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Research Article

Fixed Points for (α,β)-Admissible Mappings via Simulation Functions

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Abstract. In this paper, by using the concept of (α,β)-admissible mappings with respect to Z-contraction, we prove some fixed point results in complete metric-like spaces. Our results generalize and extend several well-known results on literature. An example is given to support the obtained results.

Keywords. Fixed point, Metric-like space, Simulation function, (α,β)-admissible mapping

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1. Introduction

Hitzler and Seda [10] introduced the concept of metric-like (or dislocated metric) spaces, which is a generalized version of metric spaces. Later, Amini-harandi [2] established some fixed point results in the class of metric-like space. Several authors proved the existence of fixed and common fixed point in metric-like space (for instance see [1], [4–7], [18]).

In 2012, Samet *et al*. [17] introduced the concept of α-contraction and α-admissible mappings and proved various fixed point theorems for such class of mappings defined on complete metric spaces. There after several authors have proved fixed point theorems for α-admissible mappings in complete metric space (see [11], [12], [13], [15] and [16]).

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Recently, Chandok [8] introduced the notion of (α,β)-admissible mappings and obtained some fixed point theorems. Then various authors studied in this direction (see [1] and [18]). In [14], Khojasteh *et al*. proposed the notion of simulation function to unify the several existing fixed point results in the literature. There are many fixed point results in the setting of simulation function. For instance, (see [1], [3], [9], [18]).

In this paper, we use the concept of (α,β)-admissible Z-contraction with respect to ζ and establish the existence of fixed points for this class of mappings in metric-like spaces. Our result generalizes and extends some existing theorems in the literature. One illustrated example is given to support the obtained results.

2. Preliminaries

Definition 2.1 ([2])**.** Let X be a nonempty set. A function $\sigma : X\times X \to \mathbb{R}^+$ is said to be a metric-like (or a dislocated metric) on X, if for any $x,y,z\in X$, the following conditions hold:

(σ_1) $\sigma(x,y)=0 \Rightarrow x=y$;

(σ_2) $\sigma(x,y)=\sigma(y,x)$;

(σ_3) $\sigma(x,z)\le \sigma(x,y)+\sigma(y,z)$.

The pair (X,σ) is called a metric-like space. Then a metric-like on X satisfies all of the conditions of a metric except that $\sigma(x,x)$ may be positive for $x\in X$. Each metric-like σ on X generates a topology τ_σ on X, whose base is the family of open σ-balls, then for all $x\in X$ and $\epsilon>0$

$$B_\sigma(X,\varepsilon)=y\in X:\sigma(x,y)-\sigma(x,x)<\epsilon.$$

Now, let (X,σ) be a metric-like space. A sequence $\{x_n\}$ in X converges to $x\in X$, if and only if

$$\lim_{n\to\infty}\sigma(x_n,x)=\sigma(x,x).$$

Let (X,σ) be metric-like space and let $T:X\to X$ be a continuous mapping. Then

$$\lim_{n\to\infty}x_n=x\Rightarrow \lim_{n\to\infty}T(x_n)=T(x).$$

A sequence $\{x_n\}$ is Cauchy in (X,σ), if and only if $\lim_{n,m\to\infty}\sigma(x_n,x_m)$ exists and is finite. Moreover, (X,σ) is complete, if and only is for every Cauchy sequence $\{x_n\}$ in X, there exists $x\in X$ such that

$$\lim_{n\to+\infty}\sigma(x,x_n)=\sigma(x,x)=\lim_{n,m\to\infty}\sigma(x_n,x_m).$$

Every partial metric space and metric space is a metric-like space, but the converse is not true.

Example 2.2 ([1])**.** Let $X=\{0,1\}$ and

$$\sigma(x,y)=\begin{cases}2, & \text{if } x=y=0,\\ 1, & \text{otherwise.}\end{cases}$$

Then (X,σ) is a metric-like space. It is neither a partial metric space ($\sigma(0,0)\nleq\sigma(0,1)$), nor a metric space ($\sigma(0,0)=2\neq 0$).

Remark 2.3 ([1])**.** A subset A of a metric-like space (X,σ) is bounded if there is a point $b\in X$ and a positive constant k such that $\sigma(a,b)\le k$, for all $a\in A$.

Remark 2.4 ([1,2]). Let $X=\{0,1\}$ such that $\sigma(x,y)=1$ for each $x,y\in X$ and let $x_n=1$ for $n\in N$. Then it is easy to see that $x_n\to 0$ and $x_n\to 1$ and so in metric-like space, the limit of a convergence sequence is not necessarily unique.

Lemma 2.5 ([2,5,9]). *Let (X,σ) be a metric-like space. Let $\{x_n\}$ be a sequence in X such that $x_n\to x$, where $x\in X$ and $\sigma(x,y)=0$. Then for all $y\in X$ we have $\lim_{n\to\infty}\sigma(x_n,y)=\sigma(x,y)$.*

Definition 2.6 ([17]). For a nonempty set X, let $T:X\to X$ and $\alpha:X\times X\to[0,\infty)$ be given mappings. We say that T is α-admissible, if for all $x,y\in X$, we have

$$\alpha(x,y)\geq 1 \quad\Longrightarrow\quad \alpha(Tx,Ty)\geq 1.$$

Chandok [8] introduced the concept of (α,β)-admissible Geraghty type contractive mapping, which sufficient condition for the existence of a fixed point for such class of generalized nonlinear contractive mapping in metric space.

Definition 2.7 ([8]). Let X be a nonempty set $T:X\to X$ and $\alpha,\beta:X\times X\to\mathbb{R}^+$, we say that T is an (α,β)-admissible mapping if $\alpha(x,y)\geq 1$ and $\beta(x,y)\geq 1$ implies $\alpha(Tx,Ty)\geq 1$ and $\beta(Tx,Ty)\geq 1$, for all $x,y\in X$.

Khojasteh *et al*. [14] introduced a new class of mappings called simulation functions. They proved many fixed point theorems and showed that several results in the literature are simple consequences of their obtained results.

Definition 2.8 ([14]). A function $\zeta:[0,\infty)\times[0,\infty)\to\mathbb{R}$ is called a simulation function if ζ satisfies the following conditions:

(ζ_1) $\zeta(0,0)=0$;

(ζ_2) $\zeta(t,s)<s-t$, for all $t,s>0$;

(ζ_3) if $\{t_n\}$ and $\{s_n\}$ are sequences in $(0,\infty)$ such that

$$\lim_{n\to\infty}t_n=\lim_{n\to\infty}s_n=l\in(0,\infty),$$

then

$$\lim_{n\to\infty}\sup\zeta(t_n,s_n)<0.$$

In [14], the following unique fixed point theorem is established.

Theorem 2.9 ([14]). *Let (X,d) be a metric space and $T:X\to X$ be a Z-contraction with respect to a simulation function ζ, that is,*

$$\zeta(d(Tx,Ty),d(x,y))\geq 0,\quad \text{for all } x,y\in X.$$

Then T has a unique fixed point.

It is worth mentioning that the Banach contraction is an example of Z-contractions by defining $\zeta:[0,\infty)\times[0,\infty)\to\mathbb{R}$ via

$$\zeta(t,s)=\lambda s-t,\quad \text{for all } s,t\in[0,\infty),$$

where $\lambda\in[0,1)$.

Argoubi *et al*. [3] modified Definition 2.8 as follows.

Definition 2.10 ([3])**.** A simulation function is a function $\zeta:[0,\infty)\times[0,\infty)\to\mathbb{R}$ that satisfies the following conditions:

(i) $\zeta(t,s)<s-t$, for all $t,s>0$;

(ii) if $\{t_n\}$ and $\{s_n\}$ are sequences in $(0,\infty)$ such that

$$\lim_{n\to\infty} t_n = \lim_{n\to\infty} s_n = l \in (0,\infty),$$

then

$$\limsup_{n\to\infty} \zeta(t_n,s_n)<0.$$

It is clear that any simulation function in the sense of Khojasteh *et al*. [14] (Definition 2.8) is also a simulation function in the sense of Argoubi *et al*. [3] (Definition 2.10). The converse is not true.

Example 2.11 ([3])**.** Define a function $\zeta:[0,\infty)\times[0,\infty)\to\mathbb{R}$ by

$$\zeta(t,s)=\begin{cases}1, & \text{if } (s,t)=(0,0),\\ \lambda s-t, & \text{otherwise,}\end{cases}$$

where $\lambda\in(0,1)$. Then ζ is a simulation function in the sense of Argoubi *et al*. [3].

In the following, some other examples of simulation functions in the sense of Definition 2.8.

(i) $\zeta(t,s)=cs-t$, for all $t,s\in[0,\infty)$ where $c\in[0,1)$,

(ii) $\zeta(t,s)=s-\phi(s)-t$, for all $t,s\in[0,\infty)$,

where $\phi:\mathbb{R}_+\to\mathbb{R}_+$ is a lower semicontinuous function such that $\phi(t)=0$ if and only if $t=0$.

3. Main Results

First, we introduce the following:

Definition 3.1. Let (X,σ) be a metric-like space. Given $T:X\to X$ and $\alpha,\beta:X\times X\to\mathbb{R}^+$. Then T is said an (α,β)-admissible Z-contraction with respect to ζ if

$$\zeta(\alpha(Tx,Ty)\beta(Tx,Ty)\sigma(Tx,Ty),M(x,y))\geq 0$$

for all $x,y\in X$, where ζ is a simulation function in the sense of Definition 2.8. Here

$$M(x,y)=\max\left\{\sigma(x,y),\sigma(x,Tx),\sigma(y,Ty),\frac{\sigma(x,Ty)+\sigma(y,Tx)}{4}\right\}.$$

Now, we introduce our main theorem.

Theorem 3.2. *Let (X,σ) be a complete metric-like space and a continuous self mapping $T:X\to X$ be a (α,β)-admissible Z-contraction with respect to ζ simulation function satisfying as*

$$\zeta(\alpha(Tx,Ty)\beta(Tx,Ty)\sigma(Tx,Ty),M(x,y))\geq 0, \tag{3.1}$$

for all $x,y\in X$, where

$$M(x,y)=\max\left\{\sigma(x,y),\sigma(x,Tx),\sigma(y,Ty),\frac{\sigma(x,Ty)+\sigma(y,Tx)}{4}\right\}$$

and there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$, $\beta(x_0, Tx_0) \geq 1$.
Then T has a unique fixed point $u \in X$ such that $\sigma(u,u) = 0$.

Proof. Let $\{x_n\}$ be a sequence in X such that $x_{n+1} = Tx_n$, for all $= 0, 1, 2, \dots$. If $x_n = x_{n+1}$ then $Tx_n = x_{n+1} = x_n$, i.e., x_n is a fixed point of T. So proof is trivial. Now, we consider

$$x_n \neq x_{n+1}, \quad \text{for all } n \in \mathbb{N} \cup \{0\}.$$

Since $\alpha(x_0, Tx_0) \geq 1 \Rightarrow \alpha(x_0, x_1) \geq 1$ and T is an (α,β)-admissible, so

$$\alpha(Tx_0, Tx_1) \geq 1 \Rightarrow \alpha(x_1, x_2) \geq 1.$$

Continuing, we have for all $n \geq 0$

$$\alpha(x_{n,}x_{n+1}) \geq 1. \tag{3.2}$$

Similarly, for all $n \geq 0$, we obtain

$$\beta(x_{n,}x_{n+1}) \geq 1. \tag{3.3}$$

From (3.1), we have

$$\begin{aligned} 0 &\leq \zeta(\alpha(Tx_{n-1}, Tx_n)\beta(Tx_{n-1}, Tx_n)\sigma(Tx_{n-1}, Tx_n), M(x_{n-1}, x_n)) \\ &= \zeta(\alpha(x_n, x_{n+1})\beta(x_n, x_{n+1})\sigma(x_n, x_{n+1}), M(x_{n-1}, x_n)). \end{aligned} \tag{3.4}$$

Since

$$\begin{aligned} M(x_{n-1}, x_n) &= \max\left\{\sigma(x_{n-1}, x_n), \sigma(x_{n-1}, Tx_{n-1}), \sigma(x_n, Tx_n), \frac{\sigma(x_{n-1,}Tx_n) + \sigma(x_n, Tx_{n-1})}{4}\right\} \\ &= \max\left\{\sigma(x_{n-1}, x_n), \sigma(x_{n-1}, x_n), \sigma(x_n, x_{n+1}), \frac{\sigma(x_{n-1,}x_{n+1}) + \sigma(x_n, x_n)}{4}\right\}. \end{aligned}$$

By a triangular inequality, we have

$$\frac{\sigma(x_{n-1}, x_{n+1}) + \sigma(x_n, x_n)}{4} \leq \max\{\sigma(x_{n-1}, x_n), \sigma(x_n, x_{n+1})\}.$$

Thus

$$M(x_{n-1}, x_n) = \max\{\sigma(x_{n-1}, x_n), \sigma(x_n, x_{n+1})\}.$$

Therefore from (3.4), we have

$$\begin{aligned} 0 &\leq \zeta(\alpha(x_n, x_{n+1})\beta(x_n, x_{n+1})\sigma(x_n, x_{n+1}), \max\{\sigma(x_{n-1}, x_n), \sigma(x_n, x_{n+1})\}) \\ &< \max\{\sigma(x_{n-1}, x_n), \sigma(x_n, x_{n+1})\} - \alpha(x_n, x_{n+1})\beta(x_n, x_{n+1})\sigma(x_n, x_{n+1}) \text{ (by } (\zeta_2)) \end{aligned}$$

Then

$$\alpha(x_n, x_{n+1})\beta(x_n, x_{n+1})\sigma(x_n, x_{n+1}) < \max\{\sigma(x_{n-1}, x_n), \sigma(x_n, x_{n+1})\}. \tag{3.5}$$

Necessarily, we have

$$\max\{\sigma(x_{n-1}, x_n), \sigma(x_n, x_{n+1})\} = \sigma(x_{n-1}, x_n), \quad \text{for all } n \geq 1. \tag{3.6}$$

Consequently, we obtain

$$\alpha(x_n, x_{n+1})\beta(x_n, x_{n+1})\sigma(x_n, x_{n+1}) < \sigma(x_{n-1}, x_n), \quad \text{for all } n \geq 1. \tag{3.7}$$

We know

$$\sigma(x_n, x_{n+1}) \leq \alpha(x_n, x_{n+1})\beta(x_n, x_{n+1})\sigma(x_n, x_{n+1}). \tag{3.8}$$

Since $\alpha(x_n,x_{n+1}) \geq 1$ and $\beta(x_n,x_{n+1}) \geq 1$.
From (3.7) and (3.8) for all $n \geq 0$, we have

$$\sigma(x_n,x_{n+1}) \leq \alpha(x_n,x_{n+1})\beta(x_n,x_{n+1})\sigma(x_n,x_{n+1}) < \sigma(x_{n-1},x_n) \tag{3.9}$$

i.e.

$$\sigma(x_n,x_{n+1}) < \sigma(x_{n-1},x_n). \tag{3.10}$$

The sequence $\{\sigma(x_n,x_{n+1})\}$ is non increasing. So there exist $r \geq 0$ such that

$$\lim_{n\to\infty} \sigma(x_{n-1},x_n) = r.$$

We prove that

$$\lim_{n\to\infty} \sigma(x_{n-1},x_n) = 0. \tag{3.11}$$

Now, we assume on the contrary such that $r > 0$. By (3.9) we have

$$\lim_{n\to\infty} \{\alpha(x_n,x_{n+1})\beta(x_n,x_{n+1})\sigma(x_n,x_{n+1})\} = r.$$

Since $r > 0$ and letting $s_n = \alpha(x_n,x_{n+1})\beta(x_n,x_{n+1})\sigma(x_n,x_{n+1})$ and $t_n = \sigma(x_n,x_{n+1})$ such that $\lim\limits_{n\to\infty} s_n = \lim\limits_{n\to\infty} t_n = r$, then by (ζ_3)

$$\lim_{n\to\infty} \sup \zeta(s_n,t_n) < 0.$$

Since $\zeta(s_n,t_n) \geq 0$, so

$$0 \leq \lim_{n\to\infty} \sup \zeta(s_n,t_n) < 0,$$

which is contradiction. So our assumption is false. Hence $r = 0$. Again we show that $\{x_n\}$ is a Cauchy sequence in (X,σ) i.e.

$$\lim_{n,m\to\infty} \sigma(x_n,x_m) = 0. \tag{3.12}$$

Suppose on the contrary that is $\{x_n\}$ is not a Cauchy sequence. Then there exist $\epsilon > 0$ for which we can assume subsequences $x_{n(k)}$ and $x_{m(k)}$ of x_n with $n(k) > m(k) > k$ such that for every k

$$\sigma(x_{n(k)},x_{m(k)}) \geq \epsilon \tag{3.13}$$

and $n(k)$ is the smallest number such that (3.13) holds. From (3.13), we get

$$\sigma(x_{n(k)-1},x_{m(k)}) < \epsilon. \tag{3.14}$$

Then by triangular inequality and (3.12), we have

$$\begin{aligned}\epsilon &\leq \sigma(x_{n(k)},x_{m(k)}) \leq \sigma(x_{n(k)},x_{n(k)-1}) + \sigma(x_{n(k)-1},x_{m(k)}) \\ &< \sigma(x_{n(k)},x_{n(k)-1}) + \epsilon.\end{aligned}$$

Taking $n \to \infty$ in above equation and applying (3.11), we get

$$\lim_{n\to\infty} \sigma(x_{n(k)},x_{m(k)}) = \epsilon. \tag{3.15}$$

From the triangular inequality, we have

$$\sigma(x_{n(k)+1},x_{m(k)}) \leq \sigma(x_{n(k)+1},x_{n(k)}) + \sigma(x_{n(k)},x_{m(k)}).$$

Taking limit $n \to \infty$ and using (3.11), (3.13) and (3.15), we have

$$\lim_{n\to\infty} \sigma(x_{n(k)+1},x_{m(k)}) = \epsilon. \tag{3.16}$$

Similarly, it is easy to show that

$$\lim_{n\to\infty}\sigma(x_{n(k)+1},x_{m(k)+1})=\epsilon. \tag{3.17}$$

Since T is an (α,β)-admissible Z-contraction with respect to ζ and using (ζ_3)

$$0\le \lim_{n\to\infty}\sup\zeta(\alpha(Tx_{n(k)},Tx_{m(k)})\beta(Tx_{n(k)},Tx_{m(k)})\sigma(Tx_{n(k)},Tx_{m(k)}),M(x_{n(k)},x_{m(k)}))$$

$$0\le \lim_{n\to\infty}\sup\zeta(\alpha(x_{n(k)+1},x_{m(k)+1})\beta(x_{n(k)+1},x_{m(k)+1})\sigma(x_{n(k)+1},x_{m(k)+1}),M(x_{n(k)},x_{m(k)})). \tag{3.18}$$

Since

$$\begin{aligned}M(x_{n(k)},x_{m(k)})&=\max\left\{\sigma(x_{n(k)},x_{m(k)}),\sigma(x_{n(k)},Tx_{n(k)}),\sigma(x_{m(k)},Tx_{m(k)}),\right.\\&\qquad\left.\frac{\sigma(x_{n(k),}Tx_{m(k)})+\sigma(x_{m(k)},Tx_{n(k)})}{4}\right\}\\&=\max\left\{\sigma(x_{n(k)},x_{m(k)}),\sigma(x_{n(k)},x_{n(k)+1}),\sigma(x_{m(k)},x_{m(k)+1}),\right.\\&\qquad\left.\frac{\sigma(x_{n(k),}x_{m(k)+1})+\sigma(x_{m(k)},x_{n(k)+1})}{4}\right\}.\end{aligned}$$

From (3.11), (3.15), (3.16) and (3.17)

$$\lim_{n\to\infty}\sigma(x_{n(k)+1},x_{m(k)+1})=\lim_{n\to\infty}M(x_{n(k)},x_{m(k)})=\epsilon. \tag{3.19}$$

From (3.18) and (3.19), we have

$$0\le \lim_{n\to\infty}\sup\zeta(\alpha(x_{n(k)+1},x_{m(k)+1})\beta(x_{n(k)+1},x_{m(k)+1})\sigma(x_{n(k)+1},x_{m(k)+1}),M(x_{n(k)},x_{m(k)}))<0,$$

which is contradict due to our assumption. So $\{x_n\}$ is a Cauchy sequence.
Since (X,σ) be a complete metric-like space, then there exist $x\in X$ and using (3.12) such that

$$\lim_{n\to\infty}\sigma(x_n,x)=\sigma(x,x)=\lim_{n,m\to\infty}\sigma(x_n,x_m)=0. \tag{3.20}$$

We show that x is a fixed point of T. Since T is continuous and $x_n\to x$ as $n\to\infty$. So from (3.20),

$$\lim_{n\to\infty}\sigma(x_{n+1},Tx)=\lim_{n\to\infty}\sigma(Tx_n,Tx)=\sigma(Tx,Tx)=0. \tag{3.21}$$

Using Lemma 2.5 and (3.21), we have

$$\lim_{n\to\infty}\sigma(x_{n+1},Tx)=\sigma(x,Tx). \tag{3.22}$$

From (3.21) and (3.22), we have

$$\sigma(x,Tx)=\sigma(Tx,Tx)=0. \tag{3.23}$$

Hence $Tx=x$, that is x is a fixed point of T.
Now, we shall show that the uniqueness of fixed point of x. We argue by contrary. Assume that there exist $u\in X$ such that $Tu=u$ and $x\ne u$. Now,

$$0\le\zeta(\alpha(Tx,Tu)\beta(Tx,Tu)\sigma(Tx,Tu),M(x,u)) \tag{3.24}$$

where

$$\begin{aligned}M(x,u)&=\max\left\{\sigma(x,u),\sigma(x,Tu),\sigma(u,Tu),\frac{\sigma(x,Tu)+\sigma(u,Tx)}{4}\right\}\\&=\max\left\{\sigma(x,u),\sigma(x,u),\sigma(u,u),\frac{\sigma(x,u)+\sigma(u,x)}{4}\right\}\\&=\sigma(x,u).\end{aligned} \tag{3.25}$$

From (3.24) and (3.25), we have

$$\begin{aligned}0 &\le \zeta(\alpha(Tx,Tu)\beta(Tx,Tu)\sigma(Tx,Tu),\sigma(x,u))\\ &\le \sigma(x,u)-\alpha(Tx,Tu)\beta(Tx,Tu)\sigma(Tx,Tu)\\ &= \sigma(x,u)-\alpha(x,u)\beta(x,u)\sigma(x,u)\\ &= \sigma(x,u)[1-\alpha(x,u)\beta(x,u)] < 0.\end{aligned}$$

Since $\alpha(x,u)\ge 1$, $\beta(x,u)\ge 1$, which is a contradiction. So $x=u$. Hence T has a unique fixed point. □

Corollary 3.3. *In Theorem 3.2, if we have choose any one of the ζ simulation given below, we have the same result and proof are similar to these corollary:*

$$\zeta(\alpha(x,Tx)\beta(y,Ty)\sigma(Tx,Ty),M(x,y))\ge 0, \tag{3.26}$$

$$\zeta(\alpha(x,y)\beta(Tx,Ty)\sigma(Tx,Ty),M(x,y))\ge 0, \tag{3.27}$$

$$\zeta(\alpha(x,y)\beta(x,y)\sigma(Tx,Ty),M(x,y))\ge 0, \tag{3.28}$$

$$\zeta(\alpha(Tx,Ty)\beta(x,y)\sigma(Tx,Ty),M(x,y))\ge 0. \tag{3.29}$$

Next, we apply Theorem 3.2 to obtain different results in literature. The first one is Banach type.

Corollary 3.4. *Let (X,σ)be a complete metric-like space and let T be a self-mapping on X satisfying the following conditions:*

(i) *T is (α,β)-admissible Z-contraction;*

(ii) *there exists $x_0\in X$ such that $\alpha(x_0,Tx_0)\ge 1$ and $\beta(x_0,Tx_0)\ge 1$;*

(iii) *$\alpha(Tx,Ty)\beta(Tx,Ty)\sigma(Tx,Ty)\le \lambda M(x,y)$, for all $x,y\in X$ and $\lambda\in[0,1)$;*

(iv) *T is σ continuous.*

Then T has a unique fixed point $u\in X$ with $\sigma(u,u)=0$.

Proof. Following the steps of Theorem 3.2, by taking as a ζ-simulation function,

$$\zeta(t,s)=\lambda s-t.$$

□

Corollary 3.5. *Let (X,σ) be a complete metric-like space and let T be a self-mapping on X satisfying the following conditions:*

(i) *T is (α,β)-admissible Z-contraction;*

(ii) *there exists $x_0\in X$ such that $\alpha(x_0,Tx_0)\ge 1$ and $\beta(x_0,Tx_0)\ge 1$;*

(iii) *there exists a lower semi-continuous $\varphi:\mathbb{R}_+\to\mathbb{R}_+$ with $\varphi^{-1}=\{0\}$ such that*

$$\alpha(Tx,Ty)\beta(Tx,Ty)\sigma(Tx,Ty)\le M(x,y)-\varphi(M(x,y)),\quad \text{for all } x,y\in X;$$

(iv) *T is σ continuous.*

Then T has a unique fixed point $u\in X$ with $\sigma(u,u)=0$.

Proof. It is sufficient to take $\zeta(t,s)=s-\varphi(s)-t$. □

Example 3.6. Take $X = [0,\infty)$ endowed with the metric like $\sigma(x,y) = x+y$. Consider the mapping $T:X\to X$ given by $Tx = \begin{cases} \frac{x}{3}, & \text{if } 0\le x\le 1 \\ 3x, & \text{otherwise.}\end{cases}$

Note that (X,σ) is complete metric-like space. Define mappings $\alpha,\beta : X\times X \to \mathbb{R}^+$ by

$$\alpha(x,y) = \beta(x,y) = \begin{cases} 1, & \text{if } x,y\in[0,1] \\ 0, & \text{otherwise.}\end{cases}$$

Note that T is an (α,β)-admissible if $\alpha(x,y)\ge 1$ and $\beta(x,y)\ge 1 \Rightarrow \alpha(Tx,Ty)\ge 1$ and $\beta(Tx,Ty)\ge 1$, for all $x,y\in X$.

By definition of α,β and $x,y\in[0,1]$, we have $\alpha(Tx,Ty) = \alpha\left(\frac{x}{3},\frac{y}{3}\right) = 1$.

Similarly, $\beta(Tx,Ty) = 1$.

From above, it is clear that T is an (α,β)-admissible mapping. Let $\zeta(t,s) = \lambda s - t$, $\lambda\in[0,1]$, for all $s,t\ge 0$. Also, for $x,y\in X$ such that $\alpha(x,y)\ge 1$ and $\beta(x,y)\ge 1$. So, $x,y\in[0,1]$. In this case we have

$$\alpha(Tx,Ty)\beta(Tx,Ty)\sigma(Tx,Ty), M(x,y) = \left(\left(\frac{x}{3}+\frac{y}{3}\right), M(x,y)\right). \tag{3.30}$$

Now

$$\begin{aligned} M(x,y) &= \max\left\{\sigma(x,y),\sigma(x,Tx),\sigma(y,Ty),\frac{\sigma(x,Ty)+\sigma(y,Tx)}{4}\right\} \\ &= \max\left\{(x+y),\left(x+\frac{x}{3}\right),\left(y+\frac{y}{3}\right),\frac{\left(x+\frac{y}{3}\right)+\left(y+\frac{x}{3}\right)}{4}\right\}. \end{aligned}$$

Since $x,y\in[0,1]$, therefore

$$M(x,y) = x+y. \tag{3.31}$$

From (3.30) and (3.31), we have

$$(\alpha(Tx,Ty)\beta(Tx,Ty)\sigma(Tx,Ty), M(x,y)) = \left(\left(\frac{x}{3}+\frac{y}{3}\right), x+y\right).$$

It follows that

$$\begin{aligned} \zeta(\alpha(Tx,Ty)\beta(Tx,Ty)\sigma(Tx,Ty), M(x,y)) &= \zeta\left(\left(\frac{x}{3}+\frac{y}{3}\right), x+y\right) \\ &= \lambda(x+y) - \left(\frac{x}{3}+\frac{y}{3}\right). \end{aligned}$$

If we take $\lambda = \frac{1}{2}$, we get

$$\left(\frac{x+y}{2}\right) - \left(\frac{x}{3}+\frac{y}{3}\right) \ge 0$$

i.e.

$$\zeta(\alpha(Tx,Ty)\beta(Tx,Ty)\sigma(Tx,Ty), M(x,y)) \ge 0.$$

Also, let $\{x_n\}$ be a sequence in X such that $\alpha(x_n,x_{n+1})\ge 1$, $\beta(x_n,x_{n+1})\ge 1$ for all n and $x_n\to x\in X$. Then, $\{x_n\}\subset[0,1]$ and $x_n^2 + x^2 \to 2x^2$ as $n\to\infty$. Thus, $x_n\to x$ as $n\to\infty$ in $(X,|\cdot|)$. This implies that $x\in[0,1]$ and so $\alpha(x_n,x) = 1$, $\beta(x_n,x) = 1$ for all n. Moreover, there exists $x_0\in X$ such that $\alpha(x_0,Tx_0)\ge 1$, $\beta(x_0,Tx_0)\ge 1$. In fact, for $x_0 = 1$, we have $\alpha(1,T1) = \alpha\left(1,\frac{1}{3}\right) = 1$. Similarly, $\beta(1,T1) = \beta\left(1,\frac{1}{3}\right) = 1$. Thus, all the conditions of Theorem 3.2 are verified. Here $x = 0$ is the unique fixed point of T.

4. Conclusion

In this attempt, we studied (α,β)-admissible mappings with respect to Z-contraction and proved some fixed point results in complete metric-like spaces. Our results are generalization and extension of many existing results in the literature. Finally, we show one example to support the obtained results.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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