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Research Article

On Commutativity of Near-Rings with Outer (σ,τ)-n-derivations

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Abstract. In this paper, we investigate some appropriate conditions involving outer (σ,τ)-n-derivations for a near-ring to be a commutative ring.

Keywords. Derivations; Rings; Near-rings; Prime rings; Commutative

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1. Introduction

By a left near-ring, we mean a nonempty set N equipped with two binary operations addition $(+)$ and multiplication $(\cdot)$ satisfying the following conditions: (i) $(N,+)$ is a group(not necessarily abelian), (ii) $(N,\cdot)$ is a semigroup, and (iii) $x\cdot(y+z)=x\cdot y+x\cdot z$ for all $x,y,z\in N$. Analogously, if instead of (iii), N satisfies the right distributive law, then N is said to be a right near-ring. Throughout this paper, by the term "near-ring N" we will mean only a left near-ring. Further, we will write xy for $x\cdot y$ just for simplicity of notation. Therefore, near rings are generalized rings, need not be commutative. For further details about the concepts and other results in near rings, we refer to the treatises [4, 5, 7–11]. For any $x,y\in N$ the symbol $[x,y]$ will denote the commutator $xy-yx$; while the symbol $x\circ y$ will stand for the anti-commutator $xy+yx$. The symbol Z will represent the multiplicative center of N, that is, $Z=\{x\in N \mid xy=yx \text{ for all } y\in N\}$. A near-ring N is said to be prime if $xNy=\{0\}$ with $x,y\in N$ implies $x=0$ or $y=0$, and semiprime if $xNx=\{0\}$ with $x\in N$ implies $x=0$.

An additive mapping $d: N \to N$ is said to be a derivation if $d(xy) = xd(y) + d(x)y$ for all $x, y \in N$. Bell and Mason [6] initiated the study of derivations in near rings. Wang [12] showed that condition $d(xy) = xd(y) + d(x)y$ for all $x, y \in N$ is equivalent to $d(xy) = d(x)y + xd(y)$ for all $x, y \in N$.

Following [3], let $\sigma, \tau: N \to N$ be two near-ring automorphisms of N. An additive mapping $d: N \to N$ is called a (σ,τ)-derivation if $d(xy) = \sigma(x)d(y) + d(x)\tau(y)$ for all $x, y \in N$. It is straightforward that an $(1,1)$-derivation is an ordinary derivation.

Ashraf and Siddeeque [2] introduced the notion of (σ,τ)-n-derivation in near-ring N, where n is a positive integer. Let $\sigma, \tau: N \to N$ be two near-ring automorphisms of N. An n-additive (i.e. additive in each argument) mapping $D: N \times N \times \ldots \times N \to N$ is called a (σ,τ)-n-derivation of N if

$$
\begin{aligned}
D(x_1y_1, x_2, \ldots, x_n) &= D(x_1, x_2, \ldots, x_n)\sigma(y_1) + \tau(x_1)D(y_1, x_2, \ldots, x_n), \\
D(x_1, x_2y_2, \ldots, x_n) &= D(x_1, x_2, \ldots, x_n)\sigma(y_2) + \tau(x_2)D(x_1, y_2, \ldots, x_n), \\
&\vdots \\
D(x_1, x_2, \ldots, x_ny_n) &= D(x_1, x_2, \ldots, x_n)\sigma(y_n) + \tau(x_n)D(x_1, x_2, \ldots, y_n),
\end{aligned}
$$

hold for all $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n \in N$.

Recently, Aroonruviwat and Leerawat [1] introduced a notion of outer (σ,τ)-n-derivation in a near-ring and investigated commutativity of prime near-rings admitting suitably constrained outer (σ,τ)-n-derivations. In this paper we investigate some further properties involving outer (σ,τ)-n-derivations of near-ring which force near-ring to be commutative ring.

2. Preliminaries

Throughout this paper, let N denote a (left) near-ring with center Z. Let n be a fixed positive integer and N^n denotes $N \times N \times \ldots \times N$ (n terms). We first recall the definitions and lemmas which are essential for developing the proofs of our main results.

Definition 2.1. A map $D: N^n \to N$ is called an n-additive mapping if

$$
\begin{aligned}
D(x_1 + y_1, x_2, \ldots, x_n) &= D(x_1, x_2, \ldots, x_n) + D(y_1, x_2, \ldots, x_n), \\
D(x_1, x_2 + y_2, \ldots, x_n) &= D(x_1, x_2, \ldots, x_n) + D(x_1, y_2, \ldots, x_n), \\
&\vdots \\
D(x_1, x_2, \ldots, x_n + y_n) &= D(x_1, x_2, \ldots, x_n) + D(x_1, x_2, \ldots, y_n),
\end{aligned}
$$

for all $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n \in N$.

Remark. Let D be an n-additive mapping.

(i) If $x_i = 0$ for some $0 \le i \le n$ then $D(x_1, x_2, \ldots, x_i, \ldots, x_n) = 0$.

(ii) $D(x_1, x_2, \ldots, -x_i, \ldots, x_n) = -D(x_1, x_2, \ldots, x_i, \ldots, x_n)$, for all $0 \le i \le n$.

Definition 2.2 ([1, Definition 3.1])**.** Let N be a near-ring and let $\sigma,\tau : N \to N$ be automorphisms. A mapping $D : N^n \to N$ is call an outer (σ,τ)-n-derivation of N if D is an n-additive mapping satisfying the relations

$$\begin{aligned}
D(x_1y_1,x_2,\dots,x_n) &= \sigma(x_1)D(y_1,x_2,\dots,x_n)+D(x_1,x_2,\dots,x_n)\tau(y_1),\\
D(x_1,x_2y_2,\dots,x_n) &= \sigma(x_2)D(x_1,y_2,\dots,x_n)+D(x_1,x_2,\dots,x_n)\tau(y_2),\\
&\vdots\\
D(x_1,x_2,\dots,x_ny_n) &= \sigma(x_n)D(x_1,x_2,\dots,y_n)+D(x_1,x_2,\dots,x_n)\tau(y_n),
\end{aligned}$$

for all $x_1,x_2,\dots,x_n,y_1,y_2,\dots,y_n \in N$.

Note that if $n=1$ then D is call an outer (σ,τ)-derivation.

Lemma 2.3 ([1, Lemma 3.5])**.** *Let N be a prime near-ring, D a nonzero outer (σ,τ)-n-derivation on N and $x \in N$.*

(i) *If $D(N,N,\dots,N)x = \{0\}$ then $x=0$.*

(ii) *If $xD(N,N,\dots,N) = \{0\}$ then $x=0$.*

Corollary 2.4. *Let N be a prime near-ring, D a nonzero outer (σ,τ)-n-derivation of N and $x \in N$. If $D(N,N,\dots,N)\tau(x) = \{0\}$ then $x=0$.*

Corollary 2.5. *Let N be a prime near-ring, d a nonzero outer (σ,τ) derivation on N and $x \in N$.*

(i) *If $d(N)x = \{0\}$ then $x=0$.*

(ii) *If $xd(N) = \{0\}$ then $x=0$.*

Corollary 2.6. *Let N be a prime near-ring, d a nonzero outer (σ,τ) derivation of N and $x \in N$. If $d(N)\tau(x) = \{0\}$ then $x=0$.*

Lemma 2.7. *Let N be a prime near-ring, D a nonzero outer (σ,τ)-n-derivation of N and $x \in N$.*

(i) *If $\sigma(D(N,N,\dots,N))x = \{0\}$ then $x=0$.*

(ii) *If $x\sigma(D(N,N,\dots,N)) = \{0\}$ then $x=0$.*

Proof. (i): Suppose that $\sigma(D(N,N,\dots,N))x = \{0\}$.
Let $x_1,x_2,\dots,x_n \in N$. Since $\sigma : N \to N$ is an automorphism, there exists $a \in N$ such that $\sigma(a)=x$. We have

$$\sigma(D(x_1,x_2,\dots,x_n)a) = \sigma(D(x_1,x_2,\dots,x_n))\sigma(a) = 0.$$

Since σ is an injective, we get

$$D(x_1,x_2,\dots,x_n)a = 0, \quad \text{for all } x_1,x_2,\dots,x_n \in N.$$

By Lemma 2.3, we get $a=0$ therefore $x=0$.

(ii): It can be proved in a similar manner. □

Lemma 2.8 ([6, Lemma 3]). *Let N be a prime near-ring.If $Z-\{0\}$ contains an element z for which $z+z\in Z$, then $(N,+)$ is abelian.*

3. Main Results

In this section, let σ and τ be automorphisms of N.

Theorem 3.1. *Let D be nonzero outer (σ,τ)-n-derivation of N and d be nonzero outer (σ,τ) derivation of N. If $dD(N,N,\dots,N)=\{0\}$, then $(N,+)$ is an abelian group.*

Proof. Suppose that $dD(N,N,\dots,N)=\{0\}$.

Let $x,y,x_1,x_2,\dots,x_n\in N$. Since $\sigma:N\to N$ is an automorphism, there exists $a\in N$ such that $\sigma(a)=x_1$.

Then $dD(ax,x_2,\dots,x_n)=0$, it follows that

$$\sigma(D(a,x_2,\dots,x_n))d(\tau(x))=d(x_1)\tau(D(-x,x_2,\dots,x_n)).$$

Similarly,

$$\sigma(D(a,x_2,\dots,x_n))d(\tau(y))=d(x_1)\tau(D(-y,x_2,\dots,x_n)).$$

By hypothesis, $dD(a(x+y),x_2,\dots,x_n)=0$. This implies that

$$d(x_1)\tau(D(x+y,x_2,\dots,x_n))+\sigma(D(a,x_2,\dots,x_n))d)\tau(x+y)=0.$$

Combining the above three eqations, we have

$$d(x_1)\tau(D(x+y-x-y,x_2,\dots,x_n))=0.$$

By Corollary 2.6, we get $D(x+y-x-y,\dots,x_n)=0$.

Next, $D(x_1(x+y-x-y),\dots,x_n)=0$ implies that

$$D(x_1,\dots,x_n)\tau(x+y-x-y)=0.$$

By Corollary 2.4, we have $x+y-x-y=0$.

Therefore $x+y=y+x$ for all $x,y\in N$. Thus, $(N,+)$ is abelian. □

Theorem 3.2. *Let σ and τ be automorphisms such that $\sigma^2=\sigma$ and $\tau^2=\tau$. Let D be nonzero outer (σ,τ)-n-derivation of N and d be nonzero outer (σ,τ) derivation of N. If dD is an outer (σ,τ)-n-derivation of N then $(N,+)$ is an abelian group.*

Proof. Since dD is an outer (σ,τ)-n-derivation of N,

$$dD(x_1y_1,x_2,\dots,x_n)=\sigma(x_1)dD(y_1,x_2,\dots,x_n)+dD(x_1,x_2,\dots,x_n)\tau(y_1)$$

for all $x_1,y_1,x_2,\dots,x_n\in N$. On the other hand, we also have

$$\begin{aligned}dD(x_1y_1,x_2,\dots,x_n)&=d\{D(x_1y_1,x_2,\dots,x_n)\}\\&=\sigma(x_1)dD(y_1,x_2,\dots,x_n)+d(\sigma(x_1))\tau(D(y_1,x_2,\dots,x_n))\\&\quad+\sigma(D(x_1,x_2,\dots,x_n))d(\tau(y_1))+dD(x_1,x_2,\dots,x_n)\tau(y_1),\end{aligned}$$

for all $x_1,y_1,x_2,\dots,x_n \in N$. By comparing the above two values of $dD(x_1y_1,x_2,\dots,x_n)$, we find that

$$\sigma(D(x_1,x_2,\dots,x_n))d(\tau(y_1)) = -d(\sigma(x_1))\tau(D(y_1,x_2,\dots,x_n)), \tag{3.2.1}$$

for all $x_1,y_1,x_2,\dots,x_n \in N$.

Let $x,y \in N$. Since $\tau : N \to N$ is an automorphism, there exist $a,b \in N$ such that $\tau(a) = x$ and $\tau(b) = y$. Replacing y_1 by $a+b$ in (3.2.1), we get

$$\sigma(D(x_1,x_2,\dots,x_n))d(\tau(a+b)) = -d(\sigma(x_1))\tau(D(a+b,x_2,\dots,x_n)). \tag{3.2.2}$$

Replacing y_1 by $-a$ and $-b$, respectively in (3.2.1), we conclude that

$$\sigma(D(x_1,x_2,\dots,x_n))d(\tau(-a-b)) = d(\sigma(x_1))\tau(D(a+b,x_2,\dots,x_n)). \tag{3.2.3}$$

By combining (3.2.2) and (3.2.3), we obtain

$$\sigma(D(x_1,x_2,\dots,x_n))d(\tau(a+b-a-b)) = 0, \quad \text{for all } x_1,x_2,\dots,x_n \in N.$$

Hence

$$\sigma(D(x_1,x_2,\dots,x_n))d(x+y-x-y) = 0, \quad \text{for all } x_1,x_2,\dots,x_n,x,y \in N.$$

By Lemma 2.7, we have

$$d(x+y-x-y) = 0, \quad \text{for all } x,y \in N.$$

For $w \in N$, $0 = d(wx+wy-wx-wy = d(w(x+y-x-y))$ and so we obtain $d(w)\tau(x+y-x-y) = 0$.

By Corollary 2.6, we get $x+y = y+x$ for all $x,y \in N$. Therefore $(N,+)$ is abelian. □

Theorem 3.3. *Let σ and τ be automorphisms such that $\sigma^2 = \sigma$ and $\tau^2 = \tau$. Let D be an outer (σ,τ)-n-derivation of N and d be an outer (σ,τ) derivation of N. If $(N,+)$ is non-abelian and dD is an outer (σ,τ)-n-derivation of N, then $D = 0$ or $d = 0$.*

Proof. Suppose that $(N,+)$ is non-abelian and dD is an outer (σ,τ)-n-derivation of N. If $D = 0$, then nothing to do. Assume that $D \neq 0$. Now, using similar arguments as used in the proof of Theorem 3.2, we conclude that

$$d(N)\tau(x+y-x-y) = \{0\} \tag{3.3.1}$$

for all $x,y \in N$.

If $d \neq 0$, then by Corollary 2.6 we have $(N,+)$ is abelian, a contradiction to the assumption. Hence $d = 0$. The proof is complete. □

Theorem 3.4. *Let D be nonzero outer (σ,τ)-n-derivation of N. Suppose that for any positive integer $i \in \{1,2,\dots,n\}$, $D(x_1,x_2,\dots,[x_i,y_i],\dots,x_n) = 0$ for all $x_1,x_2,\dots,x_i,y_i,\dots,x_n \in N$. Then N is a commutative ring.*

Proof. For any $x_1,x_2,\dots,x_i,y_i,\dots,x_n \in N$, we have

$$D(x_1,x_2,\dots,[x_i,x_iy_i],\dots,x_n) = 0.$$

But

$$D(x_1,x_2,\dots,[x_i,x_iy_i],\dots,x_n)=\sigma(x_i)D(x_1,x_2,\dots,[x_i,y_i],\dots,x_n)$$
$$+D(x_1,x_2,\dots,x_i,\dots,x_n)\tau([x_i,y_i]).$$

By hypothesis, we get

$$D(x_1,x_2,\dots,x_i,\dots,x_n)\tau(x_iy_i)=D(x_1,x_2,\dots,x_i,\dots,x_n)\tau(y_ix_i), \tag{3.4.1}$$

for all $x_1,x_2,\dots,x_i,y_i,\dots,x_n\in N$.

Let $x_1,x_2,\dots,x_i,y_i,\dots,x_n,z\in N$. Since τ is an automorphism, there exits $a\in N$ such that$\tau(a)=z$. By using (3.4.1), Then

$$D(x_1,x_2,\dots,x_i,\dots,x_n)\tau(x_ia)=D(x_1,x_2,\dots,x_i,\dots,x_n)\tau(ax_i)$$

and

$$D(x_1,x_2,\dots,x_i,\dots,x_n)z\tau(x_iy_i-y_ix_i)=D(x_1,x_2,\dots,x_i,\dots,x_n)\tau(ax_iy_i)-\tau(ay_ix_i)$$
$$=D(x_1,x_2,\dots,x_i,\dots,x_n)\tau(x_iay_i)-\tau(ay_ix_i)=0.$$

Therefore

$$D(x_1,x_2,\dots,x_i,\dots,x_n)z\tau(x_iy_i-y_ix_i)=0,$$

for all $x_1,x_2,\dots,x_i,y_i,\dots,x_n,z\in N$.

That is

$$D(x_1,x_2,\dots,x_i,\dots,x_n)N\tau(x_iy_i-y_ix_i)=\{0\}$$

for all $x_1,x_2,\dots,x_i,\dots,x_n,z\in N$.

By the primeness of N and D is a nonzero outer (σ,τ)-n-derivation of N, we have

$$\tau(x_iy_i-y_ix_i)=0,\quad \text{for all } x_i,y_i\in N.$$

Since τ is an injective function, $x_iy_i=y_ix_i$ for all $x_i,y_i\in N$. Hence N is a commutative near-ring. Then $N=Z$. Applying Lemma 2.8, we can conclude that $(N,+)$ is an abelian group. So N is a commutative ring. □

Theorem 3.5. *Let D be nonzero outer (σ,τ)-n-derivation of N. Suppose that for any positive integer $i\in\{1,2,\dots,n\}$. Assume that*

$$D(x_1,x_2,\dots,[x_i,y_i],\dots,x_n)=\pm[\sigma(x_i),\sigma(y_i)],$$

for all $x_1,x_2,\dots,x_i,y_i,\dots,x_n\in N$. Then N is a commutative ring.

Proof. For any $x_1,x_2,\dots,x_i,y_i,\dots,x_n\in N$, we have

$$D(x_1,x_2,\dots,[x_i,x_iy_i],\dots,x_n)=\pm\sigma(x_i)[\sigma(x_i),\sigma(y_i)].$$

In the other hand, we get

$$D(x_1,x_2,\dots,[x_i,x_iy_i],\dots,x_n)=\sigma(x_i)D(x_1,x_2,\dots,[x_i,y_i],\dots,x_n)$$
$$+D(x_1,x_2,\dots,x_i,\dots,x_n)\tau([x_i,y_i]).$$

Therefore

$$\sigma(x_i)D(x_1,x_2,\ldots,[x_i,y_i],\ldots,x_n)+D(x_1,x_2,\ldots,x_i,\ldots,x_n)\tau([x_i,y_i])=\pm\sigma(x_i)[\sigma(x_i),\sigma(y_i)],$$

for all $x_1,x_2,\ldots,x_i,y_i,\ldots,x_n\in N$.

By hypothesis, we have

$$D(x_1,x_2,\ldots,x_i,\ldots,x_n)\tau([x_i,y_i])=0,$$

for all $x_1,x_2,\ldots,x_i,y_i,\ldots,x_n\in N$.

Then, by using the same technique in the proof of Theorem 3.4, we can conclude that N is a commutative ring. □

Theorem 3.6. *Let D be nonzero outer (σ,τ)-n-derivation of N. Suppose that for any positive integer $i\in\{1,2,\ldots,n\}$. Assume that*

$$D(x_1,x_2,\ldots,(x_i\circ y_i),\ldots,x_n)=\pm(\sigma(x_i)\circ\sigma(y_i)),$$

for all $x_1,x_2,\ldots,x_i,y_i,\ldots,x_n\in N$. Then N is a commutative ring.

Proof. For any $x_1,x_2,\ldots,x_i,y_i,\ldots,x_n\in N$, we have

$$D(x_1,x_2,\ldots,(x_i\circ x_iy_i),\ldots,x_n)=\pm\sigma(x_i)(\sigma(x_i)\circ\sigma(y_i)).$$

On the other hand, we get

$$\begin{aligned}D(x_1,x_2,\ldots,(x_i\circ x_iy_i),\ldots,x_n)&=\sigma(x_i)D(x_1,x_2,\ldots,(x_i\circ y_i),\ldots,x_n)\\&\quad+D(x_1,x_2,\ldots,x_i,\ldots,x_n)\tau(x_i\circ y_i).\end{aligned}$$

Therefore

$$\sigma(x_i)D(x_1,x_2,\ldots,(x_i\circ y_i)+D(x_1,x_2,\ldots,x_i,\ldots,x_n)\tau(x_i\circ y_i)=\pm\sigma(x_i)(\sigma(x_i)\circ\sigma(y_i)),$$

for all $x_1,x_2,\ldots,x_i,y_i,\ldots,x_n\in N$.

By hypothesis, we have

$$D(x_1,x_2,\ldots,x_i,\ldots,x_n)\tau(x_i\circ y_i)=0, \tag{3.6.1}$$

for all $x_1,x_2,\ldots,x_i,y_i,\ldots,x_n\in N$.

Replacing x_i by $-x_i$ in (3.6.1), we have

$$D(x_1,x_2,\ldots,-x_i,\ldots,x_n)\tau(-x_iy_i)=D(x_1,x_2,\ldots,-x_i,\ldots,x_n)\tau(y_ix_i). \tag{3.6.2}$$

Then, by (3.6.1) and (3.6.2), we obtain

$$D(x_1,x_2,\ldots,-x_i,\ldots,x_n)N\tau([x_i,y_i])=\{0\},$$

for all $x_1,x_2,\ldots,x_i,y_i,\ldots,x_n\in N$.

By using the same technique in the proof of Theorem 3.4, we can conclude that N is a commutative ring. □

Corollary 3.7. *Let N be a 2-torsion free prime near-ring. Then there exists no nonzero outer (σ,τ)-n-derivation D such that*

$$D(x_1,x_2,\dots,(x_i\circ y_i),\dots,x_n)=\pm(\sigma(x_i)\circ\sigma(y_i)),$$

for all $x_1,x_2,\dots,x_i,y_i,\dots,x_n\in N$, and for $i\in\{1,2,\dots,n\}$.

Proof. Suppose that there exists a nonzero outer (σ,τ)-n-derivation D such that

$$D(x_1,x_2,\dots,(x_i\circ y_i),\dots,x_n)=\pm(\sigma(x_i)\circ\sigma(y_i)),$$

for all $x_1,x_2,\dots,x_i,y_i,\dots,x_n\in N$.

By Theorem 3.6, we get N is a commutative ring. Since N is 2-torsion free, $D(x_1,x_2,\dots,x_n)y_ix_i=0$, for all $x_1,x_2,\dots,x_i,y_i,\dots,x_n\in N$.

Hence $D(x_1,x_2,\dots,x_n)Nx_i=\{0\}$, for all $x_1,x_2,\dots,x_i,\dots,x_n\in N$.

Since N is prime and $D\neq 0$, $x_i=0$. This implies that $D=0$, a contradiction. The proof of the corollary is complete. □

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