Communications in Mathematics and Applications
Vol. 13, No. 2, pp. 725–735, 2022
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications
DOI: 10.26713/cma.v13i2.1796

http://www.rgnpublications.com

Research Article

Applying Hessian Matrix Techniques to Obtain the Efficient Optimal Order Quantity Using Fuzzy Parameters

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Received: January 15, 2022 **Accepted:** May 9, 2022

Abstract. In the field of applied mathematics, optimization techniques formulate to the maximizing and minimizing for an objective function. The purpose of the optimization problems plays a vital role in the field of inventory management. The aim is to minimize the total cost, which comprises many fluctuating costs such as shortage, ordering, and holding cost. In this paper, the defective items were under the classification synchronous and asynchronous under a rework strategy process. The rework strategy is separating and accumulating the imperfect items at the time of completion of the process. This study considered asynchronous defective items and tried to minimize the total cost incurred. The optimality of the non-linear programming was achieved by the Hessian matrix, which results in the minimization of the total cost incurred. Furthermore, the usage of hexagonal fuzzy numbers formulates many real-life problems that arise due to flawed knowledge. There might be several situations in decision-making problems where optimization techniques require six parameters or more. The inclusion of Python coding has further made numerical working simpler. Furthermore, sensitivity analysis is carried out.

Keywords. Optimization, Fuzzy, EOQ, Python, Hessian matrix, Hexagonal fuzzy numbers

Mathematics Subject Classification (2020). 03B52, 03E72, 90B05

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1. Introduction

For strong and successful management, there requires an ideal inventory model that helps to minimize the total cost and maximize the profit. In 1913, Harris [5] was the first to introduce the EOQ (economic order quantity) model by assuming the demand, ordering, and holding costs to be constant. The literature shows that many optimization techniques were implemented by several researchers emphasizing various real-life cases. But in further studies, demand was set as a fluctuating parameter. Fuzzy set theory was first introduced by Zadeh [24] in the year 1965 which was carried the meaning of vagueness or uncertainty. Further in 1983, Zimmermann [25] developed wide-reaching mathematical tools for modeling and casting a classical problem. The EPQ (economic production quantity) was first developed by Taft [14] in 1918. It was the extension of EOQ (economic order quantity) that helps the company to minimize the total inventory costs by controlling the storage costs and ordering costs. Fuzzy theory on inventory models was framed in 1987 by Park [10]. Many researchers had extended the study of fuzzy under inventory models. In 1996, Chen *et al*. [2] studied the backorder in a fuzzy environment. Roy and Maiti [12] developed a model with limited storage capacity and geometric programming. In the year 2000, Yao *et al*. [23] considered an inventory problem without backorder and applying a triangular fuzzy number for order and demand parameters. Yao and Wu [22] applied signed-distance with a new ranking method for fuzzy methods. Yao and Chiang [21] studied an inventory model by fuzzifying the total cost by using the centroid and signed distance method for defuzzification.

In 2004, Chang [1] proposed an inventory model with a fuzzy defective rate by considering a screening process to determine the defective items. Hsieh [6] introduced two production inventory models for crisp and fuzzy total production quantity. Zimmermann [26] had suggested several applications on fuzzy sets. In 2020a, Kalaiarasi *et al*. [8] had done a comparative study between Lagrangian and Kuhn-tucker optimization methods in minimizing the total cost function. Further in 2020b, Kalaiarasi *et al*. [7] a non-linear programming problem was solved by geometric programming technique with numerical calculations using Python coding. Rosenblatt and Lee [11] in 1986 had studied EOQ for imperfect quality items. In 2007, Wang *et al*. [19] studied an EOQ problem where the percentage of flawed items was specified as a fuzzy variable. Salameh and Jaber [13] determined the stochastic models in production inventory for defective items. Vujošević *et al*. [17] had examined the EOQ in four different fuzzy sense aspects by using trapezoidal fuzzy numbers. Lee and Yao [9] determined a computing way for EPQ with demand rate in a fuzzy sense. Donaldson [3] was the first to give an alternative for the conventional constant demand with linear demand. Wagner and Whitin [18] proposed a generalization for the demand of the product in the basic EOQ model with a dynamic version lot-size model. Durai and Karpagam [4] had given a new membership function using α-cuts for hexagonal fuzzy numbers.

In the year 2009, Tokgöz *et al*. [15] had applied Hessian matrix techniques for solving queueing system problems for continuous variables. Yang and Wee [20] studied an integrated vendor-buyer model and used Hessian matrix for the convexity of the objective function. Velmurugan and Uthayakumar [16] in the year 2015, considered and inventory model and using matrix operations for obtaining the optimal order quantity.

Python is a multi-purpose language created by Guido van Rossum in the year 1991 which helps programmers to apply in projects under integrated environments. Coding is a process to instruct the computers to recognize the task by using the programming languages. Python, the most powerful programming language in web development like data science and creating software prototypes. It is a high-level language that uses the object-oriented programming style.

The objective of this paper is to optimize the non-linear programming problem with backorder for non-defective items, by using the Hessian matrix. Hessian matrix is a two-dimensional optimization technique that uses second order partial derivatives as its entries. If the principal minors of the matrix are positive then the stationary point is minimum. Further, scrutinizing the fuzzy inventory model using hexagonal fuzzy numbers and the signed-distance method is considered for the fuzzification and defuzzification process. The complete numerical calculations are executed using Python coding with self-generating hexagonal fuzzy numbers required by the user. The programming procedure had been explained through a flow chart.

Motivation

Fuzzy sets have numerous applications in various fields of study. To understand the circumstances that involve six different situations that involve vagueness, hexagonal fuzzy numbers can be used. Hessian matrix contains the second-order partial derivatives of multivariable functions. It can be used for optimization problems in real-life for maximizing or minimizing the output or utility needed. The actual purpose of the inventory management problem is to minimize the total annual inventory cost.

Outlines of the Work

Section 2 contains the definition of fuzzy sets and hexagonal fuzzy sets followed by the arithmetical operations defined for hexagonal fuzzy sets. Second 3 starts with the notations and assumptions followed by the optimal order quantity of the crisp inventory model. Section 4 formulates the total cost for the inventory model, Section 5 discusses the Hessian matrix optimization technique that was applied in the model. A fuzzy inventory model is done in Section 6. In Section 7, the Python coding process is described through a flowchart. At last, the numerical illustrations, discussions and conclusions are accomplished.

2. Preliminaries

2.1 Definition (Fuzzy set [24])**.** Let X be a space of points (objects). A fuzzy set R in X is an object of the form $R = \{(x, \mu_R(x)) : x \in X\}$ where $\mu_R : X \to [0,1]$ is called the membership function of the fuzzy set R.

2.2 Definition (Hexagonal fuzzy numbers [4])**.** A hexagonal fuzzy number $\tilde{A}_H$ denoted by is $\tilde{A}_H = (h_1, h_2, h_3, h_4, h_5, h_6)$ where $h_1, h_2, h_3, h_4, h_5, h_6$ are real numbers having a membership function as

$$\mu_{\tilde{A}_H}(x) = \begin{cases} \frac{1}{2}\left(\frac{x-h_1}{h_2-h_1}\right), & h_1 \le x \le h_2, \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-h_2}{h_3-h_2}\right), & h_2 \le x \le h_3, \\ 1-\frac{1}{2}\left(\frac{x-h_4}{h_5-h_4}\right), & h_4 \le x \le h_5, \\ \frac{1}{2}\left(\frac{h_6-x}{h_6-h_5}\right), & h_5 \le x \le h_6, \\ 0 & \text{otherwise.} \end{cases}$$

2.3 Arithmetic Operations on Hexagonal Fuzzy Numbers. Let $\tilde{R} = (r_1, r_2, r_3, r_4, r_5, r_6)$, $\tilde{S} = (s_1, s_2, s_3, s_4, s_5, s_6)$ be two hexagonal fuzzy numbers, the arithmetic operations are performed as

Addition: $\tilde{R} \otimes \tilde{S} = (r_1+s_1, r_2+s_2, r_3+s_3, r_4+s_3, r_5+s_5, r_6+s_6)$

Subtraction: $\tilde{R} \ominus \tilde{S} = (r_1-s_6, r_2-s_5, r_3-s_4, r_4-s_3, r_5-s_2, r_6-s_1)$

Multiplication: $\tilde{R} \otimes \tilde{S} = (r_1 * s_1, r_2 * s_2, r_3 * s_3, r_4 * s_4, r_5 * s_5, r_6 * s_6)$

2.4 Hessian Matrix. The Hessian matrix of a function of n variables $f(y_1, y_2, \dots, y_n)$ is as follows:

$$\nabla^2 f(y) = \left[\frac{\partial^2 f}{\partial y_i \partial y_i}\right], \quad i, j = 1, 2, \dots, n$$
$$= \begin{bmatrix} \frac{\partial^2 f(y^0)}{\partial y_1 \partial y_1} & \frac{\partial^2 f(y^0)}{\partial y_1 \partial y_2} & \cdots & \frac{\partial^2 f(y^0)}{\partial y_1 \partial y_n} \\ \frac{\partial^2 f(y^0)}{\partial y_2 \partial y_1} & \frac{\partial^2 f(y^0)}{\partial y_2 \partial y_2} & \cdots & \frac{\partial^2 f(y^0)}{\partial y_2 \partial y_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f(y^0)}{\partial y_n \partial y_1} & \frac{\partial^2 f(y^0)}{\partial y_n \partial y_2} & \cdots & \frac{\partial^2 f(y^0)}{\partial y_n \partial y_n} \end{bmatrix}.$$

3. Notations

The parameters used in the total cost function are

- H inventory holding cost
- O lot size of the items
- β backorder items
- D demand rate of items per time
- δ backorder cost per item with demand
- γ production setup cost with demand
- s the rework rate for the defective items
- μ the inventory cost for the non-defective items

3.1 Assumptions Made in the Model

(i) Demand is considered to be constant.

(ii) The production rate is a constant.

(iii) The manufactured items go through a screening process and classified as defective or non-defective under synchronous and nonsynchronous.

(iv) Assuming that the asynchronous rework rate is greater than the demand.

(v) Rework is planned and backorders are allowed to manage productions.

4. Formulation of the Inventory Model

The total inventory of non-defective items. The total cost function consisting of the various costs under asynchronous classification is minimized

$$T_c(O) = \alpha - H\beta + \frac{\mu\beta^2}{O} + \frac{\delta\beta}{O} + sO \tag{4.1}$$

partially differentiating and solving for 'O',

$$\frac{\partial T_c}{\partial O} = -\frac{\mu\beta^2}{O^2} - \frac{\delta\beta}{O^2} - \frac{\gamma}{O^2} + s \tag{4.2}$$

Equating $\frac{\partial T_c}{\partial O} = 0$.

The optimal order quantity is hence derived,

$$\Rightarrow \quad O = \sqrt{\frac{\beta^2\mu + \beta\delta + \gamma}{s}}. \tag{4.3}$$

5. The Hessian Optimization Technique for Solving Crisp EOQ Model

Considering the total cost function, we observe it is strictly convex and the entries of the Hessian matrix are positive and the determinant is positive.

$$\frac{\partial T_c}{\partial O} = -\frac{\mu\beta^2}{O^2} - \frac{\delta\beta}{O^2} - \frac{\gamma}{O^2} + s \tag{5.1}$$

$O > 0$, $\beta > 0$, also $\frac{\partial^2 T_c}{\partial O^2} > 0$.

Applying the Hessian matrix,

$$\Rightarrow \quad \frac{12854127}{40O^2}\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = \frac{34295O^2 + 187868801O + 2056660320}{6400O^2}. \tag{5.2}$$

The resulting matrix is positive definite since the principal diagonal minors of the matrix are positive.

6. The Fuzzy Inventory Model

Fuzzifying the inventory model using hexagonal fuzzy number defuzzifying by applying signed-distance method for the parameters $= (s_1, s_2, s_3, s_4, s_5, s_6)$

$$T_O(O) = \left[\alpha - H\beta + \frac{\mu\beta^2}{O} + \frac{\delta\beta}{O} + \frac{\gamma}{O}\right] \oplus [\tilde{s} \oplus O], \tag{6.1}$$

$$T_O(O) = \left[\alpha - H\beta + \frac{\mu\beta^2}{O} + \frac{\delta\beta}{O} + \frac{\gamma}{O}\right] \oplus [(s_1, s_2, s_3, s_4, s_5, s_6) \oplus O], \tag{6.2}$$

$$T_O(O) = \left[\alpha - H\beta + \frac{\mu\beta^2}{O} + \frac{\delta\beta}{O} + \frac{\gamma}{O}\right] \oplus [s_1 O, s_2 O, s_3 O, s_4 O, s_5 O, s_6 O], \tag{6.3}$$

$$\begin{aligned}\tilde{T}_O = \Bigg[&\alpha - H\beta + \frac{\mu\beta^2}{O} + \frac{\delta\beta}{O} + \frac{\gamma}{O} + s_1 O,\\ &\alpha - H\beta + \frac{\mu\beta^2}{O} + \frac{\delta\beta}{O} + \frac{\gamma}{O} + s_2 O,\\ &\alpha - H\beta + \frac{\mu\beta^2}{O} + \frac{\delta\beta}{O} + \frac{\gamma}{O} + s_3 O,\\ &\alpha - H\beta + \frac{\mu\beta^2}{O} + \frac{\delta\beta}{O} + \frac{\gamma}{O} + s_4 O,\\ &\alpha - H\beta + \frac{\mu\beta^2}{O} + \frac{\delta\beta}{O} + \frac{\gamma}{O} + s_5 O,\\ &\alpha - H\beta + \frac{\mu\beta^2}{O} + \frac{\delta\beta}{O} + \frac{\gamma}{O} + s_6 O\Bigg].\end{aligned} \tag{6.4}$$

The defuzzification is done using signed-distance method,

$$\tilde{T}_O(O) = \alpha - \frac{\mu\beta^2}{O} + \frac{\delta\beta}{O} + \frac{\gamma}{O} + \left(\frac{s_1 + 2s_2 + s_3 + s_4 + 2s_5 + s_6}{8}\right) O = f(O) \tag{6.5}$$

differentiating with respect to 'O' and computing the values

$$\frac{\partial f}{\partial O} = -\frac{\mu\beta^2}{O^2} - \frac{\delta\beta}{O^2} - \frac{\gamma}{O^2} + \frac{s_1 + 2s_2 + s_3 + s_4 + 2s_5 + s_6}{8} \tag{6.6}$$

$$O = \sqrt{\frac{\beta^2\mu + \beta\delta + \gamma}{\left(\frac{s_1 + 2s_2 + s_3 + s_4 + 2s_5 + s_6}{8}\right)}} \tag{6.7}$$

$$\implies \quad O = \sqrt{\frac{8\left[\beta^2\mu + \beta\delta + \gamma\right]}{(s_1 + 2s_2 + s_3 + s_4 + 2s_5 + s_6)}} \tag{6.8}$$

Clearly, $\frac{\partial^2 f}{\partial O^2} > 0$ depicts that $f(O)$ is minimum.

7. Flowchart and Importance of Python Coding

PYTHON is the most user-friendly programming language which can be downloaded freely. The version of Python used 3.7.5 [MSC v.1916 64 bit (AMD64)] on Win32. Figure 1 displays the flowchart using Python for self-generation of hexagonal fuzzy numbers by performing the fuzzification and defuzzification by signed-distance method. In Figure 2, the graph shows the comparison between the crisp and fuzzy values from Table 1. The sensitivity analysis between the fuzzy and crisp values is depicted in Figure 2 and a 3D-plot using Python coding between backorder items, backorder cost per item with demand and the rework rate for the defective items is shown in Figure 3.

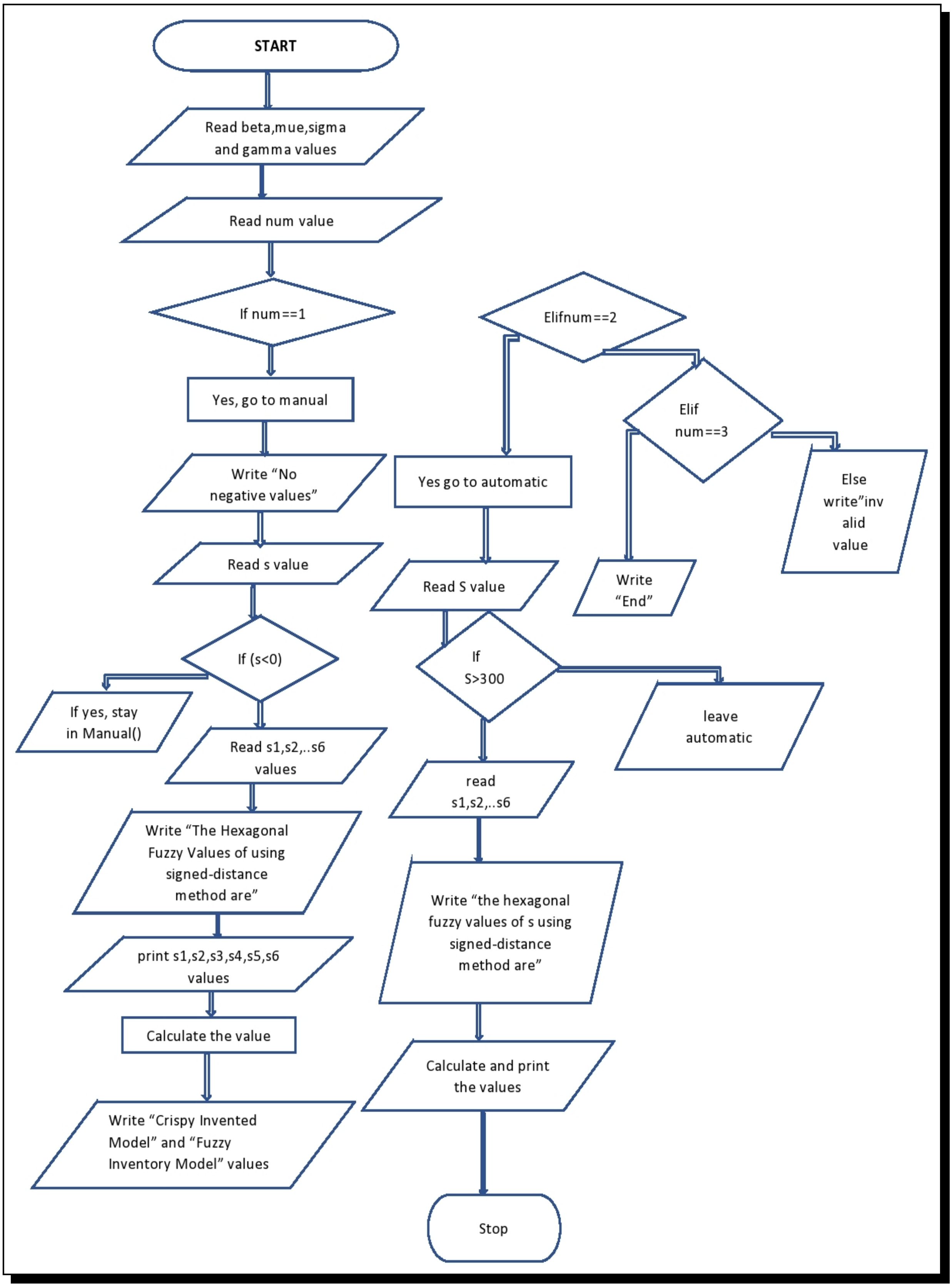

Figure 1. Flowchart of the Python programming to generate the machine-driven fuzzy numbers

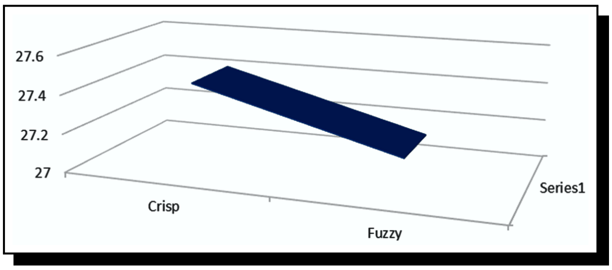

Figure 2. Crisp values opposed to fuzzy values

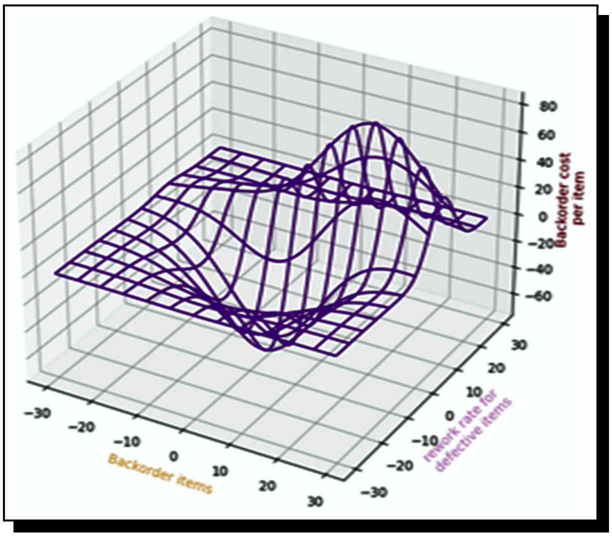

Figure 3. 3D plot for economic order quantity

8. Numerical Illustrations

The numerical calculations with parameters backorder items, backorder cost per item with demand, the inventory cost for the non-defective items, production setup cost with demand and the rework rate for the defective items is compared between the crisp and fuzzy inventory model in Table 1. Sensitivity analysis of 's' the rework rate for the defective items is executed to contrast between the crisp and fuzzy cases in Table 2.

Table 1. Numerical comparison between the crisp and fuzzy inventory model

Parameters	Crisp inventory model	Fuzzy inventory model
$\beta = 70$ units	27.433783	27.15146
$\delta = 200$ units		
$\mu = 120$ units		
$\gamma = 90$ units		
$s = 800$ units (500, 600, 700, 900, 1000, 1100)		

Table 2. Sensitivity Analysis of the parameter in crisp and fuzzy sense

Variations	Parameters	Crisp inventory model	Fuzzy inventory model
+ 50%	$\beta = 105$ units	40.98918	41.58234
	$\delta = 300$ units		
	$\mu = 180$ units		
	$\gamma = 135$ units		
	$s = 1200$ units (900, 1000, 1100, 1200, 1300, 1400)		
+ 25%	$\beta = 87.5$ units	34.21147	33.92913
	$\delta = 250$ units		
	$\mu = 150$ units		
	$\gamma = 112.5$ units		
	$s = 1000$ units (700, 800, 900, 1100, 1200, 1300)		
− 25%	$\beta = 52.5$ units	20.65611	20.37381
	$\delta = 150$ units		
	$\mu = 90$ units		
	$\gamma = 67.5$ units		
	$s = 600$ units (300, 400, 500, 700, 800, 900)		
− 50%	$\beta = 35$ units	13.87849	13.59624
	$\delta = 100$ units		
	$\mu = 60$ units		
	$\gamma = 45$ units		
	$s = 400$ units (100, 200, 300, 500, 600, 700)		

9. Discussions

The program allows to enter the values for the inventory model and splits the hexagonal fuzzy values if it's machine choice. Moreover, there can also be a manual entry for the fuzzified values. Figure 3 displays a Python 3D graph plotted using MATPLOTLIB.PYPLOT coding in Python that considers the backorder items, backorder cost per item with demand and the rework rate for the defective items. Table 1 shows the obtained optimal values of the crisp inventory model with the fuzzy inventory model. It is observed the output values of both the models are correspond. Table 2 tabulates the sensitivity analysis between the varying parameters. It shows the connection between the backorder items, backorder cost per item, and rework rate for defective items. Python coding helps to reach the optimal values of the fuzzy and crisp more accurately. The program has been designed to perform numerical calculations with ease. The coding is framed in such a way that negative values are restricted since it results in a complex value.

10. Conclusion

The optimum order quantity was derived using the total cost function. The aim of the model allows us to maximize the total profit and minimizing the total cost. The fuzzification using hexagonal fuzzy numbers for the parameter s and signed distance method was applied for the defuzzification process and outputs were compared between the crisp and the fuzzy terms. Additionally, Python coding was done to generate the hexagonal numbers for any crisp inputs given that enables a clear trouble-free calculation. In the future, the programming can be extended for varying fuzzy numbers.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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