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Research Article

Synthesizing a Planar Four Bar Linkage and Coupler Curve

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Abstract. Synthesis of planar four bar mechanism is an important area in robotics and mechanical engineering. The analysis of the lengths of the four-bar mechanism and the associated angle helps in determining the coupler curve. In this paper, we present the problem of synthesizing a planar four-bar linkages whose coupler curve passes through five precision points that points are chosen from quadratic polynomial function. Also, we analyze its solution and to find out suitable solution for a chosen coupler curve.

Keywords. Mechanism of four bar linkages, Coupler curve, Displacement matrix, Homotopy continuation method

Mathematics Subject Classification (2020). 13P15; 65H04, 65H10, 70B15

1. Introduction

Maximum performance, while, minimizing the mechanism is an important objective in the fields of mechanical engineering, medicine and robotics industry. In particular the movement of robotics in a particular direction or position is an important task in the engineering industry. For this problem to be solved, we should design tools with particular specifications. Currently, most real-life problems and mathematical models are formulated in terms of polynomial equations and solving polynomial equation is a fundamental problem. Nahon [7], Manocha [5],

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Montazeri [6] proposed new methods to solve system of polynomial equations. In kinematics, based on the four bar linkages, many researchers have developed the concepts and tools to meet various specifications [4, 12–14, 17]. The proper selection of the lengths of linkages and joints is an important task in the synthesis of a four-bar mechanism. Also, the relationship between the shapes of coupler curves and the position tracing points on the coupler plane is yet a unsolved problem. Some questions are till now open in inverse kinematics problems. What shape will a special point in the coupler plane trace? Is there any point tracing a curve of a given shape? and if any, where are they located on the four-bar mechanism? The four-bar linkage consists of four links namely crank, coupler, follower and a link attached with ground. The follower and crank are attached to the ground link. One end of coupler link is joined with crank and another end is attached with follower [1, 8, 15]. A sensor is attached in coupler and it captures the image when the coupler is moved on target position. Also, we assume that the position of the sensor is fixed in the coupler link. Through ground link, we define global co-ordinate system, the fixed positions are denoted by O_1 and O_2 and its co-ordinates are (o_{1x}, o_{1y}), (o_{2x}, o_{2y}). We defined local co-ordinate system for each joint. Let $P_1(p_{1x}, p_{1y})$, $P_2(p_{2x}, p_{2y})$, $P_3(p_{3x}, p_{3y})$, $P_4(p_{4x}, p_{4y})$ and $P_5(p_{5x}, p_{5y})$ be the target positions which are not lie on the same line. We know that, from kinematics theory, with given lengths of linkages, the constraint equations corresponding to constraint length of each linkage are

$$(D_{1i}O_3 - O_1)^T(D_{1i}O_3 - O_1) - (O_3 - O_1)^T(O_3 - O_1) = 0, \tag{1.1}$$

$$(D_{1i}O_4 - O_2)^T(D_{1i}O_4 - O_2) - (O_4 - O_2)^T(O_4 - O_2) = 0, \tag{1.2}$$

where $O_3 = \begin{pmatrix} o_{3x} \\ o_{3y} \\ 1 \end{pmatrix}$, $O_4 = \begin{pmatrix} o_{4x} \\ o_{4y} \\ 1 \end{pmatrix}$ and displacement matrix

$$D_{1i} = \begin{bmatrix} c_{1i} & -s_{1i} & p_{ix} - p_{1x}c_{1i} + p_{1y}s_{1i} \\ s_{1i} & c_{1i} & p_{iy} - p_{1x}s_{1i} - p_{1y}c_{1i} \\ 0 & 0 & 1 \end{bmatrix}, \quad i = 2, 3, 4, 5$$

where

$$c_{1i} = \cos\theta_{1i},\ s_{1i} = \sin\theta_{1i} \text{ and } \theta_{1i} \text{ is relative angle.} \tag{1.3}$$

From these equations (1.1), (1.2) and (1.3), we will get twelve non-linear equations.
These twelve non-Linear equations can be computed in the following manner.

Procedure algorithm.

Step 1:

Begin

For $i : 2$ to 5

$(c_{1i})^2 + (s_{1i})^2 = 1$

End

Step 2:

Begin

For $i:2$ to 5

$$(c_{1i}o_{3x}-s_{1i}o_{3y}+p_{ix}-c_{1i}p_{1x}+s_{1i}p_{1y}-o_{1x})^2+(s_{1i}o_{3x}+c_{1i}o_{3y}+p_{iy}-s_{1i}p_{1x}-c_{1i}p_{1y}-o_{1y})^2 -[(o_{3x}-o_{1x})^2+(o_{3y}-o_{1y})^2]=0$$

End

Step 3:

Begin

For $i:2$ to 5

$$(c_{1i}o_{4x}-s_{1i}o_{4y}+p_{ix}-c_{1i}p_{1x}+s_{1i}p_{1y}-o_{2x})^2+(s_{1i}o_{4x}+c_{1i}o_{4y}+p_{iy}-s_{1i}p_{1x}-c_{1i}p_{1y}-o_{2y})^2 -[(o_{4x}-o_{2x})^2+(o_{4y}-o_{2y})^2]=0$$

End

2. Numerical Example

Now, we consider the polynomial $f(x)=1+x+x^2$. The shape of coupler curve changes when we change our target points. We assume our target points chosen from this polynomial function is $(5,31)$, $(4,21)$, $(3,13)$, $(2,7)$ and $(1,3)$. Also, take $(o_{1x}=0, o_{1y}=0)$ and $(o_{2x}=6, o_{2y}-0)$. Using the above algorithm, we will generate twelve non-linear equations and we use the following notations: a,b,c,d,e,f,g,h,i,j,k,l instead of $c_{12},s_{12},c_{13},s_{13},c_{14},s_{14},c_{15},s_{15},o_{1x},o_{1y},o_{2x},o_{2y}$. Then

$$a^2+b^2-1=f_1 \quad \text{(say)}$$
$$e^2+f^2-1=f_3 \quad \text{(say)}$$
$$c^2+d^2-1=f_2 \quad \text{(say)}$$
$$g^2+h^2-1=f_4 \quad \text{(say)}$$
$$8ai+42aj-1342a-10i-62j+42bi-8bj+38b+1443=f_5 \quad \text{(say)}$$
$$6ci+26cj-836c-10i-62j+26di-6dj+56d+1164=f_6 \quad \text{(say)}$$
$$4ei+14ej-454e-10i-62j+14fi-4fj+54f+1039=f_7 \quad \text{(say)}$$
$$2gi+64gj-196g-10i-62j+6hi-2hj+32h+996=f_8 \quad \text{(say)}$$
$$2k+42\,al-4ak-1282a-62l+4bl+42bk-334b+1395=f_9 \quad \text{(say)}$$
$$2k+26cl-6ck-776c-62l+6dl+26dk-316d+1128=f_{10} \quad \text{(say)}$$
$$2k+14el-8ek-394e-62l+8fl+14fk-318f+1015=f_{11} \quad \text{(say)}$$
$$2k+6gl-10gk-136g-62l+10hl+6hk-340h+984=f_{12} \quad \text{(say)}$$

Solving the system of equations involving many variables is one of the challenging tasks in this area. Many methods are available [2,3,10,11], in particular numerical methods give the solution approximately but practically many solutions are not suitable for designing problem. So here we are proposing an algebraic method, using homotopy continuation method [16] and obtain the following results:

Fifty six non-singular solutions exist, and ten solutions are real solution

Table 1. Possible real solutions

Solution	a	b	c	d	e	f	g	h	i	j	k	l
1	−0.9995	−0.0298	−0.9869	−0.1614	−0.8998	−0.4363	0.5384	0.8426	−85.734	42.737	−1083.26	−23.5659
2	−0.5772	0.8165	0.9998	0.0183	0.9362	0.3514	−0.9934	0.1143	−48.8324	13.8798	−22.7101	11.8772
3	0.9982	0.0595	0.9949	0.1004	0.9909	0.1338	0.8278	−0.5609	65.1589	6.6818	67.6576	6.5394
4	0.8902	0.4555	0.7065	0.7077	0.4499	0.8931	0.0240	0.9997	10.9347	15.6905	12.7131	14.2591
5	0.5162	0.8565	0.9053	−0.4247	0.9946	−0.1035	−0.3415	0.9398	−5.5191	13.0732	1.1508	13.2407
6	0.9993	0.0376	0.9965	0.0835	0.9829	0.1838	0.8894	0.4572	150.9830	1.5344	151.0535	1.5477
7	0.9871	−0.1598	0.9675	−0.2528	0.9720	−0.2347	−0.3680	−0.9298	−25.0615	17.2925	−23.7272	18.3357
8	0.3554	−0.9347	0.2168	−0.9762	0.3312	−0.9435	−0.4465	0.89948	10.3007	11.4248	11.6515	15.7608
9	0.9938	0.1110	0.9772	0.2125	0.9393	0.3432	0.6435	−0.7654	47.2827	11.2118	48.9265	10.8682
10	0.9997	0.0246	0.9993	0.0372	0.9989	0.0468	0.9890	0.1475	113.0129	−0.3425	117.0880	−0.3829

Since the system does not produce unique solution, so we will compare two solutions for same prescription points. Assume these two solutions are two different samples from same population. Using principle of least square method we can analysis how much deviate from trend lines. Clearly, from the above solution table, solutions 4 and 6 are suitable practically.

Table 2. Suitable solutions

Solution	a	b	c	d	e	f	g	h	i	j	k	l
4	0.8902	0.4555	0.7065	0.7077	0.4499	0.8931	0.0240	0.9997	10.9347	15.6905	12.7131	14.2591
6	0.9993	0.0376	0.9965	0.0835	0.9829	0.1838	0.8894	0.4572	150.9830	1.5344	151.0535	1.5477

Suppose we take the precision points on the circular path say $(1,0)$, $(0,1)$, $(-1,0)$, $(0,-1)$ and again take original position $(1,0)$. Also, take $(o_{1x}=0, o_{1y}=0)$ and $(o_{2x}=6, o_{2y}=0)$. Then using the above procedure, we will get the following equations:

$$a^2+b^2-1=f_1 \quad \text{(say)}$$
$$e^2+f^2-1=f_3 \quad \text{(say)}$$
$$c^2+d^2-1=f_2 \quad \text{(say)}$$
$$g^2+h^2-1=f_4 \quad \text{(say)}$$
$$2aj+2ib-2i-2b+2=f_5 \quad \text{(say)}$$
$$2jd-2ci-2i+2c+2=f_6 \quad \text{(say)}$$
$$2if-2ej-2i+2f+2=f_7 \quad \text{(say)}$$
$$2gi-2jh-2g-2i+2=f_8 \quad \text{(say)}$$
$$10k-12ak+12a+2al+12bl+2kb-2b+2=f_9 \quad \text{(say)}$$
$$10k-14ck+14c+14dl+14=f_{10} \quad \text{(say)}$$
$$10k-12ek+12e-2el+12fl-2kf+2f+2=f_{11} \quad \text{(say)}$$
$$10k-10kg+10g+10hl-10=f_{12} \quad \text{(say)}$$

Solving the above equations, no feasible solution exists for the construction of four bar mechanism, using the method of Homotopy continuation technique, for this sample points. Further investigation is required to solve this system to arrive at a mechanically feasible system which passes through the given points. The same concept is extended to five bar mechanisms [9] but the suitable solution for practical problem is difficult task.

3. Conclusion

Homotopy continuation method is very superior method, which helps in the solution of a large system of equations in many variables, which can be used to solve the inverse kinematics problem. It has limitations as regards the generation of mechanically feasible solutions to all system of equations.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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