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Research Article

Generalized Arithmetic Graphs With Equal and Unequal Powers of Annihilator Domination Number

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Abstract. Current work is carried out in Generalized Arithmetic Graphs to explore the theory of conquest by the Annihilator Dominion Number of Upper bound. Kulli and Janakiram [8] first demonstrated split domination while Suryanarayana Rao and Vangipuram [12] introduced the domination of Annihilator and obtained several interesting results in Arithmetic graphs. There are few significant and important studies on Annihilator's domination being examined in the current paper.

Keywords. Split dominance; Array and number of annihilator domination; Arithmetic graphs

MSC. 05C69

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1. Introduction

The origins of graph theory date back to the 18th century, with a mathematician called Leonhard Euler. He solved the problem known today as the Seven Bridges of Konigsberg in 1735. Back then, Konigsberg was a city in Prussia, divided into four bodies of land by the Pregl River. Seven bridges connected these separate bodies of land, posing a specific question to Euler. It was possible to walk around the city, finish back at the starting point and cross each of the seven bridges exactly once. Euler proved not to be. What is more influential is that, in order to do so, he used graphic representation, laying the foundations for studying graphs as mathematical structures, and modeling real-world graph problems. Graph theory is one of modern mathematics and computer applications most flourishing branches. Thanks to its broad applications to discrete optimization problems, combinatorial problems and classical algebraic problems, the last 30 years have seen spectacular growth of Graph theory. It has a very wide range of applications in many fields such as engineering, physical, social and biological sciences, linguistics, etc. In recent times, the theory of domination has been the core of research activity in graph theory [4].

The rigorous study of dominant graph theory sets began around 1960, although the subject has historical roots from 1862 when de Jaenisch studied the problems of determining the minimum number of queens needed to cover or dominate a $n \times n$ chessboard. In 1958, Berge established the definition of a graph's dominance number, calling it an "external stability coefficient". In 1962, for the same definition, Ore used the name "dominating array" and "domination number". The $\gamma(G)$ domination number, which is the most commonly used domination number, is the minimum cardinality among all G dominant sets. The $\gamma(G)$ parameter will be referred to as the G [7] dominance number of the vertex-vertex. A graph uses the basic idea of using vertices to create node pair relationships. Many real world interactions are better modeled with graphic constructs in terms of applications.

A social network like Twitter, a virus that spreads during an influenza outbreak, a highway system which links different cities, and molecular representations are just a few examples of real-world systems that are often graphically modeled. Graphs are also omnipresent in aviation research: a graph of linking airports and trains, flights and connecting passengers, alliance-building airlines, etc. Many real-world situations can be easily represented through a diagram consisting of a series of dots along with lines connecting such dot pairs. Suryanarayana Rao and Sreeenivasan ([10, 11]) demonstrated parameters of domination in arithmetic and product graphs as well as obtaining a formula for constructing the same using parameter of domination. Current research on the Annihilator dominating array and the Annihilator dominating number and their properties are complicated by the study of dominance and split domination.

Applications of Graph Theory

Graph theory is the study of relationships. With a collection of nodes & links capable of abstracting anything from city designs to computer data, graph theory provides a useful method for quantifying and simplifying the many moving parts of dynamic systems. The analysis of graphs through a system provides answers to many problems of structure, networking, coordination, matching and operation. In the field of electrical engineering, a whole discipline revolves around designing, measuring, and maintaining multi-part electrical circuits often diagram according to the principles of graph theory. While, scientists extrapolate prediction models in the field of molecular biology to monitor disease spread or breeding trends. And finally, in the controversial world of social media network analysis, we are witnessing graph theory leveraged to create now-standard features such as LinkedIn's degree-of-separation & Face book's friend-recommendation features.

2. Basic Definitions

Definition 2.1 (Split Dominant Set)**.** The dominant set D of graph G is considered as the dominant split set, if the induced subgraph $\langle V-D\rangle$ is disconnected. The split dominant number $\gamma_s(G)$ of G is the minimum cardinality of the split dominant set.

Definition 2.2 (Arithmetic Graph)**.** The Arithmetic graph V_m is defined as a graph with its vertex set as the set of all the divisors m (excluding 1), where$m$ is a natural number and $m=p_1^{a_1}\cdot p_2^{a_2}\cdot p_3^{a}$, a canonical representation of m, where p_i are distinct primes and $a_i \geq 1$ and two distinct vertexes a,b which are not of the same parity are adjacent to this graph if $(a,b) = p_i$, for $1 \leq i \leq r$. Vertices a and b are said to have the same parity if both a and b are the same prime powers, e.g. $a=P^2$ and $b=P^5$.

Definition 2.3 (Array of Annihilator Domination)**.** The dominant set D of graph G is said to be the Array of annihilator dominant, if the induced subgraph $\langle V-D\rangle$ is a graph with isolated vertices or a graph with independent vertices.

Definition 2.4 (Annihilator Domination Number)**.** The annihilator domination number $\gamma_a(G)$ of G is the minimum cardinality of an annihilator dominating set.

3. Preliminary Results

Suryanarayana Rao and Sreeenivasan [10] have obtained an interesting result on the annihilation domination of the Arithmetic Graph given below and have also established a construction method using annihilation dominant number. The following are the two results of the arithmetic graphs

Theorem 3.1. *If* $m=p_1^{a_1}p_2^{a_2}$ *where* p_1,p_2 *dissimilar primes* $a_1,a_2 \geq 1$*, then*

(i) $\gamma_a[V_m] \leq 2a_1+1$ *if* $a_1<a_2$,

(ii) $\gamma_a[V_m] \leq 2a_1$, *if* $a_1=a_2$.

Theorem 3.2. *If* $m=p_1^{a_1}p_2^{a_2}p_3^{a_3}$ *where* p_1p_2,p_3*-dissimilar primes and* $a_1,a_2,a_3 \geq 1$*, then* $\gamma_a[V_m] \leq 3a_1$, *if* $a_1=a_2=a_3$.

4. Main Results

The results of Annihilator domination in arithmetic graph are as follows:

In this paper, the following results are obtained from Generalized Arithmetic Graphs with equal and unequal powers of Annihilator Domination Number (Upper bound)

Theorem 4.1. *If* $m=p_1^{a_1}p_2^{a_2}p_3^{a_3}p_4^{a_4}$ *where* p_1,p_2,p_3,p_4 *are distinct primes and* $a_1,a_2,a_3,a_4 \geq 1$*, then* $\gamma_a[v_m] \leq 4a_1+6$ *if* $a_1<a_2<a_3<a_4$.

Proof. Given $m=p_1^{a_1}p_2^{a_2}p_3^{a_3}p_4^{a_4}$. The vertex set V_m is
$\{p_1,p_1^2\cdots p_1^{a_1};p_2,p_2^2,\cdots,p_2^{a_2};p_3,p_3^2,\cdots,p_3^{a_3};p_4,p_4^2,\cdots,p_4^{a_4};$

$p_1p_2, p_1p_2^2, \cdots, p_1p_2^{a_2}; p_1^2p_2, p_1^2p_2^2, \cdots, p_1^2p_2^{a_2}; \cdots; p_1^{a_1}p_2, p_1^{a_1}p_2^2, \cdots, p_1^{a_1}p_2^{a_2};$
$p_1p_3, p_1p_3^2, \cdots, p_1p_3^{a_3}; p_1^2p_3, p_1^2p_3^2, \cdots, p_1^2p_3^{a_3}; \cdots; p_1^{a_1}p_3, p_1^{a_1}p_3^2, \ldots, p_1^{a_1}p_3^{a_3};$
$p_1p_4, p_1p_4^2, \cdots, p_1p_4^{a_4}; p_1^2p_4, p_1^2p_4^2, \cdots, p_1^2p_4^{a_4}; \cdots; p_1^{a_1}p_4, p_1^{a_1}p_4^2, \cdots, p_1^{a_1}p_4^{a_4};$
$p_2p_3, p_2p_3^2, \cdots, p_2p_3^{a_3}; p_2^2p_3, p_2^2p_3^2, \cdots, p_2^2p_3^{a_3}; \ldots; p_2^{a_2}p_3, p_2^{a_2}p_3^2, \ldots, p_2^{a_2}p_3^{a_3};$
$p_2p_4, p_2p_4^2, \cdots, p_2p_4^{a_4}; p_2^2p_4, p_2^2p_4^2, \ldots, p_2^2p_4^{a_4}; \ldots p_2^{a_2}p_4, p_2^{a_2}p_4^2, \ldots, p_2^{a_2}p_4^{a_4};$
$p_3p_4, p_3p_4^2, \ldots, p_3p_4^{a_4}; p_3^2p_4, p_3^2p_4^2, \ldots, p_3^2p_4^{a_4}; \ldots; p_3^{a_3}p_4, p_3^{a_3}p_4^2, \ldots, p_3^{a_3}p_4^{a_4};$
$p_1p_2p_3, p_1p_2p_3^2, \ldots, p_1p_2p_3^{a_3}; p_1p_2^2p_3, p_1p_2^2p_3^2, \ldots, p_1p_2^2p_3^{a_3}; \ldots; p_1p_2^{a_2}p_3,$
$p_1p_2^{a_2}p_3^2, \ldots, p_1p_2^{a_2}p_3^{a_3}; p_1^2p_2p_3, p_1^2p_2p_3^2, \ldots, p_1^2p_2p_3^{a_3}; p_1^2p_2^2p_3, p_1^2p_2^2p_3^2, \ldots, p_1^2p_2^2p_3^{a_3}; \ldots;$
$p_1^2p_2^{a_2}p_3, p_1^2p_2^{a_2}p_3^2, \ldots, p_1^2p_2^{a_2}p_3^{a_3}; \ldots p_1^{a_1}p_2p_3, p_1^{a_1}p_2p_3^2, \ldots, p_1^{a_1}p_2p_3^{a_3}; p_1^{a_1}p_2^2p_3, p_1^{a_1}p_2^2p_3^2, \ldots, p_1^{a_1}p_2^2p_3^{a_3};$
$\ldots; p_1^{a_1}p_2^{a_2}p_3, p_1^{a_1}p_2^{a_2}p_3^2, \ldots, p_1^{a_1}p_2^{a_2}p_3^{a_3}; p_1p_2p_4, p_1p_2p_4^2, \ldots, p_1p_2p_4^{a_4}; p_1p_2^2p_4, p_1p_2^2p_4^2, \ldots, p_1p_2^2p_4^{a_4};$
$\ldots; p_1p_2^{a_2}p_4, p_1p_2^{a_2}p_4^2, \ldots, p_1p_2^{a_2}p_4^{a_4}; p_1^2p_2p_4, p_1^2p_2p_4^2, \ldots, p_1^2p_2p_4^{a_4}; p_1^2p_2^2p_4, p_1^2p_2^2p_4^2, \ldots, p_1^2p_2^2p_4^{a_4};$
$\ldots; p_1^2p_2^{a_2}p_4, p_1^2p_2^{a_2}p_4^2, \ldots, p_1^2p_2^{a_2}p_4^{a_4}; \ldots p_1^{a_1}p_2p_4, p_1^{a_1}p_2p_4^2, \ldots, p_1^{a_1}p_2p_4^{a_4};$
$p_1^{a_1}p_2^2p_4, p_1^{a_1}p_2^2p_4^2, \ldots, p_1^{a_1}p_2^2p_4^{a_4}; \ldots; p_1^{a_1}p_2^{a_2}p_4, p_1^{a_1}p_2^{a_2}p_4^2, \ldots, p_1^{a_1}p_2^{a_2}p_4^{a_4}; p_1p_3p_4, p_1p_3p_4^2, \ldots, p_1p_3p_4^{a_4};$
$p_1p_3^2p_4, p_1p_3^2p_4^2, \ldots, p_1p_3^2p_4^{a_4}; \ldots; p_1p_3^{a_3}p_4, p_1p_3^{a_3}p_4^2, \ldots, p_1p_3^{a_3}p_4^{a_4}; p_1^2p_3p_4, p_1^2p_3p_4^2, \ldots, p_1^2p_3p_4^{a_4};$
$p_1^2p_3^2p_4, p_1^2p_3^2p_4^2, \ldots, p_1^2p_3^2p_4^{a_4}; \ldots; p_1^2p_3^{a_3}p_4, p_1^2p_3^{a_3}p_4^2, \ldots, p_1^2p_3^{a_3}p_4^{a_4}; \cdots$
$p_1^{a_1}p_3p_4, p_1^{a_1}p_3p_4^2, \ldots, p_1^{a_1}p_3p_4^{a_4}; p_1^{a_1}p_3^2p_4, p_1^{a_1}p_3^2p_4^2, \ldots, p_1^{a_1}p_3^2p_4^{a_4}; \ldots; p_1^{a_1}p_3^{a_3}p_4, p_1^{a_1}p_3^{a_3}p_4^2, \ldots, p_1^{a_1}p_3^{a_3}p_4^{a_4};$
$p_2p_3p_4, p_2p_3p_4^2, \ldots, p_2p_3p_4^{a_4}; p_2p_3^2p_4, p_2p_3^2p_4^2, \ldots, p_2p_3^2p_4^{a_4}; \ldots; p_2p_3^{a_3}p_4, p_2p_3^{a_3}p_4^2, \ldots, p_2p_3^{a_3}p_4^{a_4};$
$p_2^2p_3p_4, p_2^2p_3p_4^2, \ldots, p_2^2p_3p_4^{a_4}; p_2^2p_3^2p_4, p_2^2p_3^2p_4^2, \ldots, p_2^2p_3^2p_4^{a_4};$
$\ldots; p_2^2p_3^{a_3}p_4, p_2^2p_3^{a_3}p_4^2, \ldots, p_2^2p_3^{a_3}p_4^{a_4};$
$\ldots p_2^{a_2}p_3p_4, p_2^{a_2}p_3p_4^2, \ldots, p_2^{a_2}p_3p_4^{a_4}; p_2^{a_2}p_3^2p_4, p_2^{a_2}p_3^2p_4^2, \ldots, p_2^{a_2}p_3^2p_4^{a_4}; \ldots;$
$p_2^{a_2}p_3^{a_3}p_4, p_2^{a_2}p_3^{a_3}p_4^2, \ldots, p_2^{a_2}p_3^{a_3}p_4^{a_4}; p_1p_2p_3p_4, p_1p_2p_3p_4^2, \ldots, p_1p_2p_3p_4^{a_4}; p_1p_2^2p_3p_4, p_1p_2^2p_3p_4^2, \ldots, p_1p_2^2p_3p_4^{a_4};$
$\ldots; p_1p_2^{a_2}p_3p_4, p_1p_2^{a_2}p_3p_4^2, \ldots, p_1p_2^{a_2}p_3p_4^{a_4}; p_1p_2p_3^2p_4, p_1p_2p_3^2p_4^2, \ldots, p_1p_2p_3^2p_4^{a_4};$
$p_1p_2^2p_3^2p_4, p_1p_2^2p_3^2p_4^2, \ldots, p_1p_2^2p_3^2p_4^{a_4}; \ldots; p_1p_2^{a_2}p_3^2p_4, p_1p_2^{a_2}p_3^2p_4^2, \ldots, p_1p_2^{a_2}p_3^2p_4^{a_4}; \ldots$
$p_1p_2p_3^{a_3}p_4, p_1p_2p_3^{a_3}p_4^2, p_1p_2p_3^{a_3}p_4^{a_4}; p_1p_2^2p_3^{a_3}p_4, p_1p_2^2p_3^{a_3}p_4^2, \ldots,$
$p_1p_2^2p_3^{a_3}p_4^{a_4}; \ldots; p_1p_2^{a_2}p_3^{a_3}p_4, p_1p_2^{a_2}p_3^{a_3}p_4^2, \ldots, p_1p_2^{a_2}p_3^{a_3}p_4^{a_4};$
$p_1^{a_1}p_2p_3p_4, p_1^{a_1}p_2p_3p_4^2, \ldots, p_1^{a_1}p_2p_3p_4^{a_4}; p_1^{a_1}p_2^2p_3p_4, p_1^{a_1}p_2^2p_3p_4^2, .., p_1^{a_1}p_2^2p_3p_4^{a_4},$
$p_1^{a_1}p_2^{a_2}p_3p_4, p_1^{a_1}p_2^{a_2}p_3p_4^2, \ldots, p_1^{a_1}p_2^{a_2}p_3p_4^{a_4}; p_1^{a_1}p_2p_3^2p_4, p_1^{a_1}p_2p_3^2p_4^2, \ldots, p_1^{a_1}p_2p_3^2p_4^{a_4};$
$p_1^{a_1}p_2^2p_3^2p_4, p_1^{a_1}p_2^2p_3^2p_4^2, p_1^{a_1}p_2^2p_3^2p_4^{a_4}; p_1^{a_1}p_2^{a_2}p_3^2p_4, p_1^{a_1}p_2^{a_2}p_3^2p_4^2, p_1^{a_1}p_2^{a_2}p_3^2p_4^{a_4};$
$p_1^{a_1}p_2p_3^{a_3}p_4, p_1^{a_1}p_2p_3^{a_3}p_4^2, p_1^{a_1}p_2p_3^{a_3}p_4^{a_4};$
$p_1^{a_1}p_2^2p_3^{a_3}p_4, p_1^{a_1}p_2^2p_3^{a_3}p_4^2, p_1^{a_1}p_2^2p_3^{a_3}p_4^{a_4};$
$p_1^{a_1}p_2^{a_2}p_3^{a_3}p_4, p_1^{a_1}p_2^{a_2}p_3^{a_3}p_4^2, \ldots, p_1^{a_1}p_2^{a_2}p_3^{a_3}p_4^{a_4}\}$

We obtain $(a_1+1)(a_2+1)(a_3+1)(a_4+1)-1$ number of vertices.

The set of vertices $D = \{p_1, p_1^2, \ldots, p_1^{a_1}, p_2, p_2^2, \ldots, p_2^{a_2}, p_3, p_3^2, \ldots, p_3^{a_3}, p_4, p_4^2, \ldots, p_4^{a_4}\}$ is an annihilator dominating set.

For, if V is any vertex in V-D then V is of the form
$p_1^{i_1}p_2^{i_2}, p_1^{i_1}p_3^{i_3}, p_1^{i_1}p_4^{i_4}, p_2^{i_2}p_3^{i_3}, p_2^{i_2}p_4^{i_4}, p_3^{i_3}p_4^{i_4}, p_1^{i_1}p_2^{i_2}p_3^{i_3}, p_1^{i_1}p_2^{i_2}p_4^{i_4}, p_1^{i_1}p_3^{i_3}p_4^{i_4}, p_2^{i_2}p_3^{i_3}p_4^{i_4}, p_1^{i_1}p_2^{i_2}p_3^{i_3}p_4^{i_4}$
where $1 < i_1 < a_1, 1 < i_2 < a_2, 1 < i_3 < a_3$ and $1 < i_4 < a_4$.

We observe that

(i) If $i_1 > 1$ (then for all i_2), the vertex of the form v=$p_1^{i_1}, p_2^{i_2}$ is adjacent to p_1 and p_2 in $D$and if i_1=0 (then for all i_2), the vertex V is adjacent to p_2 in D.

(ii) If $i_1 > 1$ (then for all i_3), the vertex of the form $v{=}p_1^{i_1}, p_2^{i_3}$ is adjacent to p_1 and p_3 in D and if i_1=0 (then for all i_3), the vertex V is adjacent to p_3 in D.

(iii) If $i_1 > 1$ (then for all i_4), the vertex of the form $v{=}p_1^{i_1}, p_2^{i_4}$ is adjacent to p_1 and p_4 in D and if i_1=0 (then for all i_4), the vertex V is adjacent to p_4 in D.

(iv) If $i_2 > 1$ (then for all i_3), the vertex of the form $v{=}p_2^{i_2}, p_3^{i_3}$ is adjacent to p_2 and p_3 in D and if i_2=0 (then for all i_3), the vertex V is adjacent to p_3 in D.

(v) If $i_2 > 1$ (then for all i_4), the vertex of the form $v{=}p_2^{i_2}, p_4^{i_4}$ is adjacent to p_2 and p_4 in D and if i_2=0 (then for all i_4), the vertex V is adjacent to p_4 in D.

(vi) If $i_3 > 1$ (then for all i_4), the vertex of the form $v{=}p_3^{i_2}, p_4^{i_4}$ is adjacent to p_3 and p_4 in D and if i_3=0 (then for all i_4), the vertex V is adjacent to p_4 in D.

(vii) If $i_1 > 1$ (then for all i_2 and i_3), the vertex of the form $v{=}p_1^{i_1}, p_2^{i_2}, p_3^{i_3}$ is adjacent to p_1,p_2 and p_3 in D and if i_1=0 (then for all $i_2$and i_3), the vertex V is adjacent to p_2 and p_3 in D.

(viii) If $i_1 > 1$ (then for all i_2 and i_4), the vertex of the form $v{=}p_1^{i_1}, p_2^{i_2}, p_4^{i_4}$ is adjacent to p_1,p_2 and p_4 in D and if i_1=0 (then for all $i_2$and i_4), the vertex V is adjacent to p_2 and p_4 in D.

(ix) If $i_1 > 1$ (then for all i_3 and i_4), the vertex of the form $v{=}p_1^{i_1}, p_3^{i_3}, p_4^{i_4}$ is adjacent to p_1,p_3 and p_4 in D and if i_1=0 (then for all $i_3$and i_4), the vertex V is adjacent to p_3 and p_4 in D.

(x) If $i_2 > 1$ (then for all i_3 and i_4), the vertex of the form $v{=}p_2^{i_2}, p_3^{i_3}, p_4^{i_4}$ is adjacent to p_2,p_3 and p_4 in D and if i_2=0 (then for all $i_3$and i_4), the vertex V is adjacent to p_3 and p_4 in D.

(xi) If $i_1 > 1$ (then for all i_2, i_3 and i_4), the vertex of the form $v{=}p_1^{i_1}, p_2^{i_2}, p_3^{i_3}, p_4^{i_4}$ is adjacent to p_1,p_2 and p_3 in D and if i_1=0 (then for all i_2, i_3 and i_4), the vertex V is adjacent to p_2, p_3 and p_4 in D.

Similarly, if $i_1 = 1$ or $i_2 = 1$ or $i_3 = 1$ or $i_4 = 1$ respectively, the corresponding vertex V is adjacent to p_1 and $p_1^{i_1}$ $(1 < i_1 < a_1)$, p_2 and $p_2^{i_2}$ $(1 < i_2 < a_2)$, p_3 and $p_3^{i_3}$ $(1 < i_3 < a_3)$, p_4 and $p_4^{i_4}$ $(1 < i_4 < a_4)$, respectively. In all the above cases it is proved that the vertices in V-D are having adjacency with atleast one vertex in D. Thus D is a Dominating set.

In V-D, any vertices will be of the form $p_1^{d_1}p_2^{d_2}, p_1^{e_1}p_2^{e_2}, p_1^{f_1}p_3^{f_2}, p_1^{g_1}p_3^{g_2}, p_1^{k_1}p_4^{k_2}, p_1^{l_1}p_4^{l_2}$, $p_2^{m_1}p_3^{m_2}, p_2^{n_1}p_3^{n_2}, p_2^{o_1}p_4^{o_2}, p_2^{q_1}p_4^{q_2}, p_3^{r_1}p_4^{r_2}, p_3^{s_1}p_4^{s_2}, p_1^{t_1}p_2^{t_2}p_3^{t_3}, p_1^{u_1}p_2^{u_2}p_3^{u_3}, p_1^{v_1}p_2^{v_2}p_4^{v_3}, p_1^{w_1}p_2^{w_2}p_4^{w_3}$ $p_1^{x_1}p_3^{x_2}p_4^{x_3}, p_1^{y_1}p_3^{y_2}p_4^{y_3}, p_2^{z_1}p_3^{z_2}p_4^{z_3}, p_2^{z'_1}p_3^{z'_2}p_4^{z'_3}$ $p_1^{z''_1}p_2^{z''_2}p_3^{z''_3}p_4^{z''_4}, p_1^{z'''_1}p_2^{z'''_2}p_3^{z'''_3}p_4^{z'''_4}$

Now, we have to verify the following cases:

Case (I) If $d_1, e_1, f_1, g_1, k_1, l_1, m_1, n_1, o_1, q_1, r_1, s_1, t_1, u_1, v_1, w_1, x_1, y_1, z_1 z'_1, z''_1, z'''_1 > 1$, since $d_2, e_2, f_2, g_2, k_2, l_2, m_2, n_2, o_2, q_2, r_2, s_2, t_2, u_2, v_2, w_2, x_2, y_2, z_2 z'_2, z''_2, z'''_2$ are also > 1, then the vertices

(i) $(p_1^{d_1}, p_2^{d_2}, p_1^{e_1}, p_2^{e_2}) = p_1^{b_1}, p_2^{b_2}$, where $b_1, b_2 > 1$and hence the vertices are not adjacent.

(ii) $(p_1^{f_1}, p_3^{f_2}, p_1^{g_1}, p_3^{g_2}) = p_1^{b_3}, p_3^{b_4}$, where $b_3, b_4 > 1$ and hence the vertices are not adjacent.

(iii) $(p_1^{k_1}, p_4^{k_2}, p_1^{i_1}, p_4^{i_2}) = p_1^{b_5}, p_4^{b_6}$, where $b_5, b_6 > 1$ and hence the vertices are not adjacent.

(iv) $(p_2^{m_1}, p_3^{m_2}, p_2^{n_1}, p_3^{n_2}) = p_2^{c_1}, p_3^{c_2}$, where $c_1, c_2 > 1$ and hence the vertices are not adjacent.

(v) $(p_2^{o_1}, p_4^{o_2}, p_2^{q_1}, p_4^{q_2}) = p_2^{c_3}, p_4^{c_4}$, where $c_3, c_4 > 1$ and hence the vertices are not adjacent.

(vi) $(p_3^{r_1}, p_4^{r_2}, p_3^{s_1}, p_4^{s_2}) = p_3^{c_5}, p_4^{c_6}$, where $c_5, c_6 > 1$ and hence the vertices are not adjacent.

(vii) $(p_1^{t_1}, p_2^{t_2}, p_3^{t_3}, p_1^{u_1}, p_2^{u_2}, p_3^{u_3}) = p_1^{h_1}, p_2^{h_2}, p_3^{h_3}$, where $h_1, h_2, h_3 > 1$ and hence the vertices are not adjacent.

(viii) $(p_1^{v_1}, p_2^{v_2}, p_4^{v_3}, p_1^{w_1}, p_2^{w_2}, p_4^{w_3}) = p_1^{h_4}, p_2^{h_5}, p_4^{h_6}$, where $h_4, h_5, h_6 > 1$ and hence the vertices are not adjacent.

(ix) $(p_1^{x_1}, p_3^{x_2}, p_4^{x_3}, p_1^{y_1}, p_3^{y_2}, p_4^{y_3}) = p_1^{h_7}, p_3^{h_8}, p_4^{h_9}$, where $h_7, h_8, h_9 > 1$ and hence the vertices are not adjacent.

(x) $(p_2^{z_1}, p_3^{z_2}, p_4^{z_3}, p_2^{z'_1}, p_3^{z'_2}, p_4^{z'_3}) = p_2^{j_1}, p_3^{j_2}, p_4^{j_3}$, where $j_1, j_2, j_3 > 1$ and hence the vertices are not adjacent.

(xi) $(p_1^{z'_1}, p_2^{z'_2}, p_3^{z'_3}, p_4^{z'_4}, p_1^{z''_1}, p_2^{z''_2}, p_3^{z''_3}, p_4^{z''_4}) = p_1^{j_4}, p_2^{j_5}, p_3^{j_6}, p_4^{j_7}$, where $j_4, j_5, j_6, j_7 > 1$ and hence the vertices are not adjacent.

Case (II) If $d_1 = f_1 = k_1 = m_1 = o_1 = r_1 = t_1 = v_1 = x_1 = z_1 = z'_1 = 0$, $e_1, g_1, l_1, n_1, q_1, s_1, u_1, w_1, y_1, z'_1, z''_1 > 0$ since $d_2, e_2, f_2, g_2, k_2, l_2, m_2, n_2, o_2, q_2, r_2, s_2, t_2, u_2, v_2, w_2, x_2, y_2, z_2, z'_2, z''_2, z'''_2 > 1$, then we have

(i) $(p_1^{d_1} p_2^{d_2}, p_1^{e_1} p_2^{e_2}) = (p_2^{d_2}, p_1^{e_1} p_2^{e_2}) = p_2^{b'_2}$, where $b'_2 > 1$ and hence the vertices are not adjacent.

(ii) $(p_1^{f_1} p_3^{f_2}, p_1^{g_1} p_3^{g_2}) = (p_3^{f_2}, p_1^{g_1} p_2^{g_2}) = p_3^{b'_4}$, where $b'_4 > 1$ and hence the vertices are not adjacent.

(iii) $(p_1^{k_1} p_4^{k_2}, p_1^{l_1} p_4^{l_2}) = (p_4^{k_2}, p_1^{l_1} p_4^{l_2}) = p_4^{b'_6}$, where $b'_6 > 1$ and hence the vertices are not adjacent.

(iv) $(p_2^{m_1} p_3^{m_2}, p_2^{n_1} p_3^{n_2}) = (p_3^{m_2}, p_2^{n_1} p_3^{n_2}) = p_3^{c'_2}$, where $c'_2 > 1$ and hence the vertices are not adjacent.

(v) $(p_2^{o_1} p_4^{o_2}, p_2^{q_1} p_4^{q_2}) = (p_4^{o_2}, p_2^{q_1} p_4^{q_2}) = p_4^{c'_4}$, where $c'_4 > 1$ and hence the vertices are not adjacent.

(vi) $(p_3^{r_1} p_4^{r_2}, p_3^{s_1} p_4^{s_2}) = (p_4^{r_2}, p_3^{s_1} p_4^{s_2}) = p_4^{c'_6}$, where $c'_6 > 1$ and hence the vertices are not adjacent.

(vii) $(p_1^{t_1} p_2^{t_2} p_3^{t_3}, p_1^{u_1} p_2^{u_2} p_3^{u_3}) = (p_2^{t_2} p_3^{t_3}, p_1^{u_1} p_2^{u_2} p_3^{u_3}) = (p_2^{h'_2}, p_3^{h'_3})$, where $h'_2, h'_3 > 1$ and hence the vertices are not adjacent.

(viii) $(p_1^{v_1} p_2^{v_2} p_4^{v_3}, p_1^{w_1} p_2^{w_2} p_4^{w_3}) = (p_2^{v_2} p_4^{v_3}, p_1^{w_1} p_2^{w_2} p_4^{w_3}) = (p_2^{h'_5}, p_4^{h'_6})$, where $h'_5, h'_6 > 1$ and hence the vertices are not adjacent.

(ix) $(p_1^{x_1} p_3^{x_2} p_4^{x_3}, p_1^{y_1} p_3^{y_2} p_4^{y_3}) = (p_3^{x_2} p_4^{x_3}, p_1^{y_1} p_3^{y_2} p_4^{y_3}) = (p_3^{h'_7}, p_4^{h'_8})$, where $h'_7, h'_8 > 1$ and hence the vertices are not adjacent.

(x) $(p_2^{z_1} p_3^{z_2} p_4^{z_3}, p_2^{z'_1} p_3^{z'_2} p_4^{z'_3}) = (p_3^{z_2} p_4^{z_3}, p_2^{z'_1} p_3^{z'_2} p_4^{z'_3}) = (p_3^{j'_2}, p_4^{j'_3})$, where $j'_2, j'_3 > 1$ and hence the vertices are not adjacent.

(xi) $(p_1^{z''_1} p_2^{z''_2} p_3^{z''_3} p_4^{z''_4}, p_1^{z'''_1} p_2^{z'''_2} p_3^{z'''_3} p_4^{z'''_4}) = (p_2^{z''_2} p_3^{z''_3} p_4^{z''_4}, p_1^{z'''_1} p_2^{z'''_2} p_3^{z'''_3} p_4^{z'''_4}) = (p_2^{j'_5}, p_3^{j'_6}, p_4^{j'_7})$, where $j'_5, j'_6, j'_7 > 1$ and hence the vertices are not adjacent.

Case (III) If $d_1, f_1, k_1, m_1, o_1, r_1, t_1, v_1, x_1, z_1, z''_1 > 0$ and $e_1 = g_1 = l_1 = n_1 = q_1 = s_1 = u_1 = w_1 = y_1 = z'_1 = z''_1 = 0$, since $d_2, e_2, f_2, g_2, k_2, l_2, m_2, n_2, o_2, q_2, r_2, s_2, t_2, u_2, v_2, w_2, x_2, y_2, z_2, z'_2, z''_2, z'''_2 > 1$, then we have

(i) $(p_1^{d_1}p_2^{d_2}, p_1^{e_1}p_2^{e_2}) = (p_1^{d_1}p_2^{d_2}, p_2^{e_2}) = p_2^{b_2''}$, where $b_2'' > 1$ and hence the vertices are not adjacent.

(ii) $(p_1^{f_1}p_3^{f_2}, p_1^{g_1}p_3^{g_2}) = (p_1^{f_1}p_3^{f_2}, p_3^{g_2}) = p_3^{b_4''}$, where $b_4'' > 1$ and hence the vertices are not adjacent.

(iii) $(p_1^{k_1}p_4^{k_2}, p_1^{l_1}p_4^{l_2}) = (p_1^{k_1}p_4^{k_2}, p_4^{l_2}) = p_4^{b_6''}$, where $b_6'' > 1$ and hence the vertices are not adjacent.

(iv) $(p_2^{m_1}p_3^{m_2}, p_2^{n_1}p_3^{n_2}) = (p_2^{m_1}p_3^{m_2}, p_3^{n_2}) = p_3^{c_2''}$, where $c_2'' > 1$ and hence the vertices are not adjacent.

(v) $(p_2^{o_1}p_4^{o_2}, p_2^{q_1}p_4^{q_2}) = (p_2^{o_1}p_4^{o_2}, p_4^{q_2}) = p_4^{c_4''}$, where $c_4'' > 1$ and hence the vertices are not adjacent.

(vi) $(p_3^{r_1}p_4^{r_2}, p_3^{s_1}p_4^{s_2}) = (p_3^{r_1}p_4^{r_2}, p_4^{s_2}) = p_4^{c_6''}$, where $c_6'' > 1$ and hence the vertices are not adjacent.

(vii) $(p_1^{t_1}p_2^{t_2}p_3^{t_3}, p_1^{u_1}p_2^{u_2}p_3^{u_3}) = (p_1^{t_1}p_2^{t_2}p_3^{t_3}, p_2^{u_2}p_3^{u_3}) = p_2^{h_2''}p_3^{h_3''}$, where $h_2'', h_3'' > 1$ and hence the vertices are not adjacent.

(viii) $(p_1^{v_1}p_2^{v_2}p_4^{v_3}, p_1^{w_1}p_2^{w_2}p_4^{w_3}) = (p_1^{v_1}p_2^{v_2}p_4^{v_3}, p_2^{w_2}p_4^{w_3}) = p_2^{h_5''}p_4^{h_6''}$, where $h_5'', h_6'' > 1$ and hence the vertices are not adjacent.

(ix) $(p_1^{x_1}p_3^{x_2}p_4^{x_3}, p_1^{y_1}p_3^{y_2}p_4^{y_3}) = (p_1^{x_1}p_3^{x_2}p_4^{x_3}, p_3^{y_2}p_4^{y_3}) = p_3^{h_8''}p_4^{h_9''}$, where $h_8'', h_9'' > 1$ and hence the vertices are not adjacent.

(x) $(p_2^{z_1}p_3^{z_2}p_4^{z_3}, p_2^{z_1'}p_3^{z_2'}p_4^{z_3'}) = (p_2^{z_1}p_3^{z_2}p_4^{z_3}, p_3^{z_2'}p_4^{z_3'}) = p_3^{j_2''}p_4^{j_3''}$, where $j_2'', j_3'' > 1$ and hence the vertices are not adjacent.

(xi) $(p_1^{z_1''}p_2^{z_2''}p_3^{z_3''}p_4^{z_4''}, p_1^{z_1'''}p_2^{z_2'''}p_3^{z_3'''}p_4^{z_4'''}) = (p_1^{z_1''}p_2^{z_2''}p_3^{z_3''}p_4^{z_4''}, p_2^{z_2'''}p_3^{z_3'''}p_4^{z_4'''}) = p_2^{j_5''}p_3^{j_6''}p_4^{j_7''}$, where j_5'', j_6'', j_7'' > 1 and hence the vertices are not adjacent.

Case (IV) If $d_1 = f_1 = k_1 = m_1 = o_1 = r_1 = t_1 = v_1 = x_1 = z_1 == 0$ and $e_1 = g_1 = l_1 = n_1 = q_1 = s_1 = u_1 = w_1 = y_1 = z_1' = z_1''' = 0$, since $d_2, e_2, f_2, g_2, k_2, l_2, m_2, n_2, o_2, q_2, r_2, s_2, t_2, u_2, v_2, w_2, x_2, y_2, z_2, , z_2', z_2'', z_2''' > 1$, then we have

(i) $(p_1^{d_1}p_2^{d_2}, p_1^{e_1}p_2^{e_2}) = (p_2^{d_2}, p_2^{e_2}) = p_2^{b_2'''}$, where $b_2''' > 1$ and hence the vertices are not adjacent.

(ii) $(p_1^{f_1}p_3^{f_2}, p_1^{g_1}p_3^{g_2}) = (p_3^{f_2}, p_3^{g_2}) = p_3^{b_4'''}$, where $b_4''' > 1$ and hence the vertices are not adjacent.

(iii) $(p_1^{k_1}p_4^{k_2}, p_1^{l_1}p_4^{l_2}) = (p_4^{k_2}, p_4^{l_2}) = p_4^{b_6'''}$, where $b_6''' > 1$ and hence the vertices are not adjacent.

(iv) $(p_2^{m_1}p_3^{m_2}, p_2^{n_1}p_3^{n_2}) = (p_3^{m_2}, p_3^{n_2}) = p_3^{c_2'''}$, where $c_2''' > 1$ and hence the vertices are not adjacent.

(v) $(p_2^{o_1}p_4^{o_2}, p_2^{q_1}p_4^{q_2}) = (p_4^{o_2}, p_4^{q_2}) = p_4^{c_4'''}$, where $c_4''' > 1$ and hence the vertices are not adjacent.

(vi) $(p_3^{r_1}p_4^{r_2}, p_3^{s_1}p_4^{s_2}) = (p_4^{r_2}, p_4^{s_2}) = p_4^{c_6'''}$, where $c_6''' > 1$ and hence the vertices are not adjacent.

(vii) $(p_1^{t_1}p_2^{t_2}p_3^{t_3}, p_1^{u_1}p_2^{u_2}p_3^{u_3}) = (p_2^{t_2}p_3^{t_3}, p_2^{u_2}p_3^{u_3}) = p_2^{h_2'''}p_3^{h_3'''}$, where $h_2''', h_3''' > 1$ and hence the vertices are not adjacent.

(viii) $(p_1^{v_1}p_2^{v_2}p_4^{v_3}, p_1^{w_1}p_2^{w_2}p_4^{w_3}) = (p_2^{v_2}p_4^{v_3}, p_2^{w_2}p_4^{w_3}) = p_2^{h_5'''}p_4^{h_6'''}$, where $h_5''', h_6''' > 1$ and hence the vertices are not adjacent.

(ix) $(p_1^{x_1}p_3^{x_2}p_4^{x_3}, p_1^{y_1}p_3^{y_2}p_4^{y_3}) = (p_3^{x_2}p_4^{x_3}, p_3^{y_2}p_4^{y_3}) = p_3^{h_8'''}p_4^{h_9'''}$, where $h_8''', h_9''' > 1$ and hence the vertices are not adjacent.

(x) $(p_2^{z_1}p_3^{z_2}p_4^{z_3}, p_2^{z_1'}p_3^{z_2'}p_4^{z_3'}) = (p_3^{z_2}p_4^{z_3}, p_3^{z_2'}p_4^{z_3'}) = p_3^{j_2'''}p_4^{j_3'''}$, where $j_2''', j_3''' > 1$ and hence the vertices are not adjacent

(xi) $(p_1^{z_1''}p_2^{z_2''}p_3^{z_3''}p_4^{z_4''}, p_1^{z_1'''}p_2^{z_2'''}p_3^{z_3'''}p_4^{z_4'''}) = (p_2^{z_2''}p_3^{z_3''}p_4^{z_4''}, p_2^{z_2'''}p_3^{z_3'''}p_4^{z_4'''}) = p_2^{j_5'''}p_3^{j_6'''}p_4^{j_7'''}$, where $j_5''', j_6''', j_7''' > 1$ and hence the vertices are not adjacent

In all the above cases by the definition of V_m it is proved that the vertices are not adjacent in the induced subgraph $\langle V - D\rangle$. So that D is an annihilator dominating set of V_m. D is also minimal annihilator dominating set.

If we remove any vertex V in D then $D - \{V\}$ is not annihilator dominating set.

If V is of the form $p_1^{i_1}$ where $1 \le i_1 \le a_1$ (or) $p_2^{i_2}$ where $1 \le i_2 \le a_2$ (or) $p_3^{i_3}$ where $1 \le i_3 \le a_3$ (or) $p_4^{i_4}$ where $1 \le i_4 \le a_4$ then

(i) the vertex $p_1^{i_1}(1 \le i_1 \le a_1)$ is having adjacency with all the vertices $p_1p_2^{b_2}, p_1p_3^{b_3}, p_1p_4^{b_4}, p_1p_2^{b_2}p_3^{b_3}, p_1p_2^{b_2}p_4^{b_4}, p_1p_3^{b_3}p_4^{b_4}, p_1p_2^{b_2}p_3^{b_3}p_4^{b_4}$; where $0 \le b_2 \le a_2; 0 \le b_3 \le a_3, 0 \le b_4 \le a_4$; in the induced subgraph of $\langle V - \{D - \{V\}\}\rangle$.

(ii) the vertex $p_2^{i_2}(1 \le i_2 \le a_2)$ is having adjacency with all the vertices $p_1^{b_1}p_2, p_2p_3^{b_3}, p_2p_4^{b_4}, p_1^{b_1}p_2p_3^{b_3}, p_1^{b_1}p_2p_4^{b_4}, p_2p_3^{b_3}p_4^{b_4}, p_1^{b_1}p_2p_3^{b_3}p_4^{b_4}$, where $0 \le b_1 \le a_1; 0 \le b_3 \le a_3, 0 \le b_4 \le a_4$; in the induced subgraph of $\langle V - \{D - \{V\}\}\rangle$.

(iii) the vertex $p_3^{i_3}(1 \le i_3 \le a_3)$ is having adjacency with all the vertices $p_1^{b_1}p_3, p_2^{b_2}p_3, p_3p_4^{b_4}, p_1^{b_1}p_2^{b_2}p_3, p_2^{b_2}p_3p_4^{b_4}, p_1^{b_1}p_3p_4^{b_4}, p_1^{b_1}p_2^{b_2}p_3p_4^{b_4}$; where $0 \le b_1 \le a_1; 0 \le b_2 \le a_2, 0 \le b_4 \le a_4$ in the induced subgraph of $\langle V - \{D - \{V\}\}\rangle$.

(iv) the vertex $p_4^{i_4}(1 \le i_4 \le a_4)$ is having adjacency with all the vertices $p_1^{b_1}p_4, p_2^{b_2}p_4, p_3^{b_3}p_4, p_1^{b_1}p_2^{b_2}p_4, p_1^{b_1}p_3^{b_3}p_4, p_2^{b_2}p_3^{b_3}p_4, p_1^{b_1}p_2^{b_2}p_3^{b_3}p_4$; where $0 \le b_1 \le a_1; 0 \le b_2 \le a_2 and 0 \le b_3 \le a_3$; in the induced subgraph of $\langle V - \{D - \{V\}\}\rangle$.

Then $D - \{V\}$ is not an annihilator dominating set.

Therefore D is a minimal annihilator dominating set, it follows that $\gamma_a[V_m] \le |D| = 4a_1 + 6$ Hence the proof. □

Note. The above theorem is proved for four distinct primes(unequal powers) and it can be generalized for n distinct primes(unequal powers) as follows.

Theorem 4.2. *If $m = p_1^{a_1}p_2^{a_2}p_3^{a_3}p_4^{a_4}\dots p_n^{a_n}$, then $\gamma_a[v_m] \le na_1 + \frac{n(n-1)}{2}$, where $p_1, p_2, p_3, p_4, \dots, p_n$ are distinct primes & $a_1, a_2, a_3, a_4, \dots, a_n \ge 1$; ($a_1 < a_2 < a_3 < a_4 < \dots < a_n$)*

Proof. Given $m = p_1^{a_1}p_2^{a_2}p_3^{a_3}p_4^{a_4}\dots p_n^{a_n}$ and the vertex set V_m

$p_1, p_1^2, \cdots, p_1^{a_1}; p_2, p_2^2, \dots, p_2^{a_2}; \cdots; p_n, p_n^2, \dots, p_n^{a_n}$

$p_1p_2, p_1p_2^2, \cdots p_1p_2^{a_2}; p_1^2p_2, p_1^2p_2^2, \dots p_1^2p_2^{a_2}; \cdots; p_1^{a_1}p_2, p_1^{a_1}p_2^2, \cdots p_1^{a_1}p_2^{a_2}$

$p_1p_3, p_1p_3^2, \dots p_1p_3^{a_3}; p_1^2p_3, p_1^2p_3^2, \dots p_1^2p_3^{a_3} \cdots; p_1^{a_1}p_3, p_1^{a_1}p_3^2, \dots p_1^{a_1}p_3^{a_3}$

$\cdots p_1p_n, p_1p_n^2, \dots p_1p_n^{a_n}; p_1^2p_n, p_1^2p_n^2, \dots p_1^2p_n^{a_n} \cdots$

$p_1^{a_1}p_n, p_1^{a_1}p_n^2, \dots p_1^{a_1}p_n^{a_n}; p_2p_3, p_2p_3^2, \dots p_2p_3^{a_3}; p_2^2p_3, p_2^2p_3^2, \dots p_2^2p_3^{a_3}$

$p_2^{a_2}p_3, p_2^{a_2}p_3^2, \dots p_2^{a_2}p_3^{a_3}; \cdots; p_2p_n, p_2p_n^2, \dots p_2p_n^{a_n}; p_2^2p_n, p_2^2p_n^2, \dots p_2^2p_n^{a_n}$
$p_2^{a_2}p_n, p_2^{a_2}p_n^2, \dots p_2^{a_2}p_n^{a_n}; \cdots p_{r-1}p_n, p_{r-1}p_n^2, \dots p_{r-1}p_n^{a_n}; p_{r-1}^2p_n, p_{r-1}^2p_n^2$
$p_{r-1}^{a_{r-1}}p_n, p_{r-1}^{a_{r-1}}p_n^2, \dots p_{r-1}^{a_{r-1}}p_n^{a_n}, r = 2,3,4,\dots$
$p_1p_2p_n, p_1p_2p_n^2, \dots p_1p_2p_n^{a_n}; p_1p_2^2p_n, p_1p_2^2p_n^2, \dots p_1p_2^2p_n^{a_n}; \dots; p_1p_2^{a_2}p_n, p_1p_2^{a_2}p_n^2, \dots p_1p_2^{a_2}p_n^{a_n}$
$p_1^2p_2p_n, p_1^2p_2p_n^2, \dots p_1^2p_2p_n^{a_n}; p_1^2p_2^2p_n, p_1^2p_2^2p_n^2, \dots p_1^2p_2^2p_n^{a_n}; \dots; p_1^2p_2^{a_2}p_n, p_1^2p_2^{a_2}p_n^2, \dots p_1^2p_2^{a_2}p_n^{a_n}$
$p_1^{a_1}p_2p_n, p_1^{a_1}p_2p_n^2, \dots p_1^{a_1}p_2p_n^{a_n}; p_1^{a_1}p_2^2p_n, p_1^{a_1}p_2^2p_n^2, \dots p_1^{a_1}p_2^2p_n^{a_n}; \dots; p_1^{a_1}p_2^{a_2}p_n, p_1^{a_1}p_2^{a_2}p_n^2, \dots p_1^{a_1}p_2^{a_2}p_n^{a_n}$
$p_{r-2}p_{r-1}p_n, p_{r-2}p_{r-1}p_n^2, \dots p_{r-2}p_{r-1}p_n^{a_n}; p_{r-2}p_{r-1}^2p_n, p_{r-2}p_{r-1}^2p_n^2, \dots p_{r-2}p_{r-1}^2p_n^{a_n}$
$p_{r-2}p_{r-1}^{a_{r-1}}p_n, p_{r-2}p_{r-1}^{a_{r-1}}p_n^2, \dots p_{r-2}p_{r-1}^{a_{r-1}}p_n^{a_n}; p_{r-2}^2p_{r-1}p_n, p_{r-2}^2p_{r-1}p_n^2, \dots p_{r-2}^2p_{r-1}p_n^{a_n}$
$p_{r-2}^2p_{r-1}^2p_n, p_{r-2}^2p_{r-1}^2p_n^2, \dots p_{r-2}^2p_{r-1}^2p_n^{a_n}; \dots; p_{r-2}^2p_{r-1}^{a_{r-1}}p_n, p_{r-2}^2p_{r-1}^{a_{r-1}}p_n^2, \dots p_{r-2}^2p_{r-1}^{a_{r-1}}p_n^{a_n}$
$p_{r-2}^{a_{r-2}}p_{r-1}p_n, p_{r-2}^{a_{r-2}}p_{r-1}p_n^2, \dots p_{r-2}^{a_{r-2}}p_{r-1}p_n^{a_n}; p_{r-2}^{a_{r-2}}p_{r-1}^2p_n, p_{r-2}^{a_{r-2}}p_{r-1}^2p_n^2, \dots p_{r-2}^{a_{r-2}}p_{r-1}^2p_n^{a_n}$
$p_{r-2}^{a_{r-2}}p_{r-1}^{a_{r-1}}p_n, p_{r-2}^{a_{r-2}}p_{r-1}^{a_{r-1}}p_n^2, \dots p_{r-2}^{a_{r-2}}p_{r-1}^{a_{r-1}}p_n^{a_n}; r = 3,4,5,\dots p_1p_2p_3p_4, p_1p_2p_3p_4^2, \dots p_1p_2p_3p_4^{a_4}$
$p_1p_2^2p_3p_4, p_1p_2^2p_3p_4^2, \dots p_1p_2^2p_3p_4^{a_4}; \dots; p_1p_2^{a_2}p_3p_4, p_1p_2^{a_2}p_3p_4^2, \dots p_1p_2^{a_2}p_3p_4^{a_4}$
$p_1p_2p_3^2p_4, p_1p_2p_3^2p_4^2, \dots p_1p_2p_3^2p_4^{a_4}; p_1p_2^2p_3^2p_4, p_1p_2^2p_3^2p_4^2, \dots p_1p_2^2p_3^2p_4^{a_4}$
$p_1p_2^{a_2}p_3^2p_4, p_1p_2^{a_2}p_3^2p_4^2, \dots p_1p_2^{a_2}p_3^2p_4^{a_4}; \dots p_1p_2p_3^{a_3}p_4, p_1p_2p_3^{a_3}p_4^2, \dots p_1p_2p_3^{a_3}p_4^{a_4}$
$p_1p_2^2p_3^{a_3}p_4, p_1p_2^2p_3^{a_3}p_4^2, \dots p_1p_2^2p_3^{a_3}p_4^{a_4}; \dots; p_1p_2^{a_2}p_3^{a_3}p_4, p_1p_2^{a_2}p_3^{a_3}p_4^2, \dots p_1p_2^{a_2}p_3^{a_3}p_4^{a_4}$
$p_{r-3}p_{r-2}p_{r-1}p_n, p_{r-3}p_{r-2}p_{r-1}p_n^2, \dots p_{r-3}p_{r-2}p_{r-1}p_n^{a_n};$
$p_{r-3}p_{r-2}^2p_{r-1}p_n, p_{r-3}p_{r-2}^2p_{r-1}p_n^2, \dots p_{r-3}p_{r-2}^2p_{r-1}p_n^{a_n}$
$p_{r-3}p_{r-2}^{a_{r-2}}p_{r-1}p_n, p_{r-3}p_{r-2}^{a_{r-2}}p_{r-1}p_n^2, \dots p_{r-3}p_{r-2}^{a_{r-2}}p_{r-1}p_n^{a_n};$
$p_{r-3}p_{r-2}p_{r-1}^2p_n, p_{r-3}p_{r-2}p_{r-1}^2p_n^2, \dots p_{r-3}p_{r-2}p_{r-1}^2p_n^{a_n}$
$p_{r-3}p_{r-2}^2p_{r-1}^2p_n, p_{r-3}p_{r-2}^2p_{r-1}^2p_n^2, \dots p_{r-3}p_{r-2}^2p_{r-1}^2p_n^{a_n}; \dots;$
$p_{r-3}p_{r-2}^{a_{r-2}}p_{r-1}^2p_n, p_{r-3}p_{r-2}^{a_{r-2}}p_{r-1}^2p_n^2, \dots p_{r-3}p_{r-2}^{a_{r-2}}p_{r-1}^2p_n^{a_n}$
$p_{r-3}p_{r-2}p_{r-1}^{a_{r-1}}p_n, p_{r-3}p_{r-2}p_{r-1}^{a_{r-1}}p_n^2, \dots p_{r-3}p_{r-2}p_{r-1}^{a_{r-1}}p_n^{a_n}$
$p_{r-3}p_{r-2}^2p_{r-1}^{a_{r-1}}p_n, p_{r-3}p_{r-2}^2p_{r-1}^{a_{r-1}}p_n^2, \dots p_{r-3}p_{r-2}^2p_{r-1}^{a_{r-1}}p_n^{a_n}$
$p_{r-3}p_{r-2}^{a_{r-2}}p_{r-1}^{a_{r-1}}p_n, p_{r-3}p_{r-2}^{a_{r-2}}p_{r-1}^{a_{r-1}}p_n^2, \dots p_{r-3}p_{r-2}^{a_{r-2}}p_{r-1}^{a_{r-1}}p_n^{a_n}\} r = 4,5,6,\dots$

There are $(a_1+1)(a_2+1)\dots(a_n+1)-1$ vertices.

The set of vertices $D=\{p_1, p_1^2, \cdots, p_1^{a_1}; p_2, p_2^2, \dots, p_2^{a_2}; \cdots; p_n, p_n^2, \dots, p_n^{a_n}\}$ is an annihilator dominating set.

For, if V is any vertex in $V-D$ then V is of the form
$p_1^{i_1}p_2^{i_2}, p_1^{i_1}p_3^{i_3}, p_1^{i_1}p_4^{i_4}, \cdots, p_1^{i_1}p_n^{i_n}$
$p_2^{i_2}p_3^{i_3}, p_2^{i_2}p_4^{i_4}, \cdots, p_2^{i_2}p_n^{i_n}$
$p_3^{i_3}p_4^{i_4}, p_3^{i_3}p_5^{i_5}, \cdots, p_3^{i_3}p_n^{i_n}$
$\cdots p_n^{i_n}p_1^{i_1}, p_n^{i_n}p_2^{i_2}, \cdots, p_n^{i_n}p_{n-1}^{i_{n-1}}$
$p_1^{i_1}p_2^{i_2}p_3^{i_3}, p_1^{i_1}p_2^{i_2}p_4^{i_4}, \cdots, p_1^{i_1}p_2^{i_2}p_n^{i_n}$
$p_1^{i_1}p_3^{i_3}p_4^{i_4}, p_1^{i_1}p_3^{i_3}p_5^{i_5}, \cdots, p_1^{i_1}p_3^{i_3}p_n^{i_n}$
$p_2^{i_2}p_3^{i_3}p_4^{i_4}, p_2^{i_2}p_3^{i_3}p_5^{i_5}, \dots, p_2^{i_2}p_3^{i_3}p_n^{i_n}$
$\cdots\ p_1^{i_1}p_2^{i_2}p_3^{i_3}p_4^{i_4}\dots p_1^{i_1}p_2^{i_2}p_3^{i_3}\dots p_n^{i_n}.$

For different values of $i_1, i_2, i_3, \dots, i_n \geq 1$.

The above values are having adjacency with the vertices $p_1p_1^{a_1}; p_2p_2^{a_2}; p_3p_3^{a_3}; \dots, p_np_n^{a_n}$, respectively. So that it is evident that all the vertices in $V-D$ are having adjacency that atleast one vertex in D. Thus D is a dominating set.

In $V-D$ any vertex will be of the form
$p_1^{d_1}p_2^{d_2}, p_1^{e_1}p_2^{e_2}, p_1^{f_1}p_3^{f_2}, p_1^{g_1}p_3^{g_2}, p_1^{k_1}p_4^{k_2}, p_1^{l_1}p_4^{l_2}, \ldots p_2^{m_1}p_3^{m_2}, p_2^{n_1}p_3^{n_2},$
$p_2^{o_1}p_4^{o_2}, p_2^{q_1}p_4^{q_2}, \ldots p_3^{r_1}p_4^{r_2}, p_3^{s_1}p_4^{s_2}, \ldots p_1^{t_1}p_2^{t_2}p_3^{t_3}, p_1^{u_1}p_2^{u_2}p_3^{u_3}, \ldots p_1^{v_1}p_2^{v_2}p_4^{v_3}, p_1^{w_1}p_2^{w_2}p_4^{w_3},$
$p_1^{x_1}p_3^{x_2}p_4^{x_3}$
$p_1^{y_1}p_3^{y_2}p_4^{y_3}, \ldots p_2^{z_1}p_3^{z_2}p_4^{z_3}, p_2^{z'_1}p_3^{z'_2}p_4^{z'_3}, \ldots p_1^{z''_1}p_2^{z''_2}p_3^{z''_3}p_4^{z''_4}, p_1^{z'''_1}p_2^{z'''_2}p_3^{z'''_3}p_4^{z'''_4}$
i.e. the product of $2,3,4\cdots(n-1)n$ primes with different primes

Case I: If all power of primes >1, then the vertices of product of $2,3,4\cdots(n-1)n$ primes are not adjacent.

Case II: If all first powers of each prime >0 and second powers of each prime$=0$ and the remaining powers of each prime >1, we have the vertices of product of $2,3,4\cdots(n-1)n$ primes are not adjacent.

Case III: If all first primes of each prime$=0$and the remaining are >1; then we can establish that the vertices of product of $2,3,4\cdots(n-1)n$ primes are not adjacent.

In all the above cases, by the definition of V_m it is proved that the vertices are not adjacent in the induced subgraph of$\langle V-D\rangle$. So that D is an Annihilator Dominating set. If we remove any vertex V in D then $D-\{V\}$ is not an annihilator dominating set, If V is of the form

$p_1^{i_1}$ where $1\le i_1\le a_1$ (or) $p_2^{i_2}$ where $1\le i_2\le a_2$ (or) $p_3^{i_3}$ where $1\le i_3\le a_3$ $\cdots$ (or) $p_n^{i_n}$ where $1\le i_n\le a_n$

then the vertex $p_1^{i_1}(1\le i_1\le a_1)$ is having adjacency with all the vertices of product of various primes such as
$p_1p_2^{b_2}, p_1p_3^{b_3}, p_1p_4^{b_4}, \cdots p_1p_n^{b_n}; p_1p_2^{b_2}p_3^{b_3}, p_1p_2^{b_2}p_4^{b_4}, \ldots p_1p_{n-1}^{b_{n-1}}p_n^{b_n}; \cdots; p_1p_2^{b_2}p_3^{b_3}\ldots p_n^{b_n}$; where $0\le b_2\le a_2; 0\le b_3\le a_3; \ldots; 0\le b_n\le a_n$ in the induced sub graph of $\langle V-\{D-\{V\}\}\rangle$.

Similarly, $p_2^{i_2}(1\le i_2\le a_2)$ $p_3^{i_3}(1\le i_3\le a_3)$ $\cdots$ $p_n^{i_n}(1\le i_n\le a_n)$ also establishes the same adjacency with the vertices of various product of primes respectively.

Then $D-\{V\}$ is not an annihilator dominating set, therefore D is a minimal annihilator dominating set, it follows that $\gamma_a[V_m]\le |D| = na_1+\frac{n(n-1)}{2}$.
Hence the proof. □

Construction for Theorem 4.1. If n is any number, which is the sum of a_1,a_2,a_3,a_4 with $a_1<a_2<a_3<a_4$; Then we have $m = p_1^{\frac{n-6}{4}}.p_2^{a_2}.p_3^{a_3}.p_4^{a_4}$ where p_1,p_2,p_3,p_4 are distinct primes and $D=\{p_1,p_1^2,\ldots p_1^{\frac{n-6}{4}}; p_2,p_2^2,\ldots p_2^{a_2}; p_3,p_3^2,\ldots p_3^{a_3}; p_4,p_4^2,\ldots p_4^{a_4}\}$ is the Annihilator dominating set.

Construction for Theorem 4.2. If n is any number, which is the sum of $a_1,a_2,a_3,\cdots a_n$when $a_1<a_2<a_3<\ldots<a_n$; Then we have $m = p_1^{\frac{N-\frac{n(n-1)}{2}}{n}}.p_2^{a_2}.p_3^{a_3}\cdots p_n^{a_n}$ where $p_1,p_2,p_3,\cdots p_n$ are distinct primes and 'm' is the number of distinct primes
$D=\{p_1,p_1^2,\ldots p_1^{\frac{N-\frac{n(n-1)}{2}}{n}}; p_2,p_2^2,\ldots p_2^{a_2}; p_3,p_3^2,\ldots p_3^{a_3}; \cdots p_n,p_n^2,\ldots p_n^{a_n}\}$ is the Annihilator dominating set.

Illustration. Given $n = 14$ (Even) Let $m = p_1^2p_2^3p_3^4p_4^5$ (with $a_1 = 2, a_2 = 3, a_3 = 4, a_4 = 5$ and $a_1 < a_2 < a_3 < a_4$) where p_1, p_2, p_3, p_4 are four distinct primes.
the vertex set be v_m is the set of divisors of m (except 1) then

$v_m = \{1-p_1p_2, 2-p_1p_2^2, 3-p_1p_2^3; 4-p_1^2p_2, 5-p_1^2p_2^2, 6-p_1^2p_2^3; 7-p_1p_3, 8-p_1p_3^2, 9-p_1p_3^3, 10-p_1p_3^4;$
$11-p_1^2p_3, 12-p_1^2p_3^2, 13-p_1^2p_3^3, 14-p_1^2p_3^4; 15-p_1p_4, 16-p_1p_4^2, 17-p_1p_4^3, 18-p_1p_4^4, 19-p_1p_4^5;$
$20-p_1^2p_4, 21-p_1^2p_4^2, 22-p_1^2p_4^3, 23-p_1^2p_4^4, 24-p_1^2p_4^5; 25-p_2p_3, 26-p_2p_3^2, 27-p_2p_3^3, 28-p_2p_3^4;$
$29-p_2^2p_3, 30-p_2^2p_3^2, 31-p_2^2p_3^3, 32-p_2^2p_3^4; 33-p_2^3p_3, 34-p_2^3p_3^2, 35-p_2^3p_3^3, 36-p_2^3p_3^4; 37-p_2p_4,$
$38-p_2p_4^2, 39-p_2p_4^3, 40-p_2p_4^4, 41-p_2p_4^5; 42-p_2^2p_4, 43-p_2^2p_4^2, 44-p_2^2p_4^3, 45-p_2^2p_4^4, 46-p_2^2p_4^5;$
$47-p_2^3p_4, 48-p_2^3p_4^2, 49-p_2^3p_4^3, 50-p_2^3p_4^4, 51-p_2^3p_4^5; 52-p_3p_4, 53-p_3p_4^2, 54-p_3p_4^3, 55-p_3p_4^4, 56-p_3p_4^5;$
$57-p_3^2p_4, 58-p_3^2p_4^2, 59-p_3^2p_4^3, 60-p_3^2p_4^4, 61-p_3^2p_4^5; 62-p_3^3p_4, 63-p_3^3p_4^2, 64-p_3^3p_4^3, 65-p_3^3p_4^4,$
$66-p_3^3p_4^5; 67-p_3^4p_4, 68-p_3^4p_4^2, 69-p_3^4p_4^3, 70-p_3^4p_4^4, 71-p_3^4p_4^5; 72-p_1p_2p_3, 73-p_1p_2p_3^2, 74-p_1p_2p_3^3,$
$75-p_1p_2p_3^4; 76-p_1p_2^2p_3, 77-p_1p_2^2p_3^2, 78-p_1p_2^2p_3^3, 79-p_1p_2^2p_3^4; 80-p_1p_2^3p_3, 81-p_1p_2^3p_3^2, 82-p_1p_2^3p_3^3,$
$83-p_1p_2^3p_3^4; 84-p_1^2p_2p_3, 85-p_1^2p_2p_3^2, 86-p_1^2p_2p_3^3, 87-p_1^2p_2p_3^4; 88-p_1^2p_2^2p_3, 89-p_1^2p_2^2p_3^2, 90-p_1^2p_2^2p_3^3,$
$91-p_1^2p_2^2p_3^4; 92-p_1^2p_2^3p_3, 93-p_1^2p_2^3p_3^2, 94-p_1^2p_2^3p_3^3, 95-p_1^2p_2^3p_3^4; 96-p_1p_2p_4, 97-p_1p_2p_4^2, 98-p_1p_2p_4^3,$
$99-p_1p_2p_4^4, 100-p_1p_2p_4^5; 101-p_1p_2^2p_4, 102-p_1p_2^2p_4^2, 103-p_1p_2^2p_4^3, 104-p_1p_2^2p_4^4, 105-p_1p_2^2p_4^5;$
$106-p_1p_2^3p_4, 107-p_1p_2^3p_4^2, 108-p_1p_2^3p_4^3, 109-p_1p_2^3p_4^4, 110-p_1p_2^3p_4^5; 111-p_1^2p_2p_4, 112-p_1^2p_2p_4^2,$
$113-p_1^2p_2p_4^3, 114-p_1^2p_2p_4^4, 115-p_1^2p_2p_4^5; 116-p_1^2p_2^2p_4, 117-p_1^2p_2^2p_4^2, 118-p_1^2p_2^2p_4^3, 119-p_1^2p_2^2p_4^4,$
$120-p_1^2p_2^2p_4^5; 121-p_1^2p_2^3p_4, 122-p_1^2p_2^3p_4^2, 123-p_1^2p_2^3p_4^3, 124-p_1^2p_2^3p_4^4, 125-p_1^2p_2^3p_4^5; 126-p_1p_3p_4,$
$127-p_1p_3p_4^2, 128-p_1p_3p_4^3, 129-p_1p_3p_4^4, 130-p_1p_3p_4^5; 131-p_1p_3^2p_4, 132-p_1p_3^2p_4^2, 133-p_1p_3^2p_4^3,$
$134-p_1p_3^2p_4^4, 135-p_1p_3^2p_4^5; 136-p_1p_3^3p_4, 137-p_1p_3^3p_4^2, 138-p_1p_3^3p_4^3, 139-p_1p_3^3p_4^4, 140-p_1p_3^3p_4^5;$
$141-p_1p_3^4p_4, 142-p_1p_3^4p_4^2, 143-p_1p_3^4p_4^3, 144-p_1p_3^4p_4^4, 145-p_1p_3^4p_4^5; 146-p_1^2p_3p_4, 147-p_1^2p_3p_4^2,$
$148-p_1^2p_3p_4^3, 149-p_1^2p_3p_4^4, 150-p_1^2p_3p_4^5; 151-p_1^2p_3^2p_4, 152-p_1^2p_3^2p_4^2, 153-p_1^2p_3^2p_4^3, 154-p_1^2p_3^2p_4^4,$
$155-p_1^2p_3^2p_4^5; 156-p_1^2p_3^3p_4, 157-p_1^2p_3^3p_4^2, 158-p_1^2p_3^3p_4^3, 159-p_1^2p_3^3p_4^4, 160-p_1^2p_3^3p_4^5; 161-p_1^2p_3^4p_4,$
$162-p_1^2p_3^4p_4^2, 163-p_1^2p_3^4p_4^3, 164-p_1^2p_3^4p_4^4, 165-p_1^2p_3^4p_4^5; 166-p_2p_3p_4, 167-p_2p_3p_4^2, 168-p_2p_3p_4^3,$
$169-p_2p_3p_4^4, 170-p_2p_3p_4^5; 171-p_2p_3^2p_4, 172-p_2p_3^2p_4^2, 173-p_2p_3^2p_4^3, 174-p_2p_3^2p_4^4, 175-p_2p_3^2p_4^5;$
$176-p_2p_3^3p_4, 177-p_2p_3^3p_4^2, 178-p_2p_3^3p_4^3, 179-p_2p_3^3p_4^4, 180-p_2p_3^3p_4^5; 181-p_2p_3^4p_4, 182-p_2p_3^4p_4^2,$
$183-p_2p_3^4p_4^3, 184-p_2p_3^4p_4^4, 185-p_2p_3^4p_4^5; 186-p_2^2p_3p_4, 187-p_2^2p_3p_4^2, 188-p_2^2p_3p_4^3, 189-p_2^2p_3p_4^4,$
$190-p_2^2p_3p_4^5; 191-p_2^2p_3^2p_4, 192-p_2^2p_3^2p_4^2, 193-p_2^2p_3^2p_4^3, 194-p_2^2p_3^2p_4^4, 195-p_2^2p_3^2p_4^5; 196-p_2^2p_3^3p_4,$
$197-p_2^2p_3^3p_4^2, 198-p_2^2p_3^3p_4^3, 199-p_2^2p_3^3p_4^4, 200-p_2^2p_3^3p_4^5,$
$201-p_2^2p_3^4p_4, 202-p_2^2p_3^4p_4^2, 203-p_2^2p_3^4p_4^3, 204-p_2^2p_3^4p_4^4, 205-p_2^2p_3^4p_4^5; 206-p_2^3p_3p_4, 207-p_2^3p_3p_4^2,$
$208-p_2^3p_3p_4^3, 209-p_2^3p_3p_4^4, 210-p_2^3p_3p_4^5; 211-p_2^3p_3^2p_4, 212-p_2^3p_3^2p_4^2, 213-p_2^3p_3^2p_4^3, 214-p_2^3p_3^2p_4^4,$
$215-p_2^3p_3^2p_4^5; 216-p_2^3p_3^3p_4, 217-p_2^3p_3^3p_4^2, 218-p_2^3p_3^3p_4^3, 219-p_2^3p_3^3p_4^4, 220-p_2^3p_3^3p_4^5; 221-p_2^3p_3^4p_4,$
$222-p_2^3p_3^4p_4^2, 223-p_2^3p_3^4p_4^3, 224-p_2^3p_3^4p_4^4, 225-p_2^3p_3^4p_4^5; 226-p_1p_2p_3p_4, 227-p_1p_2p_3p_4^2, 228-p_1p_2p_3p_4^3,$
$229-p_1p_2p_3p_4^4, 230-p_1p_2p_3p_4^5; 231-p_1p_2^2p_3p_4, 232-p_1p_2^2p_3p_4^2, 233-p_1p_2^2p_3p_4^3, 234-p_1p_2^2p_3p_4^4,$
$235-p_1p_2^2p_3p_4^5; 236-p_1p_2^3p_3p_4, 237-p_1p_2^3p_3p_4^2, 238-p_1p_2^3p_3p_4^3, 239-p_1p_2^3p_3p_4^4, 240-p_1p_2^3p_3p_4^5;$
$241-p_1p_2p_3^2p_4, 242-p_1p_2p_3^2p_4^2, 243-p_1p_2p_3^2p_4^3, 244-p_1p_2p_3^2p_4^4, 245-p_1p_2p_3^2p_4^5; 246-p_1p_2^2p_3^2p_4,$
$247-p_1p_2^2p_3^2p_4^2, 248-p_1p_2^2p_3^2p_4^3, 249-p_1p_2^2p_3^2p_4^4, 250-p_1p_2^2p_3^2p_4^5; 251-p_1p_2^3p_3^2p_4, 252-p_1p_2^3p_3^2p_4^2,$
$253-p_1p_2^3p_3^2p_4^3, 254-p_1p_2^3p_3^2p_4^4, 255-p_1p_2^3p_3^2p_4^5; 256-p_1p_2p_3^3p_4, 257-p_1p_2p_3^3p_4^2, 258-p_1p_2p_3^3p_4^3,$
$259-p_1p_2p_3^3p_4^4, 260-p_1p_2p_3^3p_4^5; 261-p_1p_2^2p_3^3p_4, 262-p_1p_2^2p_3^3p_4^2, 263-p_1p_2^2p_3^3p_4^3, 264-p_1p_2^2p_3^3p_4^4,$
$265-p_1p_2^2p_3^3p_4^5; 266-p_1p_2^3p_3^3p_4, 267-p_1p_2^3p_3^3p_4^2, 268-p_1p_2^3p_3^3p_4^3, 269-p_1p_2^3p_3^3p_4^4, 270-p_1p_2^3p_3^3p_4^5;$

$271 - p_1p_2p_3^4p_4, 272 - p_1p_2p_3^4p_4^2, 273 - p_1p_2p_3^4p_4^3, 274 - p_1p_2p_3^4p_4^4, 275 - p_1p_2p_3^4p_4^5; 276 - p_1p_2^2p_3^4p_4,$
$277 - p_1p_2^2p_3^4p_4^2, 278 - p_1p_2^2p_3^4p_4^3, 279 - p_1p_2^2p_3^4p_4^4, 280 - p_1p_2^2p_3^4p_4^5; 281 - p_1p_2^3p_3^4p_4, 282 - p_1p_2^3p_3^4p_4^2,$
$283 - p_1p_2^3p_3^4p_4^3, 284 - p_1p_2^3p_3^4p_4^4, 285 - p_1p_2^3p_3^4p_4^5; 286 - p_1^2p_2p_3p_4, 287 - p_1^2p_2p_3p_4^2, 288 - p_1^2p_2p_3p_4^3,$
$289 - p_1^2p_2p_3p_4^4, 290 - p_1^2p_2p_3p_4^5; 291 - p_1^2p_2^2p_3p_4, 292 - p_1^2p_2^2p_3p_4^2, 293 - p_1^2p_2^2p_3p_4^3, 294 - p_1^2p_2^2p_3p_4^4,$
$295 - p_1^2p_2^2p_3p_4^5; 296 - p_1^2p_2^3p_3p_4, 297 - p_1^2p_2^3p_3p_4^2, 298 - p_1^2p_2^3p_3p_4^3, 299 - p_1^2p_2^3p_3p_4^4, 300 - p_1^2p_2^3p_3p_4^5;$
$301 - p_1^2p_2p_3^2p_4, 302 - p_1^2p_2p_3^2p_4^2, 303 - p_1^2p_2p_3^2p_4^3, 304 - p_1^2p_2p_3^2p_4^4, 305 - p_1^2p_2p_3^2p_4^5; 306 - p_1^2p_2^2p_3^2p_4,$
$307 - p_1^2p_2^2p_3^2p_4^2, 308 - p_1^2p_2^2p_3^2p_4^3, 309 - p_1^2p_2^2p_3^2p_4^4, 310 - p_1^2p_2^2p_3^2p_4^5; 311 - p_1^2p_2^3p_3^2p_4, 312 - p_1^2p_2^3p_3^2p_4^2,$
$313 - p_1^2p_2^3p_3^2p_4^3, 314 - p_1^2p_2^3p_3^2p_4^4, 315 - p_1^2p_2^3p_3^2p_4^5; 316 - p_1^2p_2p_3^3p_4, 317 - p_1^2p_2p_3^3p_4^2, 318 - p_1^2p_2p_3^3p_4^3,$
$319 - p_1^2p_2p_3^3p_4^4, 320 - p_1^2p_2p_3^3p_4^5; 321 - p_1^2p_2^2p_3^3p_4, 322 - p_1^2p_2^2p_3^3p_4^2, 323 - p_1^2p_2^2p_3^3p_4^3, 324 - p_1^2p_2^2p_3^3p_4^4,$
$325 - p_1^2p_2^2p_3^3p_4^5; 326 - p_1^2p_2^3p_3^3p_4, 327 - p_1^2p_2^3p_3^3p_4^2, 328 - p_1^2p_2^3p_3^3p_4^3, 329 - p_1^2p_2^3p_3^3p_4^4, 330 - p_1^2p_2^3p_3^3p_4^5;$
$331 - p_1^2p_2p_3^4p_4, 332 - p_1^2p_2p_3^4p_4^2, 333 - p_1^2p_2p_3^4p_4^3, 334 - p_1^2p_2p_3^4p_4^4, 335 - p_1^2p_2p_3^4p_4^5; 336 - p_1^2p_2^2p_3^4p_4,$
$337 - p_1^2p_2^2p_3^4p_4^2, 338 - p_1^2p_2^2p_3^4p_4^3, 339 - p_1^2p_2^2p_3^4p_4^4, 340 - p_1^2p_2^2p_3^4p_4^5; 341 - p_1^2p_2^3p_3^4p_4, 342 - p_1^2p_2^3p_3^4p_4^2,$
$343 - p_1^2p_2^3p_3^4p_4^3, 344 - p_1^2p_2^3p_3^4p_4^4, 345 - p_1^2p_2^3p_3^4p_4^5; 346 - p_1, 347 - p_1^2, 348 - p_2, 349 - p_2^2, 350 - p_2^3,$
$351 - p_3, 352 - p_3^2, 353 - p_3^3, 354 - p_3^4, 355 - p_4, 356 - p_4^2, 357 - p_4^3, 358 - p_4^4, 359 - p_4^5\}$

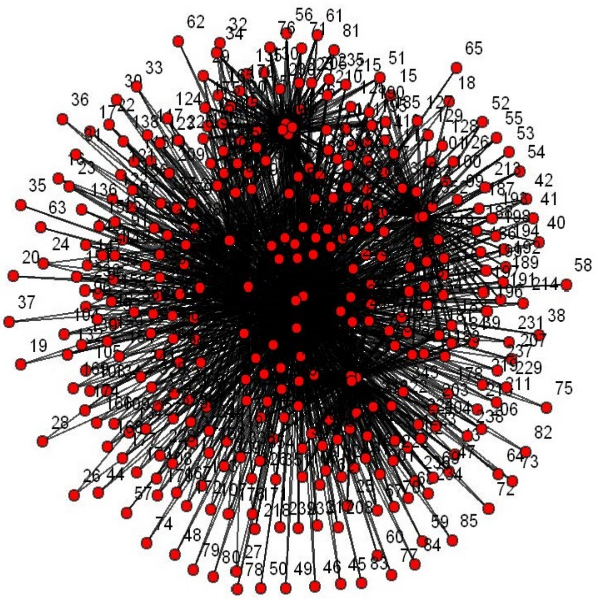

Figure 1. The graph of V_m with $m = p_1^2p_2^3p_3^4p_4^5$

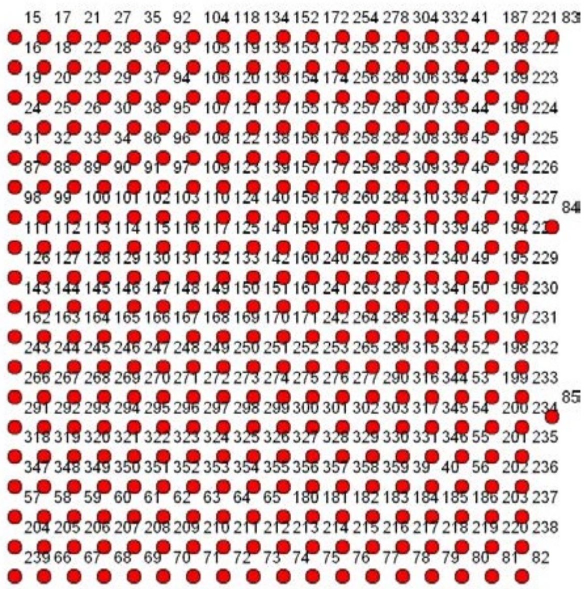

Figure 2. The induced sub graph $\langle V - D\rangle$

The Annihilator dominating set is $D = \{p_1, p_1^2, p_2, p_2^2, p_2^3, p_3, p_3^2, p_3^3, p_3^4, p_4, p_4^2, p_4^3, p_4^4, p_4^5\}$.
The induced Sub Graph $\langle V - D\rangle$ is an independent graph with isolated vertices.
Therefore $\gamma_a[V_m] \le |D| = 4a_1 + 6 = 4 \times 2 + 6 = 14$.

Theorem 4.3. *If* $m = p_1^{a_1}.p_2^{a_2}.p_3^{a_3}\dots p_n^{a_n}$, *then* $\gamma[v_m] \le na_1$ *if* $a_1 = a_2 = a_3 = \dots = a_n$, *where* $p_1, p_2, p_3, \dots p_n$ *are distinct primes and* $a_1, a_2, a_3, \dots a_n \ge 1$.

Proof. Given $m = p_1^{a_1}.p_2^{a_2}.p_3^{a_3}\dots p_n^{a_n}$,
where $p_1, p_2, p_3, \cdots p_n$ are distinct primes and $a_1, a_2, a_3, \dots a_n \ge 1$
Now we have to prove that $\gamma[v_m] \le na_1$, if $a_1 = a_2 = a_3 = \dots = a_n$

The Vertex set of V_m is

$p_1, p_1^2, \ldots, p_1^{a_1}; p_2, p_2^2, \ldots, p_2^{a_2}; \cdots; p_n, p_n^2, \ldots, p_n^{a_n};$
$p_1p_2, p_1p_2^2, \ldots p_1p_2^{a_2}; p_1^2p_2, p_1^2p_2^2, \ldots p_1^2p_2^{a_2}; \ldots; p_1^{a_1}p_2, p_1^{a_1}p_2^2, \ldots p_1^{a_1}p_2^{a_2};$
$p_1p_3, p_1p_3^2, \ldots p_1p_3^{a_3}; p_1^2p_3, p_1^2p_3^2, \ldots p_1^2p_3^{a_3}; \ldots; p_1^{a_1}p_3, p_1^{a_1}p_3^2, \ldots p_1^{a_1}p_3^{a_3};$
$p_1p_n, p_1p_n^2, \ldots p_1p_n^{a_n}; p_1^2p_n, p_1^2p_n^2, \ldots p_1^2p_n^{a_n}; \ldots; p_1^{a_1}p_n, p_1^{a_1}p_n^2, \ldots p_1^{a_1}p_n^{a_n};$
$p_2p_3, p_2p_3^2, \ldots p_2p_3^{a_3}; p_2^2p_3, p_2^2p_3^2, \ldots p_2^2p_3^{a_3}; \ldots; p_2^{a_2}p_3, p_2^{a_2}p_3^2, \ldots p_2^{a_2}p_3^{a_3};$
$p_2p_n, p_2p_n^2, \ldots p_2p_n^{a_n}; p_2^2p_n, p_2^2p_n^2, \ldots p_2^2p_n^{a_n}; \ldots; p_2^{a_2}p_n, p_2^{a_2}p_n^2, \ldots p_2^{a_2}p_n^{a_n};$
$p_{r-1}p_n, p_{r-1}p_n^2, \ldots p_{r-1}p_n^{a_n}; p_{r-1}^2p_n, p_{r-1}^2p_n^2, \cdots p_{r-1}^{a_{r-1}}p_n, p_{r-1}^{a_{r-1}}p_n^2, \ldots p_{r-1}^{a_{r-1}}p_n^{a_n}, r = 2,3,4,\cdots$
$p_1p_2p_n, p_1p_2p_n^2, \ldots p_1p_2p_n^{a_n}; p_1p_2^2p_n, p_1p_2^2p_n^2, \ldots p_1p_2^2p_n^{a_n}; \ldots; p_1p_2^{a_2}p_n, p_1p_2^{a_2}p_n^2, \ldots p_1p_2^{a_2}p_n^{a_n};$
$p_1^2p_2p_n, p_1^2p_2p_n^2, \ldots p_1^2p_2p_n^{a_n}; p_1^2p_2^2p_n, p_1^2p_2^2p_n^2, \ldots p_1^2p_2^2p_n^{a_n}; \ldots; p_1^2p_2^{a_2}p_n, p_1^2p_2^{a_2}p_n^2, \ldots p_1^2p_2^{a_2}p_n^{a_n}; \cdots$
$p_1^{a_1}p_2p_n, p_1^{a_1}p_2p_n^2, \ldots p_1^{a_1}p_2p_n^{a_n}; p_1^{a_1}p_2^2p_n, p_1^{a_1}p_2^2p_n^2, \ldots p_1^{a_1}p_2^2p_n^{a_n}; \ldots; p_1^{a_1}p_2^{a_2}p_n, p_1^{a_1}p_2^{a_2}p_n^2, \ldots p_1^{a_1}p_2^{a_2}p_n^{a_n}; \ldots$
$p_{r-2}p_{r-1}p_n, p_{r-2}p_{r-1}p_n^2, \ldots p_{r-2}p_{r-1}p_n^{a_n}; p_{r-2}p_{r-1}^2p_n, p_{r-2}p_{r-1}^2p_n^2, \ldots p_{r-2}p_{r-1}^2p_n^{a_n}; \cdots;$
$p_{r-2}p_{r-1}^{a_{r-1}}p_n, p_{r-2}p_{r-1}^{a_{r-1}}p_n^2, \ldots p_{r-2}p_{r-1}^{a_{r-1}}p_n^{a_n}; p_{r-2}^2p_{r-1}p_n, p_{r-2}^2p_{r-1}p_n^2, \ldots p_{r-2}^2p_{r-1}p_n^{a_n};$
$p_{r-2}^2p_{r-1}^2p_n, p_{r-2}^2p_{r-1}^2p_n^2, \ldots p_{r-2}^2p_{r-1}^2p_n^{a_n}; \ldots; p_{r-2}^2p_{r-1}^{a_{r-1}}p_n, p_{r-2}^2p_{r-1}^{a_{r-1}}p_n^2, \cdots p_{r-2}^2p_{r-1}^{a_{r-1}}p_n^{a_n}; \cdots$
$p_{r-2}^{a_{r-2}}p_{r-1}p_n, p_{r-2}^{a_{r-2}}p_{r-1}p_n^2, \ldots p_{r-2}^{a_{r-2}}p_{r-1}p_n^{a_n}; p_{r-2}^{a_{r-2}}p_{r-1}^2p_n, p_{r-2}^{a_{r-2}}p_{r-1}^2p_n^2, \ldots p_{r-2}^{a_{r-2}}p_{r-1}^2p_n^{a_n}; \cdots;$
$p_{r-2}^{a_{r-2}}p_{r-1}^{a_{r-1}}p_n, p_{r-2}^{a_{r-2}}p_{r-1}^{a_{r-1}}p_n^2, \ldots p_{r-2}^{a_{r-2}}p_{r-1}^{a_{r-1}}p_n^{a_n}; r = 3,4,5, \cdots p_1p_2p_3p_4, p_1p_2p_3p_4^2, \cdots p_1p_2p_3p_4^{a_4};$
$p_1p_2^2p_3p_4, p_1p_2^2p_3p_4^2, \cdots p_1p_2^2p_3p_4^{a_4}; \ldots; p_1p_2^{a_2}p_3p_4, p_1p_2^{a_2}p_3p_4^2, \cdots p_1p_2^{a_2}p_3p_4^{a_4};$
$p_1p_2p_3^2p_4, p_1p_2p_3^2p_4^2, \ldots p_1p_2p_3^2p_4^{a_4}; p_1p_2^2p_3^2p_4, p_1p_2^2p_3^2p_4^2, \ldots p_1p_2^2p_3^2p_4^{a_4}; \cdots;$
$p_1p_2^{a_2}p_3^2p_4, p_1p_2^{a_2}p_3^2p_4^2, \ldots p_1p_2^{a_2}p_3^2p_4^{a_4}; \cdots p_1p_2p_3^{a_3}p_4, p_1p_2p_3^{a_3}p_4^2, \cdots p_1p_2p_3^{a_3}p_4^{a_4};$
$p_1p_2^2p_3^{a_3}p_4, p_1p_2^2p_3^{a_3}p_4^2, \ldots p_1p_2^2p_3^{a_3}p_4^{a_4}; \cdots; p_1p_2^{a_2}p_3^{a_3}p_4, p_1p_2^{a_2}p_3^{a_3}p_4^2, \ldots p_1p_2^{a_2}p_3^{a_3}p_4^{a_4}; \cdots$
$p_{r-3}p_{r-2}p_{r-1}p_n, p_{r-3}p_{r-2}p_{r-1}p_n^2, \ldots p_{r-3}p_{r-2}p_{r-1}p_n^{a_n};$
$p_{r-3}p_{r-2}^2p_{r-1}p_n, p_{r-3}p_{r-2}^2p_{r-1}p_n^2, \ldots p_{r-3}p_{r-2}^2p_{r-1}p_n^{a_n};$
$p_{r-3}p_{r-2}^{a_{r-2}}p_{r-1}p_n, p_{r-3}p_{r-2}^{a_{r-2}}p_{r-1}p_n^2, \ldots p_{r-3}p_{r-2}^{a_{r-2}}p_{r-1}p_n^{a_n};$
$p_{r-3}p_{r-2}p_{r-1}^2p_n, p_{r-3}p_{r-2}p_{r-1}^2p_n^2, \ldots p_{r-3}p_{r-2}p_{r-1}^2p_n^{a_n};$
$p_{r-3}p_{r-2}^2p_{r-1}^2p_n, p_{r-3}p_{r-2}^2p_{r-1}^2p_n^2, \ldots p_{r-3}p_{r-2}^2p_{r-1}^2p_n^{a_n}; \cdots;$
$p_{r-3}p_{r-2}^{a_{r-2}}p_{r-1}^2p_n, p_{r-3}p_{r-2}^{a_{r-2}}p_{r-1}^2p_n^2, \ldots p_{r-3}p_{r-2}^{a_{r-2}}p_{r-1}^2p_n^{a_n};$
$p_{r-3}p_{r-2}p_{r-1}^{a_{r-1}}p_n, p_{r-3}p_{r-2}p_{r-1}^{a_{r-1}}p_n^2, \ldots p_{r-3}p_{r-2}p_{r-1}^{a_{r-1}}p_n^{a_n};$
$p_{r-3}p_{r-2}^2p_{r-1}^{a_{r-1}}p_n, p_{r-3}p_{r-2}^2p_{r-1}^{a_{r-1}}p_n^2, \ldots p_{r-3}p_{r-2}^2p_{r-1}^{a_{r-1}}p_n^{a_n};$
$p_{r-3}p_{r-2}^{a_{r-2}}p_{r-1}^{a_{r-1}}p_n, p_{r-3}p_{r-2}^{a_{r-2}}p_{r-1}^{a_{r-1}}p_n^2, \ldots p_{r-3}p_{r-2}^{a_{r-2}}p_{r-1}^{a_{r-1}}p_n^{a_n}\}$, $r = 4,5,6,\cdots$

The number of vertices of V_m is $[(a_1+1).(a_2+1).(a_3+1)\cdots(a_n+1)-1]$

Let us consider the set of vertices

$D = \{p_1, p_1^2, \ldots, p_1^{a_1}; p_2, p_2^2, \ldots, p_2^{a_2}; p_3, p_3^2, \ldots, p_3^{a_3}; \ldots p_n, p_n^2, \ldots, p_n^{a_n}\}$ is an annihilator dominating set of V_m.

Now, we have to prove that D is an annihilator dominating set.

For any vertex in $V - D$ is of the form

$p_1^{i_1}p_2^{i_2}, p_1^{i_1}p_2^{i_2}p_3^{i_3}, \cdots, p_1^{i_1}p_2^{i_2}p_3^{i_3}\ldots p_n^{i_n}$ where $1 \le \{i_1, i_2, i_3, \ldots, i_n\} \le a_1$.

These vertices are adjacent with $p_1, p_2, p_3, \cdots, p_n$ in D, then D is a dominating set.

Moreover, if $u_1, v_1, \ldots, w_1$ are any vertices in $V - D$ then $u_1, v_1, \cdots, w_1$ are of the form

$u_1 = p_1^{i_1}p_2^{i_2}(or)p_1^{i_1}p_2^{i_2}p_3^{i_3}\cdots(or)p_1^{i_1}p_2^{i_2}p_3^{i_3}\cdots p_n^{i_n}$

$v_1 = p_1^{j_1} p_2^{j_2} (or) p_1^{j_1} p_2^{j_2} p_3^{j_3} . \cdots (or) p_1^{j_1} p_2^{j_2} p_3^{j_3} \cdots p_n^{j_n}$
$w_1 = p_1^{k_1} p_2^{k_2} (or) p_1^{k_1} p_2^{k_2} p_3^{k_3} \cdots (or) p_1^{k_1} p_2^{k_2} p_3^{k_3} \cdots p_n^{k_n}$
Where $1 \le \{i_1, i_2, i_3, \dots, i_n, j_1, j_2, j_3, \dots, j_n, k_1, k_2, k_3, \cdots, k_n\} \le a_1$
But $u_1, v_1, \cdots, w_1$ are not adjacent in the induced subgraph $\langle V - D\rangle$
Since $(p_1^{i_1} p_2^{i_2}, p_1^{j_1} p_2^{j_2}) = p_1^{b_1} p_2^{b_2}$ where $b_1, b_2 \ge 1$
$(p_1^{j_1} p_2^{j_2}, p_1^{k_1} p_2^{k_2}) = p_1^{b_1'} p_2^{b_2'}$ where $b_1', b_2' \ge 1$
$(p_1^{i_1} p_2^{i_2}, p_1^{k_1} p_2^{k_2}) = p_1^{b_1''} p_2^{b_2''}$ where $b_1'', b_2'' \ge 1$
or $(p_1^{i_1} p_2^{i_2} p_3^{i_3}, p_1^{j_1} p_2^{j_2} p_3^{j_3}) = p_1^{c_1} p_2^{c_2} p_3^{c_3}$ where $c_1, c_2, c_3 \ge 1$
$(p_1^{j_1} p_2^{j_2} p_3^{j_3}, p_1^{k_1} p_2^{k_2} p_3^{k_3}) = p_1^{c_1'} p_2^{c_2'} p_3^{c_3'}$ where $c_1', c_2', c_3' \ge 1$
$(p_1^{i_1} p_2^{i_2} p_3^{i_3}, p_1^{k_1} p_2^{k_2} p_3^{k_3}) = p_1^{c_1''} p_2^{c_2''} p_3^{c_3''}$ where $c_1'', c_2'', c_3'' \ge 1$
(or) Continuing like this ... we obtain
$(p_1^{i_1} p_2^{i_2} p_3^{i_3} \dots p_n^{i_n}, p_1^{j_1} p_2^{j_2} p_3^{j_3} \dots p_n^{j_n}) = p_1^{d_1} p_2^{d_2} p_3^{d_3} \dots p_n^{d_n}$ where $d_1, d_2, d_3, \dots d_n \ge 1$
$(p_1^{j_1} p_2^{j_2} p_3^{j_3} \dots p_n^{j_n}, p_1^{k_1} p_2^{k_2} p_3^{k_3} \dots p_n^{k_n}) = p_1^{d_1'} p_2^{d_2'} p_3^{d_3'} \dots p_n^{d_n'}$ where $d_1', d_2', d_3', \dots d_n' \ge 1$
$(p_1^{i_1} p_2^{i_2} p_3^{i_3} \dots p_4^{i_4}, p_1^{k_1} p_2^{k_2} p_3^{k_3} \dots p_4^{k_4}) = p_1^{d_1''} p_2^{d_2''} p_3^{d_3''} \dots p_n^{d_n''}$ where $d_1'', d_2'', d_3'', \dots d_n'' \ge 1$.

Thus D is an annihilator dominating set. Further, it is an annihilator dominating set of minimal cardinality.

For, if we remove any vertex v_r in D, then
v_r is of the form $p_1^{i_1}$(or) $p_2^{i_2}$(or) $p_3^{i_3} \dots$ (or)$p_n^{i_n}$ where $1 \le \{i_1, i_2, i_3, \dots i_n\} \le a_1$.

If v_r is of the form $p_1^{i_1}$, for $1 \le i_1 \le a_1$ then v_r is adjacent with all the vertices in which it must contain $p_1$as one of the prime with degree one in $\langle V - \{D - \{v_r\}\}\rangle$
On the other hand, if v_r is of the form $p_2^{i_2}$for $1 \le i_2 \le a_2$, then v_r is adjacent with all the vertices in which it must contain $p_2$as one of the prime with degree one in$\langle V - \{D - \{v_r\}\}\rangle$.

Also, if v_r is of the form $p_3^{i_3}$for $1 \le i_3 \le a_3$, then v_r is adjacent with all the vertices in which it must contain $p_3$as one of the prime with degree one in $\langle V - \{D - \{v_r\}\}\rangle$ and so on.

Also, if v_r is of the form $p_n^{i_n}$for $1 \le i_n \le a_n$, then v_r is adjacent with all the vertices in which it must contain $p_n$as one of the prime with degree one in $\langle V - \{D - \{v_r\}\}\rangle$.

Thus $\{D - \{v\}_r\}$ is not an annihilator dominating set.

Therefore D is a minimal annihilator dominating set, it follows that $\gamma_a[v_m] \le |D| = na_1$, if $a_1 = a_2 = a_3 = \ldots = a_n$. □

Construction for Theorem 4.3. If N is any number, which is the sum of $a_1, a_2, a_3, \cdots a_n$with $a_1 = a_2 = a_3 = \cdots = a_n$; Then we have $m = p_1^{\frac{N}{4}} . p_2^{\frac{N}{4}} . p_3^{\frac{N}{4}} \dots p_n^{\frac{N}{4}}$ where $p_1, p_2, p_3, \cdots p_n$ are distinct primes and $D = \{p_1, p_1^2, \dots p_1^{\frac{N}{4}}; p_2, p_2^2, \dots p_2^{\frac{N}{4}}; p_3, p_3^2, \dots p_3^{\frac{N}{4}}; \dots; p_n, p_n^2, \dots p_n^{\frac{N}{4}}\}$ is the Annihilator dominating set.

Illustration. Let V_m be an arithmetic graph with $m = p_1^3 . p_2^3 . p_3^3 . p_4^3$; where$p_1, p_2, p_3, p_4$ are four different Primes such that $a_1 = a_2 = a_3 = a_4 = 3$.

The vertices of V_m are the divisors of m (except 1): The numbers are given for every vertex from 1 to 255.

The verticex set is given by

$v_m = \{1-p_1, 2-p_2, 3-p_3, 4-p_4, 5-p_1^2, 6-p_2^2, 7-p_3^2, 8-p_4^2, 9-p_1^3, 10-p_2^3, 11-p_3^3, 12-p_4^3, 13-p_1p_2,$
$14-p_1^2p_2^2, 15-p_1^3p_2^3, 16-p_1^2p_2^3, 17-p_1^3p_2^2, 18-p_1^2p_2, 19-p_1p_2^2, 20-p_1^3p_2, 21-p_1p_2^3, 22-p_1p_3, 23-p_1^2p_3^2,$
$24-p_1^3p_3^3, 25-p_1^2p_3^3, 26-p_1^3p_3^2, 27-p_1^2p_3, 28-p_1p_3^2, 29-p_1^3p_3, 30-p_1p_3^3, 31-p_1p_4, 32-p_1^2p_4^2, 33-p_1^3p_4^3,$
$34-p_1^2p_4^3, 35-p_1^3p_4^2, 36-p_1^2p_4, 37-p_1p_4^2, 38-p_1^3p_4, 39-p_1p_4^3, 40-p_2p_3, 41-p_2^2p_3^2, 42-p_2^3p_3^3, 43-p_2^2p_3^3,$
$44-p_2^3p_3^2, 45-p_2^2p_3, 46-p_2p_3^2, 47-p_2^3p_3, 48-p_2p_3^3, 49-p_2p_4, 50-p_2^2p_4^2, 51-p_2^3p_4^3, 52-p_2^2p_4^3, 53-p_2^3p_4^2,$
$54-p_2^2p_4, 55-p_2p_4^2, 56-p_2^3p_4, 57-p_2p_4^3, 58-p_3p_4, 59-p_3^2p_4^2, 60-p_3^3p_4^3, 61-p_3^2p_4^3, 62-p_3^3p_4^2, 63-p_3^2p_4,$
$64-p_3p_4^2, 65-p_3^3p_4, 66-p_3p_4^3; 67-p_1p_2p_3, 68-p_1^2p_2^2p_3^2, 69-p_1^3p_2^3p_3^3, 70-p_1p_2^2p_3^3, 71-p_1p_2^3p_3^2, 72-p_1^2p_2^3p_3, 73-$
$p_1^2p_2p_3^3, 74-p_1^3p_2p_3^2, 75-p_1^3p_2^2p_3, 76-p_1p_2p_3^2, 77-p_1p_2^2p_3, 78-p_1^2p_2p_3, 79-p_1^3p_2p_3, 80-p_1p_2^3p_3, 81-p_1p_2p_3^3, 82-$
$p_1^2p_2^2p_3, 83-p_1^2p_2p_3^2, 84-p_1p_2^2p_3^2, 85-p_1^2p_2^2p_3^3, 86-p_1^2p_2^3p_3^2, 87-p_1^3p_2^2p_3^2, 88-p_1^3p_2^3p_3, 89-p_1^3p_2p_3^3, 90-p_1p_2^3p_3^3, 91-$
$p_1^3p_2^3p_3^2, 92-p_1^3p_2^2p_3^3, 93-p_1^2p_2^3p_3^3; 94-p_1p_2p_4, 95-p_1^2p_2^2p_4^2, 96-p_1^3p_2^3p_4^3, 97-p_1p_2^2p_4^3, 98-p_1p_2^3p_4^2, 99-p_1^2p_2^3p_4, 100-$
$p_1^2p_2p_4^3, 101-p_1^3p_2p_4^2, 102-p_1^3p_2^2p_4, 103-p_1p_2p_4^2, 104-p_1p_2^2p_4, 105-p_1^2p_2p_4, 106-p_1^3p_2p_4, 107-p_1p_2^3p_4, 108-$
$p_1p_2p_4^3, 109-p_1^2p_2^2p_4, 110-p_1^2p_2p_4^2, 111-p_1p_2^2p_4^2, 112-p_1^2p_2^2p_4^3, 113-p_1^2p_2^3p_4^2, 114-p_1^3p_2^2p_4^2, 115-p_1^3p_2^3p_4, 116-$
$p_1^3p_2p_4^3, 117-p_1p_2^3p_4^3, 118-p_1^3p_2^3p_4^2, 119-p_1^3p_2^2p_4^3, 120-p_1^2p_2^3p_4^3; 121-p_1p_3p_4, 122-p_1^2p_3^2p_4^2, 123-p_1^3p_3^3p_4^3, 124-$
$p_1p_3^2p_4^3, 125-p_1p_3^3p_4^2, 126-p_1^2p_3^3p_4, 127-p_1^2p_3p_4^3, 128-p_1^3p_3p_4^2, 129-p_1^3p_3^2p_4, 130-p_1p_3p_4^2, 131-p_1p_3^2p_4, 132-$
$p_1^2p_3p_4, 133-p_1^3p_3p_4, 134-p_1p_3^3p_4, 135-p_1p_3p_4^3, 136-p_1^2p_3^2p_4, 137-p_1^2p_3p_4^2, 138-p_1p_3^2p_4^2, 139-p_1^2p_3^2p_4^3, 140-$
$p_1^2p_3^3p_4^2, 141-p_1^3p_3^2p_4^2, 142-p_1^3p_3^3p_4, 143-p_1^3p_3p_4^3, 144-p_1p_3^3p_4^3, 145-p_1^3p_3^3p_4^2, 146-p_1^3p_3^2p_4^3, 147-p_1^2p_3^3p_4^3; 148-$
$p_2p_3p_4, 149-p_2^2p_3^2p_4^2, 150-p_2^3p_3^3p_4^3 151-p_2p_3^2p_4^3, 152-p_2p_3^3p_4^2, 153-p_2^2p_3^3p_4, 154-p_2^2p_3p_4^3, 155-p_2^3p_3p_4^2, 156-$
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$p_2^3p_3^3p_4^2, 173-p_2^3p_3^2p_4^3, 174-p_2^2p_3^3p_4^3; 175-p_1p_2p_3^2p_4^3, 176-p_1p_2p_3^3p_4^2, 177-p_1p_2^2p_3p_4^3, 178-p_1p_2^2p_3^3p_4, 179-$
$p_1p_2^3p_3p_4^2, 180-p_1p_2^3p_3^2p_4, 181-p_1p_2p_3p_4^2, 182-p_1p_2p_3^2p_4, 183-p_1p_2p_3p_4^3, 184-p_1p_2p_3^3p_4, 185-p_1p_2p_3p_4, 186-$
$p_1p_2p_3^2p_4^2, 187-p_1p_2p_3^3p_4^3, 188-p_1p_2^2p_3p_4^2, 189-p_1p_2^2p_3^2p_4, 190-p_1p_2^2p_3^2p_4^3, 191-p_1p_2^2p_3^3p_4^2, 192-p_1p_2^2p_3p_4, 193-$
$p_1p_2^2p_3^2p_4^2, 194-p_1p_2^2p_3^3p_4^3, 195-p_1p_2^3p_3p_4^3, 196-p_1p_2^3p_3^3p_4, 197-p_1p_2^3p_3^2p_4^3, 198-p_1p_2^3p_3^3p_4^2, 199-p_1p_2^3p_3p_4, 200-$
$p_1p_2^3p_3^2p_4^2, 201-p_1p_2^3p_3^3p_4^3, 202-p_1^2p_2p_3^2p_4^3, 203-p_1^2p_2p_3^3p_4^2, 204-p_1^2p_2^2p_3p_4^3, 205-p_1^2p_2^2p_3^3p_4, 206-p_1^2p_2^3p_3p_4^2, 207-$
$p_1^2p_2^3p_3^2p_4, 208-p_1^2p_2p_3p_4^2, 209-p_1^2p_2p_3^2p_4, 210-p_1^2p_2p_3p_4^3, 211-p_1^2p_2p_3^3p_4, 212-p_1^2p_2p_3p_4, 213-p_1^2p_2p_3^2p_4^2, 214-$
$p_1^2p_2p_3^3p_4^3, 215-p_1^2p_2^2p_3p_4^2, 216-p_1^2p_2^2p_3^2p_4, 217-p_1^2p_2^2p_3^2p_4^3, 218-p_1^2p_2^2p_3^3p_4^2, 219-p_1^2p_2^2p_3p_4, 220-p_1^2p_2^2p_3^2p_4^2, 221-$
$p_1^2p_2^2p_3^3p_4^3, 222-p_1^2p_2^3p_3p_4^3, 223-p_1^2p_2^3p_3^3p_4, 224-p_1^2p_2^3p_3^2p_4^3, 225-p_1^2p_2^3p_3^3p_4^2, 226-p_1^2p_2^3p_3p_4, 227-p_1^2p_2^3p_3^2p_4^2, 228-$
$p_1^2p_2^3p_3^3p_4^3, 229-p_1^3p_2p_3^2p_4^3, 230-p_1^3p_2p_3^3p_4^2, 231-p_1^3p_2^2p_3p_4^3, 232-p_1^3p_2^2p_3^3p_4, 233-p_1^3p_2^3p_3p_4^2, 234-p_1^3p_2^3p_3^2p_4, 235-$
$p_1^3p_2p_3p_4^2, 236-p_1^3p_2p_3^2p_4, 237-p_1^3p_2p_3p_4^3, 238-p_1^3p_2p_3^3p_4, 239-p_1^3p_2p_3p_4, 240-p_1^3p_2p_3^2p_4^2, 241-p_1^3p_2p_3^3p_4^3, 242-$
$p_1^3p_2^2p_3p_4^2, 243-p_1^3p_2^2p_3^2p_4, 244-p_1^3p_2^2p_3^2p_4^3, 245-p_1^3p_2^2p_3^3p_4^2, 246-p_1^3p_2^2p_3p_4, 247-p_1^3p_2^2p_3^2p_4^2, 248-p_1^3p_2^2p_3^3p_4^3, 249-$
$p_1^3p_2^3p_3p_4^3, 250-p_1^3p_2^3p_3^3p_4, 251-p_1^3p_2^3p_3^2p_4^3, 252-p_1^3p_2^3p_3^3p_4^2, 253-p_1^3p_2^3p_3p_4, 254-p_1^3p_2^3p_3^2p_4^2, 255-p_1^3p_2^3p_3^3p_4^3\}.$

There are $256-1=255$ vertices.

$D=\{p_1,p_2,p_3,p_4,p_1^2,p_2^2,p_3^2,p_4^2,p_1^3,p_2^3,p_3^3,p_4^3\}$ is the Minimal Annihilator dominating set of V_m with Cardinality $4 \times 3 = 12$. By removing D from the vertex set V of V_m we get induced subgraph $\langle V-D\rangle$ is an independent graph with isolated vertices.

Hence $\gamma_a[v_m] \le 4a_1 = 12$.

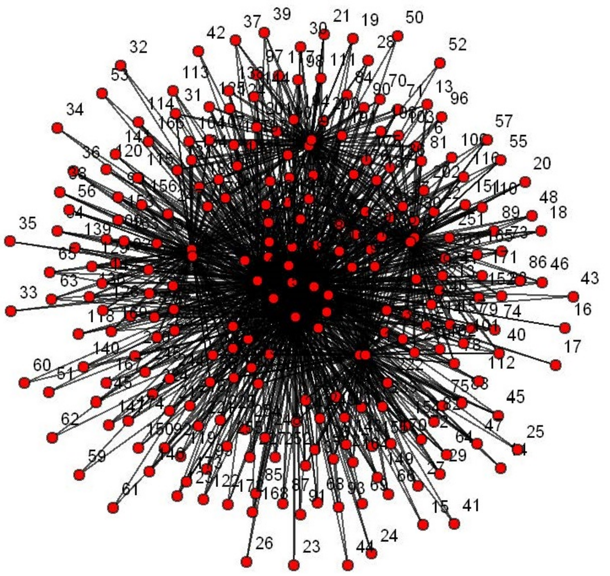

Figure 3. The Graph of v_m with $m = p_1^3.p_2^3.p_3^3.p_4^3$

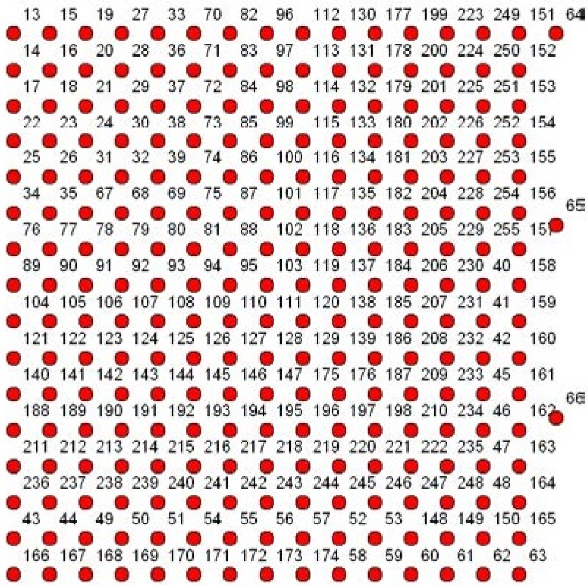

Figure 4. The induced Sub Graph $\langle V - D \rangle$

5. Conclusion

The construction of a graph using multiple annihilator dominant set is easily achieved by applying number theory tools And we have broad applications in real life situations. One of the applications of the Annihilator Conquest Collection is to eliminate pests in agriculture, to suppress viruses that cause disease in epidemic form, to maintain secrecy in transmission.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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