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Research Article

Fractional Variational Iteration Method and Adomian's Decomposition Method: Applications to Fractional Burgers Kuramoto KdV Equation via Hadamard Derivative

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Abstract. This paper presents the analytical solutions of the Fractional Burgers Kuramoto KdV equation by the variational iteration method and Adomian's decomposition method using Hadamard fractional derivative. By using initial conditions, the explicit solutions of the Burgers Kuramoto Kdv equation have been presented. The fractional derivatives are considered according to the Hadamard's approach. Two examples are given for illustrate to implement variational iteration method and Adomian's decomposition method for fractional Burgers Kuramoto KdV equation.

Keywords. Fractional Burgers Kuramoto KdV equation; Hadamard fractional; Variational iteration method; Fractional calculus; Adomian's decomposition method

Mathematics Subject Classification (2020). 26A33; 34A08; 34B15

1. Introduction

For several decades now, fractional calculus is distinguished by its many applications in various fields of physical sciences and engineering such as electromagnetics, viscoelasticity, fluid mechanics, electrochemical for example [17]. It thus became very popular with many researchers.

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The fractional differential equation solution is very much difficult to found. There is no general method which provides a exact solution for any fractional differential equations. We can obtain only approximate solutions using perturbation or linearization methods.

In this paper, we introduce the analytical approximate solutions of the fractional KdV Burgers Kuramoto equation by the variational iteration method ([9–11]) and the Adomian decomposition method ([1–3]) via Hadamard derivative. The fractional Burgers Kuramoto KdV equation with space fractional derivatives of the form:

$$\frac{\partial u}{\partial t}+u\frac{\partial^\alpha u}{\partial x^\alpha}+a\frac{\partial^2 u}{\partial x^2}+b\frac{\partial^3 u}{\partial x^3}+c\frac{\partial^4 u}{\partial x^4},\quad t>0,\ 0<\alpha\le 1,$$

where a, b, c are constants and α is a parameter which describes the fractional space derivatives order. The $u(x,t)$ function is presumed to be a causal function of space, i.e. to vanish for $t<0$ and $x<0$. The fractional derivatives are considered in Hadamard sense ([15], [16, p. 110], [18, p. 3]).

2. Preliminaries and Notations

2.1 Basic Definitions

We give some important definitions and properties of the theory of fractional calculus which are used further in this paper.

Definition 1. The Hadamard fractional integral of order $\alpha\in\mathbb{R}^+$ of a function $f(t)$, for all $t>0$, is defined as:

$${}^H I^\alpha g(t)=\frac{1}{\Gamma(\alpha)}\left(t\frac{\partial}{\partial t}\right)^n\int_0^t\left(\ln\frac{t}{s}\right)^{\alpha-1}\frac{g(s)}{s}ds.$$

provided the integral exists. where Γ is the Euler Gamma-function ([6, Section 1.1]).

Definition 2. The Hadamard derivative of fractional order α for a function $g:\to(0,+\infty)$ is defined as:

$${}^H D^\alpha g(t)=\frac{1}{\Gamma(n-\alpha)}\left(t\frac{\partial}{\partial t}\right)^n\int_0^t\left(\ln\frac{t}{s}\right)^{n-\alpha+1}\frac{g(s)}{s}ds,\quad n-1<\alpha<n,\ n=[\alpha]+1,$$

where $\log(\cdot)=\log_e(\cdot)$.

Lemma 3 (See [16, p. 113]). *Let $\alpha>0$ and $n>0$. Then the following formulas:*

$${}^H I^\alpha s^n=n^{-\alpha}t^n \quad \textit{and} \quad {}^H D^\alpha s^n=n^\alpha t^n$$

hold.

Example (See [16, Chapter 2]). With $f(x)=x^\beta$, β is a complex number, one obtains:

$$\begin{aligned}({}^H D^\alpha_{0+,\mu}t^\beta)(x)&=x^{-\mu}\delta^n(x^\mu I^{n-\alpha}_{0+,\mu}t^\beta)(x)\\&=x^{-\mu}\delta^n(x^\mu(\beta+\mu)^{\alpha-n}x^\beta)\\&=x^{-\mu}\delta^{n-1}\left(x\frac{d}{x}(\beta+\mu)^{\alpha-n}x^{\beta+\mu}\right)(x)\\&=x^{-\mu}\delta^{n-1}(x(\beta+\mu)^{\alpha-n+1}x^{\beta+\mu-1})\\&=x^{-\mu}\delta^{n-1}((\beta+\mu)^{\alpha-n+1}x^{\beta+\mu})\end{aligned}$$

$$= x^{-\mu}\delta^{n-2}((\beta+\mu)^{\alpha-n+2}x^{\beta+\mu}).$$

By recurrence, one obtains:

$$({}^{H}D^{\alpha}_{0+,\mu}t^{\beta})(x) = (\beta+\mu)^{\alpha}x^{\beta}.$$

In particular, for $\mu = 0$ and $\alpha > 0$. We find:

$$({}^{H}D^{\alpha}_{0+,\mu}t^{\beta})(x) = (\beta)^{\alpha}x^{\beta}.$$

Definition 4. Jumarie ([14, p. 2]) defined the fractional derivative as the following limit form:

$$f^{\alpha} = \lim_{t\to 0}\frac{\Delta^{\alpha}[f(x)-f(0)]}{h^{\alpha}}.$$

This definition is close to the standard definition of derivatives, and as a direct result, the α derivative of a constant, $0 < \alpha < 1$ is zero.

Definition 5. Fractional integral operator of order $\alpha \geq 0$ is defined as:

$$ {}_0I^{\alpha}_{x}f(x) = \frac{1}{\Gamma(\alpha)}\int_0^x (x-\xi)^{\alpha-1}f(\xi)d\xi, \quad \alpha > 0. \tag{1}$$

Fractional Leibnitz Product Law

$$ {}_0D^{\alpha}_{x}(uv) = u^{\alpha}v + uv^{\alpha},$$

fractional Leibnitz formulation:

$$ {}_0I^{\alpha}_{x}D^{\alpha}_{x}(uv) = f(x) - f(0).$$

The fractional integration by parts formula:

$$ {}_aI^{\alpha}_{b}(u^{\alpha}v) = (uv)|_a^b - {}_aI^{\alpha}_{b}(uv^{\alpha}).$$

Definition 6. Fractional derivative of compounded functions ([14, p. 5]) is defined as:

$$d^{\alpha}f \cong \Gamma(1+\alpha)df, \quad 0 < \alpha < 1.$$

Definition 7. The integral with respect to $(\partial t)^{\alpha}$ ([14, p. 5]) is defined as the solution of the fractional differential equation:

$$dx \cong f(x)(dt)^{\alpha}, \quad t \geq 0,\ x(0) = 0,\ 0 < \alpha < 1.$$

Lemma 8 (See [13, p. 3], [14, p. 5])**.** *Let f(x) denote a continuous function, then the solution of* (1) *is defined as:*

$$y = \int_0^x f(\tau)\partial(\tau)^{\alpha},$$

$$= \alpha\int_0^x (x-\tau)^{\alpha-1}f(\tau)\partial(\tau), \quad 0 < \alpha < 1 \tag{2}$$

that is,

$$I^{\alpha}f(x) = \left(\frac{1}{\Gamma(\alpha)}\right)\int_0^x f(\tau)\partial(\tau)^{\alpha}.$$

For example ([14, p. 6])*, with* $f(x) = x^{\beta}$*, one obtains:*

$$\int_0^x t^{\beta}\partial(\tau)^{\alpha} = \frac{\Gamma(\beta+1)\Gamma(\alpha+1)}{\Gamma(\alpha+\beta+1)}x^{\alpha+\beta}, \quad 0 < \alpha < 1. \tag{3}$$

3. Fractional Variational Iteration Method

3.1 The Basic Idea of the Variational Iteration Method

To clarify the basic ideas of VIM ([7, p. 2], [8]), we consider the general nonlinear differential equation:

$$Lu + Nu = f(t),$$

where L is a linear differential operator, N is a nonlinear operator and $f(t)$ is a given analytical function. a functional correction can be written as follows:

$$u_{n+1}(t) = u_n(t) + \int_t^\alpha \lambda(t,\tau)(Lu_n(\tau) + N\widetilde{u}_n(\tau) - f(\tau))\partial\tau,$$

where λ is a general Lagrangian multiplier that can be optimally identified by means of the variational theory. The subscript n indicates the nth approximation and $\widetilde{u}_n$ is considered as a restricted variation $\delta\widetilde{u}_n = 0$. After determining the Lagrange multiplier and selecting an appropriate initial function u_0, the successive u_n approximations of the u solution can be obtained with ease.

3.2 Implementation of VIM on the Fractional KdV Burgers Kuramoto Equation

To describe the solution procedure of fractional variational iteration method, we consider the fractional KdV Burgers Kuramoto equation:

$$\begin{cases} \dfrac{\partial u}{\partial t} + u\dfrac{\partial^\alpha u}{\partial x^\alpha} + a\dfrac{\partial^2 u}{\partial x^2} + b\dfrac{\partial^3 u}{\partial x^3} + c\dfrac{\partial^4 u}{\partial x^4}, \quad t > 0,\ 0 < \alpha \le 1, \\ u(x,0) = s(x). \end{cases} \tag{4}$$

$s(x)$ are continuous functions. According to variational iteration method we construct introduced by He ([7, p. 3]), we can construct a correction functional for eq. (4) as follows:

$$u_{n+1}(x,t) = u_n(x,t) + I_t^\alpha\left[\lambda(s)\left(\frac{\partial u}{\partial t} + u\frac{\partial^\alpha u}{\partial x^\alpha} + a\frac{\partial^2 u}{\partial x^2} + b\frac{\partial^3 u}{\partial x^3} + c\frac{\partial^4 u}{\partial x^4}\right)\right], \tag{5}$$

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t (t-s)^{\alpha-1}\lambda(s)\left(\frac{\partial u}{\partial t} + u\frac{\partial^\alpha u}{\partial x^\alpha} + a\frac{\partial^2 u}{\partial x^2} + b\frac{\partial^3 u}{\partial x^3} + c\frac{\partial^4 u}{\partial x^4}\right)ds.$$

Combining eqs. (2) and (5), we obtain a proposed correction functional:

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \left\{\lambda(s)\left(\frac{\partial u}{\partial t} + u\frac{\partial^\alpha u}{\partial x^\alpha} + a\frac{\partial^2 u}{\partial x^2} + b\frac{\partial^3 u}{\partial x^3} + c\frac{\partial^4 u}{\partial x^4}\right)\right\}(ds)^\alpha,$$

where λ is the general Lagrange multiplier which can be defined optimally via variational theory [12] and $u(x,t) = 0$ is the restricted variation, that is $\delta\widetilde{u}(x,t) = 0$.

Making the above functional stationary the following conditions can be obtained:

$$\delta u_{n+1}(x,t) = \delta u_n(x,t) + \delta\int_0^t \left\{\lambda(s)\left(\frac{\partial u}{\partial t} + u\frac{\partial^\alpha u}{\partial x^\alpha} + a\frac{\partial^2 u}{\partial x^2} + b\frac{\partial^3 u}{\partial x^3} + c\frac{\partial^4 u}{\partial x^4}\right)\right\}(ds)^\alpha. \tag{6}$$

Now, we can get the coefficients of δu to zero:

$$1 + \lambda(s) = 0, \quad \frac{\partial^\alpha \lambda(s)}{\partial s^\alpha} = 0.$$

So, the generalized Lagrange multiplier can be identified as:

$$\lambda(s) = -1. \tag{7}$$

Then, we obtain the following iteration formula by substituting (6) in (7):

$$u_{n+1}(x,t)=u_n(x,t)-\int_0^t\left\{\frac{\partial u}{\partial t}+u\frac{\partial^\alpha u}{\partial x^\alpha}+a\frac{\partial^2 u}{\partial x^2}+b\frac{\partial^3 u}{\partial x^3}+c\frac{\partial^4 u}{\partial x^4}\right\}(ds)^\alpha,$$

where $0<\alpha\le 1$ and D_x^α is the Hadamard derivative of order α ([16, p. 110]), $u_n(x,t)$ is an initial approximation which can be freely chosen if it satisfies the initial and boundary conditions of the problem. Consequently,

$$u(x)=\lim_{n\to\infty}u_n(x).$$

4. Adomian Decomposition Method

4.1 The Basic Idea of the Adomian Decomposition Method

The Adomian's decomposition method ([4, p. 2], [5, p. 1]) is a potent tool to solve linear or nonlinear equations. For each nonlinear differential equation can be decomposed into the following form:

$$Lu+N(u)+F(u)=g(t),$$

where L is the operator of the highest-ordered derivatives with respect to t and N is the remainder of the linear operator. The nonlinear term is represented by $F(u)$. Thus, we will get:

$$Lu=g(t)-N(u)-F(u). \tag{8}$$

The inverse L^{-1} is assumed to be an integral operator given by:

$$L_t^{-1}=\int_0^t(\cdot)dt,$$

operating with the operator L^{-1} on both sides of eq. (8) we have:

$$u=f_0+L^{-1}\{g(t)-N(u)-F(u)\},$$

where g_0 is the solution of homogeneous equation:

$$Lu=0, \tag{9}$$

involving the constants of integration. The constants of integration involved in the homogeneous equation solution (9) are to be determined by the initial or boundary condition, as the problem is a boundary value problem or an initial value. The Adomian's decomposition method assumes that the unknown function $u(x,t)$ may be expressed through an infinite series of the form:

$$u(x,t)=\sum_{n=0}^{\infty}u_n(x,t).$$

The nonlinear term $F(u)$ is represented by the Adomian polynomials A_n, given by:

$$F(u)=\sum_{n=0}^{\infty}A_n,$$

where $u(x,t)$ will be determined recurrently, and A_n depends on $u_0,u_1,\dots,u_n$ and can be formulated by:

$$A_n=\frac{1}{n!}\frac{d^n}{d\lambda^n}\left[F\left(\sum_{n=0}^{\infty}\lambda^i u_i\right)\right]_{\lambda=0},\quad n=0,1,2,\dots.$$

4.2 Implementation of Adomian's Decomposition Method on the Fractional KdV Burgers Kuramoto Equation

Now, let us consider the application of Adomian decomposition method for eq. (4). If eq. (4) is dealt with this method, it is formed as:

$$L_t u(x,t) = -(uL_{x^\alpha}u + aL_{xx}u + bL_{3x}u + cL_{4x}u), \tag{10}$$

where $L_t = \frac{\partial}{\partial t}$, $L_{xx} = \frac{\partial^2}{\partial x^2}$, $L_{3x} = \frac{\partial^3}{\partial x^3}$, $L_{4x} = \frac{\partial^4}{\partial x^4}$, $L_{x^\alpha} = \frac{\partial^\alpha}{\partial x^\alpha}$ is the Hadamard derivative of order α. If the invertible operator $L_t^{-1} = \int_0^t (\cdot)dt$ is applied to eq. (10), then

$$L_t^{-1}L_t u(x,t) = -L_t^{-1}(uL_{x^\alpha}u + aL_{xx}u + bL_{3x}u + cL_{4x}u)$$

is obtained. By this:

$$u(x,t) = u(x,0) - L_t^{-1}(uL_{x^\alpha}u + aL_{xx}u + bL_{3x}u + cL_{4x}u). \tag{11}$$

Here the main point is that the solution of the Adomian's decomposition method is put in the form

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t). \tag{12}$$

Substituting from eq. (12) in (11), we obtain:

$$\sum_{n=0}^{\infty} u_n(x,t) = u(x,0) - L_t^{-1}\left(\sum_{n=0}^{\infty} u_n(x,t) L_{x^\alpha} \sum_{n=0}^{\infty} u_n(x,t) + aL_{xx}\sum_{n=0}^{\infty} u_n(x,t) + bL_{3x}\sum_{n=0}^{\infty} u_n(x,t) + cL_{4x}\sum_{n=0}^{\infty} u_n(x,t)\right).$$

5. Applications

In this section of the article, we have applied fractional variational iteration method and Adomian's decomposition method to fractional Burgers Kuramoto KdV equation.

Example 1. In this example we consider the application of VIM to fractional nonlinear Burgers Kuramoto KdV equation:

$$\begin{cases} \dfrac{\partial u}{\partial t} + u\dfrac{\partial^\alpha u}{\partial x^\alpha} + a\dfrac{\partial^2 u}{\partial x^2} + b\dfrac{\partial^3 u}{\partial x^3} + c\dfrac{\partial^4 u}{\partial x^4}, \quad t > 0,\ 0 < \alpha \le 1, \\ u(x,0) = kx^4. \end{cases} \tag{13}$$

Substituting ($a = 0$, $b = 0$ and $c = 0$, $k = 1$) in eq. (13). Construction the nonlinear Burgers Kuramoto equation KdV formula as follows:

$$\frac{\partial u}{\partial t} + u\frac{\partial^\alpha u}{\partial x^\alpha}, \quad t > 0,\ 0 < \alpha \le 1. \tag{14}$$

D_x^α is the Hadamard fractional derivative of order α. Taking the initial value $u_0(x,t) = x^4$ we can derive the first approximate $u_1(x,t)$ as follows:

$$\begin{aligned} u_1(x,t) &= u_0(x,t) - \int_0^t \left(\frac{\partial u_0}{\partial t} + u_0\frac{\partial^\alpha u_0}{\partial s^\alpha}\right)(ds)^\alpha \\ &= x^4 - \int_0^t \{x^4({}^H D_{0+}^\alpha s^4)\}(ds)^\alpha . \end{aligned}$$

By applied Lemma 3, we find:

$$({}^H D^{\alpha}_{0+} v^4)(s) = 4^{\alpha} s^4.$$

formula the u_1 as follows:

$$u_1(x,t) = x^4 - \int_0^t \{x^4(4^{\alpha}s^4)\}(ds)^{\alpha}.$$

By applied eq. (3), we find:

$$u_1(x,t) = x^4 - x^4 4^{\alpha} \frac{\Gamma(5)\Gamma(1+\alpha)}{\Gamma(5+\alpha)} t^{4+\alpha}.$$

Substituting $\frac{\Gamma(5)\Gamma(1+\alpha)}{\Gamma(5+\alpha)} = \delta$, formula the u_1 as follows:

$$u_1(x,t) = x^4 - 4^{\alpha} x^4 \delta t^{4+\alpha},$$

$$u_2(x,t) = u_1 - \int_0^t \left\{ \frac{\partial u_1}{\partial t} + u_1({}^H D^{\alpha}_{0+}(u_1)) \right\} (ds)^{\alpha}.$$

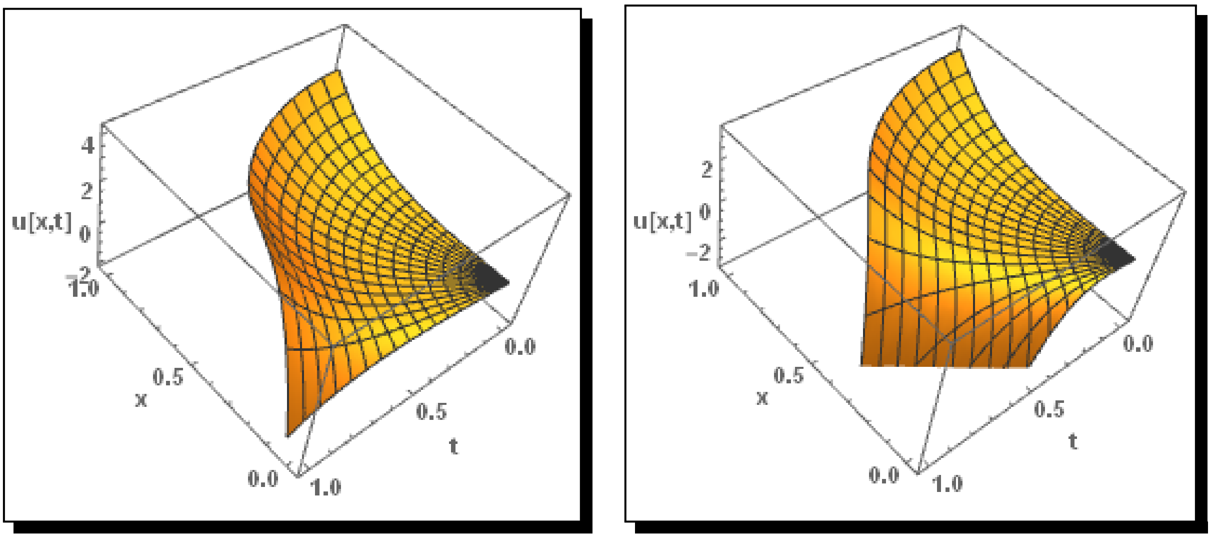
By applied eq. (3), we find:

$$u_2(x,t) = x^4 + (2+\alpha)4^{\alpha} x^4 \delta t^{4+\alpha} - 4^{2\alpha} x^4 \delta t^{8+2\alpha} + 2 \times 4^{2\alpha} x^4 \delta^2 t^{8+2\alpha},$$

$$u_3(x,t) = u_2(x,t) - \int_0^t \left\{ \frac{\partial u_2}{\partial t} + x^4({}^H D^{\alpha}_{0+} u_2) \right\} (ds)^{\alpha}.$$

Thus, the approximate solution is

$$u(x,t) = 3x^4 + (1+\alpha)4^{\alpha} x^4 \delta t^{4+\alpha} - 4^{2\alpha} x^4 \delta t^{8+2\alpha} + 2 \times 4^{2\alpha} x^4 \delta^2 t^{8+2\alpha} + \ldots .$$



(a) (b)

Figure 1. The surface shows the solution $u(x,t)$ for eq. (14) with initial condition $u_0(x,t) = x^2$: VIM results are, respectively, (a) $\alpha = 0.5$ and (b) $\alpha = 1$

Substituting ($a = 1$, $b = 1$ and $c = 1$, $k = 1$) in eq. (13). We construct the nonlinear Burgers Kuramoto equation KdV formula as follows:

$$\frac{\partial u}{\partial t} + u\frac{\partial^{\alpha} u}{\partial x^{\alpha}} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^3 u}{\partial x^3} + \frac{\partial^4 u}{\partial x^4}, \quad t > 0,\ 0 < \alpha \le 1. \tag{15}$$

Taking the initial value $u_0(x,t) = x^4$ we can derive the first approximate $u_1(x,t)$ as follows:

$$u_1(x,t) = u_0(x,t) - \int_0^t \left(\frac{\partial u_0}{\partial t} + u_0 \frac{\partial^\alpha u_0}{\partial x^\alpha} + \frac{\partial^2 u_0}{\partial x^2} + \frac{\partial^3 u_0}{\partial x^3} + \frac{\partial^4 u_0}{\partial x^4}\right)(ds)^\alpha$$

$$= x^4 - \int_0^t \{x^4({}^H D_{0+}^\alpha s^4) + 12x^2 + 24x + 24\}(ds)^\alpha .$$

By applied Lemma 3, we find:

$$({}^H D_{0+}^\alpha v^4) = 4^\alpha s^4.$$

formula the u_1 as follows

$$u_1(x,t) = x^4 - \int_0^t \left(x^4(4^\alpha s^4) + 12x^2 + 24x + 24\right)(ds)^\alpha.$$

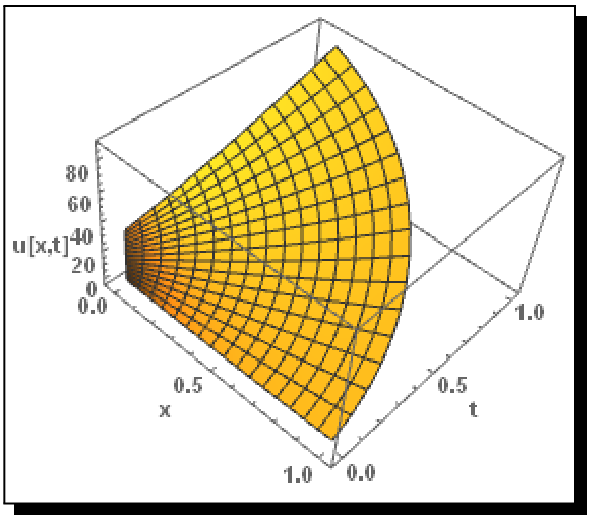
By applied eq. (3), we find:

$$u_1(x,t) = x^4 - 4^\alpha \delta x^4 t^{4+\alpha} + (12x^2 + 24x + 24)t,$$

$$u_2(x,t) = x^4 - \int_0^t \{u_1 + x^4({}^H D_{0+}^\alpha u_n) + u_1'' + u_1''' + u_1^{4\times x}\}(ds)^\alpha .$$

Thus, the approximate solution is

$$u(x,t) = 2x^4 - 4^\alpha \delta x^4 t^{4+\alpha} + (12x^2 + 24x + 24)t + \dots .$$



(c)

Figure 2. The surface shows the solution $u(x,t)$ for eq. (14) with initial condition $u_0(x,t) = x^2$: ADM results are, respectively, (c) $\alpha = 0.5$

Example 2. In this example we consider the application of Adomian's decomposition to fractional nonlinear Burgers Kuramoto equation KdV (eq. (14)).

D_x^α is the Hadamard derivative of order α. Taking the initial value $u_0(x,t) = x^4$ we can derive the first approximate $u_1(x,t)$ as follows:

$$u_1(x,t) = -\int_0^t \left(\frac{\partial u_0}{\partial t} + u_0 \frac{\partial^\alpha u_0}{\partial x^\alpha}\right) dt.$$

By applied Lemma 3, we find:

$$u_1(x,t) = -4^{\alpha}x^8 t,$$

$$u_2(x,t) = -\int_0^t \left(\frac{\partial u_1}{\partial t} + u\frac{\partial^{\alpha} u_1}{\partial x^{\alpha}}\right)dt,$$

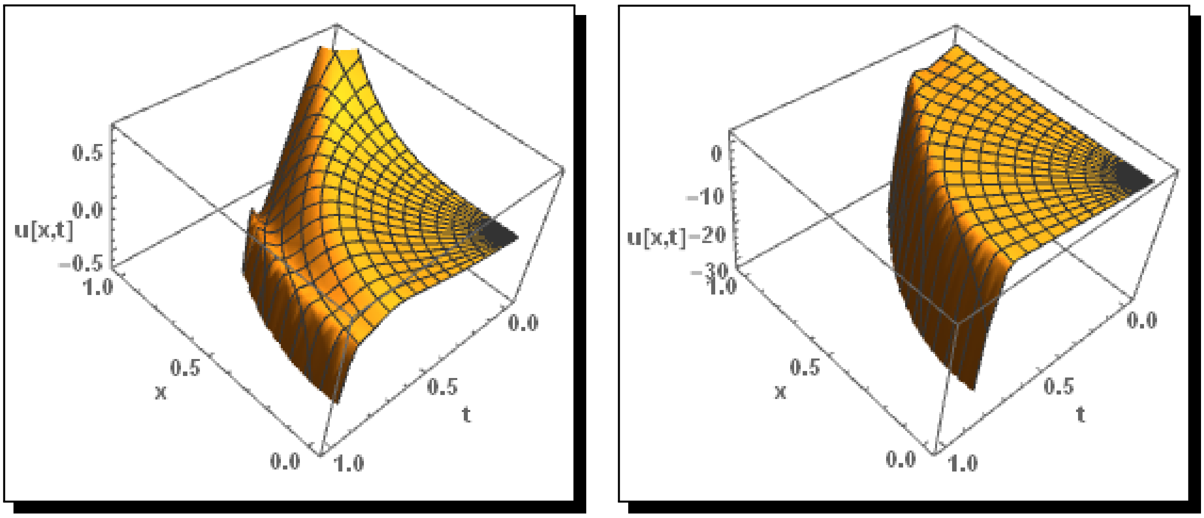
$$= 4^{\alpha}x^8 t - \frac{1}{3}4^{2\alpha}8^{\alpha}x^{16}t^3,$$

$$u_3(x,t) = -\int_0^t \left(\frac{\partial u_2}{\partial t} + u\frac{\partial^{\alpha} u_2}{\partial x^{\alpha}}\right)dt$$

$$= -4^{\alpha}x^8 t + \frac{1}{3}4^{2\alpha}8^{\alpha}x^{16}t^3 - 4^{2\alpha}8^{\alpha}x^{16}t^2 + \frac{1}{3}(4^{2\alpha}+8^{\alpha})4^{3\alpha}8^{\alpha}x^{24}t^4 - \frac{1}{9}4^{6\alpha}8^{2\alpha}x^{32}t^6.$$

Thus, the approximate solution is

$$u_n(x,t) = x^4 - 4^{\alpha}x^8 t - 4^{2\alpha}8^{\alpha}x^{16}t^2 + \frac{1}{3}(4^{2\alpha}+8^{\alpha})4^{3\alpha}8^{\alpha}x^{24}t^4 - \frac{1}{9}4^{6\alpha}8^{2\alpha}x^{32}t^6 + \dots .$$



(d) (e)

Figure 3. The surface shows the solution $u(x,t)$ for eq. (14) with initial condition $u_0(x,t) = x^2$: Adomian's decomposition method results are, respectively, (d) $\alpha = 0.5$ and (e) $\alpha = 1$

6. Conclusion

The variational iteration method and Adomian's decomposition method became regarded as an effective technique for solving certain fractional equations, such as partial differential equations, integro differential equations, and fractional KdV Burgers Kuramoto equation. In this paper, based on the variational iteration method and Adomian's decomposition method and Hadamard derivative, we have presented a general study on variational iteration method and Adomian's decomposition method in the fractional framework for analytical treatment of Fractional Burgers Kuramoto KdV equation. All of the examples concluded that the variational iteration method and Adomian's decomposition method is both effective and productive in finding analytic approximate solutions. Wolfram Mathematica has been used for presenting the graph of the solution in the present paper.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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