Communications in Mathematics and Applications
Vol. 13, No. 3, pp. 843–850, 2022
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications
DOI: 10.26713/cma.v13i3.1859

http://www.rgnpublications.com

Research Article

Semicontinuity, Semiconnected and Semicompactness in Bitopological Spaces

M. Arunmaran* and K. Kannan

Department of Mathematics and Statistics, University of Jaffna, Jaffna, Sri Lanka
***Corresponding author:** arunmaran@univ.jfn.ac.lk

Received: March 29, 2022 **Accepted:** August 8, 2022

Abstract. Let τ_1 and τ_2 be two topologies(same or distinct) which are defined in a nonempty set X. Then, the triple (X, τ_1, τ_2) is called as a bitopological space. The objective of this paper is to establish some results which are related with semi compactness in bitopological spaces and discuss the relationships between semi continuous function in bitopological space and various topological spaces. In particular, we identify the relationship between the bitopological spaces and their product space in semi compactness. Throughout this paper, we are able to get the clear understanding about the concept 'semi compactness' and how to connect this concept with topological spaces and bitopological space. In addition, we can identify how to connect the continuous maps and product spaces with semi compactness. In addition, we can identify the relationships between semi continuous function in bitopological space and various topological spaces.

Keywords. Bitopological space, $\tau_1\tau_2$-δ semi continuous, $\tau_1\tau_2$-δ semi connectedness, $\tau_1\tau_2$-δ semi compactness.

Mathematics Subject Classification (2020). 54A93, 54A96, 54A99

1. Introduction

Kelly introduced this concept "bitopological spaces" in 1963 [7]. He established the definition for bitopological space by using the theory of asymmetric metric spaces. He established this concept in his journal of London Mathematical Society in the year mentioned above. After that scholars extend the concepts which are related with bitopological space. At first, Maheswari and Prasad introduced semi open sets in bitopological spaces in 1977 [9]. In 1987, Banerjee [3] initiated the notion δ-open sets in bitopological spaces. After that, Khedr [8] introduced and studied about $\tau_1\tau_2$-δ open sets. Later, Fukutake [6] defined one kind of semi open sets in bitopological spaces and studied their properties in 1989. Recently, Samuel and Balan [13] introduced $\tau_1\tau_2$-δ semi open sets in bitopological spaces. A quasimetric on a set X is a non-negative real valued function $p(\,,\,)$ on the product $X \times X$ satisfies the following three axioms:

(i) $p(x,x) = 0$, for all $x \in X$,

(ii) $p(x,z) \leq p(x,y) + p(y,z)$, for all $x,y,z \in X$,

(iii) $p(x,y) = 0$ if and only if $x = y$, for all $x,y \in X$.

However, the symmetric property does not hold for quasi-metric. Furthermore, every metric space is a quasi-metric space. But the converse is not true. A topological space occurs for every metric space. But bitopological spaces do not exist for every metric space. It occurs only for quasi metric spaces or asymmetric metric spaces.

In this paper, we are going to discuss the following results:

(i) A bitopological space (X,τ_1,τ_2) is $\tau_1\tau_2$-δ semi connected if and only if there is no $\tau_1\tau_2$-δ semi continuous mapping $f:(X,\tau_1,\tau_2) \to (\{0,1\},\{0,1\})$ is surjective, where $(\{0,1\},\{0,1\})$ is the discrete topological space on $\{0,1\}$.

(ii) Let (X,τ_1,τ_2) be $\tau_1\tau_2$-δ semi connected space and (Y,σ) be a topological space. If $f:(X,\tau_1,\tau_2) \to (Y,\sigma)$ is surjective and $\tau_1\tau_2$-δ semi continuous mapping then (Y,σ) is a δ-connected topological space.

(iii) Let (X,τ_1,τ_2) be $\tau_1\tau_2$-δ semi connected space and $(\mathbb{R},\rho)$ be the usual topological space. If $f:(X,\tau_1,\tau_2) \to (\mathbb{R},\rho)$ is $\tau_1\tau_2$-δ semi continuous function then $f(X)$ is an interval in $\mathbb{R}$.

(iv) Let $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be a pairwise δ-continuous surjection and pairwise δ-open mapping. Then, the image of a $\tau_1\tau_2$-δ semi compact space under f is $\sigma_1\sigma_2$-δ semi compact.

(v) If a bitopological space (X,τ_1,τ_2) is $\tau_1\tau_2$-δ semi compact and topological spaces (X,τ_1) and (X,τ_2) are δ-Hausdorff space then $\tau_{1s} = \tau_{2s}$.

(vi) The product space $(X \times Y, \tau_1 \times \sigma_1, \tau_2 \times \sigma_2)$ is $\tau_1 \times \sigma_1 \tau_2 \times \sigma_2$-$\delta$ semi compact space, if both (X,τ_1,τ_2) and (Y,σ_1,σ_2) are $\tau_1\tau_2$-δ semi compact and $\sigma_1\sigma_2$-δ semi compact, respectively.

2. Preliminaries

Definition 2.1 ([7]). Let X be a nonempty set. Let τ_1 and τ_2 be two topologies (τ_1 and τ_2 may be same or distinct) on X. Then, (X,τ_1,τ_2) is called a bitopological space.

Example 2.2. Let $X = \{a,b,c\}$. Let $\tau_1 = \{\phi, X, \{a\}\}$ and $\tau_2 = \{\phi, X, \{b\}, \{a,b\}\}$. Then, (X,τ_1,τ_2) is a bitopological space.

Definition 2.3 ([7]). Let A be subset of (X,τ_1,τ_2). Then, A is said to be open, if $A \in \tau_1 \cap \tau_2$. In (X,τ_1,τ_2), complement of open set is called closed set.

Definition 2.4 ([7]). Let A be subset of (X,τ_1,τ_2). Then, A is said to be $\tau_1\tau_2$-open, if $A \in \tau_1 \cup \tau_2$. In (X,τ_1,τ_2), complement of $\tau_1\tau_2$-open set is called $\tau_1\tau_2$-closed set.

Definition 2.5 ([11]). Let A be subset of bitopological space (X,τ_1,τ_2). Then, A is called

(i) τ_{12}-regular open, if $A = \tau_1\text{-int}(\tau_2\text{-cl}(A))$.

(ii) τ_{21}-regular open, if $A = \tau_2\text{-int}(\tau_1\text{-cl}(A))$.

(iii) $\tau_1\tau_2$-semi open, if $A \subseteq \tau_2\text{-cl}(\tau_1\text{-int}(A))$.

(iv) $\tau_1\tau_2$-semi closed, if $A \supseteq \tau_2\text{-int}(\tau_1\text{-cl}(A))$.

Let A be subset of bitopological space (X,τ_1,τ_2). Then, A is called

(i) τ_{12}-regular closed, if $A = \tau_1\text{-cl}(\tau_2\text{-int}(A))$.

(ii) τ_{21}-regular closed, if $A = \tau_2\text{-cl}(\tau_1\text{-int}(A))$.

Definition 2.6 ([11]). Let A be subset of bitopological space (X,τ_1,τ_2). Then,

(i) A is said to be τ_1-δ open set, if for $x \in A$, there exists τ_{12}-regular open set G such that $x \in G \subset A$. Complement of τ_1-δ open set is called τ_1-δ closed set.

(ii) A is said to be τ_2-δ open set, if for $x \in A$, there exists τ_{21}-regular open set G such that $x \in G \subset A$. Complement of τ_2-δ open set is called τ_2-δ closed set.

(iii) Collection of all τ_1-δ open sets and τ_2-δ open sets are denoted by τ_{1s} and τ_{2s}, respectively, and also $\tau_{1s} \subset \tau_1$ and $\tau_{2s} \subset \tau_2$.

Definition 2.7 ([7]). A bitopological space (X,τ_1,τ_2) is said to be pairwise Hausdorff, if for each two distinct points x and y, there are a τ_1-neighbourhood U of x and a τ_2-neighbourhood V of y such that $U \cap V = \phi$.

Definition 2.8 ([13]). Let A be subset of bitopological space (X,τ_1,τ_2). Then, A is called $\tau_1\tau_2$-δ semi open set, if there exists an τ_1-δ open set U such that $U \subseteq A \subseteq \tau_2\text{-cl}(U)$.

Definition 2.9 ([13]). Let A be subset of bitopological space (X,τ_1,τ_2). Then, A is called $\tau_1\tau_2$-δ semi closed set, if there exists an τ_1-δ closed set F such that $\tau_2\text{-int}(F) \subseteq A \subseteq F$.

Definition 2.10 ([4]). A function $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is said to be pairwise continuous if and only if the induced functions $f:(X,\tau_1) \to (Y,\sigma_1)$ and $f:(X,\tau_2) \to (Y,\sigma_2)$ are continuous.

Definition 2.11 ([4]). Let (X,τ_1,τ_2) and (Y,σ_1,σ_2) be two bitopological spaces. Then, a function $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is called $\tau_1\tau_2$-continuous, if the inverse image of each σ_1-open set in Y is $\tau_1\tau_2$-open set in X.

Example 2.12 ([1, 2]). Let $X = Y = \{a,b,c\}$, $\tau_1 = \{\phi,X,\{b\},\{a,b\}\}$, $\tau_2 = \{\phi,X,\{b,c\}\}$, $\sigma_1 = \{\phi,Y,\{b\},\{a,b\}\}$ and $\sigma_2 = \{\phi,Y,\{a,c\}\}$.
Now, we consider $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is defined as an identity function, then f is $\tau_1\tau_2$-continuous.

Definition 2.13 ([13])**.** Let (X,τ_1,τ_2) and (Y,σ_1,σ_2) be two bitopological spaces. Then, a function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is called $\tau_1\tau_2$-δ continuous, if the inverse image of each σ_1-open set in Y is $\tau_1\tau_2$-δ open set in X.

Example 2.14 ([1, 2])**.** Let $X = Y = \{a,b,c\}$, $\tau_1 = \{\phi,X,\{a\}\}$, $\tau_2 = \{\phi,X,\{b\},\{a,b\}\}$, $\sigma_1 = \{\phi,Y,\{a\},\{b,c\}\}$ and $\sigma_2 = \{\phi,Y,\{a,b\}\}$.
Then, $\tau_1\tau_2$-δ open sets are ϕ, X, $\{a\}$, $\{b\}$, $\{a,b\}$, $\{b,c\}$. If $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is defined as $f(a)=a$, $f(b)=b$, $f(c)=b$, then f is $\tau_1\tau_2$-δ continuous.

Definition 2.15 ([13])**.** Let (X,τ_1,τ_2) and (Y,σ_1,σ_2) be two bitopological spaces. Then, a function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is called $\tau_1\tau_2$-δ semi continuous, if $f^{-1}(V)$ is $\tau_1\tau_2$-δ semi open set in X, for every σ_1-δ open set V in Y.

Example 2.16 ([1, 2])**.** Let $X = Y = \{a,b,c\}$, $\tau_1 = \{\phi,X,\{b\},\{a,b\}\}$, $\tau_2 = \{\phi,X,\{b,c\}\}$, $\sigma_1 = \{\phi,Y,\{b\},\{a,b\}\}$ and $\sigma_2 = \{\phi,Y,\{a,c\}\}$.
Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is defined as $f(a)=a$, $f(b)=c$, $f(c)=b$, then f is $\tau_1\tau_2$-δ semi continuous.

Proposition 2.17 ([2])**.** *Every $\tau_1\tau_2$-δ continuous function is $\tau_1\tau_2$-δ semi continuous function.*

Remark 2.18 ([2])**.** The converse part of the above theorem need not be true.

Definition 2.19 ([13])**.** Any subset A of a bitopological space (X,τ_1,τ_2) is called $\tau_1\tau_2$-δ semi clopen, if it is both $\tau_1\tau_2$-δ semi open and $\tau_1\tau_2$-δ semi closed.

Definition 2.20 ([13])**.** In a bitopological space, a subset Y is called a $\tau_1\tau_2$-δ semi disconnected subset of a bitopological space (X,τ_1,τ_2), if there exist two $\tau_1\tau_2$-δ semi open sets U and V such that $U\cap Y\neq\phi\neq V\cap Y$, $U\cap V\cap Y=\phi$ and $Y\subseteq U\cup V$. Otherwise Y is called a $\tau_1\tau_2$-δ semi connected subset.

Notation 2.21.

- $\delta scl(A)$ represents δ semi closed set A.
- $\delta SO(A)$ represents δ semi open set A.

Definition 2.22 ([13])**.** A bitopological space (X,τ_1,τ_2) is called $\tau_1\tau_2$-δ semi connected space, if X cannot be expressed as the union of two non-empty disjoint sets A and B such that $\{A\cap\tau_1$-$\delta scl(B)\}\cup\{\tau_2$-$\delta scl(A)\cap B\}=\phi$. Suppose X can be so expressed, then X is called $\tau_1\tau_2$-δ semi disconnected space and we write $X=A\backslash B$ and it is said to be $\tau_1\tau_2$-δ semi separation of X.

Definition 2.23 ([13])**.** A nonempty collection $C=\{A_i : i\in I\}$ is called a $\tau_1\tau_2$-δ semi open cover of a bitopological space (X,τ_1,τ_2), if $X=\bigcup_{i\in I} A_i$ and $C\subset\tau_1$-$\delta SO(X,\tau_1,\tau_2)\cup\tau_2$-$\delta SO(X,\tau_1,\tau_2)$ and C contains at least one member of τ_1-$\delta SO(X,\tau_1,\tau_2)$ and one member of τ_2-$\delta SO(X,\tau_1,\tau_2)$.

Definition 2.24 ([13]). A cover $C = \{A_i : i \in I\}$ of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-δ open cover of X, if $C \subseteq \tau_{1s} \cup \tau_{2s}$ and $C \cap \tau_{1s} \neq \phi$, $C \cap \tau_{2s} \neq \phi$ and $X = \cup C$.

Definition 2.25 ([13]). A bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2$-δ compact, if every $\tau_1\tau_2$-δ open cover of X has a finite sub cover.

Definition 2.26 ([13]). A bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2$-δ semi compact, if every $\tau_1\tau_2$-δ semi open cover of X has a finite sub cover.

3. Main Results

Proposition 3.1. *A bitopological space (X, τ_1, τ_2) is $\tau_1\tau_2$-δ semi connected if and only if there is no $\tau_1\tau_2$-δ semi continuous surjective map $f : (X, \tau_1, \tau_2) \to (\{0,1\}, \{0,1\})$, where $(\{0,1\}, \{0,1\})$ is the discrete topological space on $\{0,1\}$.*

Proof. Assume that (X, τ_1, τ_2) is $\tau_1\tau_2$-δ semi connected. Let there exist a $\tau_1\tau_2$-δ semi continuous surjective map $f : (X, \tau_1, \tau_2) \to (\{0,1\}, \{0,1\})$. Since $\{0\}$ and $\{1\}$ both are clopen sets in $(\{0,1\}, \{0,1\})$, we define two sets A, B such that $A = f^{-1}(\{0\})$ and $B = f^{-1}(\{1\})$ which are $\tau_1\tau_2$-δ semi clopen sets and also nonempty sets as f is onto. So $X = A \cup B$ is $\tau_1\tau_2$-δ semi disconnected. This contradicts with our assumption. Thus there is no $\tau_1\tau_2$-δ semi continuous mapping.

Conversely, assume that there is no $\tau_1\tau_2$-δ semi continuous surjective map $f : (X, \tau_1, \tau_2) \to (\{0,1\}, \{0,1\})$. If (X, τ_1, τ_2) is $\tau_1\tau_2$-δ semi disconnected and $X = A \cup B$ is $\tau_1\tau_2$-δ semi disconnected. Define the map $f : (X, \tau_1, \tau_2) \to (\{0,1\}, \{0,1\})$ by $f(A) = 0$ and $f(B) = 1$. Then f is $\tau_1\tau_2$-δ semi continuous surjective map, which is a contradiction. Thus, (X, τ_1, τ_2) is $\tau_1\tau_2$-δ semi connected. □

Proposition 3.2. *Let (X, τ_1, τ_2) be $\tau_1\tau_2$-δ semi connected space and (Y, σ) be a topological space. If $f : (X, \tau_1, \tau_2) \to (Y, \sigma)$ is surjective and $\tau_1\tau_2$-δ semi continuous mapping then (Y, σ) is a δ-connected topological space.*

Proof. If (Y, σ) is δ-disconnected and $Y = A \cup B$ a disconnection of Y, then $X = f^{-1}(A) \cup f^{-1}(B)$ becomes a $\tau_1\tau_2$-δ semi disconnection of X, which is a contradiction. Thus, (Y, σ) is a δ-connected topological space. □

Proposition 3.3. *Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a pairwise δ-continuous surjection and pairwise δ-open mapping. Then, the image of a $\tau_1\tau_2$-δ semi connected space under f is $\sigma_1\sigma_2$-δ semi connected.*

Proof. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a pairwise δ-continuous surjection and pairwise δ-open mapping. Let X is $\tau_1\tau_2$-δ semi connected space Suppose that Y is $\tau_1\tau_2$-δ semi disconnected. Then $Y = A \cup B$, where A is $\tau_1\tau_2$-δ semiopen set and B is τ_2-δ semi-open set in Y. Since f is pairwise-δ continuous, we have $f^{-1}(A)$ is τ_1-δ semi open and $f^{-1}(B)$ is τ_2-δ semi open in X. Also, $X = f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are two nonempty disjoint sets. Then X is $\tau_1\tau_2$-δ semi disconnected. This is a contradiction. Thus, Y is $\tau_1\tau_2$-δ semi connected. □

Proposition 3.4. *Let (X,τ_1,τ_2) be $\tau_1\tau_2$-δ semi connected space and $(\mathbb{R},\rho)$ be the usual topological space. If $f:(X,\tau_1,\tau_2)\to(\mathbb{R},\rho)$ is $\tau_1\tau_2$-δ semi continuous function then $f(X)$ is an interval in $\mathbb{R}$.*

Proof. Assume that $f(X)$ is not an interval in $\mathbb{R}$. Then there exists $a<r<b$ such that $a,b\in f(X)$, but $r\notin f(X)$, then $X=f^{-1}(-\infty,r)\cup f^{-1}(r,\infty)$ becomes disconnected space, which is a contradiction. Therefore, $f(X)$ is an interval in $\mathbb{R}$. □

Remark 3.5. The converse of the above proposition need not be true in general.

Example 3.6. Consider a bitopological space $([0,1],\tau_1,\tau_2)$, where τ_1 is a usual topology on X induced from $\mathbb{R}$, and τ_2 is an indiscrete topology on X, then $([0,1],\tau_1,\tau_2)$ is $\tau_1\tau_2$-δ semi disconnected, since the $\tau_1\tau_2$-δ semi continuous mapping $f:([0,1],\tau_1,\tau_2)\to(\mathbb{R},\rho)$ defined by $f(x)=x$, $f(X)$ is an interval.

Proposition 3.7. *Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be a pairwise δ-continuous surjective and pairwise δ-open mapping. Then, the image of a $\tau_1\tau_2$-δ semi compact space under the map f is $\sigma_1\sigma_2$-δ semi compact.*

Proof. Consider a pairwise δ-continuous surjective and pairwise δ-open mapping $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$. Let X be $\tau_1\tau_2$-δ semi compact. Let $C=\{A_i:i\in I\}$ be a $\sigma_1\sigma_2$-δ semi open cover of Y. Then, $Y=\bigcup\limits_{i\in I}A_i$ and $C\subseteq\sigma_1$-$\delta SO(Y,\sigma_1,\sigma_2)\cup\sigma_2$-$\delta SO(Y,\sigma_1,\sigma_2)$ and C contains at least one member of σ_1-$\delta SO(Y,\sigma_1,\sigma_2)$ and one member of σ_2-$\delta SO(Y,\sigma_1,\sigma_2)$.
Therefore,

$$X=f^{-1}\Big(\bigcup_{i\in I}A_i\Big)=\bigcup_{i\in I}f^{-1}(A_i)$$

and $f^{-1}(C)\subseteq\tau_1$-$\delta SO(X,\tau_1,\tau_2)\cup\tau_2$-$\delta SO(X,\tau_1,\tau_2)$ and $f^{-1}(C)$ contains at least one member of τ_1-$\delta SO(X,\tau_1,\tau_2)$ and one member of τ_2-$\delta SO(X,\tau_1,\tau_2)$.
Therefore, $f^{-1}(C)$ is a $\tau_1\tau_2$-δ semi open cover X. Since X is $\tau_1\tau_2$-δ semi compact space, we have

$$X=\bigcup_{i=1}^{n}f^{-1}A_i\,.$$

So,

$$Y=f(X)=\bigcup_{i=1}^{n}A_i\,.$$

Hence, C has a finite sub cover. Thus, Y is $\sigma_1\sigma_2$-δ semi compact. □

Proposition 3.8. *If a bitopological space (X,τ_1,τ_2) is $\tau_1\tau_2$-δ semi compact and topological spaces (X,τ_1) and (X,τ_2) are δ-Hausdorff space then $\tau_{1s}=\tau_{2s}$.*

Proof. Let (X,τ_1) and (X,τ_2) are δ-Hausdorff space and (X,τ_1,τ_2) is $\tau_1\tau_2$-δ semi compact. Since every $\tau_1\tau_2$-δ semi compact space is $\tau_1\tau_2$-δ compact, we have (X,τ_1) and (X,τ_2) are δ-Hausdorff space and (X,τ_1,τ_2) is $\tau_1\tau_2$-δ compact. Let F be τ_{1s}-δ closed in X. Then, F^c is τ_{1s}-δ open in X. Let $C=\{A_i:i\in I\}$ be τ_2-δ open cover of X. Then, $C\cup F^c$ is $\tau_1\tau_2$-δ open cover of X. Since X is

$\tau_1\tau_2$-δ compact,

$$X = F^c \cup \Big(\bigcup_{i=1}^{n} A_i\Big).$$

Hence,

$$F = \bigcup_{i=1}^{n} A_i.$$

Thus, F is τ_2-δ compact. Since (X,τ_2) is δ-Hausdorff, we have F is τ_{2s}-δ closed. Similarly, every τ_{2s}-δ closed set is τ_{1s}-δ closed. Hence, $\tau_{1s} = \tau_{2s}$. □

Proposition 3.9. *The product space $(X\times Y, \tau_1\times\sigma_1, \tau_2\times\sigma_2)$ is $\tau_1\times\sigma_1\tau_2\times\sigma_2$-$\delta$ semi compact space, if both (X,τ_1,τ_2) and (Y,σ_1,σ_2) are $\tau_1\tau_2$-δ semi compact and $\sigma_1\sigma_2$-δ semi compact, respectively.*

Proof. Suppose that (X,τ_1,τ_2) and (Y,σ_1,σ_2) are $\tau_1\tau_2$-δ semi compact and $\sigma_1\sigma_2$-δ semi compact, respectively. Let C be a $\tau_1\times\sigma_1\tau_2\times\sigma_2$-$\delta$ semi open cover of $X\times Y$, consisting of $\tau_1\times\sigma_1\tau_2\times\sigma_2$-$\delta$ semi open sets of the form $U\times V$, where U is a $\tau_1\times\sigma_1$-δ semi open set in X and V is a $\tau_2\times\sigma_2$-δ semi open set in Y.
Let $x\in X$. Then for each $y\in Y$, there exist $(x,y)\in U_x\times V_y\in C$.
The collection $\{V_y : y\in Y\}$ is a $\sigma_1\sigma_2$-δ semi open cover of Y. But Y is $\sigma_1\sigma_2$-δ semi compact, and, this collection has a finite sub cover $\{V_{y_1}, V_{y_2}, \dots, V_{y_n}\}$.
Take $U_x = \bigcap_{i=1}^{n} V_{y_i}$. Then,

$$\{x\}\times Y \subseteq U_x\times Y = U_x \times \bigcup_{i=1}^{n} V_{y_i} = \bigcup_{i=1}^{n} (U_x\times V_{y_i}).$$

Thus, $\forall\ x\in X$, $\exists\ U_x$ such that $\{x\}\times Y\subseteq U_x\times Y$ and $U_x\times Y$ is contained in a finite number of sets in $X\times Y$. But the collection $\{U_x : x\in X\}$ covers X. Since X is $\tau_1\tau_2$-δ semi compact, this collection has a finite sub cover $\{U_{x_1}, U_{x_2}, \dots, U_{x_m}\}$.
Then,

$$X\times Y \subseteq \bigcup_{i=1}^{m} U_{x_i}\times Y = \bigcup_{i=1}^{m} (U_{x_i}\times Y).$$

But $U_{x_i}\times Y$ is a subset of union of a finite number of sets in C for each i with $1\le i\le m$. It follows that $X\times Y$ = union of a finite number of sets in C. Hence, C has a finite sub cover. Therefore, $(X\times Y, \tau_1\times\sigma_1, \tau_2\times\sigma_2)$ is $\tau_1\times\sigma_1\tau_2\times\sigma_2$-$\delta$ semi compact. □

4. Conclusion

Some results related with semi connectedness and compactness in bitopological spaces have been discussed. In particular, we have proved how the product space works for semi compactness in bitopological spaces and how to connect the pairwise continuous maps in bitopological spaces with semi compactness. These results are applicable in various parts of pure mathematics, especially in ordered topology, analysis, and general topological spaces.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] M. Arunmaran and K. Kannan, Some results of $\tau_1\tau_2$-δ semiconnectedness and compactness in bitopological spaces, *Journal of Mathematics* **2018** (2018), Article ID 7863713, DOI: 10.1155/2018/7863713.

[2] M. Arunmaran and K. Kannan, Some properties of $\tau_1\tau_2$-δ semi open/closed sets in bitopological spaces, *International Journal of Pure and Applied Mathematics* **114**(5) (2017), 759 – 769, URL: https://ijpam.eu/contents/2017-115-4/9/9.pdf.

[3] G.K. Banerjee, On pairwise almost strongly θ-continuous mapping, *Bulletin of Calcutta Mathematical Society* **79** (1987), 314 – 320, URL: https://koreascience.kr/article/JAKO197125748110791.kr&sa=U.

[4] B. Bhattacharya and A. Paul, A new approach of δ-open sets in bitopological spaces, *General Mathematics Notes* **20**(2) (2014), 95 – 110, URL: https://www.emis.de/journals/GMN/yahoo_site_admin/assets/docs/7_GMN-4582-V20N2.93181604.pdf.

[5] S. Bose, Semi open sets, semi continuity and semi open mappings in bitopological spaces, *Bulletins of Calcutta Mathematical Society* **73** (1981), 237 – 246.

[6] T. Fukutake, Semi open sets on bitopological spaces, *Bulletin of Fukuoka University Education* **38**(3) (1989), 1 – 7.

[7] J.C. Kelly, Bitopological spaces, *Proceeding London Mathematical Society* **3**(13) (1963), 71 – 89, DOI: 10.1112/plms/s3-13.1.71.

[8] F.H. Khedr, Properties of $\tau_1\tau_2$-delta open sets, *Fasciculi Mathematici* **52** (2014), 65 – 81, URL: http://fasciculi-mathematici.put.poznan.pl/artykuly/52/FM52(2014)-KhedrFH.pdf.

[9] S.N. Maheshwari and R. Prasad, Semi open sets and semi continuous functions in bitopological spaces, *Mathematical Note* **26** (1977-1978), 29 – 37.

[10] T. Noiri and V. Popa, Some properties of weakly open functions in bitopological spaces,*Novi Sad Journal Mathematics* **36**(1) (2006), 47 – 54, https://eudml.org/doc/227793.

[11] K.C. Rao and V. Subha, A note on the associated bitopological space (X,τ_1^*,τ_2^*), *International Journal of Mathematical Analysis* **7** (2003), 323 – 330, DOI: 10.12988/ijma.2013.13027.

[12] I.L. Reilly, On Pairwise connected bitopological spaces, *Kyungpook Mathematical Journal* **11**(1) (1971), URL: https://koreascience.kr/article/JAKO197125748110791.kr&sa=U.

[13] A.E. Samuel and D. Balan, *δ-Interior and δ-Closure in Bitopological Spaces*, Thesis, Bharath University, Chennai, Tamilnadu, India (2015).