Communications in Mathematics and Applications
Vol. 13, No. 3, pp. 1119–1128, 2022
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications
DOI: 10.26713/cma.v13i3.1913

http://www.rgnpublications.com

Research Article

On the Study of Meromorphic Functions That Shares Small Functions Partially With the Second Order Difference Operator

Touqeer Ahmed*[1], N. Shilpa[1] and M. T. Somalatha[2]

[1] *Department of Mathematics, Presidency University, Bangalore 560064, India*
[2] *Department of Mathematics, Government Science College, Bangalore 560001, India*
***Corresponding author:** tauqeer.ahmed33@gmail.com

Received: May 29, 2022 **Accepted:** September 19, 2022

Abstract. In this paper, we looked at some problems with the uniqueness of meromorphic functions with a second order difference operator. We looked at them from the point of view of partial sharing. We have obtained two uniqueness results. In the first theorem $\Delta^2\mathfrak{g}(\mathfrak{z})$ and $\mathfrak{g}(\mathfrak{z})$ shares $\mathfrak{a}_1(\mathfrak{z})$, $\mathfrak{a}_2(\mathfrak{z})$, ∞ CM, whereas in the second theorem $\mathfrak{g}(\mathfrak{z})$ and $\Delta^2\mathfrak{g}(\mathfrak{z})$ partially share $\mathfrak{a}_1(\mathfrak{z})$, $\mathfrak{a}_2(\mathfrak{z})$ CM that generalizes the results due to Banerjee and Maity (Meromorphic function partially shares small functions or values with its linear c-shift operator, *Bulletin of the Korean Mathematical Society* **58**(5) (2021), 1175 – 1192), and Heittokangas *et al.*, Uniqueness of meromorphic functions sharing values with their shifts, *Complex Variables and Elliptic Equations* **56**(1-4) (2011), 81 – 92.

Keywords. Uniqueness, Meromorphic function, Partial sharing, Small function, Difference operator

Mathematics Subject Classification (2020). 30D35, 30D45

1. Introduction

We presume that the reader is familiar with the notations of the Nevanlinna theory and for the basic ([7, 12, 17]). We mean $\mathcal{S}(\mathfrak{r},\mathfrak{g}) = o(\mathcal{T}(\mathfrak{r},\mathfrak{g}))\ \forall\ \mathfrak{r} \in (1,\infty)$. We denote the set meromorphic functions a_i for $i = 1,2$ by $\mathcal{S}(\mathfrak{f})$.

The set of all $\mathfrak{a}$-points (counting multiplicities or CM) of $\mathfrak{f}$ is denoted by $E(\mathfrak{a},\mathfrak{f})$, and all different $\mathfrak{a}$-points of $\mathfrak{f}$ by $\overline{E}(\mathfrak{a},\mathfrak{f})$.

We use the following fundamental definitions to prove our results.

Definition 1.1 ([1]). *It is claimed that a meromorphic function* $\mathfrak{f}$ *shares* $\mathfrak{a} \in \mathcal{S}(\mathfrak{f})$ *partially CM with a meromorphic function* $\mathfrak{g}$ *if* $E(\mathfrak{a},\mathfrak{f}) \subseteq E(\mathfrak{a},\mathfrak{g})$.

Definition 1.2 ([2]). *It is claimed that a meromorphic function* $\mathfrak{f}$ *shares* $\mathfrak{a} \in \mathcal{S}(\mathfrak{f})$ *partially IM with a meromorphic function* $\mathfrak{g}$ *if* $\overline{E}(\mathfrak{a},\mathfrak{g}) \subseteq \overline{E}(\mathfrak{a},\mathfrak{f})$.

Definition 1.3 ([7]). *It is claimed that if* $\mathfrak{f}$ *and* $\mathfrak{g}$ *share the value* $\mathfrak{a}$ *CM if* $E(\mathfrak{a},\mathfrak{f}) = E(\mathfrak{a},\mathfrak{g})$. $\mathfrak{f}$ *and* $\mathfrak{g}$ *share the value* $\mathfrak{a}$ *IM if* $\overline{E}(\mathfrak{a},\mathfrak{f}) = \overline{E}(\mathfrak{a},\mathfrak{g})$.

Halburd-Korhonen [6], and Chiang-Feng [4] started the counterpart of renowned Nevanlinna's theory for difference operator. Several noteworthy results [2, 3, 10] followed, of which we would like to highlight a few.

Heittokangas *et al*. [9] looked into the relationship between a meromorphic function's shift operator and meromorphic function when they share a, ∞ CM in 2009. The outcome is as follows.

Theorem 1.1 ([9]). *Let* $\mathfrak{f}(\mathfrak{z})$ *be a meromorphic function and* $c \in \mathbb{C}$. *If* $\mathfrak{f}(\mathfrak{z}+\mathfrak{c})$ *and* $\mathfrak{f}(\mathfrak{z})$ *share* $\mathfrak{a},\infty$ *CM, where* $\mathfrak{a} \in \mathbb{C}$, *then for some constant* τ,

$$\frac{\mathfrak{f}(\mathfrak{z}+\mathfrak{c})-\mathfrak{a}}{\mathfrak{f}(\mathfrak{z})-\mathfrak{a}} = \tau .$$

In 2011 by considering three small functions CM, two small functions CM and one small function IM, Heittokangas *et al*. [8], looked into the relation between $\mathfrak{f}(\mathfrak{z})$ and $\mathfrak{f}(\mathfrak{z}+\mathfrak{c})$, and by considering the entire function, Huang-Zhang in [10] got a result as in Theorem 1.1.

Theorem 1.2 ([10]). *Let* $\mathfrak{f}(\mathfrak{z})$ *be a transcendental entire function of order* $\rho(\mathfrak{f}) < 2$. *If* $\Delta_c^k \mathfrak{f}(\mathfrak{z})$ *and* $\mathfrak{f}(\mathfrak{z})$ *share 0 CM, where* $k \in \mathbb{N}$ *and* $c \in \mathbb{C}\setminus\{0\}$ *are such that* $\Delta_c^k \mathfrak{f}(\mathfrak{z}) \neq 0$, *then*

$$\Delta_c^k \mathfrak{f}(\mathfrak{z}) \equiv \tau \mathfrak{f}(\mathfrak{z}),$$

for some constant $\mathcal{T}$.

In order to obtain a similar result for a meromorphic function corresponding to Theorem 1.2, Chen-Yi [3] researched the uniqueness of $\Delta_c \mathfrak{f}(\mathfrak{z})$ and $\mathfrak{f}(\mathfrak{z})$ as follows.

Theorem 1.3 ([3]). *Let* $\mathfrak{f}(\mathfrak{z})$ *be a transcendental meromorphic function such that the order* $\rho(\mathfrak{f})$ *is not an integer or infinite and* $\mathfrak{c} \in \mathbb{C}$ *be a constant such that* $\mathfrak{f}(\mathfrak{z}+\mathfrak{c}) \neq \mathfrak{f}(\mathfrak{z})$. *If* $\Delta_c \mathfrak{f}(\mathfrak{z})$ *and* $\mathfrak{f}(\mathfrak{z})$ *share three distinct values* $\mathfrak{a},\mathfrak{b},\infty$ *CM, then* $\mathfrak{f}(\mathfrak{z}+\mathfrak{c}) \equiv 2\mathfrak{f}(\mathfrak{z})$.

Zhang-Liao [20] worked on the entire function in 2014, he removed the restriction that "$\rho(\mathfrak{f})$ is not an integer", Zhang-Liao did this in the following way:

Theorem 1.4 ([18]). *Let* $\mathfrak{f}(\mathfrak{z})$ *be a transcendental entire function of finite order* $\mathfrak{c}$ *be a non-zero constant;* $\mathfrak{a}$, $\mathfrak{b}$ *be two distinct finite constants. If* $\Delta_\mathfrak{c} \mathfrak{f}(\mathfrak{z})$ $(\neq 0)$ *and* $\mathfrak{f}(\mathfrak{z})$ *share* $\mathfrak{a}$, $\mathfrak{b}$ *CM, then* $\Delta_\mathfrak{c} \mathfrak{f}(\mathfrak{z}) = \mathfrak{f}(\mathfrak{z})$.

Theorem 1.5 ([11]). *Let $\mathfrak{f}(\mathfrak{z})$ be a non-constant meromorphic function of finite order such that $\mathcal{N}(\mathfrak{r},\mathfrak{f}) = \mathcal{S}(\mathfrak{r},\mathfrak{f})$, let $\mathfrak{c} \in \mathbb{C}\backslash\{0\}$ be a constant such that $\Delta_{\mathfrak{c}}\mathfrak{f}(\mathfrak{z}) \neq 0$ and let $\mathfrak{a},\mathfrak{b}$ be two non-zero distinct finite complex constants. If $\Delta_c\mathfrak{f}(\mathfrak{z})$ and $\mathfrak{f}(\mathfrak{z})$ share $\mathfrak{a},\mathfrak{b}$ CM, then $\mathfrak{f}(\mathfrak{z}+\mathfrak{c}) = 2\mathfrak{f}(\mathfrak{z})$.*

In the year 2017, Lü-Lü [14] removed the order restriction from Theorem 1.5. For meromorphic functions, without any extra conditions, he proved uniqueness.

Theorem 1.6 ([14]). *Let $\mathfrak{f}(\mathfrak{z})$ be a transcendental meromorphic function of finite order and let $\mathfrak{c} \in \mathbb{C}$ be a constant such that $\mathfrak{f}(\mathfrak{z}+\mathfrak{c}) \neq \mathfrak{f}(\mathfrak{z})$. If $\Delta_{\mathfrak{c}}\mathfrak{f}(\mathfrak{z})$ and $\mathfrak{f}(\mathfrak{z})$ share three distinct values $\mathfrak{a},\mathfrak{b},\infty$ CM, then $\mathfrak{f}(\mathfrak{z}+\mathfrak{c}) \equiv 2\mathfrak{f}(\mathfrak{z})$.*

In the year 2019, Zhen [21] almost followed the same steps as the proof of Theorem 1.6, but instead of looking at value sharing, he looked at polynomial sharing. This made Theorem 1.6 better.

Theorem 1.7 ([21]). *Let $\mathfrak{f}(\mathfrak{z})$ be a transcendental meromorphic function of finite order and let $\mathfrak{c}(\neq 0)$ be a finite number. If $\Delta_{\mathfrak{c}}\mathfrak{f}(\mathfrak{z})$ and $\mathfrak{f}(\mathfrak{z})$ share three distinct polynomials $\mathcal{P}_1,\mathcal{P}_2,\infty$ CM, then $\Delta_{\mathfrak{c}}\mathfrak{f}(\mathfrak{z}) = \mathfrak{f}(\mathfrak{z})$.*

2. Lemmas

Lemma 2.1 ([16]). *Let $\mathfrak{f}$ be non-constant meromorphic function in $\mathbb{C}$. Let $\mathfrak{a}_1,\mathfrak{a}_2,\mathfrak{a}_3$ be pairwise distinct small meromorphic functions in $\mathbb{C}$ such that $\mathfrak{a}_1,\mathfrak{a}_2 \in S(\mathfrak{f})$ and*

$$\mathcal{T}(\mathfrak{r},\mathfrak{a}_3) \leq \nu\mathcal{T}(\mathfrak{r},\mathfrak{f}) + \mathcal{S}(\mathfrak{r},\mathfrak{f})$$

for some $\nu \in [0,1/3)$. Then

$$(1-3\nu-\epsilon)\mathcal{T}(\mathfrak{r},\mathfrak{f}) \leq \sum_{\mathfrak{i}=1}^{\mathfrak{q}} \overline{\mathcal{N}}\left(r,\frac{1}{\mathfrak{f}-\mathfrak{a}_i}\right) + \mathcal{S}(\mathfrak{r},\mathfrak{f}).$$

Lemma 2.2 ([13]). *Let $\mathfrak{f}$ be a meromorphic function of finite order, and let $\mathfrak{c} \in \mathbb{C}, \mathfrak{n} \in \mathbb{N}$. Then for any small periodic function $\mathfrak{a}(\mathfrak{z}) \in S(\mathfrak{f})$ with period $\mathfrak{c}$,*

$$m\left(r,\frac{\Delta_{\mathfrak{c}}^{\mathfrak{n}}\mathfrak{f}}{\mathfrak{f}(\mathfrak{z})-\mathfrak{a}(z)}\right) = \mathcal{S}(\mathfrak{r},\mathfrak{g}).$$

Lemma 2.3 ([11]). *Let $\mathfrak{f}(\mathfrak{z})$ be a meromorphic function, and let η be a fixed non-zero complex number, then for each $\epsilon > 0$, we have $\mathcal{T}(\mathfrak{r},\mathfrak{f}(\mathfrak{z}+\eta)) = \mathcal{T}(\mathfrak{r},\mathfrak{f}) + \mathcal{S}(\mathfrak{r},\mathfrak{f})$.*

Lemma 2.4 ([15]). *Let $\mathfrak{f}$ be a meromorphic function of hyper-order $\gamma(\mathfrak{f}) < 1$ and let $\mathfrak{c} \in \mathbb{C}\backslash\{0\}$. Let $\mathfrak{a}_1,\mathfrak{a}_2,\mathfrak{a}_3 \in S(\mathfrak{f})$ be three distinct periodic functions with period $\mathfrak{c}$. Assume that $\mathfrak{f}(\mathfrak{z})$ and $\mathfrak{f}(\mathfrak{z}+\mathfrak{c})$ share partially $\mathfrak{a}_1,\mathfrak{a}_2$ CM and share partially $\mathfrak{a}_3$ IM, i.e.,*

$$E(\mathfrak{a}_1,\mathfrak{f}(\mathfrak{z})) \subseteq E(\mathfrak{a}_1,\mathfrak{f}(\mathfrak{z}+\mathfrak{c})),\quad E(\mathfrak{a}_2,\mathfrak{f}(\mathfrak{z})) \subseteq E(\mathfrak{a}_2,\mathfrak{f}(\mathfrak{z}+\mathfrak{c}))$$

and

$$\overline{E}(\mathfrak{a}_3,\mathfrak{f}(\mathfrak{z})) \subseteq \overline{E}(\mathfrak{a}_3,\mathfrak{f}(\mathfrak{z}+\mathfrak{c})).$$

If $\mathfrak{p}(\mathfrak{a},\mathfrak{f}) > 0$ for some $\mathfrak{a} \in S(\mathfrak{f})\backslash\{\mathfrak{a}_3\}$, then $\mathfrak{f}(\mathfrak{z}) = \mathfrak{f}(\mathfrak{z}+\mathfrak{c})$ for all $\mathfrak{z} \in \mathbb{C}$.

Lemma 2.5 ([15]). *Let* $\mathfrak{T}:\mathbb{R}^+\to\mathbb{R}^+$ *be an increasing function and let* $\mathfrak{s}\in(0,+\infty)$ *such that hyper-order of* $\mathfrak{T}$ *is strictly less than one, i.e.,*

$$\gamma=\limsup_{\mathfrak{r}\to\infty}\frac{\log^+\log^+\mathfrak{T}(\mathfrak{r})}{\log\mathfrak{r}}<1.$$

Then

$$\mathfrak{T}(\mathfrak{r}+\mathfrak{s})=\mathfrak{T}(\mathfrak{r})+o\left(\frac{\mathfrak{T}(\mathfrak{r})}{\mathfrak{r}^{1-\gamma-\epsilon}}\right),$$

where $\epsilon>0$ *and* $\mathfrak{r}\to\infty$ *outside a subset of finite logarithmic measure.*

Lemma 2.6 ([17]). *Suppose that* $\mathfrak{f}(\mathfrak{z})$ *is a non-constant meromorphic function and* $\mathcal{P}(\mathfrak{f})=\mathfrak{a}_0\mathfrak{f}^{\mathfrak{p}}+\mathfrak{a}_1\mathfrak{f}^{\mathfrak{p}-1}+\cdots+\mathfrak{a}_{\mathfrak{p}}$ $(\mathfrak{a}_0\neq0)$ *is a polynomial in* $\mathfrak{f}$ *with degree* $\mathfrak{p}$ *and coefficients* $\mathfrak{a}_{\mathfrak{j}}$ $(\mathfrak{j}=0,1,\ldots,\mathfrak{p})$ *are constants, suppose furthermore that* $\mathfrak{b}_{\mathfrak{j}}(\mathfrak{j}=1,2,\ldots,\mathfrak{q})$ $(\mathfrak{q}>\mathfrak{p})$ *are distinct finite values. Then*

$$\mathfrak{m}\left(\mathfrak{r},\frac{\mathcal{P}(\mathfrak{f})\mathfrak{f}'}{(\mathfrak{f}-\mathfrak{b}_1)(\mathfrak{f}-\mathfrak{b}_2)\cdots(\mathfrak{f}-\mathfrak{b}_{\mathfrak{q}})}\right)=\mathcal{S}(\mathfrak{r},\mathfrak{f}).$$

3. Main Results

Theorem 3.1. *Considering* $\mathfrak{g}(\mathfrak{z})$ *as a non-constant meromorphic function. Suppose that* $\mathfrak{c}\in\mathbb{C}\setminus\{0\}$, $\mathfrak{b}_0\neq0$ *and* $\mathfrak{a}_1(\mathfrak{z}),\mathfrak{a}_2(\mathfrak{z})\in\mathcal{S}(\mathfrak{f})$ *are two small functions. If* $\Delta^2\mathfrak{g}(\mathfrak{z})\not\equiv0$ *and* $\Delta^2\mathfrak{g}(\mathfrak{z})$, $\mathfrak{g}$ *share* $\mathfrak{a}_1,\mathfrak{a}_2,\infty$ *CM, then* $\Delta^2\mathfrak{g}(\mathfrak{z})\equiv\mathfrak{g}(\mathfrak{z})$.

Proof. Due to the fact that $\Delta^2\mathfrak{g}(\mathfrak{z})$ and $\mathfrak{g}$ share ∞ CM, we have

$$\begin{aligned}\mathfrak{T}(\mathfrak{r},\Delta^2\mathfrak{g}(\mathfrak{z}))&\le\mathfrak{m}\left(\mathfrak{r},\frac{\Delta^2\mathfrak{g}(\mathfrak{z})}{\mathfrak{f}}\right)+\mathfrak{m}(\mathfrak{r},\mathfrak{g})+\mathcal{N}(\mathfrak{r},L_{\mathfrak{c}}\mathfrak{g})+O(1)\\&=\mathfrak{m}(\mathfrak{r},\mathfrak{g})+\mathcal{N}(\mathfrak{r},\mathfrak{g})+\mathcal{S}(\mathfrak{r},\mathfrak{g})\\&=4\mathfrak{T}(\mathfrak{r},\mathfrak{g})+\mathcal{S}(\mathfrak{r},\mathfrak{g}).\end{aligned}$$

Thus

$$S(\mathfrak{r},\Delta^2\mathfrak{g}(\mathfrak{z}))=\mathcal{S}(\mathfrak{r},\mathfrak{g}). \tag{3.1}$$

In the same way that $\Delta^2\mathfrak{g}(\mathfrak{z})$ and $\mathfrak{g}$ share $\mathfrak{a}_1,\mathfrak{a}_2,\infty$ CM. As such two polynomials $\mathfrak{p}_1(\mathfrak{z}),\mathfrak{p}_2(\mathfrak{z})$ exists, such that

$$\frac{\Delta^2\mathfrak{g}(\mathfrak{z})-\mathfrak{a}_1}{\mathfrak{g}-\mathfrak{a}_1}=\mathfrak{e}^{\mathfrak{p}_1(\mathfrak{z})}, \tag{3.2}$$

and

$$\frac{\Delta^2\mathfrak{g}(\mathfrak{z})-\mathfrak{a}_2}{\mathfrak{g}-\mathfrak{a}_2}=\mathfrak{e}^{\mathfrak{p}_2(\mathfrak{z})}. \tag{3.3}$$

Case 1: Presuming $\mathfrak{e}^{\mathfrak{p}_1(\mathfrak{z})}\equiv1$ or $\mathfrak{e}^{\mathfrak{p}_2(\mathfrak{z})}\equiv1$, then $\Delta^2\mathfrak{g}(\mathfrak{z})\equiv\mathfrak{g}(\mathfrak{z})$.

Case 2: Presuming $\mathfrak{e}^{\mathfrak{p}_1(\mathfrak{z})}\neq1$ and $\mathfrak{e}^{\mathfrak{p}_2(\mathfrak{z})}\neq1$, however if suppose $\mathfrak{e}^{\mathfrak{p}_1(\mathfrak{z})}\equiv\mathfrak{e}^{\mathfrak{p}_2(\mathfrak{z})}$, then

$$\frac{\Delta^2\mathfrak{g}(\mathfrak{z})-\mathfrak{a}_1}{\mathfrak{g}(\mathfrak{z})-\mathfrak{a}_1}=\frac{\Delta^2\mathfrak{g}(\mathfrak{z})-\mathfrak{a}_2}{\mathfrak{g}(\mathfrak{z})-\mathfrak{a}_2},$$

we obtain via easy computation that $\Delta^2\mathfrak{g}(\mathfrak{z})\equiv\mathfrak{g}(\mathfrak{z})$.

Case 3: In this case we presume that $\mathfrak{e}^{\mathfrak{p}_1(\mathfrak{z})} \not\equiv 1$ and $\mathfrak{e}^{\mathfrak{p}_2(\mathfrak{z})} \not\equiv 1$ with $\mathfrak{e}^{\mathfrak{p}_1(\mathfrak{z})} \not\equiv \mathfrak{e}^{\mathfrak{p}_2(\mathfrak{z})}$. By (3.2) and (3.3)

$$\mathfrak{g}(\mathfrak{z})\mathfrak{e}^{\mathfrak{p}_1(\mathfrak{z})} = \Delta^2\mathfrak{g}(\mathfrak{z}) - \mathfrak{a}_1 + \mathfrak{a}_1\mathfrak{e}^{\mathfrak{p}_1(\mathfrak{z})}. \tag{3.4}$$

Similarly,

$$\mathfrak{g}(\mathfrak{z})\mathfrak{e}^{\mathfrak{p}_2(\mathfrak{z})} = \Delta^2\mathfrak{g}(\mathfrak{z}) - \mathfrak{a}_2 + \mathfrak{a}_2\mathfrak{e}^{\mathfrak{p}_2(\mathfrak{z})}. \tag{3.5}$$

Now by (3.4) and (3.5), we get

$$\mathfrak{g}(\mathfrak{z}) = \frac{\mathfrak{a}_2 - \mathfrak{a}_1 + \mathfrak{a}_1\mathfrak{e}^{\mathfrak{p}_1(\mathfrak{z})} - \mathfrak{a}_2\mathfrak{e}^{\mathfrak{p}_2(\mathfrak{z})}}{\mathfrak{e}^{\mathfrak{p}_1(\mathfrak{z})} - \mathfrak{e}^{\mathfrak{p}_2(\mathfrak{z})}}. \tag{3.6}$$

Subcase 3.1: Presuming that $\mathfrak{p}_1(\mathfrak{z})$ and $\mathfrak{p}_2(\mathfrak{z})$ both polynomials are constants. Now from (3.6) we see that $\mathfrak{g}(\mathfrak{z})$ is also a constant, so is not true.

Subcase 3.2: Now, for this case, without loss of generality we will assume that $\mathfrak{p}_2(\mathfrak{z})$ is constant between $\mathfrak{p}_1(\mathfrak{z})$ and $\mathfrak{p}_2(\mathfrak{z})$. Now, using (3.6) we get

$$\mathcal{T}(\mathfrak{r},\mathfrak{g}) = \mathcal{T}(r,\mathfrak{e}^{\mathfrak{p}_1(\mathfrak{z})}) + S(r,\mathfrak{e}^{\mathfrak{p}_1(\mathfrak{z})}) \tag{3.7}$$

and

$$T(r,\mathfrak{e}^{p_2}) = S(r,\mathfrak{e}^{\mathfrak{p}_1(\mathfrak{z})}). \tag{3.8}$$

Now from (3.3) let $\mathcal{P}(\mathfrak{z},\mathfrak{g}) = (\Delta^2\mathfrak{g}(\mathfrak{z}) - \mathfrak{a}_2) - \mathfrak{e}^{\mathfrak{p}_2}(\mathfrak{g} - \mathfrak{a}_2)$. Because $\mathfrak{e}^{\mathfrak{p}_2}$ is constant, $\mathcal{P}(\mathfrak{z},\mathfrak{g})$ is a polynomial in $\mathfrak{g}(\mathfrak{z})$ and its shifts whose coefficients are small functions of $\mathfrak{g}(\mathfrak{z})$. From (3.3), we have $\mathcal{P}(\mathfrak{z},\mathfrak{g}) = 0$. So $\mathcal{P}(\mathfrak{z},\mathfrak{a}_1) = \Delta^2\mathfrak{a}_1(\mathfrak{z}) - \mathfrak{a}_2 - \mathfrak{e}^{\mathfrak{p}_2}(\mathfrak{a}_1 - \mathfrak{a}_2)$. We assert that $\mathcal{P}(\mathfrak{z},\mathfrak{a}_1) \neq 0$, on the other hand, presuming $\mathcal{P}(\mathfrak{z},\mathfrak{a}_1) = 0$ then $\mathfrak{e}^{\mathfrak{p}_2} = \frac{-\mathfrak{a}_2}{\mathfrak{a}_1 - \mathfrak{a}_2}$. From (3.6) we obtain

$$\mathfrak{g}(\mathfrak{z}) - \mathfrak{a}_1 = \frac{\mathfrak{a}_2 - a_1}{\mathfrak{e}^{\mathfrak{p}_1(\mathfrak{z})} - \mathfrak{e}^{\mathfrak{p}_2(\mathfrak{z})}}. \tag{3.9}$$

Combining (3.9) with (3.2) we get

$$\Delta^2\mathfrak{g}(\mathfrak{z}) = \frac{\mathfrak{a}_2\mathfrak{e}^{\mathfrak{p}_1} - \mathfrak{a}_1\mathfrak{e}^{\mathfrak{p}_2(\mathfrak{z})}}{\mathfrak{e}^{\mathfrak{p}_1(\mathfrak{z})} - \mathfrak{e}^{\mathfrak{p}_2(\mathfrak{z})}}. \tag{3.10}$$

From (3.9), we get

$$\left.\begin{aligned} \Delta^2(\mathfrak{g}(\mathfrak{z}) - \mathfrak{a}_1) &= \mathfrak{g}(\mathfrak{z}+2\mathfrak{c}) - \mathfrak{a}_1(\mathfrak{z}+2\mathfrak{c}) - 2\mathfrak{g}(\mathfrak{z}+\mathfrak{c}) + \mathfrak{g}(\mathfrak{z}) - \mathfrak{a}_1(\mathfrak{z}),\\ \Delta^2(\mathfrak{g}(\mathfrak{z}) - \mathfrak{a}_1) + \mathfrak{a}_1(\mathfrak{z}+2\mathfrak{c}) + \mathfrak{a}_1(\mathfrak{z}) &= \mathfrak{g}(\mathfrak{z}+2\mathfrak{c}) - 2\mathfrak{g}(\mathfrak{z}+\mathfrak{c}) + \mathfrak{g}(\mathfrak{z}),\\ &= \Delta^2\mathfrak{g}(\mathfrak{z}). \end{aligned}\right\} \tag{3.11}$$

From (3.10) and (3.11)

$$\Delta^2(\mathfrak{g}(\mathfrak{z}) - \mathfrak{a}_1) + \mathfrak{a}_1(\mathfrak{z}+2\mathfrak{c}) + \mathfrak{a}_1(\mathfrak{z}) = \frac{\mathfrak{a}_2\mathfrak{e}^{\mathfrak{p}_1(\mathfrak{z})} - \mathfrak{a}_1\mathfrak{e}^{\mathfrak{p}_2(\mathfrak{z})}}{\mathfrak{e}^{\mathfrak{p}_1(\mathfrak{z})} - \mathfrak{e}^{\mathfrak{p}_2(\mathfrak{z})}}, \tag{3.12}$$

$$\mathcal{T}\left(\mathfrak{r}, \Delta^2(\mathfrak{g}(\mathfrak{z}) - \mathfrak{a}_1) + \mathfrak{a}_1(\mathfrak{z}+2\mathfrak{c}) + \mathfrak{a}_1(\mathfrak{z})\right) = \mathcal{T}\left(r, \frac{\mathfrak{a}_2\mathfrak{e}^{\mathfrak{p}_1} - \mathfrak{a}_1\mathfrak{e}^{\mathfrak{p}_2}}{\mathfrak{e}^{\mathfrak{p}_1(\mathfrak{z})} - \mathfrak{e}^{\mathfrak{p}_2(\mathfrak{z})}}\right), \tag{3.13}$$

$$\mathcal{T}(\mathfrak{r},\mathfrak{g}) = \mathcal{S}(\mathfrak{r},\mathfrak{g}),$$

we arrive at a contradiction.

Subcase 3.3: If both $\mathfrak{p}_1(\mathfrak{z})$ and $\mathfrak{p}_2(\mathfrak{z})$ are non-constant, then $\mathfrak{p}_1'(\mathfrak{z}) \neq 0$ and $\mathfrak{p}_2'(\mathfrak{z}) \neq 0$. By (3.2) we write

$$(\Delta^2\mathfrak{g}(\mathfrak{z}) - \mathfrak{a}_1) = \mathfrak{e}^{\mathfrak{p}_1(\mathfrak{z})}(\mathfrak{g} - \mathfrak{a}_1). \tag{3.14}$$

Differentiating (3.14), we get

$$(\Delta^2 \mathfrak{g}(\mathfrak{z}))' = e^{\mathfrak{p}_1(\mathfrak{z})}\mathfrak{p}_1'(\mathfrak{z})(\mathfrak{g}-\mathfrak{a}_1) + e^{\mathfrak{p}_1(\mathfrak{z})}\mathfrak{g}',$$

$$\frac{(\Delta^2 \mathfrak{g}(\mathfrak{z}))'}{\Delta^2 \mathfrak{g}(\mathfrak{z})-\mathfrak{a}_1} = \frac{e^{\mathfrak{p}_1(\mathfrak{z})}\mathfrak{p}_1'(\mathfrak{z})(\mathfrak{g}-\mathfrak{a}_1) + e^{\mathfrak{p}_1(\mathfrak{z})}\mathfrak{g}'}{\Delta^2 \mathfrak{g}(\mathfrak{z})-\mathfrak{a}_1}.$$

Now, we get

$$\frac{(\Delta^2 \mathfrak{g}(\mathfrak{z}))'}{\Delta^2 \mathfrak{g}(\mathfrak{z})-\mathfrak{a}_1} = \mathfrak{p}_1'(\mathfrak{z}) + \frac{\mathfrak{g}'}{\mathfrak{g}-\mathfrak{a}_1}.$$

This implies

$$\mathfrak{p}_1'(\mathfrak{z}) = \frac{\left(\Delta^2 \mathfrak{g}(\mathfrak{z})\right)'}{\Delta^2 \mathfrak{g}(\mathfrak{z})-\mathfrak{a}_1} - \frac{\mathfrak{g}'}{\mathfrak{g}-\mathfrak{a}_1}, \tag{3.15}$$

where $\mathfrak{p}_1'(\mathfrak{z})$ is an entire function, since $\mathfrak{p}_1(\mathfrak{z})$ is a polynomial. By (3.15) and (3.1) we obtain

$$\mathcal{T}(\mathfrak{r},\mathfrak{p}_1'(\mathfrak{z})) = m(r,\mathfrak{p}_1'(\mathfrak{z})) \le S(r,\Delta^2\mathfrak{g}(\mathfrak{z})) + \mathcal{S}(\mathfrak{r},\mathfrak{g}) = \mathcal{S}(\mathfrak{r},\mathfrak{g}). \tag{3.16}$$

From (3.15) we obtain

$$\begin{aligned}\frac{\mathfrak{p}_1'(\mathfrak{z})}{\mathfrak{g}-\mathfrak{a}_2} &= \frac{\left(\Delta^2 \mathfrak{g}(\mathfrak{z})\right)'}{(\mathfrak{g}-\mathfrak{a}_2)(\Delta^2 \mathfrak{g}(\mathfrak{z})-\mathfrak{a}_1)} - \frac{\mathfrak{g}'}{(\mathfrak{g}-\mathfrak{a}_2)(\mathfrak{g}-\mathfrak{a}_1)}\\ &= \frac{\Delta^2 \mathfrak{g}(\mathfrak{z})}{(\mathfrak{g}-\mathfrak{a}_2)}\frac{(\Delta^2 \mathfrak{g}(\mathfrak{z}))'}{\Delta^2 \mathfrak{g}(\mathfrak{z})(\Delta^2 \mathfrak{g}(\mathfrak{z})-\mathfrak{a}_1)} - \frac{\mathfrak{g}'}{(\mathfrak{g}-\mathfrak{a}_2)(\mathfrak{g}-\mathfrak{a}_1)}.\end{aligned}$$

From the equation (3.1), Lemma 2.2 and Lemma 2.6, we obtain

$$m\left(\mathfrak{r},\frac{\mathfrak{p}_1'(\mathfrak{z})}{\mathfrak{g}-\mathfrak{a}_2}\right) = \mathcal{S}(\mathfrak{r},\mathfrak{g}). \tag{3.17}$$

From (3.16) and (3.17) we get

$$\begin{aligned}\mathfrak{m}\left(r,\frac{1}{\mathfrak{g}-\mathfrak{a}_2}\right) &\le \mathfrak{m}\left(\mathfrak{r},\frac{\mathfrak{p}_1'(\mathfrak{z})}{\mathfrak{g}-\mathfrak{a}_2}\right) + \mathfrak{m}\left(\mathfrak{r},\frac{1}{\mathfrak{p}_1'(\mathfrak{z})}\right)\\ &\le \mathcal{S}(\mathfrak{r},\mathfrak{g}) + \mathcal{T}\left(\mathfrak{r},\mathfrak{p}_1'(\mathfrak{z})\right).\end{aligned}$$

Therefore

$$\mathfrak{m}\left(\mathfrak{r},\frac{1}{\mathfrak{g}-\mathfrak{a}_2}\right) \le \mathcal{S}(\mathfrak{r},\mathfrak{g}). \tag{3.18}$$

Also, in the similar way, we obtain

$$\mathfrak{m}\left(\mathfrak{r},\frac{1}{\mathfrak{g}-\mathfrak{a}_1}\right) \le \mathcal{S}(\mathfrak{r},\mathfrak{g}). \tag{3.19}$$

By (3.2), (3.19) and the Lemma 2.2, we obtain

$$\begin{aligned}\mathcal{T}(\mathfrak{r},e^{\mathfrak{p}_1(\mathfrak{z})}) &= \mathfrak{m}(\mathfrak{r},e^{\mathfrak{a}_1(\mathfrak{z})}) \qquad (3.20)\\ &\le \mathfrak{m}\left(\mathfrak{r},\frac{\Delta^2\mathfrak{g}(\mathfrak{z})}{\mathfrak{g}-\mathfrak{a}_1}\right) + m\left(\mathfrak{r},\frac{1}{\mathfrak{g}-\mathfrak{a}_1}\right) = \mathcal{S}(\mathfrak{r},\mathfrak{g}).\end{aligned}$$

Also for $\mathcal{T}(\mathfrak{r},e^{\mathfrak{p}_2(\mathfrak{z})})$, we obtain

$$\mathcal{T}(\mathfrak{r},e^{\mathfrak{p}_2(\mathfrak{z})}) = \mathcal{S}(\mathfrak{r},\mathfrak{g}). \tag{3.21}$$

By (3.6), (3.20) and (3.21), we obtain $\mathcal{T}(\mathfrak{r},\mathfrak{g}) = \mathcal{S}(\mathfrak{r},\mathfrak{g})$, we arrive at a contradiction. Therefore by Cases 1, 2 and 3, concluding that $\Delta^2\mathfrak{g}(\mathfrak{z}) \equiv \mathfrak{g}(\mathfrak{z})$. □

Theorem 3.2. *Let $\mathfrak{f}(\mathfrak{z})$ be a non-constant meromorphic function. Let $\mathfrak{a}_1, \mathfrak{a}_2 \in S(\mathfrak{f})$ such that $\mathfrak{f}(\mathfrak{z})$ and $\Delta\mathfrak{f}(\mathfrak{z})$ partially share $\mathfrak{a}_1, \mathfrak{a}_2$ CM, $\mathfrak{a}_3 = \tau$, If*

$$\overline{\mathfrak{E}}(\mathfrak{a}_i, \mathfrak{f}(\mathfrak{z})) \subseteq \overline{\mathfrak{E}}(\mathfrak{a}_i, \Delta\mathfrak{f}(\mathfrak{z})), \quad \textit{for } \mathfrak{j} = 1,2,$$

and

$$\textit{for all } \nu \in \left[0, \frac{1}{3}\right) \textit{ and } 0 < \epsilon < \frac{1}{4}, \; \theta(\mathfrak{a}_1(\mathfrak{z}), \mathfrak{g}) + \theta(\mathfrak{a}_2(\mathfrak{z}), \mathfrak{g}) + \theta(\tau^2, \mathfrak{g}) > (3 - 3\nu - \epsilon),$$

then $\Delta\mathfrak{f}(\mathfrak{z}) \equiv \mathfrak{f}(\mathfrak{z})$.

Proof. By the assumption that $\Delta\mathfrak{f}(\mathfrak{z})$ and $\mathfrak{f}(\mathfrak{z})$ share $\mathfrak{a}$ CM, we have that

$$\mathcal{N}\left(\mathfrak{r}, \frac{1}{\Delta\mathfrak{f} - \mathfrak{a}_1(\mathfrak{z})}\right) = \mathcal{N}\left(\mathfrak{r}, \frac{1}{\mathfrak{f} - \mathfrak{a}_1(\mathfrak{z})}\right) = \mathcal{S}(\mathfrak{r}, \mathfrak{f}). \tag{3.22}$$

Similarly, we can write for $\Delta\mathfrak{f}(\mathfrak{z})$ and $\mathfrak{f}(\mathfrak{z})$ share a_2 CM as

$$\mathcal{N}\left(\mathfrak{r}, \frac{1}{\Delta\mathfrak{f} - \mathfrak{a}_2(\mathfrak{z})}\right) = \mathcal{N}\left(\mathfrak{r}, \frac{1}{\mathfrak{f} - \mathfrak{a}_2(\mathfrak{z})}\right) = \mathcal{S}(\mathfrak{r}, \mathfrak{f}). \tag{3.23}$$

It suffices to show that $\Delta\mathfrak{g}(\mathfrak{z}) = \mathfrak{g}(\mathfrak{z})$. Since $\mathfrak{g}(\mathfrak{z})$ and $\Delta\mathfrak{g}(\mathfrak{z})$ share $0, \infty$ CM

$$\frac{\Delta\mathfrak{g}(\mathfrak{z})}{\mathfrak{g}(\mathfrak{z})} = \tau.$$

By Lemma 2.4 and Definition 1.1 we have

$$\overline{E}(\mathfrak{a}_1, \mathfrak{g}(\mathfrak{z})) \subseteq \overline{E}(\mathfrak{a}_1, \Delta\mathfrak{g}(\mathfrak{z}))$$

and

$$\overline{E}(\mathfrak{a}_2, \mathfrak{g}(\mathfrak{z})) \subseteq \overline{E}(\mathfrak{a}_2, \Delta\mathfrak{g}(\mathfrak{z})).$$

Hence by Lemma 2.5, for any $\mathfrak{a} \in S(\mathfrak{f})$ we have

$$\begin{aligned}
\overline{\mathcal{N}}\left(\mathfrak{r}, \frac{1}{\Delta\mathfrak{g} - \mathfrak{a}}\right) &\le \overline{\mathcal{N}}\left(\mathfrak{r} + |\mathfrak{c}|, \frac{1}{\mathfrak{g} - \mathfrak{a}}\right) \\
&= \overline{\mathcal{N}}\left(\mathfrak{r}, \frac{1}{\mathfrak{g} - \mathfrak{a}}\right) + o\left(\overline{\mathcal{N}}\left(\mathfrak{r}, \frac{1}{\mathfrak{g} - \mathfrak{a}}\right)\right) \\
&= \overline{\mathcal{N}}\left(\mathfrak{r}, \frac{1}{\mathfrak{g} - \mathfrak{a}}\right) + \mathcal{S}(\mathfrak{r}, \mathfrak{f}).
\end{aligned}$$

Case 1: If $\tau = 1$. Then, it is clear that

$$\Delta\mathfrak{g}(\mathfrak{z}) \equiv \mathfrak{g}(\mathfrak{z}). \tag{3.24}$$

Case 2: $\tau \neq 1$. By Lemma 2.1, Lemma 2.4 and Definition 1.1, we get

$$\begin{aligned}
(1 - 3\nu - \epsilon)\mathcal{T}(\mathfrak{r}, \mathfrak{g}) &\le \sum_{j=1}^{3} \overline{\mathcal{N}}\left(r, \frac{1}{\mathfrak{g} - \mathfrak{a}_j}\right) + \mathcal{S}(\mathfrak{r}, \mathfrak{g}) \\
&\le \overline{\mathcal{N}}\left(r, \frac{1}{\mathfrak{g} - \mathfrak{a}_1}\right) + \overline{\mathcal{N}}\left(r, \frac{1}{\mathfrak{g} - \mathfrak{a}_2}\right) + \overline{\mathcal{N}}\left(r, \frac{1}{\mathfrak{g} - \mathfrak{a}_3}\right) + \mathcal{S}(\mathfrak{r}, \mathfrak{g}) \\
&\le 1 - \frac{\overline{\mathcal{N}}\left(r, \frac{1}{\mathfrak{g} - \mathfrak{a}_1}\right)}{\mathcal{T}(\mathfrak{r}, \mathfrak{g})} + 1 - \frac{\overline{\mathcal{N}}\left(r, \frac{1}{\mathfrak{g} - \mathfrak{a}_2}\right)}{\mathcal{T}(\mathfrak{r}, \mathfrak{g})} + 1 - \frac{\overline{\mathcal{N}}\left(r, \frac{1}{\mathfrak{g} - \mathfrak{a}_3}\right)}{\mathcal{T}(\mathfrak{r}, \mathfrak{g})} - 3 + \mathcal{S}(\mathfrak{r}, \mathfrak{g})
\end{aligned}$$

$$\leq 1-\lim_{r\to\infty}\frac{\overline{\mathcal{N}}\left(r,\frac{1}{\mathfrak{g}-\mathfrak{a}_1}\right)}{\mathcal{T}(\mathfrak{r},\mathfrak{g})}+1-\lim_{r\to\infty}\frac{\overline{\mathcal{N}}\left(r,\frac{1}{\mathfrak{g}-\mathfrak{a}_2}\right)}{\mathcal{T}(\mathfrak{r},\mathfrak{g})}+1-\lim_{r\to\infty}\frac{\overline{\mathcal{N}}\left(r,\frac{1}{\mathfrak{g}-\mathfrak{a}_3}\right)}{\mathcal{T}(\mathfrak{r},\mathfrak{g})}-3+\mathcal{S}(\mathfrak{r},\mathfrak{g})$$

$$\leq 1-\lim_{r\to\infty}\frac{\overline{\mathcal{N}}\left(r,\frac{1}{\mathfrak{g}-\mathfrak{a}_1}\right)}{\mathcal{T}(\mathfrak{r},\mathfrak{g})}+1-\lim_{r\to\infty}\frac{\overline{\mathcal{N}}\left(r,\frac{1}{\mathfrak{g}-\mathfrak{a}_2}\right)}{\mathcal{T}(\mathfrak{r},\mathfrak{g})}+1-\lim_{r\to\infty}\frac{\overline{\mathcal{N}}\left(r,\frac{1}{\mathfrak{g}-\tau}\right)}{\mathcal{T}(\mathfrak{r},\mathfrak{g})}-3+\mathcal{S}(\mathfrak{r},\mathfrak{g})$$

$$\leq 1-\lim_{r\to\infty}\frac{\overline{\mathcal{N}}\left(r,\frac{1}{\mathfrak{g}-\mathfrak{a}_1}\right)}{\mathcal{T}(\mathfrak{r},\mathfrak{g})}+1-\lim_{r\to\infty}\frac{\overline{\mathcal{N}}\left(r,\frac{1}{\mathfrak{g}-\mathfrak{a}_2}\right)}{\mathcal{T}(\mathfrak{r},\mathfrak{g})}+1-\lim_{r\to\infty}\frac{\overline{\mathcal{N}}\left(r,\frac{1}{\frac{\Delta\mathfrak{g}(\mathfrak{z})-\tau^2}{\tau}}\right)}{\mathcal{T}(\mathfrak{r},\mathfrak{g})}-3+\mathcal{S}(\mathfrak{r},\mathfrak{g})$$

$$\leq 1-\lim_{r\to\infty}\frac{\overline{\mathcal{N}}\left(r,\frac{1}{\mathfrak{g}-\mathfrak{a}_1}\right)}{\mathcal{T}(\mathfrak{r},\mathfrak{g})}+1-\lim_{r\to\infty}\frac{\overline{\mathcal{N}}\left(r,\frac{1}{\mathfrak{g}-\mathfrak{a}_2}\right)}{\mathcal{T}(\mathfrak{r},\mathfrak{g})}+1-\lim_{r\to\infty}\frac{\overline{\mathcal{N}}\left(r,\frac{\tau}{\Delta\mathfrak{g}(\mathfrak{z})-\tau^2}\right)}{\mathcal{T}(\mathfrak{r},\mathfrak{g})}-3+\mathcal{S}(\mathfrak{r},\mathfrak{g})$$

$$\leq 1-\lim_{r\to\infty}\frac{\overline{\mathcal{N}}\left(r,\frac{1}{\mathfrak{g}-\mathfrak{a}_1}\right)}{\mathcal{T}(\mathfrak{r},\mathfrak{g})}+1-\lim_{r\to\infty}\frac{\overline{\mathcal{N}}\left(r,\frac{1}{\mathfrak{g}-\mathfrak{a}_2}\right)}{\mathcal{T}(\mathfrak{r},\mathfrak{g})}+1-\lim_{r\to\infty}\frac{\overline{\mathcal{N}}\left(r,\frac{1}{\Delta\mathfrak{g}(\mathfrak{z})-\tau^2}\right)}{\mathcal{T}(\mathfrak{r},\mathfrak{g})}-3+\mathcal{S}(\mathfrak{r},\mathfrak{g}),$$

$$(3-3\nu-\epsilon)\leq\theta(\mathfrak{a}_1(\mathfrak{z}),\mathfrak{g})+\theta(\mathfrak{a}_2(\mathfrak{z}),\mathfrak{g})+\theta(\tau^2,\mathfrak{g})+\mathcal{S}(\mathfrak{r},\mathfrak{g}), \tag{3.25}$$

which contradicts our assumption. Hence the proof. □

Acknowledgement

This work is supported by Directorate of Minorities' Fellowship to minority students pursuing Ph.D 2019-20 (DOM/FELLOWSHIP/CR-11/2019-20).

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] A. Banerjee and S. Maity, Meromorphic function partially shares small functions or values with its linear c-shift operator, *Bulletin of the Korean Mathematical Society* **58**(5) (2021), 1175 – 1192, DOI: 10.4134/BKMS.b200840.

[2] K. S. Charak, R. J. Korhonen and G. Kumar, A note on partial sharing of values of meromorphic functions with their shifts, *Journal of Mathematical Analysis and Applications* **435**(2) (2016), 1241 – 1248, DOI: 10.1016/j.jmaa.2015.10.069.

[3] Z. X. Chen and H. X. Yi, On sharing values of meromorphic functions and their differences, *Results in Mathematics* **63**(1) (2013), 557 – 565, DOI: 10.1007/s00025-011-0217-7.

[4] Y.-M. Chiang and S.-J. Feng, On the Nevanlinna characteristic of $f(z+\eta)$ and difference equations in the complex plane, *The Ramanujan Journal* **16**(1) (2008), 105 – 29, DOI: 10.1007/s11139-007-9101-1.

[5] G. G. Gundersen, Meromorphic functions that share three values IM and a fourth value CM, *Complex Variables, Theory and Application: An International Journal* **20**(1-4) (1992), 99 – 106, DOI: 10.1080/17476939208814590.

[6] R. G. Halburd and R. J. Korhonen, Difference analogue of the lemma on the logarithmic derivative with applications to difference equations, *Journal of Mathematical Analysis and Applications* **314**(2) (2006), 477 – 87, DOI: 10.1016/j.jmaa.2005.04.010.

[7] W. K. Hayman and E. F. Lingham, Meromorphic functions, in: *Research Problems in Function Theory. Problem Books in Mathematics*, pp. 1 – 22, Springer, Cham., (2019), DOI: 10.1007/978-3-030-25165-9_1.

[8] J. Heittokangas, R. Korhonen, I. Laine and J. Rieppo, Uniqueness of meromorphic functions sharing values with their shifts, *Complex Variables and Elliptic Equations: An International Journal* **56**(1-4) (2011), 81 – 92, DOI: 10.1080/17476930903394770.

[9] J. Heittokangas, R. Korhonen, I. Laine,J. Rieppo, and J. Zhang, Value sharing results for shifts of meromorphic functions and sufficient conditions for periodicity, *Journal of Mathematical Analysis and Applications* **355**(1) (2009), 352 – 363, DOI: 10.1016/j.jmaa.2009.01.053.

[10] Z.-B. Huang and R.-R. Zhang, Uniqueness of the differences of meromorphic functions, *Analysis Mathematica* **44**(4) (2018), 461 – 473, DOI: 10.1007/s10476-018-0306-x.

[11] Y. Jiang and Z. Chen, Meromorphic functions share two values with its difference operator, *Analele Stiintifice ale Universitatii Al I Cuza din Iasi - Matematica* **63**(1) (2017), 169 – 175, DOI: 10.2478/aicu-2014-0056.

[12] I. Laine, *Nevanlinna Theory and Complex Differential Equations*, De Gruyter Studies in Mathematics series, Vol. **15** De Gruyter, Berlin—New York (1993), DOI: 10.1515/9783110863147.

[13] K. Liu, Meromorphic functions sharing a set with applications to difference equations, *Journal of Mathematical Analysis and Applications* **359**(1) (2009), 384 – 393, DOI: 10.1016/j.jmaa.2009.05.061.

[14] F. Lü and W. Lü, Meromorphic functions sharing three values with their difference operators, *Computational Methods and Function Theory* **17**(3) (2017), 395 – 403, DOI: 10.1007/s40315-016-0188-5.

[15] V. Noulorvang and D. T. Pham, On partial value sharing results of meromorphic functions with their shifts and its applications, *Bulletin of the Korean Mathematical Society* **57**(5) (2020), 1083 – 1094, DOI: 10.4134/BKMS.b190483.

[16] K. Yamanoi, The second main theorem for small functions and related problems, *Acta Mathematica* **192**(2) (2004), 225 – 294, DOI: 10.1007/BF02392741.

[17] C. C. Yang and H. X. Yi, Uniqueness theory of meromorphic functions, *Springer Science and Business Media*, Springer, **557** (2003), DOI: 10.1007/978-94-017-3626-8.

[18] J. Zhang and L. Liao, Entire functions sharing some values with their difference operators, *Science China Mathematics* **57**(10) (2014), 2143 – 2152, DOI: 10.1007/s11425-014-4848-5.

[19] J. Zhang and R. Korhonen, On the Nevanlinna characteristic of $f(qz)$ and its applications, *Journal of Mathematical Analysis and Applications* **369**(2) (2010), 537 – 544, DOI: 10.1016/j.jmaa.2010.03.038.

[20] J. Zhang, Value distribution and shared sets of differences of meromorphic functions, *Journal of Mathematical Analysis and Applications* **367**(2) (2010), 401 – 408, DOI: 10.1016/j.jmaa.2010.01.038.

[21] L. Zhen, Meromorphic Functions sharing three polynomials with their difference operators, *Journal of Contemporary Mathematical Analysis (Armenian Academy of Sciences)* **54**(5) (2019), 296 – 301, DOI: 10.3103/S1068362319050066.