Communications in Mathematics and Applications
Vol. 13, No. 1, pp. 87–101, 2022
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications
DOI: 10.26713/cma.v13i1.1808

http://www.rgnpublications.com

Research Article

Performance Analysis of $M^X/M/1$ Queue With Working Vacation, N-Policy and Customer Impatience

G. Sridhar[1], V.N. Rama Devi*[2] and K. Chandan[1]

[1] *Department of Statistics, Acharya Nagarjuna University, Guntur, Andhra Pradesh, India*
[2] *Department of Mathematics, Gokaraju Rangaraju Institute of Engineering and Technology, Hyderabad, Telanagana, India*
***Corresponding author:** ramadevivn@gmail.com

Received: December 27, 2021 **Accepted:** February 15, 2022

Abstract. This paper deals the transient analysis of a queueing system with working vacation in which the server may encounter with breakdowns. Customers enter in batches according to a Poisson process and receive service in FCFS mode. Whenever the server finds nobody, the server starts a working vacation during which the server renders service at a slower rate than the normal one. Further, we considered two types of customer's impatience balking and reneging. We solved the system of differential equations to find transient state probabilities and computed various performance indices like average queue length of system, the mean waiting time etc. We then executed sensitivity analysis and observed the impact on different parameters.

Keywords. Working vacation, Balking, Reneging, Breakdowns, N-policy

Mathematics Subject Classification (2020). 60K25, 65K30, 90B22

1. Introduction

Waiting in lines or queues are most common phenomenon in our day-to-day life. Queuing theory is the mathematical study of waiting in line within the field of operations management. Queuing theory provides mathematical models to compute performance indices of waiting lines to assess as well as to improve the flow of customers through a queuing system. As queuing theory has many applications in various manufacturing and service industries, lot of research is found on this in recent eras

Breakdowns may happen for any server which leads to customers' abandonment and impatience and also loss of production, goodwill etc. Queueing theory plays a vital role with respect to the repair of the machines to optimize the system. Gupta [6], Hsieh and Wang [7], Vemuri *et al*. [13], Jain [8], Devi *et al*. [4], and Vedala *et al*. [12] have significantly contributed this domain of server breakdowns and repair facilities.

In real life, many queueing systems encounter the problems of customer's impatient behaviours which cause loss to the server end. Many researchers like Al-Seedy *et al*. [1], Ancker *et al*. [2], Jain [8], and Wu *et al*. [15] contributed a lot towards optimizing the system with respect to the customer impatient behaviour.

Vacation Queuing models have been extensively detailed by researchers like Doshi [5], and Ke [9] where the server is assumed to remain idle and becomes completely unproductive. Studies have resulted into proper use of server's idle time as working vacation time where the server renders service during vacation also instead of being idle with a slower service rate compared with normal service times. Servi and Finn [11] were the first who have explained this concept and later many researchers, namely, Baba [3], Laxmi and Jyostna [10], Wu and Takagi [14] have significantly contributed the works in this area.

To the best of our knowledge, the existing literature focus mainly on queueing system with server breakdowns, working vacations, customers impatience in steady state fashion. In our study, we detail the transient analysis of a finite capacity Markovian queue with working vacation, server breakdowns and customers' impatience. We have used Runge-Kutta (R-K) method to find transient state probability distribution.

The rest of this paper is organized as follows: In Section 2, we present the model description. In Section 3, we compute the transient state solution. In Section 4, some system performance measures through numerical results are computed and also carried sensitivity analysis. Section 5 details final conclusions.

2. Model Description

We consider the transient analysis of a queueing system with server failures, working vacation, N-policy and customer impatience in the following manner:

(1) The capacity of the system is considered as finite (S).

(2) The arrival process is a compound Poisson process (with rate λ) of independent and identically distributed random batches of customers, where each batch size X, has a probability density function $\{a_n : a_n = P(X = n), n \geq 1\}$. Batches are served on first in first out mode.

(3) The normal mean service rate is μ.

(4) The server initiates *Working Vacation* timer (WV) immediately upon the system is empty. Parameter for working vacation times is Ø and the mean service rate is η during the working vacation mode. The server renders service in this pace up to N-1 customers and it returns to a normal service state when it attains to N customers. There is also a chance that the server will come from working vacation mode to normal mode in mean time with an optional probability of P.

(5) The server failure rates are considered as α_1 and γ_1 in WV and normal modes and respective repair rates are assumed as α_2 and γ_2.

(6) Customers are expected to either balk or renege with dissatisfaction. The probabilities of balking are considered as b_1 and b_2 in WV and normal modes and they are ξ_1 and ξ_2 in case of reneging.

3. Notations

We use the following symbolizations to denote transient probabilities for the system to be in various states:

$\pi_{1,i}^{(t)} = p(i$ customers in the system when the server is in WV mode and providing service),

$\pi_{2,i}^{(t)} = p(i$ customers in the system when the server is in WV mode and broken down),

$\pi_{3,i}^{(t)} = p(i$ customers in the system when the server is in Normal service mode and providing service),

$\pi_{4,i}^{(t)} = p(i$ customers in the system when the server is in Normal service mode but broken down).

The transient state equations governing the various probabilities are detailed in the form of following differential equations:

$$\frac{d\pi_{1,0}^{(t)}}{dt} = -(\lambda b_1 + \phi)\pi_{1,0}^{(t)} + (\eta + \xi_1)\pi_{1,1}^{(t)} + (\mu + \xi_2)\pi_{3,1}^{(t)}, \tag{3.1}$$

$$\begin{aligned}\frac{d\pi_{1,x}^{(t)}}{dt} &= -(\lambda b_1 + (1-p)\phi + \eta + (x)\xi_1 + \alpha_1)\pi_{1,x}^{(t)} + (\eta + x\xi_1)\pi_{1,x+1}^{(t)}\\ &\quad + \lambda b_1 \sum_{i=1}^{x-1} a_i \pi_{1,x-i}^{(t)} + \alpha_2 \pi_{2,x}^{(t)};\ \ 2 \le x \le N-1,\end{aligned} \tag{3.2}$$

$$\begin{aligned}\frac{d\pi_{2,x}^{(t)}}{dt} &= -(\lambda b_1 + \alpha_2)\pi_{2,x}^{(t)} + (\eta + (x+1)\xi_1)\pi_{1,x+1}^{(t)}\\ &\quad + \lambda b_1 \sum_{i=1}^{x-1} a_i \pi_{2,x-i}^{(t)} + \alpha_1 \pi_{2,x}^{(t)};\ \ 2 \le x \le N-1,\end{aligned} \tag{3.3}$$

$$\frac{d\pi_{3,0}^{(t)}}{dt} = -(\lambda b_2)\pi_{3,0}^{(t)} + p\phi\pi_{1,0}^{(t)}, \tag{3.4}$$

$$\begin{aligned}\frac{d\pi_{3,x}^{(t)}}{dt} &= -(\lambda b_2 + \ \gamma_1 + \mu + (x)\xi_2)\pi_{3,x}^{(t)} + \lambda b_2 \sum_{i=1}^{x-1} a_i \pi_{3,x-i}^{(t)} + (\mu + (x+1)\xi_2)\pi_{3,x}^{(t)}\\ &\quad + \gamma_2 \pi_{4,x}^{(t)} + p\phi\pi_{1,x}^{(t)};\ \ 1 \le x \le N-1,\end{aligned} \tag{3.5}$$

$$\begin{aligned}\frac{d\pi_{3,x}^{(t)}}{dt} &= -(\lambda b_2 + \ \gamma_1 + \mu + (x)\xi_2)\pi_{3,x}^{(t)} + \lambda b_2 \sum_{i=x-N}^{x-1} a_i \pi_{3,x-i}^{(t)}\\ &\quad + (\mu + (x+1)\xi_2)\pi_{3,x}^{(t)} + \gamma_2 \pi_{4,x}^{(t)};\ \ N \le x \le s-1,\end{aligned} \tag{3.6}$$

$$\frac{d\pi_{3,S}^{(t)}}{dt} = -(\gamma_1+\mu+S\xi_2)\pi_{3,S}^{(t)} + \lambda b_2 \sum_{i=1}^{s-1} a_i \pi_{3,s-i}^{(t)} + \gamma_2 \pi_{4,S}^{(t)}, \tag{3.7}$$

$$\frac{d\pi_{4,x}^{(t)}}{dt} = -(\lambda b_2+\gamma_2)\pi_{4,x}^{(t)} + \lambda b_2 \sum_{i=1}^{x-1} a_i \pi_{4,x-i}^{(t)} + \gamma_1 \pi_{3,x}^{(t)}; \quad 1 \le x \le S-1, \tag{3.8}$$

$$\frac{d\pi_{4,S}^{(t)}}{dt} = -(\gamma_2)\pi_{4,S}^{(t)} + \lambda b_2 \sum_{i=1}^{s-1} a_i \pi_{4,s-i}^{(t)} + \gamma_1 \pi_{3,S}^{(t)}. \tag{3.9}$$

4. Performance Measures and Numerical Results

Some queueing constants that are calculated to forecast the system through Runge-Kutta method are listed below:

(1) p (the server is in WV mode and doing service).

(2) p (the server is in WV mode and broken down).

(3) p (the server is in Normal service mode and doing service).

(4) p (the server is in Normal service mode but broken down).

(5) Expected length of the system ($L_S^{(t)}$).

(6) Mean waiting time ($W_S^{(t)}$).

In this paper, we have presented mean length of the system and mean waiting times.

MATLAB software is used to explore various probabilities and constants by developing a computational programme to solve the system of differential equations through Runge-Kutta method of order 4. The influence of various parameters on system constants is studied and are shown in Tables A.1-A.14 in Appendix. For these numerical illustrations, the model parameters are considered with the following values:

$S = 10$, $\lambda = 1.4$, $\mu = 1.8$, $\eta = 1.6$, $\phi = 0.001$, $N = 4$, $b_1 = .02$, $b_2 = .01$, $\xi_1 = .03$, $\xi_2 = .02$, $\alpha_1 = .001$, $\alpha_2 = .002$, $\gamma_1 = .002$, $\gamma_2 = .003$ and $p = 0.4$

5. Conclusion

We have presented performance measures of Markovian Queue with Working Vacation, Server Failures and Customer Impatience in transient mode and explored the following inferences:

(1) With respect to the increase in some parameters like Mean arrival rate, breakdown rates, N etc., there is a rise in Mean length as well as waiting time.

(2) With respect to the increase in some parameters like service rates, repair rates, working vacation rate etc., there is a fall in Mean length as well as waiting time.

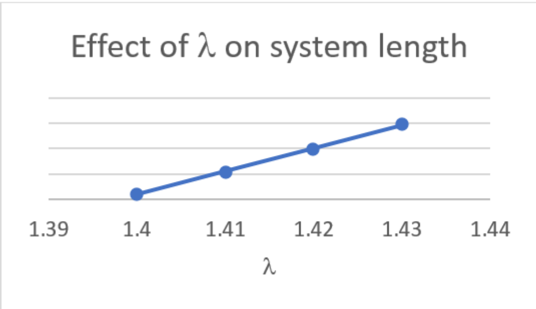
Appendix

A.1 Variation in λ

For specified range of values of λ the mean number of customers in the system ($L_s(t)$) and mean waiting time are ($W_s(t)$) are presented in Table A.1 and Figure A.1.

Table A.1. Effect of λ on $L_s(t)$ and $W_s(t)$

Parameter (λ)	t	0.5	1	1.5	2
1.40	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107090
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768747
1.41	$L_S^{(t)}$	0.000383111	0.000698370	0.000955084	0.001162356
	$W_S^{(t)}$	0.000924916	0.001726358	0.002418508	0.002847024
1.42	$L_S^{(t)}$	0.000401336	0.000731195	0.001000522	0.001218620
	$W_S^{(t)}$	0.001002118	0.001804470	0.002522145	0.003125287
1.43	$L_S^{(t)}$	0.000414681	0.000762421	0.001045911	0.001272873
	$W_S^{(t)}$	0.001046120	0.001901578	0.002624774	0.003263534



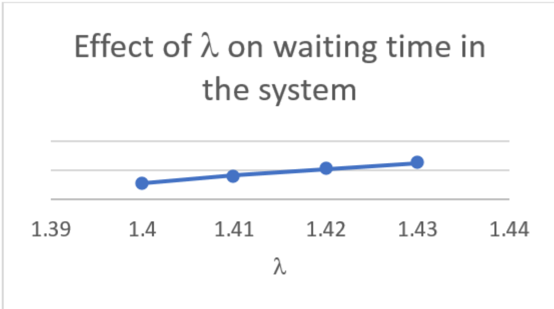


Figure A.1. Effect of λ on $L_s(t)$ and $W_s(t)$

A.2 Variation in μ

For specified range of values of μ the mean number of customers in the system ($L_s(t)$) and mean waiting time are ($W_s(t)$) are presented in Table A.2 and Figure A.2.

Table A.2. Effect of μ on $L_s(t)$ and $W_s(t)$

Parameter (μ)	t	0.5	1	1.5	2
1.80	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107090
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768747
1.81	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107089
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768747
1.82	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107089
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768746
1.83	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107089
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768746

Figure A.2. Effect of μ on $L_s(t)$ and $W_s(t)$

A.3 Variation in η

For specified range of values of the mean number of customers in the system ($L_s(t)$) and mean waiting time are ($W_s(t)$) are presented in Table A.3 and Figure A.3.

Table A.3. Effect of on $L_s(t)$ and $W_s(t)$

Parameter (η)	t	0.5	1	1.5	2
1.60	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107090
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768747
1.61	$L_S^{(t)}$	0.000364320	0.000663111	0.000905537	0.001100513
	$W_S^{(t)}$	0.000910926	0.001658174	0.002264551	0.002752291
1.62	$L_S^{(t)}$	0.000363767	0.000661092	0.000901437	0.001093984
	$W_S^{(t)}$	0.000909542	0.001653121	0.002254293	0.002735954
1.63	$L_S^{(t)}$	0.000363214	0.000659078	0.000897358	0.001087501
	$W_S^{(t)}$	0.000908160	0.001648085	0.002244088	0.002719735

Figure A.3. Effect of on $L_s(t)$ and $W_s(t)$

A.4 Variation in ϕ

For specified range of values of the mean number of customers in the system ($L_s(t)$) and mean waiting time are ($W_s(t)$) are presented in Table A.4 and Figure A.4.

Table A.4. Effect of on $L_s(t)$ and $W_s(t)$

Parameter (ϕ)	t	0.5	1	1.5	2
.0010	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107090
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768747
.0011	$L_S^{(t)}$	0.000364874	0.000665134	0.000909652	0.001107079
	$W_S^{(t)}$	0.000912310	0.001663234	0.002274845	0.002768721
.0012	$L_S^{(t)}$	0.000364873	0.000665131	0.000909645	0.001107069
	$W_S^{(t)}$	0.000912308	0.001663226	0.002274829	0.002768695
.0013	$L_S^{(t)}$	0.000364872	0.000665128	0.000909639	0.001107058
	$W_S^{(t)}$	0.000912306	0.001663218	0.002274812	0.002768669

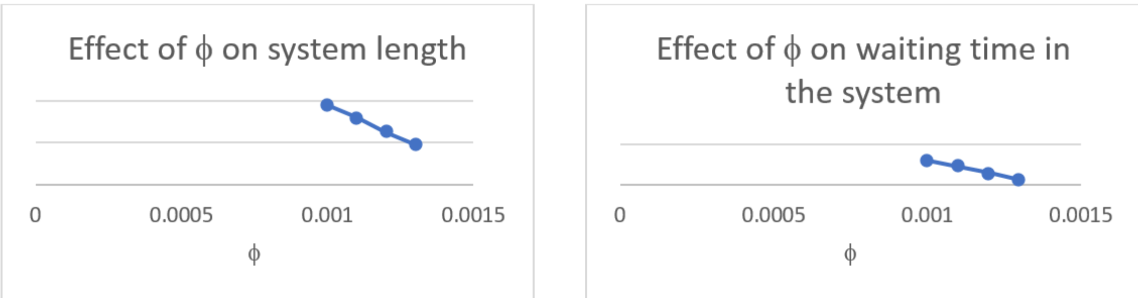


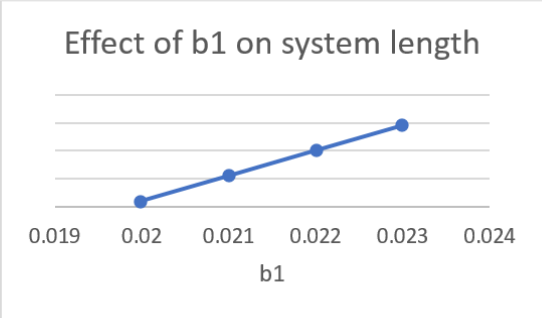
Figure A.4. Effect of on $L_s(t)$ and $W_s(t)$

A.5 Variation in b_1

For specified range of values of b_1 the mean number of customers in the system ($L_s(t)$) and mean waiting time are ($W_s(t)$) are presented in Table A.5 and Figure A.5.

Table A.5. Effect of b_1 on $L_s(t)$ and $W_s(t)$

Parameter (b_1)	t	0.5	1	1.5	2
.020	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107090
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768747
.021	$L_S^{(t)}$	0.000383111	0.000698368	0.000955088	0.001162359
	$W_S^{(t)}$	0.000957916	0.001746358	0.002388508	0.002907024
.022	$L_S^{(t)}$	0.000401346	0.000731595	0.001000512	0.001217620
	$W_S^{(t)}$	0.001003518	0.001829470	0.002502145	0.003045287
.023	$L_S^{(t)}$	0.000419581	0.000764821	0.001045931	0.001272873
	$W_S^{(t)}$	0.001049120	0.001912578	0.002615774	0.003183534



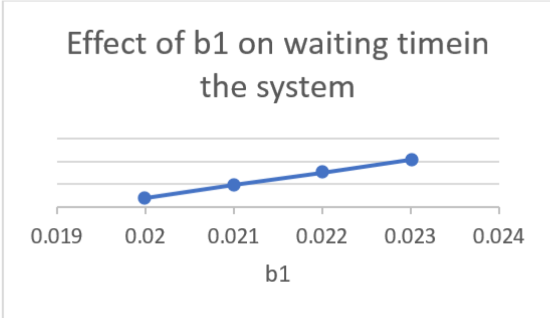


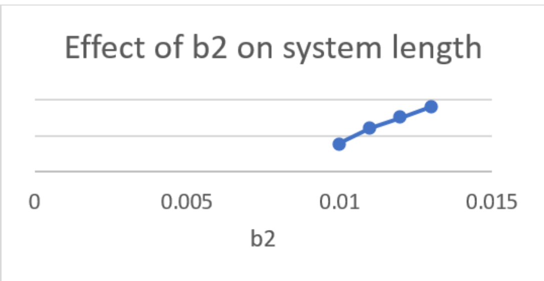
Figure A.5. Effect of b_1 on $L_s(t)$ and $W_s(t)$

A.6 Variation in b_2

For specified range of values of b_2 the mean number of customers in the system ($L_s(t)$) and mean waiting time are ($W_s(t)$) are presented in Table A.6 and Figure A.6.

Table A.6. Effect of b_2 on $L_s(t)$ and $W_s(t)$

Parameter (b_2)	t	0.5	1	1.5	2
.010	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107090
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768747
.011	$L_S^{(t)}$	0.000364875	0.000665139	0.000909662	0.001107095
	$W_S^{(t)}$	0.000912313	0.001663246	0.002274869	0.002768761
.012	$L_S^{(t)}$	0.000364875	0.000665140	0.000909665	0.001107101
	$W_S^{(t)}$	0.000912314	0.001663249	0.002274878	0.002768775
.013	$L_S^{(t)}$	0.000364876	0.000665142	0.000909668	0.001107106
	$W_S^{(t)}$	0.000912315	0.001663253	0.002274886	0.002768789



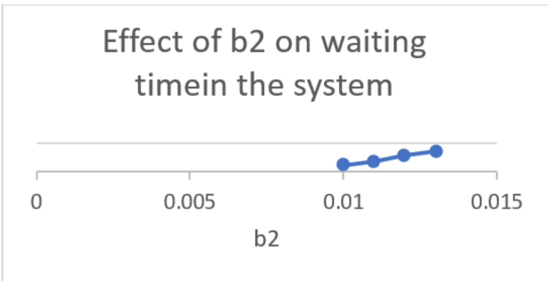


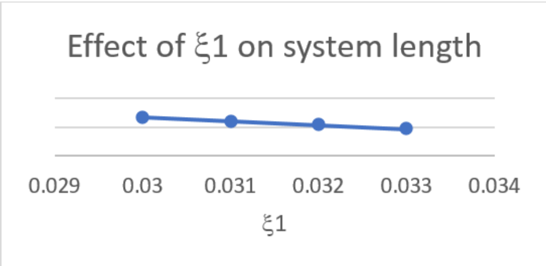
Figure A.6. Effect of b_2 on $L_s(t)$ and $W_s(t)$

A.7 Variation in ξ_1

For specified range of values of ξ_1 the mean number of customers in the system ($L_s(t)$) and mean waiting time are ($W_s(t)$) are presented in Table A.7 and Figure A.7.

Table A.7. Effect of ξ_1 on $L_s(t)$ and $W_s(t)$

Parameter (ξ_1)	t	0.5	1	1.5	2
.030	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107090
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768747
.031	$L_S^{(t)}$	0.000364865	0.000665104	0.000909591	0.001106983
	$W_S^{(t)}$	0.000912289	0.001663159	0.002274694	0.002768481
.032	$L_S^{(t)}$	0.000364856	0.000665071	0.000909524	0.001106877
	$W_S^{(t)}$	0.000912267	0.001663076	0.002274526	0.002768215
.033	$L_S^{(t)}$	0.000364847	0.000665038	0.000909458	0.001106771
	$W_S^{(t)}$	0.000912244	0.001662993	0.002274359	0.002767949



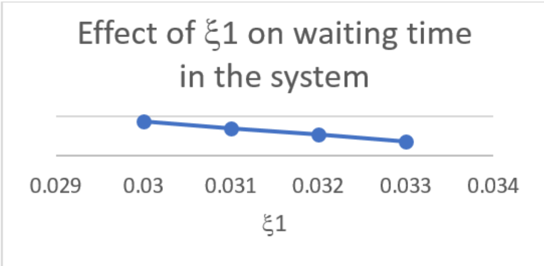


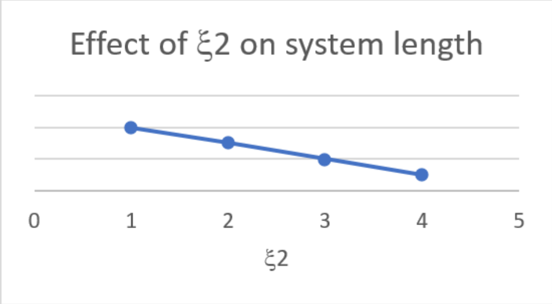
Figure A.7. Effect of ξ_1 on $L_s(t)$ and $W_s(t)$

A.8 Variation in ξ_2

For specified range of values of ξ_2 the mean number of customers in the system ($L_s(t)$) and mean waiting time are ($W_s(t)$) are presented in Table A.8 and Figure A.8.

Table A.8. Effect of ξ_2 on $L_s(t)$ and $W_s(t)$

Parameter (ξ_2)	t	0.5	1	1.5	2
.020	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107090
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768747
.021	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107090
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768747
.022	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107090
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768747
.023	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107090
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768747



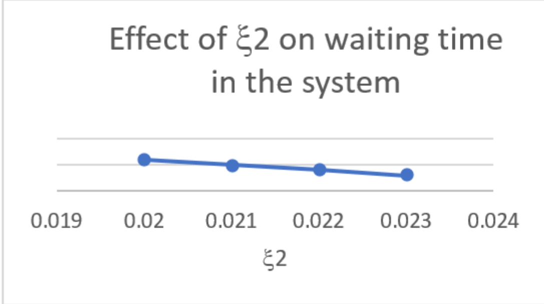


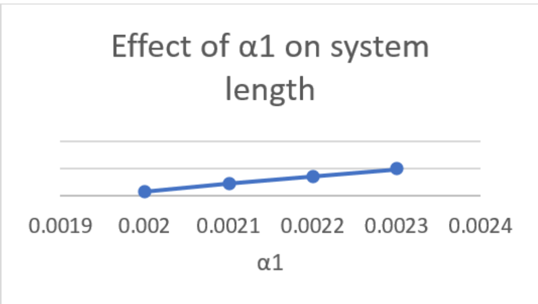
Figure A.8. Effect of ξ_2 on $L_s(t)$ and $W_s(t)$

A.9 Variation in α_1

For specified range of values of α_1 the mean number of customers in the system ($L_s(t)$) and mean waiting time are ($W_s(t)$) are presented in Table A.9 and Figure A.9.

Table A.9. Effect of α_1 on $L_s(t)$ and $W_s(t)$

Parameter (α_1)	t	0.5	1	1.5	2
.0020	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107090
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768747
.0021	$L_S^{(t)}$	0.000364875	0.000665142	0.000909672	0.001107119
	$W_S^{(t)}$	0.000912314	0.001663253	0.002274895	0.002768821
.0022	$L_S^{(t)}$	0.000364876	0.000665146	0.000909685	0.001107149
	$W_S^{(t)}$	0.000912315	0.001663263	0.002274928	0.002768895
.0023	$L_S^{(t)}$	0.000364876	0.000665150	0.000909699	0.001107178
	$W_S^{(t)}$	0.000912317	0.001663274	0.002274962	0.002768969



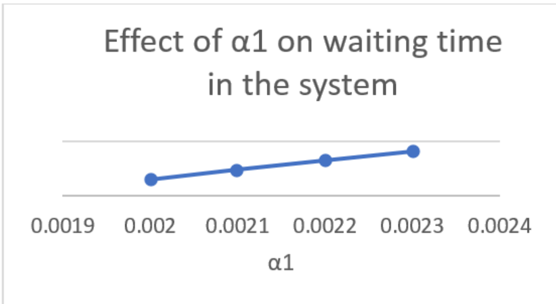


Figure A.9. Effect of α_1 on $L_s(t)$ and $W_s(t)$

A.10 Variation in α_2

For specified range of values of α_2 the mean number of customers in the system ($L_s(t)$) and mean waiting time are ($W_s(t)$) are presented in Table A.10 and Figure A.10.

Table A.10. Effect of α_2 on $L_s(t)$ and $W_s(t)$

Parameter (α_2)	t	0.5	1	1.5	2
0.0030	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107090
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768747
0.0031	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107090
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768747
0.0032	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107090
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768747
0.0033	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107090
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768747

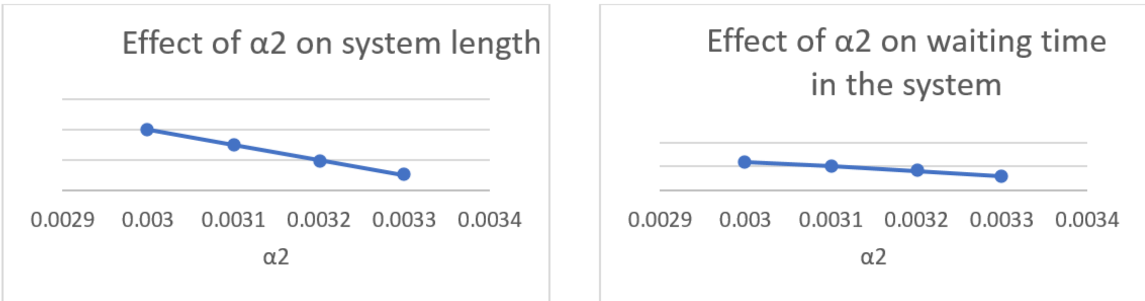


Figure A.10. Effect of α_2 on $L_s(t)$ and $W_s(t)$

A.11 Variation in γ_1

For specified range of values of γ_1 the mean number of customers in the system ($L_s(t)$) and mean waiting time are ($W_s(t)$) are presented in Table A.11 and Figure A.11.

Table A.11. Effect of γ_1 on $L_s(t)$ and $W_s(t)$

Parameter (γ_1)	t	0.5	1	1.5	2
0.0020	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107090
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768747
0.0021	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107090
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768747
0.0022	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107090
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768747
0.0023	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107090
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768747

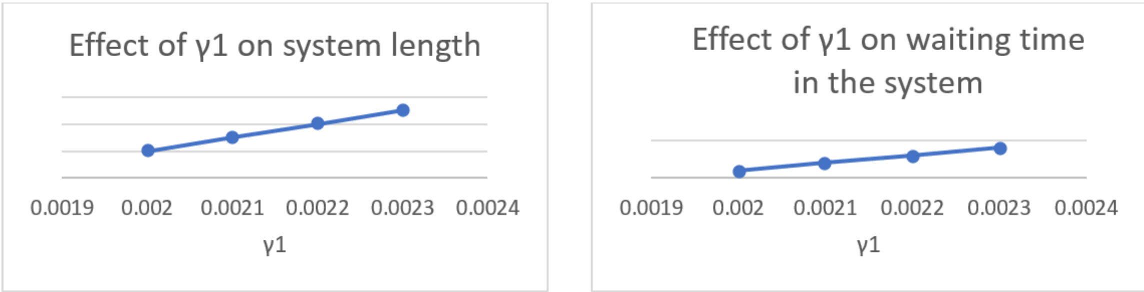


Figure A.11. Effect of γ_1 on $L_s(t)$ and $W_s(t)$

A.12 Variation in γ_2

For specified range of values of γ_2 the mean number of customers in the system ($L_s(t)$) and mean waiting time are ($W_s(t)$) are presented in Table A.12 and Figure A.12.

Table A.12. Effect of γ_2 on $L_s(t)$ and $W_s(t)$

Parameter (γ_2)	t	0.5	1	1.5	2
.0030	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107090
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768747
.0031	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107090
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768747
.0032	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107090
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768747
.0033	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107090
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768747

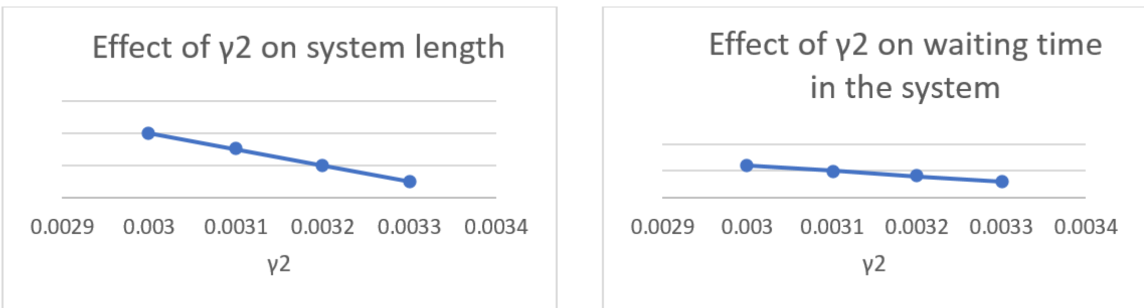


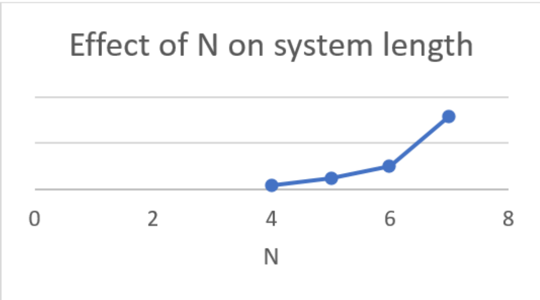
Figure A.12. Effect of γ_2 on $L_s(t)$ and $W_s(t)$

A.13 Variation in N

For specified range of values of N the mean number of customers in the system ($L_s(t)$) and mean waiting time are ($W_s(t)$) are presented in Table A.13 and Figure A.13.

Table A.13. Effect of N on $L_s(t)$ and $W_s(t)$

Parameter (N)	t	0.5	1	1.5	2
4	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107090
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768747
5	$L_S^{(t)}$	0.000934239	0.001743065	0.002436768	0.003026615
	$W_S^{(t)}$	0.002336000	0.004358967	0.006094310	0.007570026
6	$L_S^{(t)}$	0.001857265	0.003515384	0.004985186	0.006278977
	$W_S^{(t)}$	0.004644121	0.008791612	0.012468825	0.015706114
7	$L_S^{(t)}$	6.90384E-05	0.000120079	0.000157820	0.000185734
	$W_S^{(t)}$	0.000172608	0.000300232	0.000394611	0.000464422



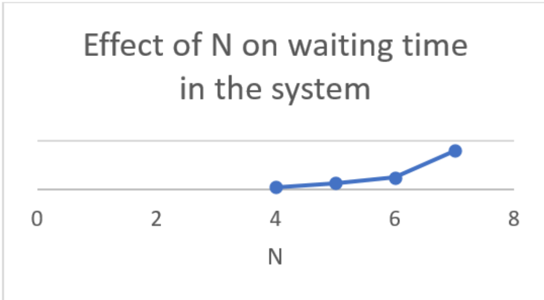


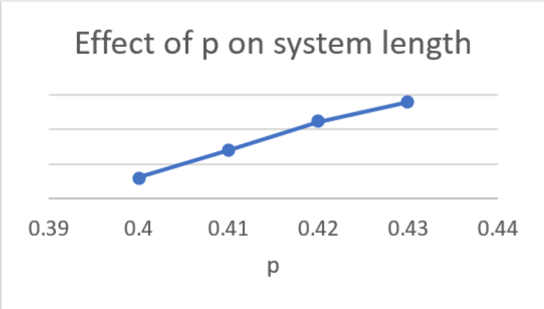
Figure A.13. Effect of N on $L_s(t)$ and $W_s(t)$

A.14 Variation in p

For specified range of values of p the mean number of customers in the system ($L_s(t)$) and mean waiting time are ($W_s(t)$) are presented in Table A.14 and Figure A.14.

Table A.14. Effect of p on $L_s(t)$ and $W_s(t)$

Parameter (p)	t	0.5	1	1.5	2
.40	$L_S^{(t)}$	0.000364875	0.000665137	0.000909658	0.001107090
	$W_S^{(t)}$	0.000912312	0.001663242	0.002274861	0.002768747
.41	$L_S^{(t)}$	0.000364875	0.000665139	0.000909662	0.001107095
	$W_S^{(t)}$	0.000912313	0.001663246	0.002274870	0.002768762
.42	$L_S^{(t)}$	0.000364876	0.000665141	0.000909666	0.001107101
	$W_S^{(t)}$	0.000912315	0.001663250	0.002274879	0.002768777
.43	$L_S^{(t)}$	0.000364876	0.000665143	0.000909669	0.001107107
	$W_S^{(t)}$	0.000912316	0.001663255	0.002274888	0.002768791



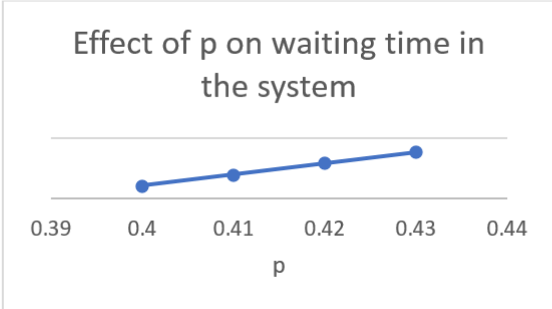


Figure A.14. Effect of p on $L_s(t)$ and $W_s(t)$

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] R.O. Al-Seedy, A.A. El-Sherbiny, S.A. El-Shehawy and S.I. Ammar, Transient solution of the M/M/c queue with balking and reneging, *Computers & Mathematics with Applications* **57**(8) (2009), 1280 – 1285, DOI: 10.1016/j.camwa.2009.01.017.

[2] C.J. Ancker (Jr.) and A.V. Gafarian, Some queueing problems with balking and reneging. I, *Operations Research* **11**(1) (1963), 88 – 100, DOI: 10.1287/opre.11.1.88.

[3] Y. Baba, Analysis of a GI/M/1 Queue with multiple working vacations, *Operations Research Letters* **33**(2) (2005), 201 – 209, DOI: 10.1016/j.orl.2004.05.006.

[4] V.N.R. Devi, A.A. Rao and K. Chandan, Analysis of a M/M/1 queueing system with two-phase, N-policy, server failure and second optional batch service with customers impatient behavior, *Journal of Physics: Conference Series* (*International Conference on Recent Inventions and Innovations in Mathematical Sciences 28 February to 1 March 2019, Andhra Pradesh, India*) **1344** (2019), 012015, DOI: 10.1088/1742-6596/1344/1/012015.

[5] B.T. Doshi, A note on stochastic decomposition in $GI/M/1$ queue with vacations or startup times, *Journal of Applied Probability* **22**(2) (1985), 419 – 428, DOI: 10.2307/3213784.

[6] S.M. Gupta, Machine interference problem with warm spares, server vacations and exhaustive service, *Performance Evaluation* **29**(3) (1997), 195 – 211, DOI: 10.1016/S0166-5316(96)00046-6.

[7] Y.C. Hsieh and K.H. Wang, Reliability of a repairable system with spares and removable repairmen, *Microelectronics and Reliability* **35**(2) (1995), 197 – 208, DOI: 10.1016/0026-2714(95)90086-6.

[8] M. Jain, N policy for redundant repairable system with additional repairmen, *Opsearch* **40** (2003), 97 – 114, DOI: 10.1007/BF03398687.

[9] J.C. Ke, The analysis of a general input queue with N policy and exponential vacations, *Queueing Systems* **45**(2) (2003), 135 – 160, DOI: 10.1023/A:1026045706255.

[10] P.V. Laxmi and K. Jyostna, Finite buffer GI/Geo/1 batch servicing queue with multiple working vacations, *RAIRO - Operations Research* **48**(4) (2014), 521 – 543, DOI: 10.1051/ro/2014022.

[11] L.D. Servi and S.G. Finn, M/M/1 queues with working vacations (M/M/1/WV), *Performance Evaluation* **50**(1) (2002), 41 – 52, DOI: 10.1016/s0166-5316(02)00057-3.

[12] N.R.D. Vedala, Y. Saritha, A.A. Rao and G. Sridhar, Study of MX/M/1 queueing system with vacation, two kinds of repair facilities and server timeout, *Advances in Science, Technology and Engineering Systems Journal* **4**(6) (2019), 339 – 342, DOI: 10.25046/aj040643.

[13] V.K. Vemuri, V.S.N.H.P. Boppana, C. Kotagiri and R.T. Bethapudi, Optimal strategy analysis of an N-policy two-phase $M^X/M/1$ queueing system with server startup and breakdowns, *Opsearch* **48** (2011), Article number: 109, DOI: 10.1007/s12597-011-0046-1.

[14] D.A. Wu and H. Takagi, M/G/1 queue with multiple working vacations, *Performance Evaluation* **63**(7) (2016), 654 – 681, DOI: 10.1016/j.peva.2005.05.005.

[15] J. Wu, J. Wang and Z. Liu, A discrete-time Geo/G/1 retrial queue with preferred and impatient customers, *Applied Mathematical Modelling* **37**(4) (2013), 2552 – 2561, DOI: 10.1016/j.apm.2012.06.011.