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Research Article

# Reliability Modeling of a System With One Main Unit and Two Associate Units Along With Sub-Unit

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**Abstract.** In this paper, we develop a stochastic reliability model of a one-unit main system with two associate units. Each associate unit also has one sub-unit. Sub-units depend upon Associate units and associate units depend upon the main unit. The system will be operable when the main unit and at least one associate unit with sub-unit are in operation. As soon as a job arrives, all the units work with load. It is assumed that only one job is taken for processing at a time. There is a single repairman who repairs the failed units on a first-come-first-served basis. Various reliability characteristics of the system effectiveness will be obtained by using semi-Markov processes and regenerative point techniques.

**Keywords.** Maximum operation time (MOT), Preventive maintenance (PM), Environmental failure (EF), Profit, Regenerative point techniques, Semi Markov process

**Mathematics Subject Classification (2020).** 90B25, 60K10



## 1. Introduction

Today's competitive world and increasing customer demand for highly reliable products makes reliability engineering a more challenging task. Reliability analysis is one of the main tools to make sure agreed delivery deadlines which in turn support certainty in real tangible factors such as customer goodwill and company reputation. The main aim of the manufacturing industries is to produce reliable products that work without failure under stated conditions. The performance study of manufacturing industries is the need for time to improve the production process. Downtime often leads to both tangible and intangible losses. These losses might be due to some unreliable components; thus, an effective strategy needs to be framed out for maintenance and repair/replacement, related to those components. In the field of reliability modeling large number of researchers such as Bashir and Joorel [1] considered "Mean time to failure with preventive maintenance", Bhardwaj and Singh [2] described "Optimum preventive maintenance policies for repairable systems", Malik and Dhankar [3] studied "Stochastic behavior of a two unit priority stand by redundant system with repair", Nakagawa [5] studied "Reliability analysis of a three unit complex system working in series-parallel configuration", Nakagawa [6] developed "Reliability modeling and cost analysis of a system with replacement of the server and unit subject to inspection", Nakagawa and Osaki [4] studied "Cost-benefit analysis of a single unit system with scheduled maintenance and variation in demand". Pathak *et al*. [8] Developed the configuration modeling and analysis of wire rod mill system". Pathak *et al*. [7] studied "Steady State Behavior of a Cold-Standby System with Server Failure and Arbitrary Repair, Replacement & Treatment", Singh and Singh [9] analyses Cost-benefit analysis of a single unit system with scheduled maintenance and variation in demand. Earlier, Taneja and Malhotra [10] studied reliability parameters of a main unit with its supporting units and compared the results with two different distributions. This model particularly differs from the other models in the sense that the concept used in this model is based on the real situation. This type of analysis is of immense help to the owners of small-scale industries. Also, the involvement of preventive maintenance in the model increases the reliability of the functioning units.

## 2. Materials and Method

In this study, the stochastic reliability of the system is analyzed by using semi-Markov process and regenerative point techniques expression for various reliability measures like Mean time to system failure. MTSF, the steady state Availability, the busy period of the server due to repair of a failed unit at $t = 0$. Busy period of the server due to preventive maintenance at $t = 0$. Expected down time at $t = 0$. Expected number of visit by server at $t = 0$. Profit incurred to the system.

### System Description

The system consists of five units namely one main unit $M$ and two associate units $U$ with sub-unit $A$ and $T$ with sub-unit $B$. Here the associate unit $U$-$A$ and $T$-$B$ dependents upon the main unit's $M$. The system is operable when the main unit and at least one associate unit with sub-unit are in operation. The main unit is employed to rotate $U$-$A$ and $T$-$B$. As soon as a job arrives, all the units work with load. It is assumed that only one job is taken for processing at a time. There is a single repairman who repairs the failed units on a first-come-first-served basis. Using regenerative point technique, several system characteristics such as transition probabilities, mean sojourn times, availability and busy period of the repairman are evaluated. In the end, the expected profit is also calculated.

## Assumptions

(a) The system consists of one main unit and two associate units with one sub-unit.

(b) The associate unit $U$-$A$ and $T$-$B$ works with the help of the main unit and sub-unit.

(c) There is a single repairman who repairs the failed units on a priority basis.

(d) After a random period of time, the whole system goes to preventive maintenance.

(e) All units work as new after repair.

(f) The failure rates of all the units are taken to be exponential whereas the repair time distributions are arbitrary.

(g) Switching devices are perfect and instantaneous.

## Notations

$p_{ij}$ = Transition probabilities from $S_i$ to $S_j$,

$\mu_i$ = Mean Sojourn time at time $t$,

$E_0$ = State of the system at epoch $t = 0$,

$E$ = set of regenerative states,

$q_{ij}(t)$ = Probability density function of transition time from $S_i$ to $S_j$,

$Q_{ij}(t)$ = Cumulative distribution function of transition time from $S_i$ to $S_j$,

$\pi_i(t)$ = Cdf of time to system failure when starting from state $E_0 = S_i \in E$,

$\mu_i(t)$ = Mean Sojourn time in the state $E_0 = S_i \in E$,

$B_i(t)$ = Repairman is busy in the repair at time $t/E_0 = S_i \in E$,

$r_1/r_2/r_3/r_4/r_5$ = Constant repair rate of Main unit $M$/Associate unit $U$/Associate unit $T$/Sub-unit $A$/Sub-unit $B$,

$\alpha/\beta/\gamma/\delta/\eta$ = Failure rate of Main unit $M$/Associate unit $U$/Associate unit $T$/Sub-unit $A$/Sub-unit $B$,

$g_1/g_2/g_3/g_4/g_5$ = Probability density function of repair time of Main unit $M$/Associate unit $U$/Associate unit $T$/Sub-unit $A$/Sub-unit $B$,

$\bar{G}_1/\bar{G}_2/\bar{G}_3/\bar{G}_4/\bar{G}_5$ = Cumulative distribution function of repair time of Main unit $M$/Associate unit $U$/Associate unit $T$/Sub-unit $A$/Sub-unit $B$,

$a(t)$ = Probability density function of preventive maintenance,

$b(t)$ = Probability density function of preventive maintenance completion time,

$\bar{A}(t)$ = Cumulative distribution functions of preventive maintenance,

$\bar{B}(t)$ = Cumulative distribution functions of preventive maintenance completion time,

$\boxed{s}$ = Symbol for Laplace-Stieltjes transforms,

$\boxed{c}$ = Symbol for Laplace-convolution.

## Symbols

$M_0/M_g/M_r$ — Main unit '$M$' under operation/good and non-operative mode/repair mode,

$U_0/U_g/U_r$ — Associate unit '$U$' under operation/good and non-operative mode/repair mode,
$T_0/T_g/T_r$ — Associate unit '$T$' under operation/good and non-operative mode/repair mode,
$A_0/A_g/A_r$ — Sub unit '$A$' under operation/good and non-operative mode/repair mode,
$B_0/B_r/B_g$ — Associate Unit '$B$' under operation/repair/good and non-operative mode,
P.M. — System under preventive maintenance.

## Up states

$S_0 = (M_0, U_0, T_0, A_0, B_0)$; $S_2 = (M_0, U_r, T_0, A_g, B_0)$; $S_3 = (M_0, U_0, T_r, A_0, B_g)$;
$S_4 = (M_0, U_g, T_0, A_r, B_0)$; $S_5 = (M_0, U_0, T_g, A_0, B_r)$.

## Down States

$S_1 = (M_r, U_g, T_g, A_g, B_g)$; $S_6 = (S.D.)$; $S_7 = $ (P.M.).

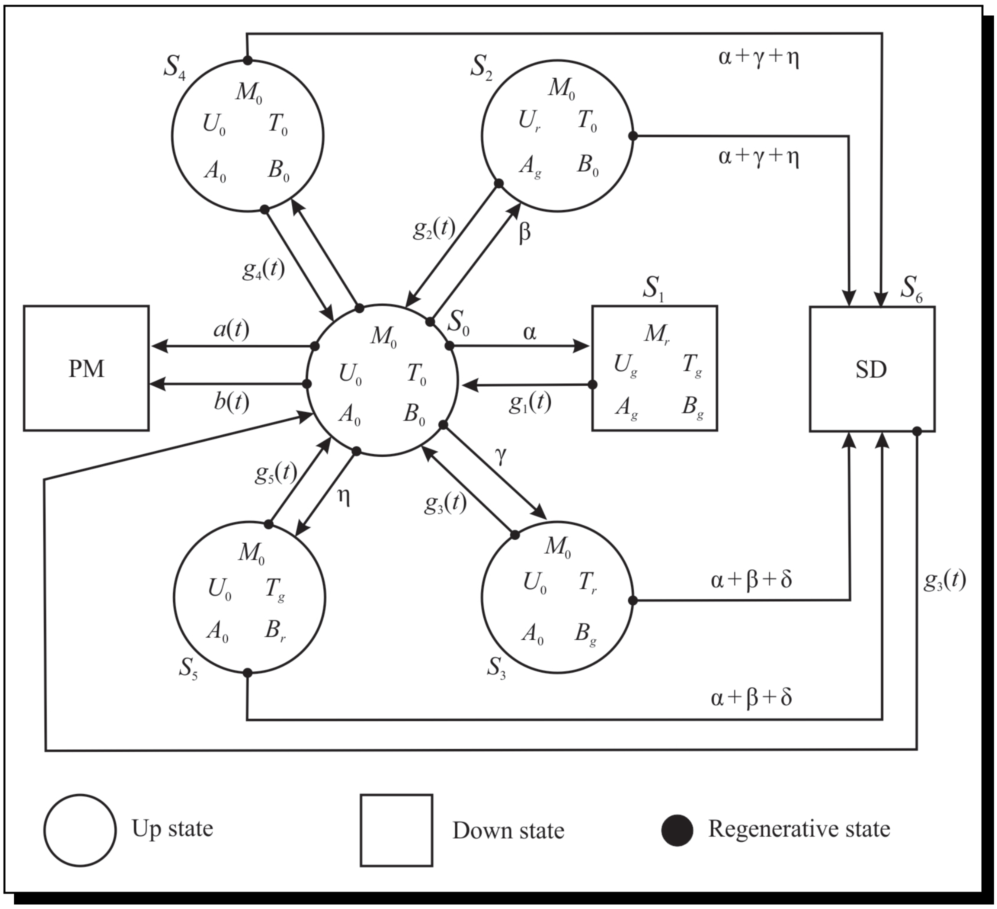


**Figure 1.** State transition diagram

# 3. Mathematical Analysis of the System

## 3.1 Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following non-zero transition probabilities:

$$Q_{01}(t) = \int_0^t \alpha e^{-(\alpha+\beta+\gamma+\delta+\eta)t} \bar{A}(t) dt, \tag{3.1}$$

$$Q_{02}(t) = \int_0^t \beta e^{-(\alpha+\beta+\gamma+\delta+\eta)t} \bar{A}(t) d, \tag{3.2}$$

$$Q_{03}(t) = \int_0^t \gamma e^{-(\alpha+\beta+\gamma+\delta+\eta)t} \bar{A}(t) dt, \tag{3.3}$$

$$Q_{04}(t) = \int_0^t \delta e^{-(\alpha+\beta+\gamma+\delta+\eta)t} \bar{A}(t) dt, \tag{3.4}$$

$$Q_{05}(t) = \int_0^t \eta e^{-(\alpha+\beta+\gamma+\delta+\eta)t} \bar{A}(t) dt, \tag{3.5}$$

$$Q_{07}(t) = \int_0^t a(t) e^{-(\alpha+\beta+\gamma+\delta+\eta)t} dt, \tag{3.6}$$

$$Q_{10}(t) = \int_0^t g_1(t) dt, \tag{3.7}$$

$$Q_{20}(t) = \int_0^t e^{-(\alpha+\gamma+\eta)t} g_2(t) dt, \tag{3.8}$$

$$Q_{26}(t) = \int_0^t (\alpha+\gamma+\eta) e^{-(\alpha+\gamma+\eta)t} \bar{G}_2(t) dt, \tag{3.9}$$

$$Q_{30}(t) = \int_0^t e^{-(\alpha+\beta+\delta)t} g_3(t) dt, \tag{3.10}$$

$$Q_{36}(t) = \int_0^t (\alpha+\beta+\delta) e^{-(\alpha+\beta+\delta)t} \bar{G}_3(t) dt, \tag{3.11}$$

$$Q_{40}(t) = \int_0^t e^{-(\alpha+\gamma+\eta)t} g_4(t) dt, \tag{3.12}$$

$$Q_{46}(t) = \int_0^t (\alpha+\gamma+\eta) e^{-(\alpha+\gamma+\eta)t} \bar{G}_4(t) dt, \tag{3.13}$$

$$Q_{50}(t) = \int_0^t e^{-(\alpha+\beta+\delta)t} g_5(t) dt, \tag{3.14}$$

$$Q_{56}(t) = \int_0^t (\alpha+\beta+\delta) e^{-(\alpha+\beta+\delta)t} \bar{G}_5(t) dt, \tag{3.15}$$

$$Q_{60}(t) = \int_0^t g_6(t) dt, \tag{3.16}$$

$$Q_{70}(t) = \int_0^t b(t) dt, \tag{3.17}$$

where $x_1 = \alpha+\beta+\gamma+\delta+\eta$.

Now, letting $t \to \infty$, we get $\lim_{t\to\infty} Q_{ij}(t) = p_{ij}$

$$p_{01} = \int_0^\infty \alpha e^{-(\alpha+\beta+\gamma+\delta+\eta)t} \bar{A}(t) dt = \frac{\alpha}{x_1}[1 - a^*(x_1)], \tag{3.18}$$

$$p_{02} = \int_0^\infty \beta e^{-(\alpha+\beta+\gamma+\delta+\eta)t} \bar{A}(t) dt = \frac{\beta}{x_1}[1 - a^*(x_1)], \tag{3.19}$$

$$p_{03} = \int_0^\infty \gamma e^{-(\alpha+\beta+\gamma+\delta+\eta)t} \bar{A}(t) dt = \frac{\gamma}{x_1}[1 - a^*(x_1)], \tag{3.20}$$

$$p_{04} = \int_0^\infty \delta e^{-(\alpha+\beta+\gamma+\delta+\eta)t} \bar{A}(t) dt = \frac{\delta}{x_1}[1 - a^*(x_1)], \tag{3.21}$$

$$p_{05}=\int_0^\infty \eta e^{-(\alpha+\beta+\gamma+\delta+\eta)t}\bar{A}(t)dt=\frac{\eta}{x_1}[1-a^*(x_1)], \tag{3.22}$$

$$p_{07}=\int_0^\infty a(t)e^{-(\alpha+\beta+\gamma+\delta+\eta)t}dt=a^*(x_1), \tag{3.23}$$

$$p_{10}=\int_0^\infty g_1(t)dt=1, \tag{3.24}$$

$$p_{20}(t)=\int_0^\infty e^{-(\alpha+\gamma+\eta)t}g_2(t)dt=g_2^*(\alpha+\gamma+\eta), \tag{3.25}$$

$$p_{26}=\int_0^\infty (\alpha+\gamma+\eta)e^{-(\alpha+\gamma+\eta)t}\bar{G}_2(t)dt=1-g_2^*(\alpha+\gamma+\eta), \tag{3.26}$$

$$p_{30}(t)=\int_0^\infty e^{-(\alpha+\beta+\delta)t}g_3(t)dt=g_3^*(\alpha+\beta+\delta), \tag{3.27}$$

$$p_{36}=\int_0^\infty (\alpha+\beta+\delta)e^{-(\alpha+\beta+\delta)t}\bar{G}_3(t)dt=1-g_3^*(\alpha+\beta+\delta), \tag{3.28}$$

$$p_{40}(t)=\int_0^\infty e^{-(\alpha+\gamma+\eta)t}g_4(t)dt=g_4^*(\alpha+\gamma+\eta), \tag{3.29}$$

$$p_{46}=\int_0^\infty (\alpha+\gamma+\eta)e^{-(\alpha+\gamma+\eta)t}\bar{G}_4(t)dt=1-g_4^*(\alpha+\gamma+\eta), \tag{3.30}$$

$$p_{50}(t)=\int_0^\infty e^{-(\alpha+\beta+\delta)t}g_5(t)dt=g_5^*(\alpha+\beta+\delta), \tag{3.31}$$

$$p_{56}=\int_0^\infty (\alpha+\beta+\delta)e^{-(\alpha+\beta+\delta)t}\bar{G}_5(t)dt=1-g_5^*(\alpha+\beta+\delta), \tag{3.32}$$

$$p_{70}(t)=\int_0^\infty b(t)dt=1, \tag{3.33}$$

$$p_{60}=\int_0^\infty g_6(t)dt=1, \tag{3.34}$$

$$p_{10}=p_{60}=p_{70}=1. \tag{3.35}$$

It is easy to see that

$$\left.\begin{array}{l} p_{01}+p_{02}+p_{03}+p_{04}+p_{05}+p_{07}=1,\;\; p_{20}+p_{26}=1,\;\; p_{30}+p_{36}=1, \\ p_{40}+p_{46}=1,\;\; p_{50}+p_{56}=1 \end{array}\right\} \tag{3.36}$$

and mean sojourn time are given by

$$\mu_0=\frac{1}{x_1}[1-a^*(x_1)], \tag{3.37}$$

$$\mu_1=\int_0^\infty \bar{G}_1(t)dt, \tag{3.38}$$

$$\mu_2=\frac{1}{\alpha+\gamma+\eta}[1-g_2^*(\alpha+\gamma+\eta), \tag{3.39}$$

$$\mu_3=\frac{1}{\alpha+\beta+\delta}[1-g_3^*(\alpha+\beta+\delta)], \tag{3.40}$$

$$\mu_4=\frac{1}{\alpha+\gamma+\eta}[1-g_4^*(\alpha+\gamma+\eta), \tag{3.41}$$

$$\mu_5=\frac{1}{\alpha+\beta+\delta}[1-g_5^*(\alpha+\beta+\delta)], \tag{3.42}$$

$$\mu_7 = \int_0^\infty \bar{B}(t)dt, \tag{3.43}$$

$$\mu_6 = \int_0^\infty \bar{G}_6(t)dt. \tag{3.44}$$

We note that the Laplace-Stieltjes transform of $Q_{ij}(t)$ is equal to Laplace transform of $q_{ij}(t)$, i.e.,

$$\tilde{Q}_{ij}(s) = \int_0^\infty e^{-st} Q_{ij}(t)dt = L\{Q_{ij}(t)\} = q^*_{ij}(s), \tag{3.45}$$

$$\tilde{Q}_{01}(s) = \int_0^\infty \alpha e^{-(s+x_1)t} \bar{A}(t)dt = \frac{\alpha}{s+x_1}[1-a^*(s+x_1)], \tag{3.46}$$

$$\tilde{Q}_{02}(s) = \int_0^\infty \beta e^{-(s+x_1)t} \bar{A}(t)dt = \frac{\beta}{s+x_1}[1-a^*(s+x_1)], \tag{3.47}$$

$$\tilde{Q}_{03}(s) = \int_0^\infty \gamma e^{-(s+x_1)t} \bar{A}(t)dt = \frac{\gamma}{s+x_1}[1-a^*(s+x_1)], \tag{3.48}$$

$$\tilde{Q}_{04}(s) = \int_0^\infty \delta e^{-(s+x_1)t} \bar{A}(t)dt = \frac{\delta}{s+x_1}[1-a^*(s+x_1)], \tag{3.49}$$

$$\tilde{Q}_{05}(s) = \int_0^\infty \eta e^{-(s+x_1)t} \bar{A}(t)dt = \frac{\eta}{s+x_1}[1-a^*(s+x_1)], \tag{3.50}$$

$$\tilde{Q}_{07}(s) = \int_0^\infty e^{-(s+x_1)t} a(t)dt = a^*(s+x_1), \tag{3.51}$$

$$\tilde{Q}_{10}(s) = \int_0^\infty e^{-st} g_1(t)dt = g^*_1(s), \tag{3.52}$$

$$\tilde{Q}_{20}(s) = \int_0^\infty e^{-(s+\alpha+\gamma+\eta)t} g_2(t)dt = g^*_2(s+\alpha+\gamma+\eta), \tag{3.53}$$

$$\tilde{Q}_{26}(s) = \int_0^\infty (\alpha+\gamma+\eta) e^{-(s+\alpha+\gamma+\eta)t} \bar{G}_2(t)dt = \frac{(\alpha+\gamma+\eta)}{s+\alpha+\gamma+\eta}[1-g^*_2(s+\alpha+\gamma+\eta)], \tag{3.54}$$

$$\tilde{Q}_{30}(s) = \int_0^\infty e^{-(s+\alpha+\beta+\delta)t} g_3(t)dt = g^*_3(s+\alpha+\beta+\delta), \tag{3.55}$$

$$\tilde{Q}_{36}(s) = \int_0^\infty (\alpha+\beta+\delta) e^{-(s+\alpha+\beta+\delta)t} \bar{G}_3(t)dt = \frac{(\alpha+\beta+\delta)}{s+\alpha+\beta+\delta}[1-g^*_3(s+\alpha+\beta+\delta)], \tag{3.56}$$

$$\tilde{Q}_{40}(s) = \int_0^\infty e^{-(s+\alpha+\gamma+\eta)t} g_4(t)dt = g^*_4(s+\alpha+\gamma+\eta), \tag{3.57}$$

$$\tilde{Q}_{46}(s) = \int_0^\infty (\alpha+\gamma+\eta) e^{-(s+\alpha+\gamma+\eta)t} \bar{G}_4(t)dt = \frac{(\alpha+\gamma+\eta)}{s+\alpha+\gamma+\eta}[1-g^*_4(s+\alpha+\gamma+\eta)], \tag{3.58}$$

$$\tilde{Q}_{50}(s) = \int_0^\infty e^{-(s+\alpha+\beta+\delta)t} g_5(t)dt = g^*_5(s+\alpha+\beta+\delta), \tag{3.59}$$

$$\tilde{Q}_{56}(s) = \int_0^\infty (\alpha+\beta+\delta) e^{-(s+\alpha+\beta+\delta)t} \bar{G}_5(t)dt = \frac{(\alpha+\beta+\delta)}{s+\alpha+\beta+\delta}[1-g^*_5(s+\alpha+\beta+\delta)], \tag{3.60}$$

$$\tilde{Q}_{70}(s) = \int_0^\infty e^{-st} b(t)dt = b^*(s), \tag{3.61}$$

$$\tilde{Q}_{60}(s) = \int_0^\infty e^{-st} g_6(t)dt = g^*_6(s). \tag{3.62}$$

We define $m_{ij}$ as follows:

$$m_{ij} = -\left[\frac{d}{ds}\tilde{Q}_{ij}(s)\right]_{s=0} = -Q'_{ij}(0). \tag{3.63}$$

It can to show that

$$\left.\begin{array}{l} m_{01}+m_{02}+m_{03}+m_{04}+m_{05}+m_{07}=\mu_0; \quad m_{20}+m_{26}=\mu_2; \quad m_{30}+m_{36}=\mu_3; \\ m_{40}+m_{46}=\mu_4; \quad m_{50}+m_{56}=\mu_5, \end{array}\right\} \tag{3.64}$$

where $\alpha+\beta+\gamma+\delta+\eta=x_1$.

### 3.2 Mean Time to System Failure

Time to system failure can be regarded as the first passage time to the failed state. To obtain it we regard the down state as absorbing. Using the argument as for the regenerative process, we obtain the following recursive relations. $\pi_i(t)$ is defined as the cumulative distribution function of first passage time from $i$th state to a failed state

$$\pi_0(t)=Q_{01}(t)+Q_{02}(t)\,\boxed{s}\,\pi_2(t)+Q_{03}(t)\,\boxed{s}\,\pi_3(t)+Q_{04}(t)\,\boxed{s}\,\pi_4(t)+Q_{05}(t)\,\boxed{s}\,\pi_5(t)+Q_{07}(t), \tag{3.65}$$

$$\pi_2(t)=Q_{20}(t)\,\boxed{s}\,\pi_0(t)+Q_{26}(t), \tag{3.66}$$

$$\pi_3(t)=Q_{30}(t)\,\boxed{s}\,\pi_0(t)+Q_{36}(t), \tag{3.67}$$

$$\pi_4(t)=Q_{40}(t)\,\boxed{s}\,\pi_0(t)+Q_{46}(t), \tag{3.68}$$

$$\pi_5(t)=Q_{50}(t)\,\boxed{s}\,\pi_0(t)+Q_{56}(t). \tag{3.69}$$

Taking Laplace-Stieltjes Transform on both sides and solving we get, The mean time to system failure when the system starts from the state $S_0$ is given by

$$E(T)=-\left[\frac{d}{ds}\tilde{\pi}_0(s)\right]_{s=0}=\frac{D'_1(0)-N'_1(0)}{D_1(0)} \tag{3.70}$$

$$=\frac{\mu_0+\mu_2p_{02}+\mu_3p_{03}+\mu_4p_{04}+\mu_5p_{05}}{1-p_{02}p_{20}-p_{03}p_{30}-p_{04}p_{40}-p_{05}p_{50}}. \tag{3.71}$$

### 3.3 Availability Analysis

Let $M_i(t)$ $(i=0,2,3,4,5)$ denote the probability that the system is initially in regenerative state $S_i \in E$ is up at time $t$ without passing through any other regenerative state or returning to itself through one or more non regenerative states, i.e., either it continues to remain in a regenerative $S_i$ or a non-regenerative state, including itself. By probabilistic arguments, we have the following recursive relations.

$$\left.\begin{array}{l} M_0(t)=e^{-(\alpha+\beta+\gamma+\delta+\eta)t}\bar{A}(t), \quad M_2(t)=e^{-(\alpha+\gamma+\eta)t}\bar{G}_2(t), \quad M_3(t)=e^{-(\alpha+\beta+\delta)t}\bar{G}_3(t), \\ M_4(t)=e^{-(\alpha+\gamma+\eta)t}\bar{G}_4(t), \quad M_5(t)=e^{-(\alpha+\beta+\delta)t}\bar{G}_5(t). \end{array}\right\} \tag{3.72}$$

Recursive relations giving point wise availability $A_i(t)$given as follows:

$$A_0(t)=M_0(t)+\sum_{i=1,2,3,4,5,7} q_{0i}(t)\,\boxed{c}\,A_i(t), \tag{3.73}$$

$$A_1(t)=q_{10}(t)\,\boxed{c}\,A_0(t); \tag{3.74}$$

$$A_2(t)=M_2(t)+\sum_{i=0,6} q_{2i}(t)\,\boxed{c}\,A_i(t), \tag{3.75}$$

$$A_3(t)=M_3(t)+\sum_{i=0,6} q_{3i}(t)\,\boxed{c}\,A_i(t), \tag{3.76}$$

$$A_4(t) = M_4(t) + \sum_{i=0,6} q_{4i}(t) \boxed{c} A_i(t), \tag{3.77}$$

$$A_5(t) = M_5(t) + \sum_{i=0,6} q_{5i}(t) \boxed{c} A_i(t), \tag{3.78}$$

$$A_6(t) = q_{60}(t) \boxed{c} A_0(t), \tag{3.79}$$

$$A_7(t) = q_{70}(t) \boxed{c} A_0(t). \tag{3.80}$$

Taking Laplace-Stieltjes transformation of above equations and solving the steady-state availability is given by

$$A_0^*(\infty) = \lim_{t\to\infty} A_0^*(t) = \lim_{S\to 0} sA_0^*(s) = \frac{N_2(0)}{D_2'(0)} = \frac{\mu_0 L_0 + \mu_2 L_2 + \mu_3 L_3 + \mu_4 L_4 + \mu_5 L_5}{\sum\limits_{i=0,1,2,3,4,5,6,7} \mu_i L_i}, \tag{3.81}$$

where

$$N_2(0) = \mu_0 + \mu_2 p_{02} + \mu_3 p_{03} + \mu_4 p_{04} + \mu_5 p_{05}, \tag{3.82}$$

where

$$\left.\begin{aligned} &1 = L_0;\ p_{01} = L_1;\ p_{02} = L_2 p_{03} = L_3;\ p_{04} = L_4;\ p_{05} = L_5; \\ &p_{02}p_{26} + p_{03}p_{36} + p_{04}p_{46} + p_{05}p_{56} = L_6;\ p_{07} = L_7 \end{aligned}\right\}$$

### 3.4 Busy Period Analysis

**(a)** Let $W_i(t)$ $(i = 1,2,3,4)$denote the probability that the repairman is busy initially with repair in a regenerative state $S_i$ and remains busy at an epoch $t$ without transiting to any other state or returning to itself through one or more regenerative states.

By probabilistic arguments, we have

$$W_1(t) = \bar{G}_1(t),\ \ W_2(t) = \bar{G}_2(t),\ \ W_3(t) = \bar{G}_3(t),\ \ W_4(t) = \bar{G}_4(t),\ \ W_5(t) = \bar{G}_5(t). \tag{3.83}$$

Developing similar recursive relations as in availability, we have

$$B_0(t) = \sum_{i=1,2,3,4,5,7} q_{0i}(t) \boxed{c} B_i(t), \tag{3.84}$$

$$B_1(t) = W_1(t) + q_{10}(t) \boxed{c} B_0(t); \tag{3.85}$$

$$B_2(t) = W_2(t) + \sum_{i=0,6} q_{2i}(t) \boxed{c} B_i(t), \tag{3.86}$$

$$B_3(t) = W_3(t) + \sum_{i=0,6} q_{3i}(t) \boxed{c} B_i(t), \tag{3.87}$$

$$B_4(t) = W_4(t) + \sum_{i=0,6} q_{4i}(t) \boxed{c} B_i(t), \tag{3.88}$$

$$B_5(t) = W_5(t) + \sum_{i=0,6} q_{5i}(t) \boxed{c} B_i(t), \tag{3.89}$$

$$B_6(t) = q_{60}(t) \boxed{c} B_0(t), \tag{3.90}$$

$$B_7(t) = q_{70}(t) \boxed{c} B_0(t). \tag{3.91}$$

Taking Laplace-Stieltjes transformation of above equations and solving this, the fraction of time for which the repairman is busy with repair of the failed unit is given

$$B_0^{1^*}(\infty) = \lim_{t\to\infty} B_0^{1^*}(t) = \lim_{s\to 0} sB_0^{1^*}(s) = \frac{N_3(0)}{D_2'(0)} = \frac{\sum\limits_{i=1,2,3,4,5} \mu_i L_i}{\sum\limits_{i=0,1,2,3,4,5,6,7} \mu_i L_i}, \tag{3.92}$$

where

$$N_3(0) = (\mu_1 p_{01} + \mu_2 p_{02} + \mu_3 p_{03} + \mu_4 p_{04} + \mu_5 p_{05})$$
$$= \mu_1 L_1 + \mu_2 L_2 + \mu_3 L_3 + \mu_4 L_4 + \mu_5 L_5 = \sum_{i=1,2,3,4,5} \mu_i L_i . \tag{3.93}$$

**(b) Busy period of the Repairman in Preventive Maintenance in Time** $(0,t]$

Preceding the similar fashion as in the steady state busy period of server due to preventive maintenance of the system is given by

$$B_0^{2^*}(\infty) = \lim_{t\to\infty} B_0^{2^*}(t) = \lim_{s\to 0} sB_0^{2^*}(s) = \frac{N_4(0)}{D_2'(0)} = \frac{\mu_7 L_7}{\sum\limits_{i=0,1,2,3,4,5,6,7} \mu_i L_i}. \tag{3.94}$$

**(c) Busy Period of the Repairman in Shut Down Repair in Time** $(0,t]$

Preceding the similar fashion as in the steady state busy period of server due to preventive maintenance of the system is given by

$$B_0^{3^*}(\infty) = \lim_{t\to\infty} B_0^{3^*}(t) = \lim_{s\to 0} sB_0^{3^*}(s) = \frac{N_5(0)}{D_2'(0)} = \frac{\mu_6 L_6}{\sum\limits_{i=0,1,2,3,4,5,6,7} \mu_i L_i}. \tag{3.95}$$

## 3.5 Particular Cases

When all repair time distributions are $n$-phase Erlangian distributions, i.e.,

$$\text{Density function } g_i(t) = \frac{nr_i(nr_i t)^{n-1} e^{-nr_i t}}{n-1!}\,; \tag{3.96}$$

$$\text{Survival function } \bar{G}_j(t) = \sum_{j=0}^{n-1} \frac{(nr_i t)^j e^{-nr_i t}}{j!}. \tag{3.97}$$

and other distributions are negative exponential

$$a(t) = \theta e^{-\theta t}, \quad b(t) = \mu e^{-\mu, t}, \quad \bar{A}(t) = e^{-\theta t}, \quad \bar{B}(t) = e^{-\mu t}. \tag{3.98}$$

For $n = 1$,

$$g_i(t) = r_i e^{-r_i t}, \quad \bar{G}_i(t) = e^{-r_i t} \text{ if } i = 1,2,3,4,5,6, \tag{3.99}$$

$$g_1(t) = r_1 e^{-r_1 t}, \tag{3.100}$$

$$g_2(t) = r_2 e^{-r_2 t}, \tag{3.101}$$

$$g_3(t) = r_3 e^{-r_3 t}, \tag{3.102}$$

$$g_4(t) = r_4 e^{-r_4 t}, \tag{3.103}$$

$$g_5(t) = r_5 e^{-r_5 t}, \tag{3.104}$$

$$g_6(t) = r_6 e^{-r_6 t}, \tag{3.105}$$

$$\bar{G}_1(t) = e^{-r_1 t}, \tag{3.106}$$

$$\bar{G}_2(t) = e^{-r_2 t}, \tag{3.107}$$

$$\bar{G}_3(t) = e^{-r_3 t}, \tag{3.108}$$

$$\bar{G}_4(t) = e^{-r_4 t}, \tag{3.109}$$

$$\bar{G}_5(t) = e^{-r_5 t}, \tag{3.110}$$

$$\bar{G}_6(t) = e^{-r_6 t}. \tag{3.111}$$

Also,

$$p_{01} = \frac{\alpha}{x_1+\theta}, \tag{3.112}$$

$$p_{02} = \frac{\beta}{x_1+\theta}, \tag{3.113}$$

$$p_{03} = \frac{\gamma}{x_1+\theta}, \tag{3.114}$$

$$p_{04} = \frac{\delta}{x_1+\theta}, \tag{3.115}$$

$$p_{05} = \frac{\eta}{x_1+\theta}, \tag{3.116}$$

$$p_{07} = \frac{\theta}{x_1+\theta}, \tag{3.117}$$

$$p_{10} = 1, \tag{3.118}$$

$$p_{20} = \frac{r_2}{\alpha+\gamma+\eta+r_2}, \tag{3.119}$$

$$p_{26} = \frac{\alpha+\gamma+\eta}{\alpha+\gamma+\eta+r_2}, \tag{3.120}$$

$$p_{30} = \frac{r_3}{\alpha+\beta+\delta+r_3}, \tag{3.121}$$

$$p_{36} = \frac{\alpha+\beta+\delta}{\alpha+\beta+\delta+r_3}, \tag{3.122}$$

$$p_{40} = \frac{r_4}{\alpha+\gamma+\eta+r_4}, \tag{3.123}$$

$$p_{46} = \frac{\alpha+\gamma+\eta}{\alpha+\gamma+\eta+r_4}, \tag{3.124}$$

$$p_{50} = \frac{r_5}{\alpha+\beta+\delta+r_5}, \tag{3.125}$$

$$p_{56} = \frac{\alpha+\beta+\delta}{\alpha+\beta+\delta+r_5}, \tag{3.126}$$

$$p_{60} = p_{70} = 1 \tag{3.127}$$

$$\mu_0 = \frac{1}{x_1+\theta}, \tag{3.128}$$

$$\mu_1 = \frac{1}{r_1}, \tag{3.129}$$

$$\mu_2 = \frac{1}{\alpha+\gamma+\eta+r_2}, \tag{3.130}$$

$$\mu_3 = \frac{1}{\alpha+\beta+\delta+r_3}, \tag{3.131}$$

$$\mu_4 = \frac{r_4}{\alpha+\gamma+\eta+r_4}, \tag{3.132}$$

$$\mu_5 = \frac{1}{\alpha+\beta+\delta+r_5}, \tag{3.133}$$

$$\mu_6 = \frac{1}{r_6}, \tag{3.134}$$

$$\mu_7 = \frac{1}{\mu}, \tag{3.135}$$

$$MTSF = \frac{\mu_0+\mu_2 p_{02}+\mu_3 p_{03}+\mu_4 p_{04}+\mu_5 p_{05}}{1-p_{02}p_{20}-p_{03}p_{30}-p_{04}p_{40}-p_{05}p_{50}}, \tag{3.136}$$

$$A_0(\infty) = \frac{\mu_0 L_0+\mu_2 L_2+\mu_3 L_3+\mu_4 L_4++\mu_5 L_5}{\sum\limits_{i=0,1,2,3,4,5,6.7}\mu_i L_i}, \tag{3.137}$$

$$B_0^{1^*}(\infty) = \frac{\sum\limits_{i=1,2,3,4.5}\mu_i L_i}{\sum\limits_{i=0,1,2,3,4,5,6,7}\mu_i L_i}, \tag{3.138}$$

$$B_0^{2^*}(\infty) = \frac{\mu_7 L_7}{\sum\limits_{i=0,1,2,3,4,5,6,7}\mu_i L_i}, \tag{3.139}$$

$$B_0^{3^*}(\infty) = \frac{\mu_6 L_6}{\sum\limits_{i=0,1,2,3,4,5,6,7}\mu_i L_i}, \tag{3.140}$$

where

$$\begin{aligned}&\alpha+\beta+\gamma+\delta+\eta = x_1;\ L_0 = 1;\ L_1 = p_{01};\ L_2 = p_{02};\ L_3 = p_{03};\ L_4 = p_{04};\ L_5 = p_{05};\\ &L_6 = p_{02}p_{26}+p_{03}p_{36}+p_{04}p_{46}+p_{05}p_{56},\ L_7 = p_{07}\end{aligned} \tag{3.141}$$

## 3.6 Profit Analysis

The profit analysis of the system can be carried out by considering the expected busy period of the repairman in repair of the unit in $(0,t]$.

Therefore,

$$\begin{aligned}G(t) &= \text{Expected total revenue earned by the system in } (0,t]\\ &\quad -\text{Expected repair cost of the failed units}\\ &\quad -\text{Expected repair cost of the repairman in preventive maintenance}\\ &\quad -\text{Expected repair cost of the Repairman in shut down}\\ &= C_1\mu_{up}(t)-C_2\mu_{b1}(t)-C_3\mu_{b2}(t)-C_4\mu_{b3}(t)\\ &= C_1 A_0 - C_2 B_0^1 - C_3 B_0^2 - C_4 B_0^3,\end{aligned} \tag{3.142}$$

where

$$\mu_{up}(t) = \int_0^t A_0(t)dt;\ \ \mu_{b1}(t) = \int_0^t B_0^1(t)dt;\ \ \mu_{b2}(t) = \int_0^t B_0^2(t)dt;\ \ \mu_{b3}(t) = \int_0^t B_0^3(t)dt. \tag{3.143}$$

$C_1$ is the revenue per unit time and $C_2$, $C_3$, $C_4$ are the cost per unit time for which the system is under simple repair, preventive maintenance and shut down repair, respectively.

## Competing Interests

The authors declare that they have no competing interests.

## Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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