Communications in Mathematics and Applications
Vol. 12, No. 3, pp. 665–676, 2021
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications

http://www.rgnpublications.com

DOI: 10.26713/cma.v12i3.1621

Research Article

Channel Assignment of Triangular and Rhombic Honeycomb Networks Using Radio Labeling Techniques

S. Gomathi* and P. Venugopal

Department of Mathematics, Sri Sivasubramaniya Nadar College of Engineering, Kalavakkam 603110, India

Received: May 12, 2021 **Accepted:** August 10, 2021

Abstract. A radio labeling of a graph $G=(V,E)$ is a function $f:V(G)\to N$ such that $d(u,v)+|f(u)-f(v)|\geq 1+\text{diam}(G)$, where $d(u,v)$ represents the shortest distance between the vertices u and v and $\text{diam}(G)$ is the diameter of G. The span of a radio labeling f is defined as $sp(f)=\max\{|f(u)-f(v)|:u,v\in V(G)\}$. A radio number of G is the minimum span of all the radio labelings of G and is denoted by $rn(G)$. The radio number is used to optimize the assignment of frequency bands to channels in wireless communication networks. The honeycomb network is considered to be one of the most important network for placement of base stations in wireless communications networks. In this paper, the upper and lower bounds for the radio number of two well-known topologies of honeycomb network namely triangular and rhombic honeycomb networks are obtained. These bounds were graphically represented for easy understanding of the minimum and maximum spectrum needed for effective communication in a network.

Keywords. Channel assignment; Radio number; Bandwidth; Triangular honeycomb network; Rhombic honeycomb network

Mathematics Subject Classification (2020). 05C12; 05C15; 05C78

1. Introduction

In a telecommunication network, the most interesting and challenging problem is the channel assignment problem. Mostly, the channel assignment problem has been studied only for connected, finite, simple, and undirected graphs. The major constraint of a channel assignment

*Corresponding author: gomathis@ssn.edu.in

problem is to design a communication network in such a way that the interference between any two transmitters is avoided or minimized [4, 5]. This problem can be converted into a graph theoretic problem where the transmitters are represented by vertices and the adjacent transmitters are connected by edges. The problem in graph theory is to assign, each vertex a non-negative integer or different colors in such a way that the adjacent vertices receive different integers or colors [12]. The process of assigning integers to the vertices or edges or both based on certain conditions,known as *graph labeling*. The graph labeling has wide range of applications in coding theory, x-ray crystallography, radar, astronomy, circuit design, communication networks, and so on [10].

The channel assignment problem was introduced by William Hale [12] in 1980. This problem motivated Griggs and Yeh [11], to introduce a new graph labeling technique called $L(2,1)$ labeling or distance two labelings. Chartrand et al. [8] introduced another labeling technicques known as Radio labeling, even though the radio labeling problem looks simple, it has been proved that finding the radio number of an arbitrary network is an NP-complete problem [1, 14]. Ali and Marinescu-Ghemeci [3] has obtained the bounds of radio number of some ladder-related graphs. The radio number of caterpillar related graphs was studied by Kang et al. [15]. Li et al. [19] investigated the optimal radio labeling of complete m-ary trees. The radio number of extended mesh was studied by Yenoke [30]. Bharathi Rajan and Yenoke [24] investigated the radio number of uniform theta graphs. Vaidya and Vihol [29] has obtained the radio labeling for some cycle related graphs. Ali et al. [2] computed the radio labeling associated with zero divisor graph of a commutative ring. Sooryanarayana et al. [27] studied the radio number of kth-transformation graphs of a Path. Bantva [9] obtained a lower bound for the radio number of certain graphs. Kchikech et al. [17] has found out radio k-labeling of trees. Cada et al. [7] has obtained radio labeling of distance graphs. Radio number for corona of paths and cycles were studied [21]. The three well-known topologies in wireless communication networks are honeycomb, square and hexagonal grids. The most studied topology so far is the hexagonal grid [16, 22, 26]. However, the honeycomb grid appears to be more convenient than the hexagonal and square grid [6]. Honeycomb networks are better in terms of degree, diameter, and the total number of links, cost, and bisection width than mesh connected planar graphs. The communication in the honeycomb network is proved to be more cient compare to other networks [5]. It is widely used in computer graphics [18], cellular phone base stations [22], image processing, and in chemistry as the representation of benzenoid hydrocarbons [20]. Stojmenovic [28] has studied the topological properties of honeycomb networks, routing in honeycomb networks and honeycomb torus networks. Honeycomb networks can be built from hexagons in various ways by recursively building using the hexagon tessellation [20]. Parhami [23] gave a unified formulation for the honeycomb and the diamond networks. Channel assignment in basic honeycomb networks has been reported in literature [16, 28]. Two well-known topologies of honeycomb networks are rhombic and triangular honeycomb networks [13, 28]. In this paper, the upper bound and lower bound for radio number of rhombic and triangular honeycomb networks were studied.

2. Radio Number of Triangular Honeycomb Network

In this section, the bounds of the radio number of triangular honeycomb network is studied.

2.1 Construction of Triangular Honeycomb Network

A honeycomb network is formed by joining a collection of hexagons [28]. A triangular honeycomb network is constructed as follows. Consider a hexagon, which is assumed to be in layer 1. Two hexagons are added to the bottom of given hexagon in such a way that each of these hexagons share a common edge with layer 1 hexagon. These two added hexagons are said to form layer 2. This structure constructed is said to of triangular honeycomb network of dimension 1. It is denoted by $THC(1)$. The first dimension honeycomb network has 3 levels (see Figure 1). In the similar way, by adding hexagons below the lowermost level, in each dimension, this network can be constructed up to nth dimension. It is denoted by $THC(n)$. It has $(n+2)$ levels and $(n+1)$ hexagons in the $(n+2)$th level.

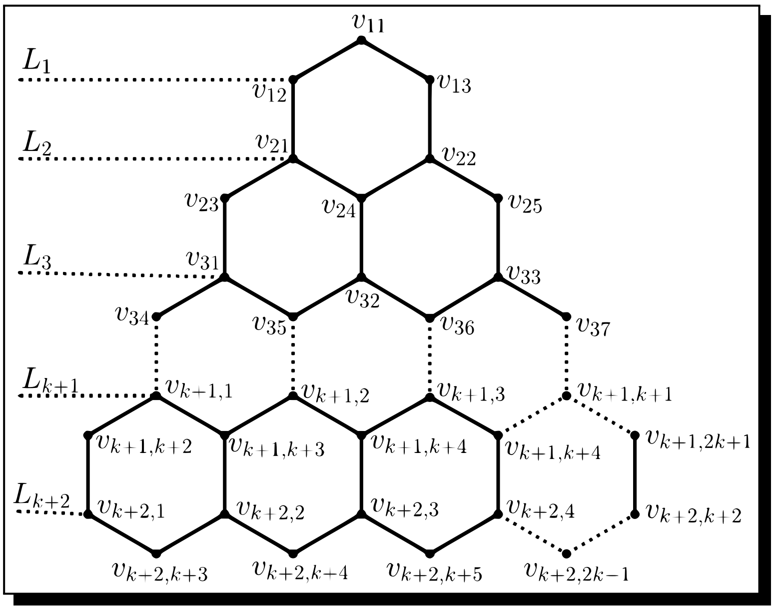

Figure 1. Different levels of $THC(n)$

The $(n+2)$ levels of triangular honeycomb network $THC(n)$ are taken as L_k and L_r, where $1 \le k \le n+1$ and $r = n+2$. The vertex set in kth level is given by $v_{k,j}$, $1 \le j \le k+1$ and vertex set in rth level is $v_{r,j}$, $1 \le j \le 2r-1$. $THC(n)$ has n^2+6n+6 vertices and $\frac{3}{2}(n^2+5n+4)$ edges. Its diameter is $2n+3$.

2.2 Lower Bound for $THC(n)$

The lower bound of radio number of graphs with small diameter can be obtained as follows. In the triangular honeycomb network $THC(n)$, there are $(n+3)$ pairs u,v such that $|f(u)-f(v)|=1$. From these $(n+3)$ pairs of vertices, only three pair of vertices are considered to apply radio labeling condition, the remaining are associated to already assigned vertex v for a radio labeling f. The lower bound of radio labeling is given by

$$rn(G) \geq 1 + x + (n+1)(k-1-x) + Y$$

where $Y = 4n+2$, x is the diametric distance vertex and k is the number of vertices in $THC(n)$.

Definition 2.1 ([25])**.** Let $S(v)$ be the sum of the distance between v and every other vertex in G. That is $S(v) = \sum_{u \in G} d(u,v)$. The minimum distance sum, $S(v)$, $\forall\ v \in G$ is called the *median* of G and is given by $M(G) = \min\{S(v) : v \in G\}$. The vertex v corresponds to $M(G)$ is said to be the *centre* of G.

Theorem 2.1. *The radio number of triangular honeycomb network of dimension one,* $rn(THC(1)) = 28$.

Proof. Let $G = THC(1)$ be a triangular honeycomb network of dimension 1. The diameter of G, $\text{diam}(G) = 5$. In $THC(1)$, the number of vertices, $k = 13$ and the number of edges is 15. From Section 2.2, for $n = 1$, $k = 13$ the lower bound of G is $rn(G) \geq 1 + 3 + (n+1)(k-1-3) + Y = 4 + 2(9) + 6 = 28$.

Any radio labeling f of G must satisfy the following radio labeling condition

$$d(u,v) + |f(u) - f(v)| \geq \text{diam}(G) + 1 = 6. \tag{2.1}$$

The vertices of G are labeled as follows. First, label the centre vertex v_7 of G as 1. Next label the vertex at maximum distance fromv_7. The vertices v_1, v_9 and v_{11} are at maximum distance. Without loss of generality, label v_1 by applying the radio labeling condition, i.e., $f(v_1) = 4$. From v_1, the vertices v_{13} and v_{12} are at maximum distance. Choose v_{13} and label it by using the radio labeling condition, i.e., $f(v_{13}) = 5$. Likewise, the remaining vertices of $THC(1)$ can be labeled. Starting from v_{13}, the vertices v_6, v_8, v_2, v_{12} and v_3 taken in this order are at distance 4 from each other and are labeled as $7, 9, 11, 13$ and 15 in such a way that they satisfy the radio labeling condition. From v_3, the vertex v_4 is the unlabeled vertex at maximum distance 3. Take $f(v_4) = 18$. Starting from v_4, label the vertices v_{11} v_9 and v_5 taken in this order, which are at distance 4 from each other. From v_5, label the remaining vertex v_{10} as 28, which is the span of G.

Hence, $rn(THC(1)) \leq 28$.

Therefore, $rn(THC(1)) = 28$. □

Theorem 2.2. *The radio number of triangular honeycomb network for all $n \geq 2$ is,* $rn(THC(n)) \geq k(n+1) + 2$.

Proof. Let $G = THC(n)$ be a triangular honeycomb network of dimension n.

From Section 2.1, $|V(G)| = n^2 + 6n + 6$ and $|E(G)| = \frac{3}{2}(n^2 + 5n + 4)$ and its diameter $2n+3$.

There are $(n+3)$ pairs of vertices at diametric distance such that $|f(u) - f(v)| = 1$.

Any radio labeling f of G must satisfy the following radio labeling condition

$$d(u,v) + |f(u) - f(v)| \geq diam(G) + 1 = 2n + 4 \Rightarrow |f(u) - f(v)| \geq 2n + 4 - d(u,v).$$

Consider any two distinct vertices $u, v \in V(G)$. Let f be an optimal radio labeling for G. Using

Definition 2.1, choose the centre vertex, v_i of G. Label it as 1, i.e., $f(v_i) = 1$. The remaining vertices of $THC(n)$ can be labeled by adopting the same technique discoursed in Theorem 2.1. From Section 2.2, the lower bound of the radio number of G is $rn(THC(n)) \geq 1+3+(n+1)(k-1-3)+Y \geq (n+1)k+2$. □

Theorem 2.3. *The radio number of triangular honeycomb network $THC(n)$ forn ≥ 2 is $rn(THC(n)) \leq n^3+23n^2-6n+34$.*

Proof. Let $\{v_1 v_2, \dots, v_{n^2+6n+6}\}$ be the vertices of $THC(n)$. These vertices are labeled as follows. Let $f(v_1) = 1$. The vertex v_{n^2+6n+6} is at a diametric distance from v_1, label it as 2, i.e., $f(v_{n^2+6n+6}) = 2$. The remaining vertices of $THC(n)$ are labeled by the following mapping:

$$f(v_i) = 2(n+1)(i-1)+2, 1 < i < n^2+6n+6. \tag{2.2}$$

Any radio labeling f of G must satisfy the following radio labeling condition:

$$d(u,v)+|f(u)-f(v)| \geq diam(G)+1 = 2n+4 \Rightarrow |f(u)-f(v)| \geq 2n+4-d(u,v). \tag{2.3}$$

Claim: The mapping (2.2) is a valid radio labeling.
To prove this, it is enough to show that equation (2.2) satisfies equation (2.3).
Let $u,v \in THC(n)$.

Case (i): Suppose $d(u,v) = 1$.
Clearly from the structure of $THC(n)$, $u(= v_{i+2}$ or $v_{i+3})$, $v(= v_i)$.
By applying (2.2) in radio labeling condition, we get,

$$|f(u)-f(v)| = 2(n+1)[i+2-1-(i-1)] = 2(n+1)(2) = 4n+4 \geq 2n+3.$$

Case (ii): Suppose the vertices u, v lie in the same level and $d(u,v) \geq 2$.
In this case, $u(= v_{i+1}$ or v_{i+2} or v_{i+3} etc.), $v(= v_i)$.
By using (2.2), in radio labeling condition, we get

$$|f(u)-f(v)| = 2(n+1)[i+1-1-(i-1)] = 2n+2,$$

by taking $u = v_{i+1}$.
Suppose $u = v_{i+4}$ then

$$|f(u)-f(v)| = 2(n+1)[i+4-1-(i-1)] = 8n+8 \geq 2n+4.$$

Similarly, the result can be verified for any $u \in THC(n)$.

Case (iii): Suppose the vertices u, v are in different levels and $d(u,v) \geq 2$.
In this case, $u(= v_{i+4}$ or v_{i+5} etc.), $v(= v_i)$.
By applying mapping in radio labeling condition, we get

$$|f(u)-f(v)| = 2(n+1)[i+4-1-(i-1)] = 8n+8 \geq 2n+4.$$

Case (iv): Suppose the vertices $u = v_1$ and $v = v_{n^2+6n+6}$.
In this case, $d(u,v) = 2n+3$.
By our assumption, $|f(u)-f(v)| = 1$.
Hence in all the cases, mapping (2.2) satisfies the radio labeling condition (2.3).
Therefore, mapping (2.2) is a valid radio labeling.

By the mapping, the vertex v_{n^2+6n+6} receives the maximum label and its label is

$$f(v_{n^2+6n+5}) = n^3 + 23n^2 - 6n + 34,$$

which is the span of $THC(n)$.
Hence, $rn(THC(n)) \leq n^3 + 23n^2 - 6n + 34$. □

Theorem 2.4. *The bounds of radio number of triangular honeycomb network lies between* $(n+1)k+2$ *and* $n^3 + 23n^2 - 6n + 34$ *for* $n \geq 2$.

Proof. The proof is obvious from Theorem 2.2 and Theorem 2.3.
Hence, $(n+1)k + 2 \leq rn(G) \leq n^3 + 23n^2 - 6n + 34$. □

Table 1. Lower and upper bounds of $THC(n)$

Dimensions (n)	2	3	4	5	6	7	8	9	10
Number of nodes (k)	22	33	46	61	78	97	118	141	166
Lower bound (y)	68	134	232	368	548	778	1064	1412	1828
Upper bound (y)	122	250	442	704	1042	1462	1970	2572	3274

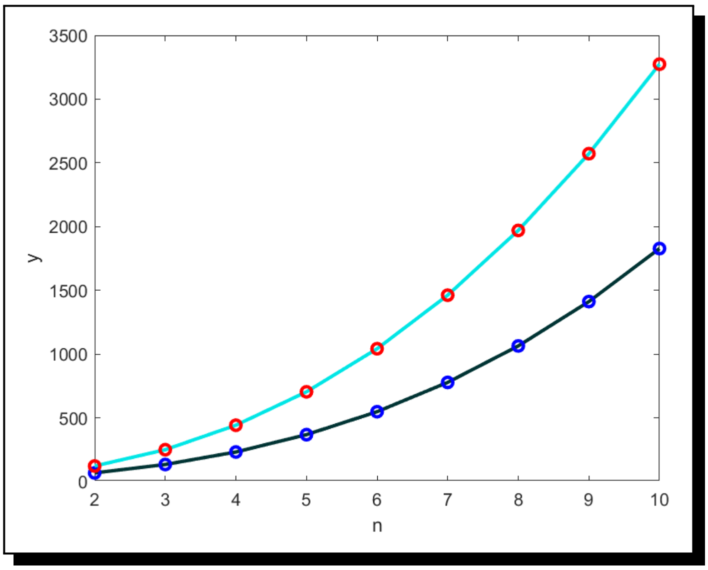

Figure 2. Lower and upper bounds of $THC(n)$

3. Radio Number of Rhombic Honeycomb Networks

In this section, the bounds of radio number of rhombic honeycomb network have been studied.

3.1 Construction of Rhombic Honeycomb Network

The rhombic honeycomb network (RHC) is constructed by placing hexagonal tessellations inside a rhombus. The rhombic honeycomb network of dimension n is denoted by $RHC(n)$. The vertices in each horizontal straight path starting from left to right side of a rhombic structure is said to be a level (L) of RHC. There are $(2n+2)$ levels in$RHC(n)$. These levels can be divided into two sets, say L_k, $1 \le k \le n+1$ and L_r, $n+2 \le r \le 2n+2$. A rhombic honeycomb mesh network of first dimension with different levels is shown in Figure 3.

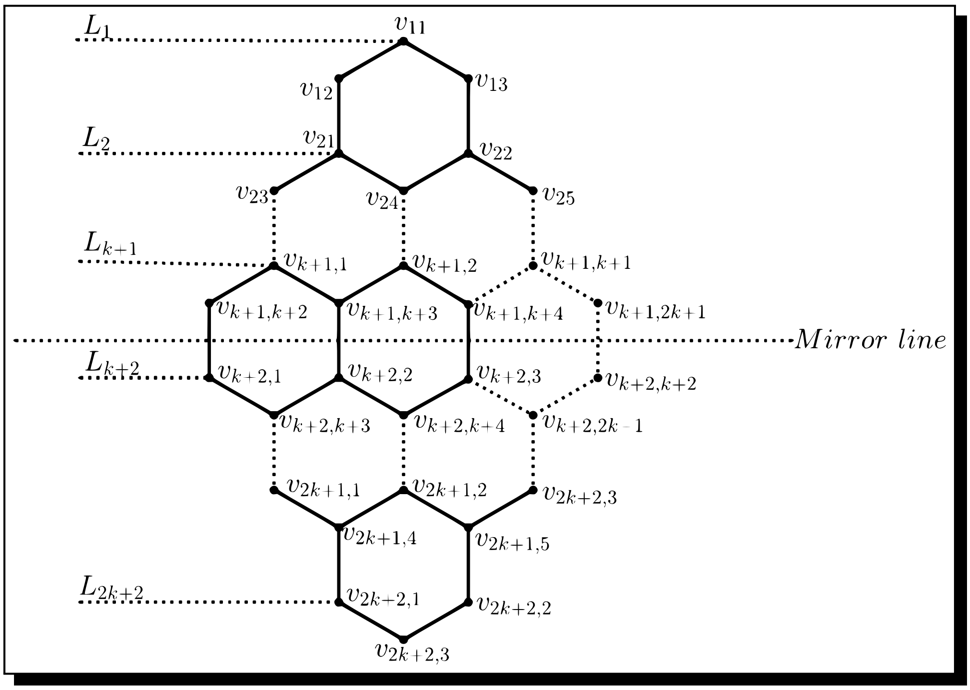

Figure 3. Different levels of $RHC(n)$

In $RHC(n)$, the number of vertices in the $(n+2)$th level is same as the number of vertices in the $(n+1)$th level, the number of vertices of $(n+3)$th level is same as the number of vertices in the nth level and so on. That is the number of vertices in the levels $L_{n+2}, L_{n+3}, \cdots L_{2n+2}$ are same as the number of vertices in the levels $L_{n+1}, L_n \cdots L_1$ taken in this order. $RHC(n)$ has $2n^2+8n+6$ vertices and $3n^2+10n+6$ edges. Its diameter is $4n+3$.

3.2 Lower Bound for $RHC(n)$

The lower bound of radio number of graphs with small diameter can be obtained as follows. In $RHC(n)$, there is only one pair of vertices (u,v) such that $|f(u)-f(v)|=1$. This pair is chosen to apply radio labeling condition. The remaining vertices are chosen for a radio labeling f in the way they are associated to already assigned vertex v. The lower bound of radio labeling is given by $rn(G) \ge 1+x+(n+3)(k-1-x)$, where x is the diametric distance vertex and k is the number of vertices in $RHC(n)$.

Theorem 3.1. *The radio number of rhombic honeycomb network of dimension one is* 58.

Proof. Let $G = RHC(1)$ be a rhombic honeycomb network of dimension 1.= The diameter of G is $diam(G) = 7$.
In $RHC(1)$ the number of vertices, $k = 16$ and the number of edges is 19.

From Section 3.2, for $n = 1$, $k = 16$ the lower bound of G is

$$rn(G) \geq 1+1+(n+3)(k-1-1) = 2+4(14) = 58.$$

Any radio labeling f of G must satisfy the following radio labeling condition $d(u,v)+|f(u)-f(v)| \geq \text{diam}(G)+1 = 8$.
The vertices of G are labeled as follows. First, label the centre vertex v_7 of G as 1. Next, label the vertex at maximum distance from v_7. The only vertex v_{16} is at maximum distance label, v_{16} by applying the radio labeling condition, i.e., $f(v_{16}) = 5$. From v_{16}, the vertex v_1 at maximum distance and label it by using the radio labeling condition, i.e., $f(v_1) = 6$. Likewise, the remaining vertices of $RHC(1)$ can be labeled. Starting from v_1, the vertices v_9, v_2, v_{13}, v_3, v_{12}, v_{14}, v_4 and v_{15} taken in this order are at distance 4 from each other and are labeled in such a way that they satisfy the radio labeling condition. From v_{15} to the vertex v_6 and v_6 to v_{11} are at maximum distance 5. Take $f(v_6) = 49$ and $f(v_{11}) = 52$. From v_{11}, label the remaining vertex v_{10} as 58, which is the span of G. Hence, $rn(RHC(1)) \leq 58$.
Therefore, $rn(RHC(1)) = 58$. □

Theorem 3.2. *For $n \geq 2$ radio number of rhombic honeycomb network is*

$$rn(RHC(n)) \geq (n+3)(k-2)+2.$$

Proof. Let $G = RHC(n)$ be a rhombic honeycomb network of dimension n.
From Section 3.1, $|V(G)| = 2n^2+8n+6$ and $|E(G)| = 3n^2+10n+6$ and its diameter $4n+3$. There is a only one pair of vertices at diametric distance such that $|f(u)-f(v)| = 1$ say (v_1, v_{2n+2}).
Any radio labeling f of G must satisfy the following radio labeling condition

$$d(u,v)+|f(u)-f(v)| \geq diam(G)+1 = 4n+4$$

$$\Rightarrow \quad |f(u)-f(v)| \geq 4n+4-d(u,v)$$

Consider any two distinct vertices $u, v \in V(G)$.
Let f be an optimal radio labeling for G. Choose the centre vertex, v_i of G. Label it as 1, i.e., $f(v_i) = 1$. The remaining vertices of $RHC(n)$ can be labeled by adopting the same technique discoursed in Theorem 3.1.
From Section 3.2, the lower bound of the radio number of G is

$$rn(RHC(n)) \geq 1+1+(n+3)(k-1-1) \geq (n+3)(k-2)+2.$$ □

Theorem 3.3. *Radio number of rhombic honeycomb network $RHC(n)$ for $n \geq 2$ is $rn(RHC(n)) \leq 13n^3+n^2+102n-30$.*

Proof. Let $\{v_1, v_2, \cdots v_{2n^2+8n+6}\}$ be the vertices of $RHC(n)$. These vertices of $RHC(n)$ is labeled as follows.
Take $f(v_1) = 1$. As the vertex $f(v_{2n^2+8n+6})$ is at diametric distance from v_1. Label it as 2, i.e., $f(v_{2n^2+8n+6}) = 2$.
The remaining vertices of $RHC(n)$ is labeled by the mapping,

$$f(v_i) = (4n+2)(i-1)+2, \quad 1 < i < 2n^2+8n+6. \tag{3.1}$$

Any radio labeling f of G must satisfy the following radio labeling condition

$$d(u,v)+|f(u)-f(v)| \geq \text{diam}(G)+1 = 4n+4$$

$$\Rightarrow \quad |f(u)-f(v)| \geq 4n+4-d(u,v) \tag{3.2}$$

Claim: The mapping (3.1) is a valid radio labeling.
To prove this, it is enough to show that the mapping (3.1) satisfies the equation (3.2)). In order to prove this claim, the following cases have been considered.
Let u,v be any two vertices of $RHC(n)$.

Case (i): Suppose $d(u,v)=1$.
Clearly, from the structure of $RHC(n)$, $u(=v_{i+2}$ or $v_{i+3})$, $v(=v_i)$.
By applying (3.1) in radio labeling condition, we get

$$|f(u)-f(v)| = (4n+2)[i+2-1-(i-1)] = 8n+4 \geq 4n+3.$$

Case (ii): Suppose the vertices u, v lie in the same level and $d(u,v) \geq 2$.
In this case, $u(=v_{i+1})$ or $(v_{i+2}$ etc.), $v(=v_i)$.
From the mapping and radio labeling condition, we get

$$|f(u)-f(v)| = (4n+2)[i+1-1-(i-1)] = 4n+2,$$

by taking $u=v_{i+1}$.
Suppose $u=v_{i+4}$, $|f(u)-f(v)| = (4n+2)[i+4-1-(i-1)] = 16n+8 \geq 4n+4$.
Similarly, the result can be verified for any $u \in RHC(n)$.

Case (iii): Suppose the vertices u, v are in different levels and $d(u,v) \geq 2$.
In this case, $u(=v_{i+4})$ or v_{i+5} etc., $v(=v_i)$.
By radio labeling condition, we get

$$|f(u)-f(v)| = (4n+2)[i+4-1-(i-1)] = 16n+8 \geq 4n+4.$$

Case (iv): Suppose the vertices $u=v_1$ and $v=v_{n^2+8n+6}$.
In this case, $d(u,v)=4n+3$.
By our assumption, $|f(u)-f(v)|=1$.
Hence in all the cases, mapping (3.1) satisfies the radio labeling condition (3.2).
By the mapping, the vertex v_{n^2+8n+5} receives the maximum label and its label is

$$f(v_{n^2+8n+5}) = 13n^3+n^2+102n-30,$$

which is the span of $RHC(n)$.
Hence, $rn(THC(n)) \leq 13n^3+n^2+102n-30$. □

Theorem 3.4. *The bounds of radio number of rhombic honeycomb network lies between* $(n+3)(k-2)+2$ *and* $13n^3+n^2+102n-30$ *for* $n \geq 2$.

Proof. The proof is obvious from Theorem 3.2 and Theorem 3.3.

$$(n+3)(k-2)+2 \leq rn(RHC(n)) \leq 13n^3+n^2+102n-30.$$ □

Table 2. Lower and upper bounds of $RHC(n)$

Dimensions (n)	2	3	4	5	6	7	8	9	10
Number of nodes (k)	30	48	70	96	126	160	198	240	286
Lower bound (y)	142	278	478	754	1118	1582	2158	2858	3694
Upper bound (y)	282	636	1226	2130	3426	5162	7506	10446	14090

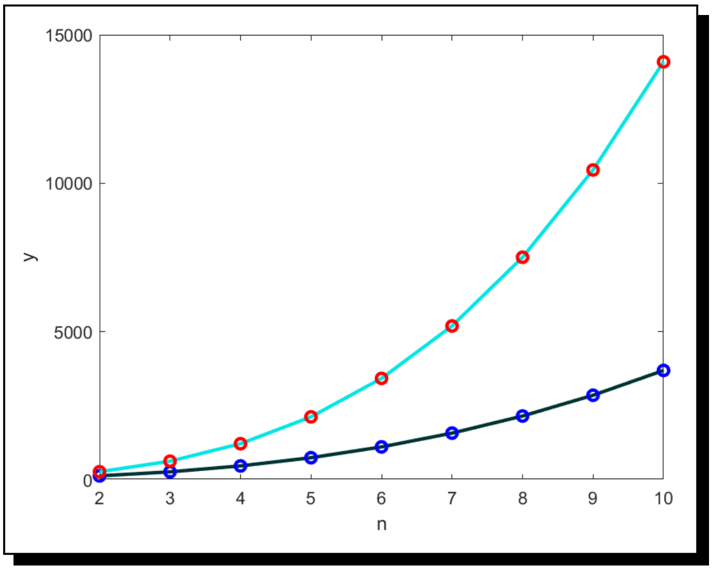

Figure 4. Lower and upper bounds of $RHC(n)$

Remark 3.1. For $n = 2,3,\cdots 10$, the values of lower and upper bound of $rn(THC(n))$ and $rn(RHC(n))$ have been displayed in Table 1 and Table 2. These table values were graphically represented in Figure 2 and Figure 4. From these figures, it is clear that as dimension n of the network increases, the lower and upper bound of the radio number increases drastically. In the figures, the curves with red dots represent the upper bound and blue dots represents the lower bound. By knowing these bounds, it is easy to estimate the minimum and maximum spectrum needed for the effective communication, without any interference in a communication network.

4. Conclusion

In communication networks, radio labeling plays a vital role in assigning the channels (frequencies) to all the transmitters in a network in such a way that the total bandwidth required for the network and the chance of interference gets minimized. In this work, the radio number of triangular and rhombic honeycomb network has been modeled and reported. The lower bound of the radio number of triangular honeycomb network $THC(n)$ and rhombic honeycomb network $RHC(n)$ for $n \geq 2$ is $rn(THC(n)) \geq k(n+1)+2$ and $rn(RHC(n)) \geq (n+3)(k-2)+2$. The upper bounds of the radio number of triangular and rhombic honeycomb networks have also been investigated and reported. Also, the graphical representation of the bounds of these labeling were presented.

Competing Interests

The author declares that she has no competing interests.

Authors' Contributions

The author wrote, read and approved the final manuscript.

References

[1] B. D. Acharya, K. A. Germina, K. L. Princy and S. B. Rao, Graph labellings, embedding and NP-completeness theorems, *J. Combin. Math. Combin. Comput* **67** (2008), 163 – 180.

[2] A. Ali and A. Haider, Computing the radio labeling associated with zero divisor graph of a commutative ring, *UPB Scientific Bulletin, Series A* **81** (2019), 65 – 72, URL: https://www.scientificbulletin.upb.ro/rev_docs_arhiva/fulldfa_904583.pdf.

[3] A. Ali and R. Marinescu-Ghemeci, Radio labeling of some ladder related graphs, *Mathematical Report* (Bucuresti) **19** (2017), 107 – 119.

[4] A. A. Bertossi, C. M. Pinotti and R. B. Tan, Channel assignment with separation for interference avoidance in wireless networks, *IEEE Transactions on Parallel and Distributed Systems* **14** (2003), 222 – 235, DOI: 10.1109/TPDS.2003.1189581.

[5] A. A. Bertossi, C. M. Pinotti, R. Rizzi and A. M. Shende, Channel assignment for interference avoidance in honeycomb wireless networks, *Journal of Parallel and Distributed Computing* **64** (2004), 1329 – 1344, DOI: 10.1016/j.jpdc.2004.06.006.

[6] A. A. Bertossi, C. M. Pinotti, R. Rizzi and A. M. Shende, Channel assignment in honeycomb networks, in: *Italian Conference on Theoretical Computer Science*, Springer, Berlin — Heidelberg (2003), 150 – 162.

[7] R. Cada, P. J. Ekstein, P. Holub and O. Togni, Radio labelings of distance graphs, *Discrete Applied Mathematics* **161** (2013), 2876 – 2884, DOI: 10.1016/j.dam.2013.06.024.

[8] G. Chartrand, D. Erwin, P. Zhang and F. Harary, Radio labelings of graphs, *Bull. Inst. Combin. Appl* **33** (2011), 77 – 85.

[9] B. Devsi, A lower bound for the radio number of graphs, in: *Conference on Algorithms and Discrete Applied Mathematics*, Springer, Cham, (2019), 161 – 173, DOI: 10.1007/978-3-030-11509-814.

[10] J. A. Gallian, A dynamic survey of graph labeling, *Electronic Journal of Combinatorics* **1** (2018), Dynamic Surveys DS6.

[11] J. R. Griggs and R. K. Yeh, Labelling graphs with a condition at distance 2, *SIAM Journal on Discrete Mathematics* **5** (1992), 586 – 595, DOI: 10.1137/0405048.

[12] W. K. Hale, Frequency assignment: theory and applications, *Proceedings of IEEE 68* (1980), DOI: 10.1109/PROC.1980.11899.

[13] F. Havet, Channel assignment and multicolouring of the induced subgraphs of the triangular lattice, *Discrete Mathematics* **233** (2001), 219 – 231, DOI: 10.1016/S0012-365X(00)00241-7.

[14] F. Havet, M. Klazar, I.J. Kratochv, D. Kratsch and M. Liedlo, Exact algorithms for $L(2,1)$-labeling of graphs, *Algorithmica* **59** (2011), 169 – 194, DOI: 10.1007/s00453-009-9302-7.

[15] S. M. Kang, S. Nazeer, W. Nazeer, I. Kousar and C. Y. Jung, Radio Labeling and radio number of caterpillar related graphs, *Mitteilungen Klosterneuburg* **65** (2015), 149 – 159.

[16] I. Katzela and M. Naghshineh, Channel assignment schemes for cellular mobile telecommunication systems: A comprehensive survey, *IEEE Personal Communications* **3** (1996), 10 – 31, DOI: 10.1109/98.511762.

[17] M. Kchikech, R. Khennoufa, O. Togni, Linear and cyclic radio k-labelings of trees, *Discussiones Mathematicae Graph Theory* **130** (2007), 105 – 123, DOI: 10.7151/dmgt.1348.

[18] L. N. Lester and J. Sandor, Computer graphics on hexagonal grid, *Computer Graphics* **8** (1984), 401 – 409, DOI: 10.1016/0097-8493(84)90038-4.

[19] X. Li, V. Mak and S. Zhou, Optimal radio labellings of complete m-ary trees, *Discrete Applied Mathematics* **158** (2010), 507 – 515, DOI: 10.1016/j.dam.2009.11.014.

[20] P. Manuel, R. Bharati and I. Rajasingh, On minimum metric dimension of honeycomb networks, *Journal of Discrete Algorithms* **6** (2008), 20 – 27, DOI: 10.1016/j.jda.2006.09.002.

[21] P. K. Niranjan and S. R. Kola, On the radio number for corona of paths and cycles, *AKCE International Journal of Graphs and Combinatorics* **102** (2020), 1 – 7, DOI: 10.1016/j.akcej.2019.06.006.

[22] F. G. Nocetti, I. Stojmenovic and J. Zhang, Addressing and routing in hexagonal networks with applications for tracking mobile users and connection rerouting in cellular networks, *IEEE Transactions on Parallel and Distributed Systems* **13** (2002), 963 – 971, DOI: 10.1109/TPDS.2002.1036069.

[23] B. Parhami and D.-M. Kwai, A unified formulation of honeycomb and diamond networks, *IEEE Transactions on Parallel and Distributed Systems* **12** (2001), 74 – 79, DOI: 10.1109/71.899940.

[24] B. Rajan and K. Yenoke, Radio number of uniform theta graphs, *Journal of Computer and Mathematical Sciences* **2** (2011), 874 – 881.

[25] A. Santhakumaran, Median of a graph with respect to edges, *Discussiones Mathematicae Graph Theory* **32** (2012), 19 – 29, DOI: 10.7151/dmgt.1582.

[26] A. Sen, T. Roxborough and S. Medidi, Upper and lower bounds of a class of channel assignment problems in cellular networks, *Proceedings of IEEE INFOCOM 1998*, San Francisco CA, March 29 – April 2, IEEE Computer Society Press (1998), 1273 – 1283, DOI: 10.1109/IN-FCOM.1998.662943.

[27] B. Sooryanarayana, N. R. Shankar and S. Yogalakshmi, Radio number of kth-transformation graphs of a Path, *International Journal of Applied Graph Theory* **3**(2) (2019), 39 – 62, DOI: 10.14317/jami.2017.059.

[28] I. Stojmenovic, Honeycomb networks: Topological properties and communication algorithms, *IEEE Transactions on Parallel and Distributed Systems* **8** (1997), 1036 – 1042, DOI: 10.1109/71.629486.

[29] S. K. Vaidya and P. L. Vihol, Radio labeling for some cycle related graphs, *International Journal of Mathematics and Soft Computing* **2** (2012), 11 – 24, DOI: 10.26708/IJMSC.2012.2.2.03.

[30] K. Yenoke, On the Radio number of extended mesh, *Journal of Computer and Mathematical Sciences* **5** (2014), 358 – 366.