Communications in Mathematics and Applications
Vol. 12, No. 3, pp. 589–594, 2021
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications

http://www.rgnpublications.com

DOI: 10.26713/cma.v12i3.1527

Research Article

On $\hat{\hat{g}}$-Closed Sets in Fuzzy Topological Spaces

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Received: January 23, 2021 **Accepted:** March 1, 2021

Abstract. In this paper, we introduce the concepts of fuzzy $\hat{\hat{g}}$-closed sets and fuzzy $\hat{\hat{g}}$-open sets. Further, we study some of their properties.

Keywords. Fuzzy topology; Fuzzy $\hat{\hat{g}}$-closed set; Fuzzy $\hat{\hat{g}}$-open set

Mathematics Subject Classification (2020). 54A40; 54J05; 03F55

1. Introduction

The concept of a fuzzy subset which was introduced and studied by L.A. Zadeh [18] in the year 1965. The subsequent research activities in this area and the related areas have found applications in many branches of science and engineering. Chang [4] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces. Many researchers like Azad [1], Sinha [3], Wong [16] and many others have contributed to the development of fuzzy topological spaces and so on. New class of fuzzy generalized closed sets namely fuzzy $\tilde{g}$-closed sets is to introduce and study in fuzzy topological spaces. Further, compare with related to various types of fuzzy generalized closed sets are investigated. More over, some properties of fuzzy $\tilde{g}$-closed sets are given in this paper.

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2. Preliminaries

Throughout this paper, (X, F_τ) (briefly, X) will denote fuzzy topological spaces or space (X, F_τ).

Definition 2.1. A fuzzy subset A of a fuzzy topological space (X, τ) is called:

(1) *fuzzy semi-open set* [1] if $A \leq \mathrm{cl}(\mathrm{int}(A))$.

(2) *fuzzy α-open set* [3] if $A \leq \mathrm{int}(\mathrm{cl}(\mathrm{int}(A)))$.

(3) *fuzzy semi-preopen set* [15] if $A \leq \mathrm{cl}(\mathrm{int}(\mathrm{cl}(A)))$.

(4) *fuzzy regular open set* [1] if $A = \mathrm{int}(\mathrm{cl}(A))$.

The complements of the above mentioned fuzzy open sets are called their respective fuzzy closed sets.

The fuzzy semi-closure [17] (resp. fuzzy α-closure [7], fuzzy semi-preclosure [17]) of a fuzzy subset A of X, denoted by $\mathrm{scl}(A)$ (resp. $\alpha\,\mathrm{cl}(A)$, $\mathrm{spcl}(A)$), is defined to be the intersection of all fuzzy semi-closed (resp. fuzzy α-closed, fuzzy semi-preclosed) sets of (X, τ) containing A. It is known that $\mathrm{scl}(A)$ (resp. $\alpha\,\mathrm{cl}(A)$, $\mathrm{spcl}(A)$) is a fuzzy semi-closed (resp. fuzzy α-closed, fuzzy semi-preclosed) set.

Definition 2.2. A fuzzy subset A of a fuzzy topological space (X, τ) is called a fuzzy generalized closed set (resp. briefly fg-closed [2], fsg-closed [6], fgs-closed [6], $fg\alpha$-closed and $f\alpha g$-closed [11], $fgsp$-closed [5], $fpsg$-closed [10], $f\omega$-closed [13], and $f\psi$-closed [8]).

The complements of the above mentioned fuzzy closed sets are called their respective fuzzy open sets.

3. Fuzzy $\hat{\hat{g}}$-closed Sets

Definition 3.1. A fuzzy subset A of X is called fuzzy $\hat{\hat{g}}$-closed set (briefly $f\hat{\hat{g}}$-closed set) if $\mathrm{cl}(A) \leq U$ whenever $A \leq U$ and U is fsg-open in (X, τ). The complement of $f\hat{\hat{g}}$-closed set is called $f\hat{\hat{g}}$-open set.
The collection of all fuzzy $\hat{\hat{g}}$-closed sets in X is denoted by $F\hat{\hat{G}}C(X)$.

Theorem 3.2. *Every fuzzy closed set is $f\hat{\hat{g}}$-closed.*

If A is any fuzzy closed set in (X, τ) and G is any fsg-open set such that $A \leq G$, then $G \geq A = \mathrm{cl}(A)$. Hence A is $f\hat{\hat{g}}$-closed.

Theorem 3.3. *Every $f\hat{\hat{g}}$-closed set is fgsp-closed.*

If A is a $f\hat{\hat{g}}$-closed subset of (X, τ) and G is any fuzzy open set such that $G \geq A$, every fuzzy open set is fsg-open, we have $G \geq \mathrm{cl}(A) \geq \mathrm{spcl}(A)$. Hence A is $fgsp$-closed in (X, τ).

Theorem 3.4. *Every $f\hat{\hat{g}}$-closed set is fuzzy ω-closed.*

Suppose that $A \leq G$ and G is fuzzy semi-open in (X, τ). Since every fuzzy semi-open set is fsg-open and A is $f\hat{\hat{g}}$-closed set therefore $\mathrm{cl}(A) \leq G$. Hence A is fuzzy ω-closed in (X, τ).

Theorem 3.5. *Every $f\hat{\hat{g}}$-closed set is fg-closed.*

If A is a $f\hat{\hat{g}}$-closed subset of (X,τ) and G is any fuzzy open set such that $G \geq A$, since every fuzzy open set is fsg-open, we have $G \geq \text{cl}(A)$. Hence A is fg-closed in (X,τ).

Theorem 3.6. *Every $f\hat{\hat{g}}$-closed set is $f\alpha g$-closed.*

If A is a $f\hat{\hat{g}}$-closed subset of (X,τ) and G is any fuzzy open set such that $G \geq A$, since every fuzzy open set is fsg-open, we have $G \geq \text{cl}(A) \geq \alpha\,\text{cl}(A)$. Hence A is $f\alpha g$-closed in (X,τ).

Theorem 3.7. *Every $f\hat{\hat{g}}$-closed set is fgs-closed.*

If A is a $f\hat{\hat{g}}$-closed subset of (X,τ) and G is any fuzzy open set such that $G \geq A$, since every fuzzy open set is fsg-open, we have $G \geq \text{cl}(A) \geq \text{scl}(A)$. Hence A is fgs-closed in (X,τ).

Definition 3.8. A fuzzy subset A of X is called a fuzzy $\hat{\hat{g}}_\alpha$-closed set (briefly $f\hat{\hat{g}}_\alpha$-closed set) if $\alpha\,\text{cl}(A) \leq U$ whenever $A \leq U$ and U is fsg-open in (X,τ). The complement of $f\hat{\hat{g}}_\alpha$-closed set is called $f\hat{\hat{g}}_\alpha$-open set. The collection of all fuzzy $\hat{\hat{g}}_\alpha$-closed sets in X is denoted by $F\hat{\hat{G}}_\alpha C(X)$.

Theorem 3.9. *Every $f\hat{\hat{g}}$-closed set is $f\hat{\hat{g}}_\alpha$-closed.*

If A is a $f\hat{\hat{g}}$-closed subset of (X,τ) and G is any fsg-open set such that $G \geq A$ then $G \geq \text{cl}(A) \geq \alpha\,\text{cl}(A)$. Hence A is $f\hat{\hat{g}}_\alpha$-closed in (X,τ).

Theorem 3.10. *Every fuzzy α-closed set is $f\hat{\hat{g}}_\alpha$-closed.*

If A is an α-closed set and G is any fsg-open such that $G \geq A$, then $G \geq A = \alpha\,\text{cl}(A)$. Hence $f\hat{\hat{g}}_\alpha$-closed.

Theorem 3.11. *Every $f\hat{\hat{g}}$-closed set is $f\psi$-closed.*

If A is a $f\hat{\hat{g}}$-closed subset of (X,τ) and G is any fsg-open set such that $G \geq A$ then $A \geq \text{cl}(A) \geq \text{scl}(A)$. Hence A is $f\psi$-closed in (X,τ).

Theorem 3.12. *Every $f\psi$-closed set is fsg-closed.*

Suppose that $A \leq G$ and G is fuzzy semi-open in (X,τ). Since every fuzzy semi-open set is fsg-open and A is $f\psi$-closed set therefore $\text{scl}(A) \leq G$. Hence A is fsg-closed in (X,τ).

Theorem 3.13. *Every fuzzy semi-closed set is $f\psi$-closed.*

If A is a fuzzy semi-closed subset of (X,τ) such that $G \geq A$, we have $G \geq A = \text{scl}(A)$. Hence A is $f\psi$-closed in (X,τ).

Definition 3.14. A fuzzy subset A of X is called a fuzzy αgs-closed set if $\alpha\,\text{cl}(A) \leq U$ whenever $A \leq U$ and U is fuzzy semi-open in (X,τ). The complement of fuzzy αgs-closed set is called fuzzy αgs-open set.

Theorem 3.15. *Every $f\omega$-closed set is fuzzy αgs-closed.*

If A is a fuzzy ω-closed subset of (X,τ) and G is any fuzzy semi-open set such that $G \geq A$, we have $G \geq \mathrm{cl}(A) \geq \alpha\,\mathrm{cl}(A)$. Hence A is fuzzy αgs-closed in (X,τ).

Theorem 3.16. *Every $f\hat{\hat{g}}$-closed set is fuzzy αgs-closed.*

If A is a $f\hat{\hat{g}}$-closed subset of (X,τ) and G is any fuzzy semi-open set such that $G \geq A$, since every fuzzy semi-open set is fsg-open, we have $G \geq \mathrm{cl}(A) \geq \alpha\,\mathrm{cl}(A)$. Hence A is fuzzy αgs-closed in (X,τ).

Theorem 3.17. *Every fuzzy αgs-closed set is fuzzy αg-closed.*

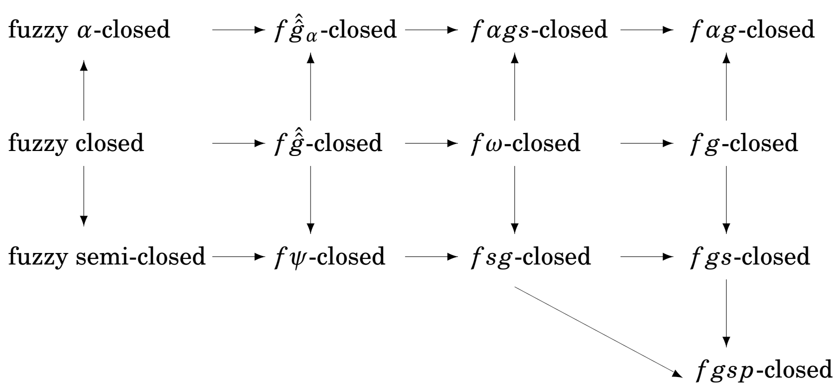
Suppose that $A \leq G$ and G is fuzzy open in (X,τ) such that $A \leq G$. Since every fuzzy open set is fuzzy semi-open. We have $G \geq \alpha\,\mathrm{cl}(A)$. Hence A is fuzzy αg-closed in (X,τ).

Remark 3.18. The following Examples show that $f\hat{\hat{g}}$-closed sets are independent of fuzzy α-closed sets and fuzzy semi-closed sets.

Example 3.19. Let $X = \{a,b\}$ and $\tau = \{0_X, \alpha, 1_X\}$, where α is a fuzzy set in X defined by $\alpha(a) = \alpha(b) = 0.5$. Then (X,τ) is a fts. Clearly, λ defined by $\lambda(a) = \lambda(b) = 0.4$ is $f\hat{\hat{g}}$-closed set but it is neither fuzzy α-closed nor fuzzy semi-closed in (X,τ).

Example 3.20. Let $X = \{a,b\}$ and $\tau = \{0_X, \alpha, 1_X\}$, where α is a fuzzy set in X defined by $\alpha(a) = 1$, $\alpha(b) = 0$. Then (X,τ) is a fts. Clearly, λ defined by $\lambda(a) = 0.5$, $\lambda(b) = 0$ is fuzzy α-closed well as fuzzy semi-closed in (X,τ) but not $f\hat{\hat{g}}$-closed set in (X,τ).

Remark 3.21. We obtain the following diagram where $A \to B$ represents A implies B but not conversely.



Acknowledgement

The authors thank the referees for their valuable comments and suggestions for improvement of this article.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] K. K. Azad, On fuzzy semi continuity fuzzy almost continuity and fuzzy weakly continuity, *Journal of Mathematical Analysis and Applications* **82**(1) (1981), 14 – 32, DOI: 10.1016/0022-247X(81)90222-5.

[2] G. Balasubramanian and P. Sundaram, On some generalizations of fuzzy continuous functions, *Fuzzy Sets and Systems* **86**(1) (1997), 93 – 100, DOI: 10.1016/0165-0114(95)00371-1.

[3] A. S. Bin Shahna, On fuzzy strong semicontinuity and fuzzy precontinuity, *Fuzzy Sets and Systems* **44**(2) (1991), 303 – 308, DOI: 10.1016/0165-0114(91)90013-G.

[4] C.L. Chang, Fuzzy topological spaces, *Journal of Mathematical Analysis and Applications* **24** (1968), 182 – 190, DOI: 10.1016/0022-247X(68)90057-7.

[5] M. E. El-Shafei and A. Zakari, Semi-generalized continuous mappings fuzzy topological spaces, *Journal of the Egyptian Mathematical Society* **15**(1) (2007), 57 – 67.

[6] H. Maki, T. Fukutaka, M. Kojima and H. Harade, Generalized closed sets in fuzzy topological spaces, *I. Meeting on Topological Spaces, Theory and Applications* (1998), 23 – 26.

[7] R. Prasad, S.S. Thakur and R.K. Saraf, Fuzzy α-irresolute mappings, *Journal of Fuzzy Mathematics* **2**(2) (1994), 335 – 339.

[8] R. K. Saraf and M. Khanna, Fuzzy generalized semi preclosed sets, *Journal of Tripura Mathematical Society* **3** (2001), 59 – 68.

[9] R. K. Saraf and M. Khanna, On gs-closed sets in fuzzy topology, *The Journal of the Indian Academy of Mathematics* **25**(1) (2003), 133 – 143, `https://www.researchgate.net/publication/265699532`.

[10] R.K. Saraf, G. Navalagi and M. Khanna, On fuzzy semi-pre-generalized closed sets, *Bulletin of the Malaysian Mathematical Sciences Society* (2) **28**(1) (2005), 19 – 30, `http://www.kurims.kyoto-u.ac.jp/EMIS/journals/BMMSS/pdf/v28n1/v28n1p3.pdf`.

[11] R. K. Saraf, M. Caldas and S. Mishra, Fαg-closed sets, *Journal of Tripura Mathematical Society* **2** (2000), 27 – 32.

[12] M. K. Singal and N. Prakash, Fuzzy preopen sets and fuzzy preseparation axioms, *Fuzzy Sets and Systems* **44** (1991), 273 – 281, DOI: 10.1016/0165-0114(91)90010-N.

[13] M. Sudha, E. Roja and M. K. Uma, Slightly fuzzy ω-continuous mappings, *International Journal of Mathematical Analysis* **5**(16) (2011), 779 – 787, `http://m-hikari.com/ijma/ijma-2011/ijma-13-16-2011/sudhaIJMA13-16-2011.pdf`.

[14] S. S. Thakur and R. Malviya, Generalized super closed sets in fuzzy topology, *Mathematical Notes* **38** (1995), 137 – 140.

[15] S. S. Thakur and S. Singh, On fuzzy semi pre-open sets and fuzzy semi pre continuity, *Fuzzy Sets and Systems* **98**(3) (1998), 383 – 391, DOI: 10.1016/S0165-0114(96)00363-6.

[16] C. K. Wong, Fuzzy points and local properties of fuzzy topology, *Journal of Mathematical Analysis and Applications* **46** (1974), 316 – 328, DOI: 10.1016/0022-247X(74)90242-X.

[17] T. H. Yalvac, Semi-interior and semi-closure of a fuzzy set, *Journal of Mathematical Analysis and Applications* **132**(2) (1988), 356 – 364, DOI: 10.1016/0022-247X(88)90067-4.

[18] L. A. Zadeh, Fuzzy sets, *Information and Control* **8** (1965), 338 – 353, DOI: 10.1016/S0019-9958(65)90241-X.