Communications in Mathematics and Applications
Vol. 12, No. 2, pp. 325–333, 2021
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications

http://www.rgnpublications.com

DOI: 10.26713/cma.v12i2.1489

Research Article

Characterizations of Some Regularities in Ordered Ternary Semigroups in Terms of Fuzzy Subsets

Nareupanat Lekkoksung*, Hataikhan Sanpan and Somsak Lekkoksung

Division of Mathematics, Faculty of Engineering, Rajamangala University of Technology Isan, Khon Kaen Campus, Khon Kaen 40000, Thailand

Received: December 1, 2020 **Accepted:** February 26, 2021

Abstract. Characterizations of some classes of ordered ternary semigroups; left (resp., right) lightly regular and generalized regular are given in terms of fuzzy left ideals, fuzzy right ideals, fuzzy lateral ideals, and fuzzy ideals of ordered ternary semigroups.

Keywords. Fuzzy left ideal; Fuzzy lateral ideal; Fuzzy right ideal; Ordered ternary semigroup

Mathematics Subject Classification (2020). 06F05; 06D72; 08A72

1. Introduction

The concept of ternary algebraic structures was introduced by Lehmer (see [7]). Moreover, the concept of regular ternary semigroups was defined. A ternary semigroup is a nonempty set together with an associative ternary operation. It is well-known that every semigroup induces a ternary semigroup, but a ternary semigroup need not necessarily reduce to a semigroup. Łoś studied the concept of ternary semigroups and showed that every semigroup can be embedded in ternary semigroups. In other words, the concept of ternary semigroups is a generalization of semigroups (see [10]). The notion of ternary semigroups was developed by Santiago and Sri Bala [12], regularity conditions in ternary semigroups were also intensively studied.

A generalization of ternary semigroups, so-called the concept of ordered ternary semigroups, was introduced by Iampan in 2009. The minimality and maximality of some ideals in ordered

*Corresponding author: nareupanat.le@rmuti.ac.th

ternary semigroups have been investigated (see [6]). After this concept was published, many notions and problems concerning ordered ternary semigroups were widely discussed (see [5,12]). The regularities of ordered ternary semigroups were extremely investigated by Pornsurat and Pibaljommee. Moreover, the characterizations of such regularities were provided in terms of several ideal-theoretical concepts (see [13]).

The notion of fuzzy sets was first introduced by Zadeh as a generalization of vague sets (see [15]). This notion was applied to ordered ternary semigroups by Chinram and Saelee in 2010. They also introduced fuzzy ideals and fuzzy filters, which are generalizations of ideals and filters, in ordered ternary semigroups (see [4]). By the fact that the concept of fuzzy sets is a generalization of vague sets, any kinds of ideals in ordered ternary semigroups can be regarded as some kinds of fuzzy ideals in ordered ternary semigroups (see [3]). Many researchers conducted on the fuzzy ordered ternary semigroups in various ways (see [1,2,8,9,14]).

In this paper, we characterize some classes of ordered ternary semigroups presented by Pornsurat and Pibaljommee in [13] via their fuzzy ideals.

2. Preliminaries

Introduced in this section are that the basic definitions and basic results that are necessary for our main results. Firstly, we recall the concept of ordered ternary semigroups.

A *ternary semigroup* $(T;g)$ is an algebra of type (3) such that

$$g(g(x_1,x_2,x_3),x_4,x_5) = g(x_1,g(x_2,x_3,x_4),x_5) = g(x_1,x_2,g(x_3,x_4,x_5)),$$

for all $x_1x_2,x_3,x_4,x_5 \in T$.

Definition 2.1. A structure $(T;g,\leq)$ is called an *ordered ternary semigroup* if $(T;g)$ is a ternary semigroup and $\leq$ is a partial order on T such that for all $x,y \in T$, $x \leq y$ implies that $g(x,x_1,x_2) \leq g(y,x_1,x_2)$, $g(x_1,x,x_2) \leq g(x_1,y,x_2)$ and $g(x_1,x_2,x) \leq g(x_1,x_2,y)$ for any $x_1,x_2 \in T$.

We can see that any ternary semigroup can be regarded as an ordered ternary semigroup by considering the usual equality relation as a partial order.

Throughout this paper, we write T for an ordered ternary semigroup, unless specify otherwise. For any $x,y,z \in T$, we will write xyz instead of $g(x,y,z)$.

Let A,B,C be nonempty subsets of T. We denote

$$ABC := \{abc \in T : a \in A, b \in B \text{ and } c \in C\}$$

and

$$(A] := \{x \in T : x \leq a \text{ for some } a \in A\}.$$

Definition 2.2. A nonempty subset I of an ordered ternary semigroup T is called a *left* (*resp., right, lateral*) *ideal* of T if $TTI \subseteq I$ (resp. $ITT \subseteq I, TIT \subseteq I$) and $I = (I]$.

If A is both a left ideal and a right ideal of T, then A is said to be a *two-sided ideal* of T. Moreover, A is called an *ideal* of T if it is both a two-sided ideal and a lateral ideal of T.

Let us consider two particular classes of ordered ternary semigroups. An ordered ternary semigroup T is

(1) *right* (*resp., left*) *lightly regular* [13] if for each $a \in T$, we have $a \in (aTaTT]$ (resp., $a \in (TTaTa]$),

(2) *generalized regular* [13] if for each $a \in T$, we have $a \in (TTaTT]$.

The above classes of ordered ternary semigroups introduced in [13] were characterized by Pornsurat and Pibaljommee as follows.

Lemma 2.3 ([13]). *Let T be an ordered ternary semigroup. Then the following statements are equivalent.*

(1) *T is left lightly regular.*

(2) *$R \cap M \cap L \subseteq (TTRML]$ for any left ideal L, right ideal R and lateral ideal M of T.*

(3) *$L \subseteq (LTL]$ for any left ideal L of T.*

(4) *$M \cap L \subseteq (LML]$ for any left ideal L and lateral ideal M of T.*

Lemma 2.4 ([13]). *Let T be an ordered ternary semigroup. Then the following statements are equivalent.*

(1) *T is right lightly regular.*

(2) *$R \cap M \cap L \subseteq (RMLTT]$ for any left ideal L, right ideal R and lateral ideal M of T.*

(3) *$R \subseteq (RTR]$ for any right ideal R of T.*

(4) *$R \cap M \subseteq (RMR]$ for any right ideal R and lateral ideal M of T.*

Lemma 2.5 ([13]). *Let T be an ordered ternary semigroup. Then the following statements are equivalent.*

(1) *T is generalized regular.*

(2) *$L \subseteq (TTLTT]$ for any left ideal L of T.*

(3) *$R \subseteq (TTRTT]$ for any right ideal R of T.*

(4) *$M \subseteq (TTMTT]$ for any lateral ideal M of T.*

(5) *$I \subseteq (TTITT]$ for any ideal I of T.*

Next, we recall the notion of fuzzy sets introduced by Zadeh. This concept is a generalization of vague sets (see [15]).

Let A be a nonempty set. A *fuzzy subset* of A is described as a function $f : A \to [0,1]$, where $[0,1]$ is a unit interval. A fuzzy subset of A whose maps every element of A to 1 is denoted by $\mathbf{1}_A$. If it is clear in the context, the subscript is omitted.

Let T be an ordered ternary semigroup, and f, g and h be fuzzy subsets of T. Denote by $f \subseteq g$ if $f(x) \le g(x)$ for all $x \in T$. Define a new fuzzy set $f \cap g \cap h$ on T by $(f \cap g \cap h)(x) := \min\{f(x), g(x), h(x)\}$ for all $x \in T$.

For any $a \in T$, we define $A_a := \{(x,y,z) \in T \times T \times T : a \leq xyz\}$. The product $f \circ g \circ h$ of fuzzy subsets f, g and h of T is defined by

$$(f \circ g \circ h)(a) := \begin{cases} \bigvee\limits_{a \leq xyz} \{\min\{f(x), g(y), h(z)\}\} & \text{if } A_a \neq \emptyset, \\ 0 & \text{otherwise,} \end{cases}$$

for all $a \in T$. A fuzzy subset f of T is called a *fuzzy subsemigroup* of T if for any $x,y,z \in T$, we have $f(xyz) \geq \min\{f(x), f(y), f(z)\}$.

The following definition generalizes left (resp., right, lateral) ideals of an ordered ternary semigroup T.

Definition 2.6 ([4])**.** A fuzzy subset f of an ordered ternary semigroup T is called a *fuzzy left* (*resp., right, lateral*) *ideal* of T if

(1) $x \leq y$ implies $f(x) \geq f(y)$,

(2) $f(xyz) \geq f(z)$ (resp., $f(xyz) \geq f(x)$, $f(xyz) \geq f(y)$),

for all $x,y,z \in T$.

A fuzzy subset f of an ordered ternary semigroup T is called a *fuzzy two-sided ideal* if it is both a fuzzy left and a fuzzy right ideal of T. Moreover, f is called a *fuzzy ideal* if it is both a fuzzy two-sided and a fuzzy lateral ideal of T.

We denote by f_A the characteristic function of a subset A of T. That is, f_A is the mapping from T into $[0,1]$ defined by

$$f_A(a) := \begin{cases} 1 & \text{if } a \in A, \\ 0 & \text{otherwise,} \end{cases}$$

for all $a \in T$.

Remark 2.7. Any left (resp., right, lateral) ideal L of an ordered ternary semigroup T can be considered as a fuzzy subset f_L of T defined by $f_L(x) := 1$ if $x \in L$ and $f_L(x) := 0$ if $x \notin L$. It is not difficult to calculate that f_L is a fuzzy left (resp., right, lateral) ideal of T. This means that Definition 2.6 is a generalized version of Definition 2.2 (see [3]).

3. Results

In this main section, we present characterizations of left (resp., right) lightly regular and generalized regular ordered ternary semigroups by their various kinds of fuzzy ideals.

The following lemma is an essential part of our main results.

Lemma 3.1 ([4])**.** *Let T be an ordered ternary semigroup, and A be a nonempty subset of T. Then the following statements are equivalent.*

(1) *A is a left ideal* (*lateral ideal, right ideal, ideal*) *of T.*

(2) *f_A is a fuzzy left ideal* (*resp., fuzzy lateral ideal, fuzzy right ideal, fuzzy ideal*) *of T.*

Firstly, we provide characterizations of left lightly regular ordered ternary semigroups.

Theorem 3.2. *Let T be an ordered ternary semigroup. Then the following statements are equivalent.*

(1) *T is left lightly regular.*

(2) *$f \cap g \cap h \subseteq \mathbf{1} \circ \mathbf{1} \circ f \circ g \circ h$ for any fuzzy right ideal f, fuzzy lateral ideal g and fuzzy left ideal h of T.*

Proof. (1) $\Rightarrow$ (2). Let f, g and h be a fuzzy right ideal, a fuzzy lateral ideal, and a fuzzy left ideal of T, respectively. Let $a \in T$. Then, there exist $x, y, z \in T$ such that $a \le xyaza \le xy(xyaza)za = (xyxya)(zaz)a$. This implies that $A_a \neq \emptyset$. Then

$$\begin{aligned}(\mathbf{1}\circ\mathbf{1}\circ f\circ g\circ h)(a) &= \bigvee_{a\le pqr}\{\min\{(\mathbf{1}\circ\mathbf{1}\circ f)(p), g(q), h(r)\}\}\\ &\ge \min\{(\mathbf{1}\circ\mathbf{1}\circ f)(xyxya), g(zaz), h(a)\}\\ &= \min\left\{\bigvee_{xyxya\le xyxya}\{\min\{\mathbf{1}(x),\mathbf{1}(yxy), f(a)\}\}, g(zaz), h(a)\right\}\\ &\ge \min\{f(a), g(a), h(a)\}\\ &= (f\cap g\cap h)(a).\end{aligned}$$

This means that $f \cap g \cap h \subseteq \mathbf{1} \circ \mathbf{1} \circ f \circ g \circ h$.

(2) $\Rightarrow$ (1). Let R, M and L be a right ideal, a lateral ideal, and a left ideal of T, respectively. Then, by Lemma 3.1, we have that f_R, f_M and f_L is a fuzzy right ideal, a fuzzy lateral ideal, and a fuzzy left ideal of T, respectively. Let $a \in R \cap M \cap L$. Then

$$\begin{aligned}1 &= \min\{f_R(a), f_M(a), f_L(a)\}\\ &= (f_R\cap f_M\cap f_L)(a)\\ &\le (\mathbf{1}\circ\mathbf{1}\circ f_R\circ f_M\circ f_L)(a)\\ &\le 1\end{aligned}$$

This implies that $(\mathbf{1}\circ\mathbf{1}\circ f_R\circ f_M\circ f_L)(a) = 1$, that is $A_a \neq \emptyset$. Thus,

$$1 = (\mathbf{1}\circ\mathbf{1}\circ f_R\circ f_M\circ f_L)(a) = \bigvee_{a\le pqr}\{\min\{(\mathbf{1}\circ\mathbf{1}\circ f_R)(p), f_M(q), f_L(r)\}\}.$$

Hence, there exist $x, y, z \in T$ such that $a \le xyz$ and $1 = (\mathbf{1}\circ\mathbf{1}\circ f_R)(x) = f_M(y) = f_L(z) = 1$, which implies that $y \in M$ and $z \in R$. Since $1 = (\mathbf{1}\circ\mathbf{1}\circ f_R)(x)$, so $A_x \neq \emptyset$. Thus,

$$1 = (\mathbf{1}\circ\mathbf{1}\circ f_R)(x) = \bigvee_{x\le uvs}\{\min\{\mathbf{1}(u), \mathbf{1}(v), f_R(s)\}\},$$

Then, there exist $x_1, y_1, z_1 \in T$ such that $x \le x_1y_1z_1$ and $f_R(z_1) = 1$, which implies that $z_1 \in R$. Hence, we have that $a \le xyz \le x_1y_1z_1yz \in TTRML$. That is, $a \in (TTRML]$. Therefore $R \cap M \cap L \subseteq (TTRML]$. By Lemma 2.3, we have that T is left lightly regular. □

Theorem 3.3. *Let T be an ordered ternary semigroup. Then the following statements are equivalent.*

(1) *T is left lightly regular.*

(2) *$f \subseteq f \circ \mathbf{1} \circ f$ for any fuzzy left ideal f of T.*

(3) *$f \cap g \subseteq f \circ g \circ f$ for any fuzzy left ideal f and fuzzy lateral ideal g of T.*

Proof. (1) $\Rightarrow$ (3). Let f and g be a fuzzy left ideal and a fuzzy lateral ideal of T, respectively. Let $a \in T$. Then, there exist $x,y,z \in T$ such that $a \le xyaza \le xyaz(xyaza) = (xya)(zxyaz)(a)$. This implies that $A_a \neq \emptyset$. Then

$$\begin{aligned}(f \circ g \circ f)(a) &= \bigvee_{a \le pqr} \{\min\{f(p), g(q), f(r)\}\} \\ &\ge \min\{f(xya), g(zxyaz), f(a)\} \\ &\ge \min\{f(a), g(a), f(a)\} \\ &= \min\{f(a), g(a)\} \\ &= (f \cap g)(a).\end{aligned}$$

This shows that $f \cap g \subseteq f \circ g \circ f$.

(3) $\Rightarrow$ (2). It is clear because $\mathbf{1}$ is also a fuzzy lateral ideal of T.

(2) $\Rightarrow$ (1). Let L be a left ideal of T. Then by Lemma 3.1, f_L is a fuzzy left ideal of T. Let $a \in L$. Then, by our presumption,

$$1 = f_L(a) \le (f_L \circ \mathbf{1} \circ f_L)(a) \le 1.$$

This implies that $(f_L \circ \mathbf{1} \circ f_L)(a) = 1$, that is, $A_a \neq \emptyset$. Thus,

$$1 = (f_L \circ \mathbf{1} \circ f_L)(a) = \bigvee_{a \le xyz} \{\min\{f_L(x), \mathbf{1}(y), f_L(z)\}\}.$$

Hence, there exist $p,q,r \in T$ such that $a \le pqr$ with $f_L(p) = 1 = f_L(r)$. It follows that $p,r \in L$. Then $a \le pqr \in LTL \subseteq (LTL]$. By Lemma 2.3, we obtain T is left lightly regular. □

Combining Theorem 3.2 and Theorem 3.3, we obtain the following corollary.

Corollary 3.4. *Let T be an ordered ternary semigroup. Then the following statements are equivalent.*

(1) *T is left lightly regular.*

(2) *$f \cap g \cap h \subseteq \mathbf{1} \circ \mathbf{1} \circ f \circ g \circ h$ for any fuzzy right ideal f, fuzzy left ideal h and fuzzy lateral ideal g of T.*

(3) *$f \subseteq f \circ \mathbf{1} \circ f$ for any fuzzy left ideal f of T.*

(4) *$f \cap g \subseteq f \circ g \circ f$ for any fuzzy left ideal f and fuzzy lateral ideal g of T.*

On the other hand, we obtain characterizations of right regular ordered ternary semigroups as follows.

Corollary 3.5. *Let T be an ordered ternary semigroup. Then the following statements are equivalent.*

(1) *T is right lightly regular.*

(2) $f \cap g \cap h \subseteq f \circ g \circ h \circ \mathbf{1} \circ \mathbf{1}$ *for any fuzzy right ideal* f*, fuzzy lateral ideal* g *and fuzzy left ideal* h *of* T.

(3) $f \subseteq f \circ \mathbf{1} \circ f$ *for any fuzzy right ideal* f *of* T.

(4) $f \cap g \subseteq f \circ g \circ f$ *for any fuzzy right ideal* f *and fuzzy lateral ideal* g *of* T.

We provide characterizations of generalized regular ordered ternary semigroups as follows.

Theorem 3.6. *Let* T *be an ordered ternary semigroup. Then the following statements are equivalent.*

(1) T *is generalized regular.*

(2) $f \subseteq \mathbf{1} \circ \mathbf{1} \circ f \circ \mathbf{1} \circ \mathbf{1}$ *for any fuzzy ideal* f *of* T.

(3) $f \subseteq \mathbf{1} \circ \mathbf{1} \circ f \circ \mathbf{1} \circ \mathbf{1}$ *for any fuzzy left ideal* f *of* T.

(4) $f \subseteq \mathbf{1} \circ \mathbf{1} \circ f \circ \mathbf{1} \circ \mathbf{1}$ *for any fuzzy right ideal* f *of* T.

(5) $f \subseteq \mathbf{1} \circ \mathbf{1} \circ f \circ \mathbf{1} \circ \mathbf{1}$ *for any fuzzy lateral ideal* f *of* T.

Proof. We show only the equivalence (1) $\Leftrightarrow$ (2). For other equivalences, (1) $\Leftrightarrow$ (3), (1) $\Leftrightarrow$ (4) and (1) $\Leftrightarrow$ (5), can be proved similarly.

(1) $\Rightarrow$ (2). Let f be a fuzzy ideal of T and $a \in T$. Then, there exist $w,x,y,z \in T$ such that $a \le wxayz$. This implies that $A_a \neq \emptyset$. Then

$$\begin{aligned}(\mathbf{1} \circ \mathbf{1} \circ f \circ \mathbf{1} \circ \mathbf{1})(a) &= \bigvee_{a \le pqr} \{\min\{(\mathbf{1} \circ \mathbf{1} \circ f)(p), \mathbf{1}(q), \mathbf{1}(r)\}\} \\ &\ge \min\{(\mathbf{1} \circ \mathbf{1} \circ f)(wxa), \mathbf{1}(y), \mathbf{1}(z)\} \\ &\ge f(a).\end{aligned}$$

This shows that $f \subseteq \mathbf{1} \circ \mathbf{1} \circ f \circ \mathbf{1} \circ \mathbf{1}$.

(2) $\Rightarrow$ (1). Let I be an ideal of T. By Lemma 3.1, f_I is a fuzzy ideal of T. Suppose that $a \in I$. Then

$$1 = f_I(a) \le (\mathbf{1} \circ \mathbf{1} \circ f_I \circ \mathbf{1} \circ \mathbf{1})(a) \le 1.$$

This implies that $(\mathbf{1} \circ \mathbf{1} \circ f_I \circ \mathbf{1} \circ \mathbf{1})(a) = 1$. That is, $A_a \neq \emptyset$. Thus,

$$1 = (\mathbf{1} \circ \mathbf{1} \circ f_I \circ \mathbf{1} \circ \mathbf{1})(a) = \bigvee_{a \le xyz} \{\min\{(\mathbf{1} \circ \mathbf{1} \circ f_I)(x), \mathbf{1}(y), \mathbf{1}(z)\}\}.$$

Hence, there exist $p,q,r \in T$ such that $a \le pqr$ with $1 = (\mathbf{1} \circ \mathbf{1} \circ f_I)(p)$, which implies that $A_p \neq \emptyset$. Then

$$1 = (\mathbf{1} \circ \mathbf{1} \circ f_I)(p) = \bigvee_{p \le uvs} \{\min\{\mathbf{1}(u), \mathbf{1}(v), f_I(s)\}\},$$

Thus, there exist $x_1, y_1, z_1 \in T$ such that $p \le x_1y_1z_1$ with $f_I(z_1) = 1$. That is, $z_1 \in I$. Then $a \le pqr \le (x_1y_1z_1)qr \in TTITT$, so $a \in (TTITT]$. By Lemma 2.5, we obtain that T is generalized regular. □

4. Conclusion

We focus on the concept of an algebraic system, so-called ordered ternary semigroups. Indeed, left lightly, right lightly, and generalized regular ordered ternary semigroups are considered. We provide some characterizations of such particular classes of ordered ternary semigroups through some kind of fuzzy ideals. For our future work, we will examine these results in some hyperalgebraic systems.

Acknowledgment

We would like to thank the referees for their valuable remarks and suggestions.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] M. Akram and N. Yaqoob, Intuitionistic fuzzy soft ordered ternary semigroups, *International Journal of Pure and Applied Mathematics* **84** (2013), 93 – 107, DOI: 10.12732/ijpam.v84i2.8.

[2] S. Bashir and X. Du, Intra-regular and weakly regular ordered ternary semigroups, *Annals of Fuzzy Mathematics and Informatics*, **13** (2017), 539 – 551, `http://www.afmi.or.kr/papers/2017/Vol-13-No-04/PDF/AFMI-13-4(539-551)-H-160823-1.pdf`.

[3] R. Bělohlávek, Fuzzy equational logic, *Archive for Mathematical Logic* **41** (2002), 83 – 90, DOI: 10.1007/s001530200006.

[4] R. Chinram and S. Saelee, Fuzzy ideals and fuzzy filters of ordered ternary semigroups, *Journal of Mathematics Research* **2** (2010), 93 – 97, DOI: 10.5539/jmr.v2n1p93.

[5] V.R. Daddi and Y.S. Pawar, On ordered ternary semigroups, *Kyungpook Mathematical Journal* **52** (2012), 375 – 381, DOI: 10.5666/KMJ.2012.52.4.375.

[6] A. Iampan, Characterizing the minimality and maximality of ordered lateral ideals in ordered ternary semigroups, *Journal of the Korean Mathematical Society* **46** (2009), 775 – 784, DOI: 10.4134/JKMS.2009.46.4.775.

[7] D.H. Lehmer, A ternary analogue of Abelian groups, *American Journal of Mathematical Society* **59** (1932), 329 – 338, DOI: 10.2307/2370997.

[8] N. Lekkoksung and P. Jampachon, Characterizations of regular ordered ternary semigroups in term of fuzzy subsets, *Southeast-Asian Journal of Sciences* **2** (2013), 141 – 153, `http://sajs.ntt.edu.vn/index.php/jst/article/view/34`.

[9] N. Lekkoksung and P. Jampachon, On right weakly regular ordered ternary semigroups, *Quasigroups and Related Systems* **22** (2014), 257 – 266, `http://www.quasigroups.eu/contents/download/2014/22_25.pdf`.

[10] J. Łoś, On the extending of models (I), *Fundamenta Mathematicae* **42** (1955), 38 – 54, https://eudml.org/doc/213372.

[11] J. Sanborisoot and T. Changphas, On pure ideals in ordered ternary semigroups, *Thai Journal of Mathematics* **12** (2014), 455 – 464, http://thaijmath.in.cmu.ac.th/index.php/thaijmath/article/view/892.

[12] M.L. Santiago and S. Sri Bala, Ternary semigroups, *Semigroup Forum* **81** (2010), 380 – 388, DOI: 10.1007/s00233-010-9254-x.

[13] P. Pornsurat and B. Pibaljommee, Regularities in ordered ternary semigroups, *Quasigroups and Related Systems* **27** (2019), 107 – 118, http://www.quasigroups.eu/contents/download/2019/27_11.pdf.

[14] N. Yaqoob, S. Abdullah, N. Rehman and M. Naeem, Roughness and fuzziness in ordered ternary semigroups, *World Applied Sciences Journal* **17** (2012), 1683 – 1693, https://www.idosi.org/wasj/wasj17(12)12/21.pdf.

[15] L.A. Zadeh, Fuzzy sets, *Information and Control* **8** (1065), 338 – 353, DOI: 10.1016/S0019-9958(65)90241-X.