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Research Article

Some Results on Conditionally Compatible and Conditionally Semi-Compatible Mappings in Probabilistic 2-Metric Space

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Abstract. The objective of this paper is to obtain two results in probabilistic 2-metric space by employing the concepts of (E.A)-property, conditionally compatible, conditionally semi-compatible and sub-sequentially continuous mappings. These findings improve the theorem proved in [5]. Further, these results are substantiated by supporting examples.

Keywords. (E.A)-property, Conditionally compatible, Conditionally semi-compatible, Sub-sequentially continuous, Probabilistic 2-metric space

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1. Introduction

Fixed point theory is a popular topic in contemporary analysis research. These findings will be valuable in the fields of science and engineering. Menger space is one category of advancement area initiated by Menger [7]. Menger introduced the theory of Statistical metric (SM/PM) space by interpreting the distance between two points using a probabilistic notion and a distribution function. By contributing essential notions such as neighborhoods, convergence and continuity Alsina *et al*. [1] enhanced these statistical metric spaces. Sehgal and Bharucha-Reid [10] using the concept of contraction to extract fixed point results to enrich the Menger spaces. Mishra [8] established the compatible idea in Menger space, which allows numerous fixed point theory findings in statistical spaces to be proved and which drew the attention of several scholars. Singh and Jain [11] used the notion of weakly compatible mappings to generate fixed points in Menger space, Xiaohong *et al*. *et al*. [13] contributed for the enrichment of SM-space in fixed point theory by employing Schweizer-Sklar t-norm in fuzzy logic system. Bisht and Shahazad [2] introduced another notion conditional compatible maps and extracted some results. Further, Jain and Khan [6] coined the similar concept conditional semi-compatible mappings and obtained some results in metric space. Some more results can be witnessed by using the concepts like sub-sequentially continuous and semi-compatible mappings in Menger space [9]. Gahler [3] employed the 2-metric space is the generalization of metric space. Further, this concepts is turned as probabilistic 2-metric space introduced by Golet [4]. Gupta *et al*. [5] employed the notion of weakly compatible mappings in 2-Menger space derived some fixed point results. In this paper, use the concepts of (E.A)-property [12] along with conditionally compatible, conditionally semi-compatible and sub-sequentially continuous in 2-Menger space and generate two fixed point theorems these are generalizations of the theorem proved by Gupta *et al*. [5].

2. Preliminaries

2.1 Definition ([5]). $\mathsf{F}:\mathsf{R}\to\mathsf{R}^+$ is distribution function if it is

(i) non-decreasing,

(ii) continuous from left,

(iii) $\inf\{\mathsf{F}(\alpha):\alpha\in R\}=0$,

(iv) $\sup\{\mathsf{F}(\alpha):\alpha\in R\}=1$.

The letter L is used to refer to a collection of all distribution functions.

2.2 Definition ([5]). A probabilistic 2-metric space (2-PM space) ia a pair (Ω,F) with$\mathsf{F}:\Omega\times\Omega\times\Omega\to\mathsf{L}$ here L stands as the set of all distribution functions and the F value at $(e,f,g)\in\Omega\times\Omega\times\Omega$ is written as $\mathsf{F}_{e,f,g}$ and fulfill the following properties:

(a) $\mathsf{F}_{e,f,g}(0)=0$,

(b) $\exists\ g\in\Omega$ such that $\mathsf{F}_{e,f,g}(t_\epsilon)<1$, $\forall\ e,f\in\Omega$, $e\neq f$, for some $t_\epsilon>0$,

(c) $\mathsf{F}_{e,f,g}(t_\epsilon)=1$, $\forall\ t_\epsilon>0$ if $e=f=g$ or $e=f$ or $f=g$ or $e=g$,

(d) $\mathsf{F}_{e,f,g}(t_\epsilon)=\mathsf{F}_{f,g,e}(t_\epsilon)=\mathsf{F}_{g,f,e,}(t_\epsilon)$,

(e) $\mathsf{F}_{e,f,g}(t_x)=\mathsf{F}_{f,g,e}(t_y)=\mathsf{F}_{g,f,e,}(t_z)=1\Rightarrow\mathsf{F}_{e,f,g}(t_x+t_y+t_z)=1$,

$\forall\ e,f,g\in\Omega$ and $t_x,t_y,t_z\geq 0$.

2.3 Definition ([5]). The mapping $t_\epsilon;[0,1]^3\to[0,1]$ is a t-norm it has the properties:

(i) $t_\epsilon(0,0,0)=0$,

(ii) $t_\epsilon(v,1,1)=v$,

(iii) $t_\epsilon(a_0,b_0,c_0)=t_\epsilon(b_0,c_0,a_0)=t_\epsilon(c_0,a_0,b_0)$,

(iv) $t_\epsilon(d,e,f)\ge t_\epsilon(d_1,e_1,f_1)$ for $d\ge d_1, e\ge e_1, f\ge f_1$,

(v) $t_\epsilon(t_\epsilon(a_0,b_0,c_0)),r,s)=t_\epsilon(a_0,t_\epsilon(b_0,c_0,r),s)=t_\epsilon(a_0,b_0,t_\epsilon(c_0,r,s))$,

$\forall\ v,a_0,b_0,c_0,d,e,f,d_1,e_1,f_1 r$ and $s\in\Omega$.

2.4 Definition ([5]). A Menger probabilistic 2-metric space is a triplet $(\Omega,\mathsf{F},t_\epsilon)$ where (Ω,F) is a 2-PM space and t_ϵ is a t-norm having triangle inequality:

$$\mathsf{F}_{u,v,w}(t_x+t_y+t_z)\ge t(\mathsf{F}_{u,v,p}(t_x),\mathsf{F}_{u,p,w}(t_y),\mathsf{F}_{p,v,w}(t_z)),\quad \forall\ w,p,v,u\in\Omega \text{ and } t_x,t_y,t_z\ge 0.$$

2.5 Definition ([5]). A sequence (p_n) in 2-Menger space $(\Omega,\mathsf{F},t_\epsilon)$:

(i) *converges* to β if for each $\epsilon>0$, $t_\epsilon>0$, $\exists\ N(\epsilon)\in N\Rightarrow F_{p_n,\beta,a}(\epsilon)>1-t_\epsilon$, $\forall\ a\in\Omega$ and $n\ge N(\epsilon)$,

(ii) *Cauchy* if for each $\epsilon>0$, $t_\epsilon>0$, $\exists\ N(\epsilon)\in N\Rightarrow F_{p_n,p_m,a}(\epsilon)>1-t_\epsilon$, $\forall\ a\in\Omega$ and $n,m\ge N(\epsilon)$,

(iii) if each Cauchy sequence converges in Ω then it is mentioned as *complete* 2-Menger space.

2.6 Definition ([12]). Two mappings P, S on 2-Menger space to itself $(\Omega,\mathsf{F},t_\epsilon)$ are having (E.A)-property means there is a sequence (c_m) such that lim $\lim_{m\to\infty} Pc_m=\lim_{m\to\infty} Sc_m=\mu$ for some $\mu\in\Omega$.

2.1 Example. Define $\forall\ t_\epsilon\in[0,1]$

$$\mathsf{F}_{v,\beta,\gamma}(t_1)=\begin{cases}\frac{t_\epsilon}{t_\epsilon+d(v,\beta)}, & \text{if } t_\epsilon>0,\\ 0, & \text{if } t_\epsilon=0,\end{cases}\tag{2.1}$$

$\forall\ v,\beta$ and fixed $\gamma=0$, $t_\epsilon>0$.

By considering $\Omega=\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ and d is usual distance on Ω then by eq. (2.1) $(\Omega,\mathsf{F},t_\epsilon)$ forms 2-Menger space.

The mappings $P,S:\Omega\to\Omega$ are defined as

$$P(a)=\tan(a),\quad \forall\ a\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right),\tag{2.2}$$

$$S(a)=\sin(a),\quad \forall\ a\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right).\tag{2.3}$$

Then there is a sequence $(c_m)=\frac{3}{m^2}$, $\forall\ m\ge 1$. Then from eq. (2.2)

$$\lim_{m\to\infty} Pc_m=\lim_{m\to\infty} p\left(\frac{3}{m^2}\right)=\lim_{m\to\infty}\tan\left(\frac{3}{m^2}\right)=0\tag{2.4}$$

and from (2.3)

$$\lim_{m\to\infty} Sc_m=\lim_{m\to\infty} S\left(\frac{3}{m^2}\right)=\lim_{m\to\infty}\sin\left(\frac{3}{m^2}\right)=0.\tag{2.5}$$

Eqs. (2.4) and (2.5) resulting

$$\lim_{m\to\infty} Pc_m=\lim_{m\to\infty} Sc_m=0.\tag{2.6}$$

Thus from eq. (2.6) the mappings P, S satisfy (E.A)-property.

2.7 Definition. Self-mappings P, S in 2-Menger space $(\Omega, \mathsf{F}, t_\epsilon)$ are known as:

(a) *Compatible* [5] if $\mathsf{F}_{PSx_n, SPx_n, a}(\beta) \to 1$, $\forall\ a \in \Omega$ and $\beta > 0$ whenever a sequence $(x_n) \in \Omega$ such that $Px_n, Sx_n \to \theta$ as $n \to \infty$ where θ is some element of Ω.

(b) *Weakly compatible* [6] if commute at their coincidence points.

(c) *Conditionally compatible* [2] if whenever the sequence (c_m) satisfying

$$\left\{(c_m) : \lim_{m\to\infty} Pc_m = \lim_{m\to\infty} Sc_m\right\} \neq \phi$$

then there exists another sequence (e_m) in Ω with $\lim_{m\to\infty} Pe_m = \lim_{m\to\infty} Se_m = \eta$ for some $\eta \in X$ such that

$$\lim_{m\to\infty} \mathsf{F}_{PSe_n, SPe_n, a}(\beta) = 1, \quad \forall\ a \in \Omega,\ \beta > 0.$$

(d) *Conditionally semi-compatible* [6] if whenever the sequence (c_m) satisfying

$$\left\{(c_m) : \lim_{m\to\infty} Pc_m = \lim_{m\to\infty} Sc_m\right\} \neq \phi$$

then there exists another sequence (e_m) in Ω with $\lim_{m\to\infty} Pe_m = \lim_{m\to\infty} Se_m = \eta$ for some $\eta \in \Omega$ such that

$$\lim_{m\to\infty} \mathsf{F}_{PSe_m, S\eta, a}(\beta) = 1 \text{ and } \lim_{m\to\infty} \mathsf{F}_{SPe_m, P\eta, a}(\beta) = 1, \quad \forall\ a \in \Omega,\ \beta > 0.$$

(e) *Sub sequentially continuous* [9] if there exists a sequence $\{c_m\}$ such that

$$\lim_{m\to\infty} Pc_m = \lim_{m\to\infty} Sc_m = \eta, \quad \text{for some } \eta \in \Omega$$

$$\Rightarrow \quad \lim_{m\to\infty} \mathsf{F}_{P\eta, PSc_m, a}(\beta) = 1 \text{ and } \lim_{m\to\infty} \mathsf{F}_{S\eta, SPc_m, a}(\beta) = 1, \quad \forall\ a \in \Omega, \text{ for some } \beta > 0.$$

2.2 Example. Let $(\Omega, \mathsf{F}, t_\epsilon)$ be a 2-Menger space where F, t_ϵ be as in eq. (2.1), choose $\Omega = R$. The mappings $P, S : \Omega \to \Omega$ are defined as:

$$P(a) = 2^{-a}, \quad \forall\ x \in R, \tag{2.7}$$

$$S(a) = 2^{-x^2}, \quad \forall\ x \in R. \tag{2.8}$$

Let $(a_m) = 1 - \frac{10}{m}$, $\forall\ m \geq 1$ then from eq. (2.7)

$$\lim_{m\to\infty} Pa_m = \lim_{m\to\infty} P\left(1 - \frac{10}{m}\right) = \lim_{m\to\infty} 2^{-(1-\frac{10}{m})} = 2^{-1}, \tag{2.9}$$

$$\lim_{m\to\infty} Sa_m = \lim_{m\to\infty} S\left(1 - \frac{10}{m}\right) = \lim_{m\to\infty} 2^{-(1-\frac{10}{m})^2} = 2^{-1}. \tag{2.10}$$

From eqs. (2.9)-(2.10), we get

$$\lim_{m\to\infty} Pa_m = \lim_{m\to\infty} Sa_m. \tag{2.11}$$

From eq. (2.11) implies

$$\left\{(a_m) : \lim_{m\to\infty} Pa_m = \lim_{m\to\infty} Sa_m\right\} \neq \phi.$$

Then there exists another sequence $c_m = \frac{-5}{m}$, $\forall\ m \geq 1$ and from eq. (2.7)

$$\lim_{m\to\infty} Pc_m = \lim_{m\to\infty} P\left(\frac{-5}{m}\right) = \lim_{m\to\infty} 2^{-\frac{-5}{m}} = 1 \tag{2.12}$$

and eq. (2.8)

$$\lim_{m\to\infty} Sc_m = \lim_{m\to\infty} S\left(\frac{-5}{m}\right) = \lim_{m\to\infty} 2^{-(\frac{-5}{m})^2} = 1. \tag{2.13}$$

From eqs. (2.12), (2.13)

$$\lim_{m\to\infty} Pc_m = \lim_{m\to\infty} Sc_m = 1. \tag{2.14}$$

Further from eqs. (2.7), (2.13)

$$\lim_{m\to\infty} PSc_m = \lim_{m\to\infty} P(2^{-(\frac{-5}{m})^2}) = \lim_{m\to\infty} 2^{-2^{-(\frac{-5}{m})^2}} = 2^{-1}, \tag{2.15}$$

from eqs. (2.8), (2.12)

$$\lim_{m\to\infty} SPc_m = \lim_{m\to\infty} S(2^{(\frac{5}{m})}) = \lim_{m\to\infty} 2^{-2^{(\frac{5}{m})^2}} = 2^{-1}. \tag{2.16}$$

Thus from eqs. (2.15), (2.16)

$$\lim_{m\to\infty} \mathsf{F}_{PSc_n,SPc_n,a}(\beta) = 1. \tag{2.17}$$

Moreover at coincidence point $a = 1$, $P(1) = S(1) = \frac{1}{2}$,

$$PS(1) = P\left(\frac{1}{2}\right) = 2^{-(\frac{1}{2})}, \tag{2.18}$$

$$SP(1) = S\left(\frac{1}{2}\right) = 2^{-(\frac{1}{4})}. \tag{2.19}$$

From eqs. (2.18)-(2.19)

$$PS(1) \neq SP(1). \tag{2.20}$$

We can conclude that from eqs. (2.17), (2.20) the pair (P,S) is conditionally compatible but not weakly compatible.

2.3 Example. Let $(\Omega, \mathsf{F}, t_\epsilon)$ be a 2-Menger space where F, t_ϵ be as in eq. (2.1) and $\Omega = R$. The mappings $P,S : \Omega \to \Omega$ are defined as

$$P(a) = 3^a, \quad \forall\ a \in R, \tag{2.21}$$

$$S(a) = 3^{a^2}, \quad \forall\ a \in R. \tag{2.22}$$

Let $(a_m) = 1 - \frac{2}{m}$, $\forall\ m \geq 1$ then from eq. (2.21)

$$\lim_{m\to\infty} Pa_m = \lim_{m\to\infty} P\left(1 - \frac{2}{m}\right) = \lim_{m\to\infty} 3^{(1-\frac{2}{m})} = 3, \tag{2.23}$$

from eq. (2.22)

$$\lim_{m\to\infty} Sa_m = \lim_{m\to\infty} S\left(1 - \frac{2}{m}\right) = 3^{(1-\frac{2}{m})^2} = 3. \tag{2.24}$$

Again from eqs. (2.23)-(2.24), we get

$$\lim_{m\to\infty} Pa_m = \lim_{m\to\infty} Sa_m. \tag{2.25}$$

From eq. (2.25)

$$\left\{(a_m) : \lim_{m\to\infty} Pa_m = \lim_{m\to\infty} Sa_m\right\} \neq \phi.$$

There exists another sequence $e_m = \frac{3}{m}$, $\forall\ m \geq 1$ then from eq. (2.21)

$$\lim_{m\to\infty} Pe_m = \lim_{m\to\infty} P\left(\frac{3}{m}\right) = \lim_{m\to\infty} 3^{\frac{3}{m}} = 1 \tag{2.26}$$

and eq. (2.22)

$$\lim_{m\to\infty} Se_m = \lim_{m\to\infty} S\left(\frac{3}{m}\right) = 3^{(\frac{3}{m})^2} = 1. \tag{2.27}$$

Resulting from eqs. (2.26)-(2.27)

$$\lim_{m\to\infty} Pe_m = \lim_{m\to\infty} Se_m = 1. \tag{2.28}$$

Further from eqs. (2.21), (2.27)

$$\lim_{m\to\infty} PSe_m = \lim_{m\to\infty} P(3^{(\frac{9}{m^2})}) = \lim_{m\to\infty} 3^{3^{(\frac{9}{m^2})}} = 3, \tag{2.29}$$

from eqs. (2.22), (2.26)

$$\lim_{m\to\infty} SPa_m = \lim_{m\to\infty} S(3^{(\frac{3}{m})}) = \lim_{m\to\infty} 3^{3^{(\frac{3}{m})^2}} = 3. \tag{2.30}$$

Thus from eqs. (2.29), (2.22), (2.30), (2.21)

$$\lim_{m\to\infty} \mathsf{F}_{PSe_n,S(1),a}(\beta) = 1 \text{ and } \lim_{m\to\infty} \mathsf{F}_{SPe_n,P(1),a}(\beta) = 1. \tag{2.31}$$

Further at coincidence point $a = 1$, $P(1) = S(1) = 3$,

$$PS(1) = P(3) = 27, \tag{2.32}$$

$$SP(1) = S(3) = 3^9. \tag{2.33}$$

From eqs. (2.32)-(2.33) implies

$$PS(1) \neq SP(1). \tag{2.34}$$

Hence we can conclude that from eqs. (2.31) and (2.34), the pair (P,S) is conditionally semi-compatible but not weakly compatible.

2.4 Example. Let $(\Omega, \mathsf{F}, t_\epsilon)$ be a 2-Menger space where F, t_ϵ be as in (2.1) and $\Omega = R$. The mappings $P,S : \Omega \to \Omega$ are defined as

$$P(a) = \begin{cases} \frac{1}{4}, & \text{if } a < 0, \\ a^2, & \text{if } a \geq 0, \end{cases} \tag{2.35}$$

$$S(a) = \begin{cases} \frac{1}{5}, & \text{if } a < 0, \\ 4a - 3, & \text{if } a \geq 0. \end{cases} \tag{2.36}$$

Let $(c_m) = 1 - \frac{\sin(\frac{\pi m}{4})}{m}$, $\forall\ m \geq 1$ then from eq. (2.35)

$$\lim_{m\to\infty} Pc_m = \lim_{m\to\infty} P\left(1 - \frac{\sin(\frac{\pi m}{4})}{m}\right) = \lim_{m\to\infty}\left(1 - \frac{\sin(\frac{\pi m}{4})}{m}\right)^2 = 1, \tag{2.37}$$

from eq. (2.36)

$$\lim_{m\to\infty} Sc_m = \lim_{m\to\infty} S\left(1 - \frac{\sin(\frac{\pi m}{4})}{m}\right) = \lim_{m\to\infty} 4\left(1 - \frac{\sin(\frac{\pi m}{4})}{m}\right) - 3 = 1. \tag{2.38}$$

From eqs. (2.37)-(2.38), we get

$$\lim_{m\to\infty} Pc_m = \lim_{m\to\infty} Sc_m = 1. \tag{2.39}$$

Further from eqs. (2.35), (2.38)

$$\lim_{m\to\infty} PSc_m = \lim_{m\to\infty} P\left(1-4\frac{\sin(\frac{\pi m}{4})}{m}\right) = \lim_{m\to\infty}\left(1-\frac{\sin(\frac{\pi m}{4})}{m}\right)^2 = 1, \tag{2.40}$$

from eqs. (2.36), (2.37)

$$\lim_{m\to\infty} SPa_m = \lim_{m\to\infty} S\left(1-\frac{\sin(\frac{\pi m}{4})}{m}\right)^2 = \lim_{m\to\infty} 4\left(1-\frac{\sin(\frac{\pi m}{4})}{m}\right)^2 - 3 = 1. \tag{2.41}$$

Thus from eqs. (2.40), (2.35), (2.41), (2.36)

$$\lim_{m\to\infty} \mathsf{F}_{PSc_m,P(1),a}(\beta) = 1 \text{ and } \lim_{n\to\infty} \mathsf{F}_{SPc_m,S(1),a}(\beta) = 1. \tag{2.42}$$

Further $a = 1,3$ are coincidence points of the mappings P, S.
At $x = 3$, $P(3) = S(3) = 9$ so that

$$PS(3) = P(9) = 81 \tag{2.43}$$

and

$$SP(3) = S(9) = 33. \tag{2.44}$$

From eqs. (2.43)-(2.44)

$$PS(3) \neq SP(3). \tag{2.45}$$

Therefore from eqs. (2.42), (2.45) the pair (P,S) is sub-sequentially continuous but not weakly compatible.

The following theorem was proved by Gupta *et al*. [5].

2.1 Theorem. *Let A, B, S and T be self-mappings on a complete probabilistic 2-metric space $(\mathsf{X},\mathsf{F},t_\epsilon)$ satisfying:*

(i) $A(\mathsf{X}) \subseteq T(\mathsf{X}), B(\mathsf{X}) \subseteq S(\mathsf{X})$,

(ii) *one of $A(\mathsf{X}), B(\mathsf{X}), T(\mathsf{X})$ or $S(\mathsf{X})$ is complete,*

(iii) *pairs (A,S) and (B,T) are weakly compatible,*

(iv) $\mathsf{F}_{Ax,By,\gamma}(t_\epsilon) \geq r(\mathsf{F}_{Sx,Ty,\gamma}(t_\epsilon))$, *for all x, y in X and $t_\epsilon > 0$,*

where $r:[0,1]\to[0,1]$ is some continuous function such that $r(t_\epsilon) > t_\epsilon$ for each $o < t_\epsilon < 1$.
Then the mappings A, B, S and T have unique common fixed point in X.

Now, we give generalization of Theorem 2.1 as under.

3. Main Results

3.1 Theorem. *Let A, B, S and T be mappings on a probabilistic 2-metric space $(\Omega,\mathsf{F},t_\epsilon)$ to itself satisfying:*

(i) *the pairs (A,S), (B,T) satisfy (E.A)-property,*

(ii) *the pairs* (A,S), (B,T) *are conditionally compatible and sub-sequentially continuous*

$$\mathsf{F}_{Aa,Bb,\gamma}(t_\epsilon) \ge r\mathsf{F}_{Sa,Tb,\gamma}(t_\epsilon), \quad \forall\, a,b \in \Omega,\, t_\epsilon > 0. \tag{3.1}$$

Then the mappings A, B, S and T have unique common fixed point in Ω.

Proof. The pairs (A,S), (B,T) satisfy (E.A)-property implies there exist two sequences (a_m),(c_m) such that

$$\lim_{m\to\infty} Aa_m = \lim_{m\to\infty} Sa_m = \theta, \tag{3.2}$$

$$\lim_{m\to\infty} Bc_m = \lim_{m\to\infty} Tc_m = \mu, \tag{3.3}$$

for some $\theta, \mu \in \Omega$.

From eqs. (3.2)-(3.3)

$$\left\{(a_m) : \lim_{m\to\infty} Aa_m = \lim_{m\to\infty} Sa_m\right\} \ne \phi.$$

Conditionally compatible of the pairs (A,S) implies there exists another sequence (b_m), with

$$\lim_{m\to\infty} Ab_m = \lim_{m\to\infty} Sb_m = \alpha, \tag{3.4}$$

such that

$$\lim_{m\to\infty} \mathsf{F}_{ASb_m,SAb_m,a}(\beta) = 1. \tag{3.5}$$

Also the sub-sequentially continuous of the pair (A,S) implies

$$\lim_{m\to\infty} \mathsf{F}_{ASb_m,A\alpha,a}(\beta) = 1 \text{ and } \lim_{m\to\infty} \mathsf{F}_{SAb_m,S\alpha,a}(\beta) = 1. \tag{3.6}$$

Using eq. (3.6) in eq. (3.5)

$$\mathsf{F}_{A\alpha,S\alpha,a}(\beta) = 1$$

$$\Rightarrow \quad A\alpha = S\alpha. \tag{3.7}$$

From eq. (3.3)

$$\left\{(c_m) : \lim_{m\to\infty} Bc_m = \lim_{m\to\infty} Tc_m\right\} \ne \phi.$$

Then conditionally compatible of the pairs (B,T) implies there exists another sequence (d_m), with

$$\lim_{m\to\infty} Bd_m = \lim_{m\to\infty} Td_m = \delta, \tag{3.8}$$

such that

$$\lim_{m\to\infty} \mathsf{F}_{BTd_m,TBd_m,a}(\beta) = 1. \tag{3.9}$$

Also, the sub-sequentially continuous of the pair (B,T) implies

$$\lim_{m\to\infty} \mathsf{F}_{BTd_m,B\delta,a}(\beta) = 1 \text{ and } \lim_{m\to\infty} \mathsf{F}_{TBd_m,T\delta,a}(\beta) = 1. \tag{3.10}$$

Using eq. (3.10) in eq. (3.9), we get

$$\mathsf{F}_{B\delta,T\delta,a}(\beta) = 1$$

$$\Rightarrow \quad B\delta = T\delta. \tag{3.11}$$

Claim $\alpha = \delta$.

Assume $\alpha \neq \delta$.
Using $a = b_m$, $b = d_m$ in eq. (3.1)

$$\mathsf{F}_{Ab_m,Bd_m,\gamma}(t_\epsilon) \geq r(\mathsf{F}_{Sb_m,Td_m,\gamma}(t_\epsilon))$$

as $m \to \infty$ from eqs. (3.4), (3.8)

$$\mathsf{F}_{\alpha,\delta,\gamma}(t_\epsilon) \geq r(\mathsf{F}_{\alpha,\delta,\gamma}(t_\epsilon)) \tag{3.12}$$

but

$$r(\mathsf{F}_{\alpha,\delta,\gamma}(t_\epsilon)) > (\mathsf{F}_{\alpha,\delta,\gamma}(t_\epsilon)) \tag{3.13}$$

since $\alpha \neq \delta$.
From eqs. (3.12)-(3.13)

$$\mathsf{F}_{\alpha,\delta,\gamma}(t_\epsilon) > \mathsf{F}_{\alpha,\delta,\gamma}(t_\epsilon). \tag{3.14}$$

This contradicts the fact $\alpha \neq \delta$. Hence $\alpha = \delta$.
Claim $A\alpha = \delta$.
Assume $A\alpha \neq \delta$.
Using $a = \alpha$, $b = d_m$ in eq. (3.1)

$$\mathsf{F}_{A\alpha,Bd_m,\gamma}(t_\epsilon) \geq r(\mathsf{F}_{S\alpha,Td_m,\gamma}(t_\epsilon))$$

as $m \to \infty$ and use eq. (3.7)

$$\mathsf{F}_{A\alpha,\delta,\gamma}(t_\epsilon) \geq r(\mathsf{F}_{A\alpha,\delta,\gamma}(t_\epsilon)) \tag{3.15}$$

but

$$r(\mathsf{F}_{A\alpha,\delta,\gamma}(t_\epsilon)) > (\mathsf{F}_{A\alpha,\delta,\gamma}(t_\epsilon)) \tag{3.16}$$

since $A\alpha \neq \delta$.
From eqs. (3.15)-(3.16)

$$\mathsf{F}_{A\alpha,\delta,\gamma}(t_\epsilon) > \mathsf{F}_{A\alpha,\delta,\gamma}(t_\epsilon). \tag{3.17}$$

This contradicts the fact $A\alpha \neq \delta$. Hence $A\alpha = \delta$.
Claim $\alpha = B\delta$.
Assume $\alpha \neq B\delta$.
Using $a = b_m$, $b = \delta$ in eq. (3.1)

$$\mathsf{F}_{Ab_m,B\delta,\gamma}(t_\epsilon) \geq r(\mathsf{F}_{Sb_m,T\delta,\gamma}(t_\epsilon))$$

as $m \to \infty$ and use eq. (3.11)

$$\mathsf{F}_{\alpha,B\delta,\gamma}(t_\epsilon) \geq r(\mathsf{F}_{\alpha,B\delta,\gamma}(t_\epsilon)) \tag{3.18}$$

but

$$r(\mathsf{F}_{\alpha,B\delta,\gamma}(t_\epsilon)) > (\mathsf{F}_{\alpha,B\delta,\gamma}(t_\epsilon)) \tag{3.19}$$

since $\alpha \neq B\delta$.
From eqs. (3.18)-(3.19)

$$\mathsf{F}_{\alpha,B\delta,\gamma}(t_\epsilon) > \mathsf{F}_{\alpha,B\delta,\gamma}(t_\epsilon). \tag{3.20}$$

This contradicts the fact $\alpha \neq B\delta$.
Hence $\alpha = B\delta$.
Combining all, we can deduce that

$$A\alpha = S\alpha = B\alpha = T\alpha = \alpha. \tag{3.21}$$

Uniqueness: Suppose α_o be another point satisfying eq. (3.21). By eq. (3.1)

$$\mathsf{F}_{A\alpha, B\alpha_o, \gamma}(t_\epsilon) \geq r\mathsf{F}_{S\alpha, T\alpha_o, \gamma}(t_\epsilon). \tag{3.22}$$

Using (3.21) gives

$$\mathsf{F}_{\alpha, \alpha_o, \gamma}(t_\epsilon) \geq r\mathsf{F}_{\alpha, \alpha_o, \gamma}(t_\epsilon) > \mathsf{F}_{\alpha, \alpha_o, \gamma}(t_\epsilon)$$

which is absurd, resulting $\alpha = \alpha_o$.
Therefore, α is the unique common fixed point for the mappings A, S, B, T. □

Now, we provide a supporting illustration to justify Theorem 3.1.

3.1 Example. Let $(\Omega, \mathsf{F}, t_\epsilon)$ be a 2-Menger space where F, t_ϵ be as in (2.1) and $\Omega = [0,1]$. The mappings $A, S, B, T : \Omega \to \Omega$ are defined as

$$A(a) = B(a) = \begin{cases} \frac{1}{4}, & \text{if } a = 0, \\ 1 - 4a, & \text{if } a \in \left(0, \frac{1}{6}\right], \\ a^3, & \text{if } x \in \left(\frac{1}{6}, 1\right], \end{cases} \tag{3.23}$$

$$S(a) = T(a) = \begin{cases} \frac{1}{5}, & \text{if } a = 0, \\ 2a, & \text{if } a \in \left(0, \frac{1}{6}\right], \\ a^2, & \text{if } a \in \left(\frac{1}{6}, 1\right]. \end{cases} \tag{3.24}$$

From eqs. (3.23) and (3.24) we have $a = (\frac{1}{6}, 1)$ are coincidence points for mappings A, S.
At $a = \frac{1}{6}$, $S\left(\frac{1}{6}\right) = A\left(\frac{1}{6}\right) = \frac{1}{3}$ and

$$AS\left(\frac{1}{6}\right) = A\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^3 = \frac{1}{27}, \tag{3.25}$$

$$SA\left(\frac{1}{6}\right) = S\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^2 = \frac{1}{9}. \tag{3.26}$$

From eq. (3.25) and eq. (3.26)

$$AS\left(\frac{1}{6}\right) \neq SA\left(\frac{1}{6}\right). \tag{3.27}$$

Hence from eq. (3.28) the mappings are not weakly compatible.
Take a sequence $(a_m) = \frac{1}{6} - \frac{\sqrt{6}}{m}$, $\forall\ m \geq 1$ then eq. (3.23)

$$\lim_{m\to\infty} Aa_m = \lim_{m\to\infty} A\left(\frac{1}{6} - \frac{\sqrt{6}}{m}\right) = \lim_{m\to\infty} 1 - 4\left(\frac{1}{6} - \frac{\sqrt{6}}{m}\right) = \frac{1}{3}, \tag{3.28}$$

from eq. (3.24)

$$\lim_{m\to\infty} Sa_m = \lim_{m\to\infty} S\left(\frac{1}{6} - \frac{\sqrt{6}}{m}\right) = \lim_{m\to\infty} 2\left(\frac{1}{6} - \frac{\sqrt{6}}{m}\right) = \frac{1}{3}. \tag{3.29}$$

From eqs. (3.28)-(3.29), we get

$$\lim_{m\to\infty} Aa_m = \lim_{m\to\infty} Sa_m = \frac{1}{3}. \tag{3.30}$$

Hence the pairs (A,S), (B,T) are satisfying (E.A)-property.
From eq. (3.30)

$$\left\{(a_m): \lim_{m\to\infty} Aa_m = \lim_{m\to\infty} Sa_m\right\} \neq \phi$$

so that there exists another sequence $(c_m) = 1 - \frac{2\sqrt{2}}{m^2}$, $\forall\ m \geq 1$. Then from eq. (3.23)

$$\lim_{m\to\infty} Ac_m = \lim_{m\to\infty} A\left(1-\frac{2\sqrt{2}}{m^2}\right) = \lim_{m\to\infty}\left(1-\frac{2\sqrt{2}}{m^2}\right)^3 = 1 \tag{3.31}$$

and from eq. (3.24)

$$\lim_{m\to\infty} Sc_m = \lim_{m\to\infty} S\left(1-\frac{2\sqrt{2}}{m^2}\right) = \lim_{m\to\infty}\left(1-\frac{2\sqrt{2}}{m^2}\right)^2 = 1. \tag{3.32}$$

Now from eqs. (3.31)-(3.32)

$$\lim_{m\to\infty} Ac_m = \lim_{m\to\infty} Sc_m = 1. \tag{3.33}$$

Further from eqs. (3.23)-(3.32)

$$\lim_{m\to\infty} ASc_m = \lim_{m\to\infty} P\left(1-\frac{2\sqrt{2}}{m^2}\right)^2 = \lim_{m\to\infty}\left(1-\frac{2\sqrt{2}}{m^2}\right)^6 = 1, \tag{3.34}$$

from eqs. (3.24), (3.31)

$$\lim_{m\to\infty} SAc_m = \lim_{m\to\infty} S\left(1-\frac{2\sqrt{2}}{m^2}\right)^3 = \lim_{m\to\infty}\left(1-\frac{2\sqrt{2}}{m^2}\right)^6 = 1. \tag{3.35}$$

Thus from eqs. (3.34)-(3.35)

$$\lim_{m\to\infty} \mathsf{F}_{ASc_m,SAc_m,a}(\beta) = 1. \tag{3.36}$$

Further from (3.23), (3.34), (3.24) and (3.35)

$$\lim_{m\to\infty} \mathsf{F}_{ASc_m,A(1),a}(\beta) = 1 \ \text{ and } \ \lim_{m\to\infty} \mathsf{F}_{SAc_m,S(1),a}(\beta) = 1. \tag{3.37}$$

From (3.30), (3.36) and (3.37) we can conclude that the pairs (A,S), (B,T) are satisfying (E.A)-property, conditionally compatible and sub-sequentially continuous properties.
Moreover, at $a = 1$, $A(1) = S(1) = B(1) = T(1) = 1$. This demonstrate that the mappings A, S, B, T met all of the conditions of Theorem 3.1 and having single common fixed point at $a = 1$.

Now, we give another generalization of Theorem 2.1 as under.

3.2 Theorem. *Let A, B, S and T be mappings on a probabilistic 2-metric space $(\Omega,\mathsf{F},t_\epsilon)$ to itself satisfying:*

(i) *the pairs (A,S), (B,T) satisfy (E.A)-property,*

(ii) *the pairs (A,S), (B,T) are conditionally semi-compatible and sub-sequentially continuous:*

$$\mathsf{F}_{Aa,Bb,\gamma}(t_\epsilon) \geq r(\mathsf{F}_{Sa,Tb,\gamma}(t_\epsilon)), \quad \forall\ a,b in \Omega,\ t_\epsilon > 0.$$

Then the mappings A, B, S and T have unique common fixed point in Ω.

Proof. The pairs (A,S), (B,T) satisfy (E.A)-property implies there exist two sequences (x_m), (y_m) such that

$$\lim_{m\to\infty} Ax_m = \lim_{m\to\infty} Sx_m = u, \tag{3.38}$$

$$\lim_{m\to\infty} By_m = \lim_{m\to\infty} Ty_m = v, \tag{3.39}$$

for some $u,v \in \Omega$.
From eq. (3.38)

$$\left\{(x_m) : \lim_{m\to\infty} Ax_m = \lim_{m\to\infty} Sx_m\right\} \neq \phi.$$

Conditionally semi-compatible of the pairs (A,S) implies there exists another sequence (u_m), with

$$\lim_{m\to\infty} Au_m = \lim_{m\to\infty} Su_m = \eta, \tag{3.40}$$

such that

$$\lim_{m\to\infty} \mathsf{F}_{ASu_m,S\eta,a}(\beta) = 1 \ \text{ and } \ \lim_{m\to\infty} \mathsf{F}_{SAu_m,A\eta,a}(\beta) = 1. \tag{3.41}$$

Also the sub-sequentially continuous of the pair (A,S) implies

$$\lim_{m\to\infty} \mathsf{F}_{ASu_m,A\eta,a}(\beta) = 1 \ \text{ and } \ \lim_{m\to\infty} \mathsf{F}_{SAb_m,S\eta,a}(\beta) = 1. \tag{3.42}$$

Using eq. (3.42) in eq. (3.41)

$$\mathsf{F}_{A\eta,S\eta,a}(\beta) = 1$$

$$\Rightarrow \qquad A\eta = S\eta. \tag{3.43}$$

From eq. (3.39)

$$\left\{(y_m) : \lim_{m\to\infty} By_m = \lim_{m\to\infty} Ty_m\right\} \neq \phi.$$

Conditionally semi-compatible of the pairs (B,T) implies there exists another sequence (v_m), with

$$\lim_{m\to\infty} Bv_m = \lim_{m\to\infty} Tv_m = \zeta, \tag{3.44}$$

such that

$$\lim_{m\to\infty} \mathsf{F}_{BTv_m,T\zeta,a}(\beta) = 1 \ \text{ and } \ \lim_{m\to\infty} \mathsf{F}_{TBv_m,B\zeta,a}(\beta) = 1. \tag{3.45}$$

Also the sub-sequentially continuous of the pair (B,T) implies

$$\lim_{m\to\infty} \mathsf{F}_{BTv_m,B\zeta,a}(\beta) = 1 \ \text{ and } \ \lim_{m\to\infty} \mathsf{F}_{TBv_m,T\zeta,a}(\beta) = 1. \tag{3.46}$$

Using eq. (3.45) in eq. (3.46)

$$\mathsf{F}_{B\zeta,T\zeta,a}(\beta) = 1$$

$$\Rightarrow \qquad B\zeta = T\zeta. \tag{3.47}$$

Claim $\eta = \zeta$.
Assume $\eta \neq \zeta$.
Using $a = u_m$, $b = v_m$ in eq. (3.1)

$$\mathsf{F}_{Au_m,Bv_m,\gamma}(t_\epsilon) \geq r(\mathsf{F}_{Su_m,Tv_m,\gamma}(t_\epsilon))$$

as $m \to \infty$ from eqs. (3.40), (3.44)

$$\mathsf{F}_{\eta,\zeta,\gamma}(t_\epsilon) \geq r(\mathsf{F}_{\eta,\zeta,\gamma}(t_\epsilon)) \tag{3.48}$$

but

$$r(\mathsf{F}_{\eta,\zeta,\gamma}(t_\epsilon)) > (\mathsf{F}_{\eta,\zeta,\gamma}(t_\epsilon)) \tag{3.49}$$

since $\alpha \neq \delta$.

From eqs. (3.48)-(3.49)

$$\mathsf{F}_{\eta,\zeta,\gamma}(t_\epsilon) > \mathsf{F}_{\eta,\zeta,\gamma}(t_\epsilon). \tag{3.50}$$

This contradicts the fact $\eta \neq \zeta$. Hence $\eta = \zeta$.

Claim $A\eta = \zeta$.

Assume $A\eta \neq \zeta$.

Using $a = \eta$, $b = v_m$ in eq. (3.1)

$$\mathsf{F}_{A\eta,Bb_m,\gamma}(t_\epsilon) \geq r(\mathsf{F}_{S\eta,Tv_m,\gamma}(t_\epsilon))$$

as $m \to \infty$ and use eq. (3.44)

$$\mathsf{F}_{A\eta,\zeta,\gamma}(t_\epsilon) \geq r(\mathsf{F}_{A\eta,\zeta,\gamma}(t_\epsilon)) \tag{3.51}$$

but

$$r(\mathsf{F}_{A\eta,\zeta,\gamma}(t_\epsilon)) > (\mathsf{F}_{A\eta,\zeta,\gamma}(t_\epsilon)) \tag{3.52}$$

since $A\eta \neq \zeta$.

From eqs. (3.51)-(3.52)

$$\mathsf{F}_{A\eta,\zeta,\gamma}(t_\epsilon) > \mathsf{F}_{A\eta,\zeta,\gamma}(t_\epsilon). \tag{3.53}$$

This contradicts the fact $A\eta \neq \zeta$. Hence $A\eta = \zeta$.

Claim $\eta = B\zeta$.

Assume $\eta \neq B\zeta$.

Using $a = u_m$, $b = \zeta$ in eq. (3.1)

$$\mathsf{F}_{Au_m,B\zeta,\gamma}(t_\epsilon) \geq r(\mathsf{F}_{Su_m,T\zeta,\gamma}(t_\epsilon))$$

as $m \to \infty$ and use eqs. (3.40), (3.47)

$$\mathsf{F}_{\eta,B\zeta,\gamma}(t_\epsilon) \geq r(\mathsf{F}_{\eta,B\zeta,\gamma}(t_\epsilon)) \tag{3.54}$$

but

$$r(\mathsf{F}_{\eta,B\zeta,\gamma}(t_\epsilon)) > (\mathsf{F}_{\eta,B\zeta,\gamma}(t_\epsilon)) \tag{3.55}$$

since $\eta \neq B\zeta$.

From eqs. (3.54)-(3.55)

$$\mathsf{F}_{\eta,B\zeta,\gamma}(t_\epsilon) > \mathsf{F}_{\eta,B\zeta,\gamma}(t_\epsilon). \tag{3.56}$$

This contradicts the fact $\eta \neq B\zeta$. Hence $\eta = B\zeta$. Combining all we can conclude that

$$A\eta = S\eta = B\eta = T\eta = \eta. \tag{3.57}$$

Uniqueness follows easily. □

Now, we justified our theorem with proper illustration.

3.2 Example. Let $(\Omega, \mathsf{F}, t_\epsilon)$ be a 2-Menger space where F, t_ϵ be as in eq. (2.1) and $\Omega = R$. The mappings $A, S, B, T : \Omega \to \Omega$ are defined as

$$A(a) = B(a) = \begin{cases} \left(\frac{1}{3}\right)^a, & \text{if } a \le 0, \\ a^2, & \text{if } a > 0, \end{cases} \tag{3.58}$$

$$S(a) = T(a) = \begin{cases} \left(\frac{1}{3}\right)^{2a}, & \text{if } a \le 0, \\ 3a - 2, & \text{if } a > 0. \end{cases} \tag{3.59}$$

From eq. (3.58) and eq. (3.59), $a = 0, 1$ and 2 are coincidence points for the mappings A, S.
At $a = 2$, $S(2) = A(2) = 4$ and

$$AS(2) = A(4) = 16, \tag{3.60}$$

$$SA(2) = S(4) = 10. \tag{3.61}$$

From eq. (3.60) and eq. (3.61)

$$AS(2) \neq SA(2). \tag{3.62}$$

Hence from eq. (3.62) the mappings are not weakly compatible.
For a sequence $(p_m) = -\frac{\sin\frac{\pi m}{2}}{m}$, $\forall \ge 1$ then eq. (3.58)

$$\lim_{m\to\infty} Aa_m = \lim_{m\to\infty} A\left(-\frac{\sin\frac{\pi m}{2}}{m}\right) = \lim_{m\to\infty}\left(\frac{1}{3}\right)^{-(\frac{\sin\frac{\pi m}{2}}{m})} = 1, \tag{3.63}$$

from eq. (3.59)

$$\lim_{m\to\infty} Sa_m = \lim_{m\to\infty} S\left(-\frac{\sin\frac{\pi m}{2}}{m}\right) = \lim_{m\to\infty}\left(\frac{1}{3}\right)^{-2(\frac{\sin\frac{\pi m}{2}}{m})} = 1. \tag{3.64}$$

From eq. (3.63)-(3.64), we get

$$\lim_{m\to\infty} Aa_m = \lim_{m\to\infty} Sa_m = 1. \tag{3.65}$$

Hence the pairs (A, S), (B, T) are satisfying (E.A)-property.
From eq. (3.65)

$$\left\{(p_m) : \lim_{m\to\infty} Ap_m = \lim_{m\to\infty} Sp_m\right\} \neq \phi$$

there exists another sequence $(q_m) = 1 - \frac{\cos\frac{\pi m}{2}}{m^2}$, $\forall\ m \ge 1$. Then from eq. (3.58)

$$\lim_{m\to\infty} Aq_m = \lim_{m\to\infty} A\left(1 - \frac{\cos\frac{\pi m}{2}}{m^2}\right) = \lim_{m\to\infty}\left(1 - \frac{\cos\frac{\pi m}{2}}{m^2}\right)^2 = 1 \tag{3.66}$$

and eq. (3.59) gives

$$\lim_{m\to\infty} Sq_m = \lim_{m\to\infty} S\left(1 - \frac{\cos\frac{\pi m}{2}}{m^2}\right) = \lim_{m\to\infty} 3\left(1 - \frac{\cos\frac{\pi m}{2}}{m^2}\right) - 2 = 1. \tag{3.67}$$

From eqs. (3.65), (3.67)

$$\lim_{m\to\infty} Aq_m = \lim_{m\to\infty} Sq_m = 1. \tag{3.68}$$

Further from eqs. (3.58), (3.67)

$$\lim_{m\to\infty} ASq_m = \lim_{m\to\infty} A\left(1-3\frac{\cos\frac{\pi m}{2}}{m^2}\right)^2 = \lim_{m\to\infty}\left(1-3\frac{\cos\frac{\pi m}{2}}{m^2}\right)^2 = 1 \tag{3.69}$$

from eqs. (3.59), (3.66)

$$\lim_{m\to\infty} SAq_m = \lim_{m\to\infty} S\left(1-\frac{\cos\frac{\pi m}{2}}{m^2}\right)^2 = \lim_{m\to\infty} 3\left(1-\frac{\cos\frac{\pi m}{2}}{m^2}\right)^2 - 2 = 1. \tag{3.70}$$

Thus from eqs. (3.58), (3.59), and from eqs. (3.69), (3.70)

$$\lim_{m\to\infty} \mathsf{F}_{ASq_m,S(1),a}(\beta) = 1 \text{ and } \lim_{m\to\infty} \mathsf{F}_{SAq_m,A(1),a}(\beta) = 1. \tag{3.71}$$

Further from eqs. (3.58), (3.59), and from eqs. (3.69), (3.70)

$$\lim_{m\to\infty} \mathsf{F}_{ASq_m,A(1),a}(\beta) = 1 \text{ and } \lim_{m\to\infty} \mathsf{F}_{SAq_m,S(1),a}(\beta) = 1. \tag{3.72}$$

From eqs. (3.65), (3.71) and (3.72) we conclude that the pairs (A,S), (B,T) are satisfying (E.A)-property, conditionally semi-compatible and sub-sequentially continuous properties. Moreover, at $a=1$, $A(1)=S(1)=B(1)=T(1)=1$. Thus the mappings A, S, B, T satisfy all the conditions of Theorem 3.2 having the single common fixed point at $a=1$.

4. Conclusions

We generalized Theorem 2.1 in two ways by using: (i) the conditions (E.A)-property, conditionally compatible and sub-sequentially continuous in place of weakly compatible mappings in Theorem 3.1 (ii) the conditions (E.A)-property, conditionally semi-compatible and sub-sequentially continuous in place of weakly compatible mappings in Theorem 3.2. Further, these two results are justified with suitable examples.

Remark. In our result it can be noticed that the condition of completeness, closed property of subspace and inclusion condition have been removed. Further (E.A)-property is admitted in place of inclusion condition. Moreover the conditions of conditionally compatible, conditionally semi-compatible and sub-sequentially continuous have been utilized. These conditions are weaker than weakly compatible condition used in Theorem 2.1.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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