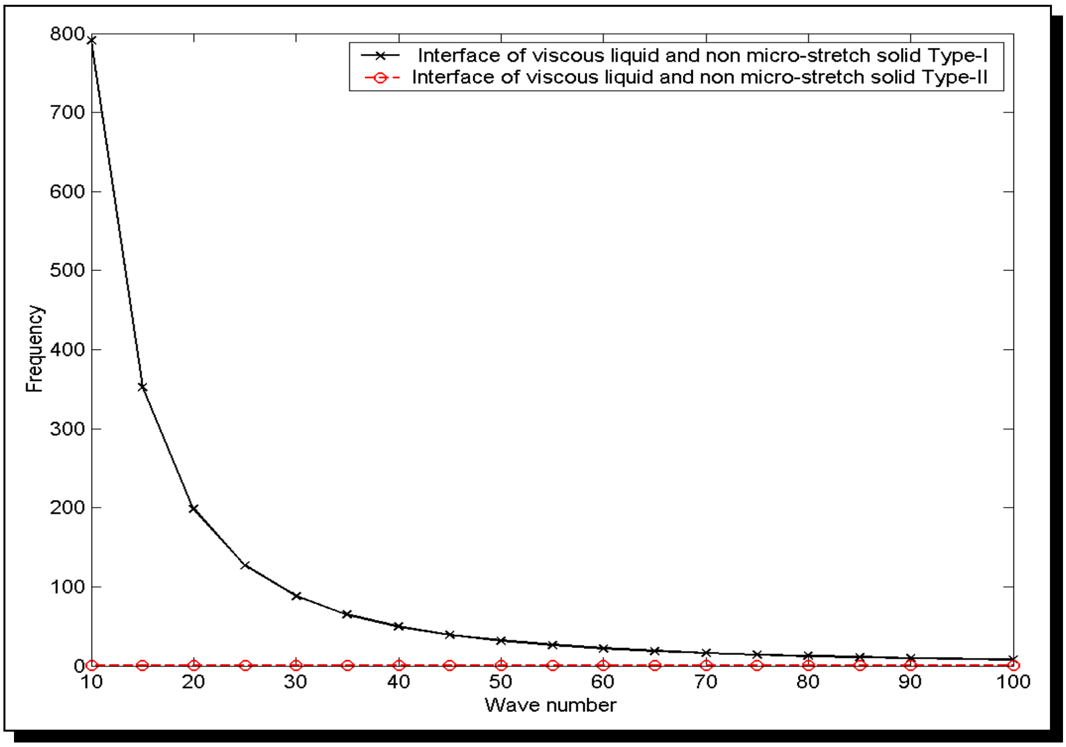

Figure 7. Frequency vs. wave number

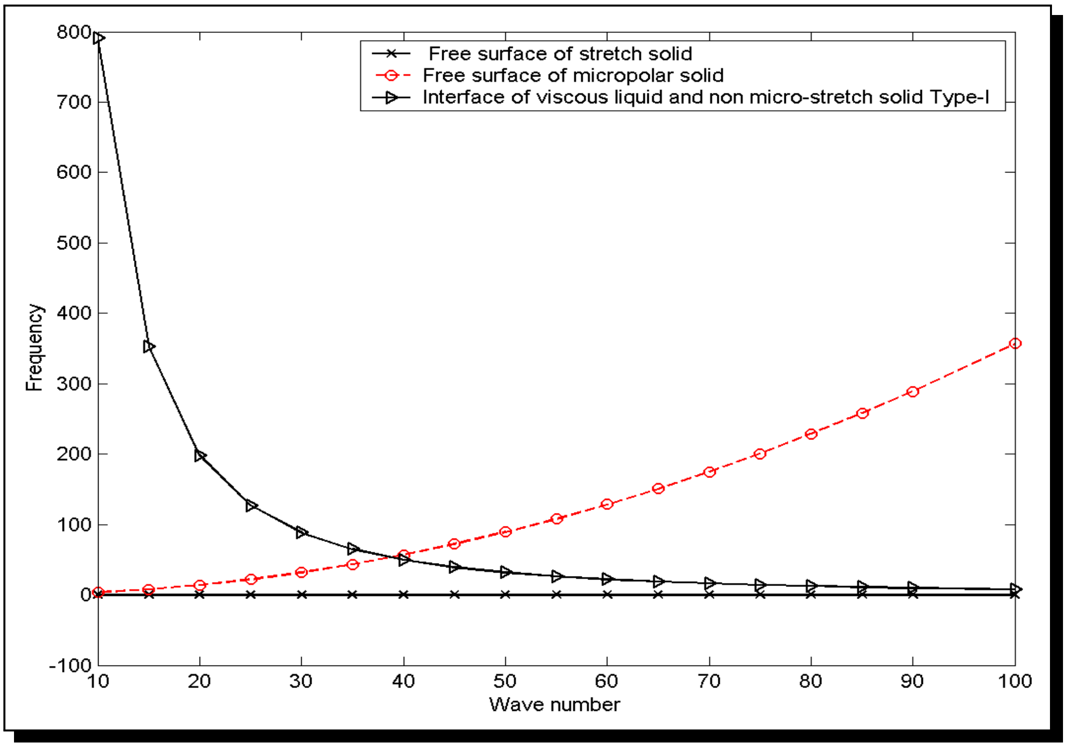

Figure 8. Frequency vs. wave number

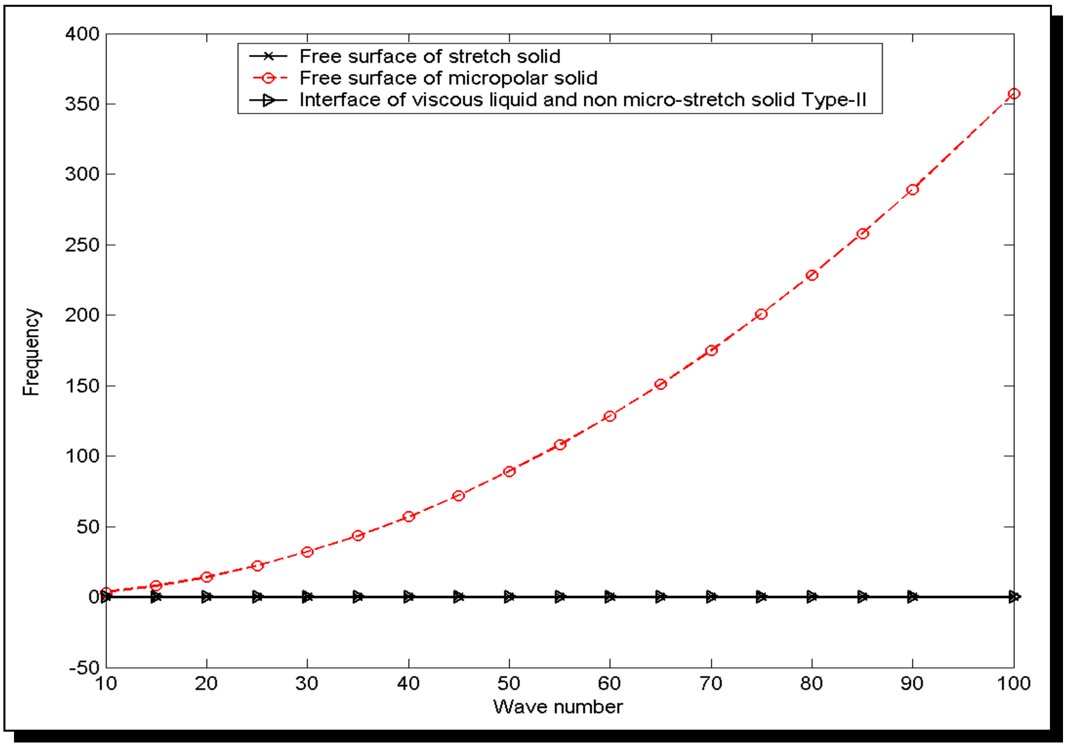

Figure 9. Frequency vs. wave number

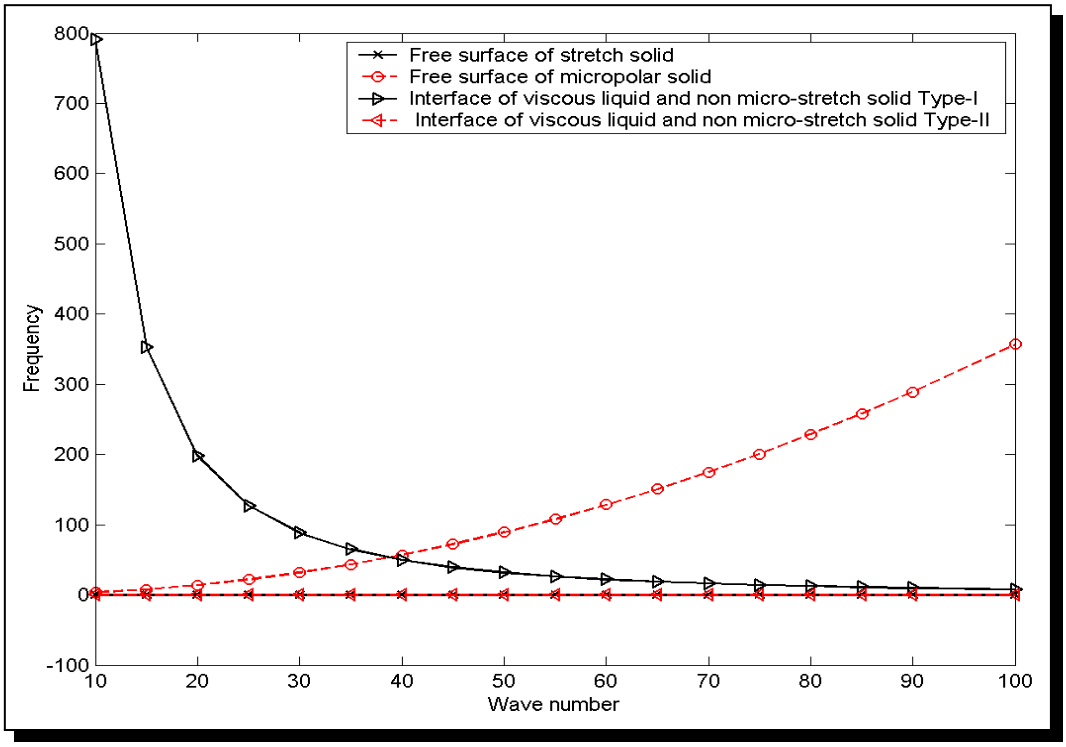

Figure 10. Frequency vs. wave number

7. Conclusion

In this present investigation, we have derived secular equations for Rayleigh waves in a homogeneous, isotropic micropolar micro-stretch elastic solid underlying viscous liquid. Under theoretical illustration and Numerical example, we conclude that

- Two Rayleigh waves along the free surface of micropolar micro-stretch elastic solid half-space and at interface of viscous liquid/micropolar micro-stretch solid are propagate. These are functions of wave number so they are dispersive in nature.
- Four special types of Rayleigh waves are propagate, in which two are at free surface of micropolar solid and micro-stretch solid and another two are at interface of viscous liquid/micropolar solid. All these waves are dispersive in nature, while the wave at free surface of micro elastic solid is non-dispersive.
- The waves at free surface of micropolar micro-stretch solid and interface of viscous liquid/micropolar solid are vanishes.
- The waves are propagate in parabolic path at free surface of micropolar solid and interface of viscous liquid/micropolar solid.

Appendix

$a_{11}=a_{12}=i\eta$, $a_{13}=r_3$, $a_{14}=r_4$, $a_{15}=-i\eta$, $a_{16}=\epsilon_2$, $a_{21}=-r_1$, $a_{22}=-r_2$, $a_{23}=i\eta$, $a_{24}=i\eta$,

$a_{25}=\epsilon_1$, $a_{26}=\epsilon_2$, $a_{31}=\lambda_0\xi_1-\lambda\eta^2-i\eta r_1C_1$, $a_{32}=\lambda_0\xi_2-\lambda\eta^2-i\eta r_2C_1$, $a_{33}=i\lambda\eta r_3-\eta^2$,

$a_{34}=i\lambda\eta r_4-\eta^2$, $a_{35}=-\left[\epsilon_1^2-(\eta^L)^2+\dfrac{2i\omega\eta^L}{3}+K^L+2i\omega\eta^L\epsilon_1^2\right]$, $a_{36}=-2\epsilon_2\omega(\eta^L)^2$,

$a_{41}=-i\eta r_1(2\mu+K)$, $a_{42}=-i\eta r_2(2\mu+K)$, $a_{43}=r_3^2C_2-\mu\eta^2-K\xi_3$, $a_{44}=r_4^2C_2-\mu\eta^2-K\xi_4$,

$a_{45}=-2\omega\epsilon_1(\eta^L)^2$, $a_{46}=i(\epsilon_2^2-(\eta^L)^2)\eta^L\omega$, $a_{51}=ib_0\xi_1\eta$, $a_{52}=ib_0\xi_2\eta$, $a_{53}=-\gamma r_3\xi_3$,

$a_{54}=-\gamma r_4\xi_4$, $a_{55}=a_{56}=0$, $a_{61}=a_{62}=0$, $a_{63}=\alpha_0r_3+ib_0\eta\xi_3$, $a_{64}=\alpha_0r_4+ib_0\eta\xi_4$,

$a_{65}=a_{66}=0$, $\delta=\left(\dfrac{\alpha_0K}{b_0\eta C_2}\right)^2$, $\delta_1=\gamma C_2\delta$, $\delta_2=\rho(\gamma-JC_2)$, $\delta_3=2C_2(K-\gamma\eta^2)$,

$\delta_4=2C_2\gamma\delta$, $\delta_5=(\gamma\eta^2-2K)\rho-C_2\eta^2J\rho$, $\delta_6=C_2(\gamma\eta^2-2K)\eta^2-\eta^2K^2$,

$$\Delta_1=\begin{vmatrix} i\eta & i\eta & r_4 & -i\eta & \epsilon_2\\ -r_1 & -r_2 & i\eta & \epsilon_1 & \epsilon_2\\ a_{31} & a_{32} & a_{34} & a_{35} & a_{36}\\ a_{41} & a_{42} & a_{44} & a_{45} & a_{46}\\ a_{51} & a_{52} & a_{54} & 0 & 0\end{vmatrix},\ \Delta_2=\begin{vmatrix} i\eta & i\eta & -i\eta & r_3 & \epsilon_2\\ -r_1 & -r_2 & \epsilon_1 & i\eta & \epsilon_2\\ a_{31} & a_{32} & a_{35} & a_{33} & a_{36}\\ a_{41} & a_{42} & a_{45} & a_{43} & a_{46}\\ a_{51} & a_{52} & 0 & a_{53} & 0\end{vmatrix}.$$

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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