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Research Article

Adjacency and Incidence Matrix of a Soft Graph

J. D. Thenge[1,*], B. Surendranath Reddy[2] and R. S. Jain[2]

[1] *School of Computational Sciences, Punyashlok Ahilyadevi Holkar Solapur University, Solapur, Maharashtra, India*
[2] *School of Mathematical Sciences, Swami Ramanand Teerth Marathwada University, Nanded, Maharashtra, India*
*__Corresponding author:__ jdthenge@sus.ac.in

Abstract. In the present paper we define the notion of adjacency matrix and incidence matrix of a soft graph and derive some results regarding these matrices. We show that some of the classical results of graph theory does not hold for soft graphs.

Keywords. Soft set; Soft graph; Adjacency matrix; Incidence matrix

MSC. 05C99

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1. Introduction

D. Molodstov [6] has introduced the concept of soft set theory in 1999, which give us new techniques for dealing with uncertainty. Ali *et al*. [3] have given the new operations on soft set theory. Ali *et al*. [4] have mentioned the representation of graphs based on neighbourhoods and soft sets. Application of soft set theory in decision making is discussed in [5, 7–9]. Akram and Nawaz [1, 2] have introduced soft graph and given some operations on soft graph. Thenge *et al*. [10] have introduced many operations on soft graph and soft tree such as tabular representation of soft graph, degree of a vertex in soft graph etc.

In this paper, we define adjacency matrix and incidence matrix of a soft graph and discuss some examples. We also derive some results and deduce that few results of graph theory are not true for soft graphs.

2. Preliminaries

Definition 2.1 (Soft set [6])**.** Let U be an universe and E be a set of parameters. Let $\mathcal{P}(U)$ denote the power set of U and C be a non-empty subset of E. A pair (H,C) is called a soft set over U, where H is a mapping given by $H : C \to \mathcal{P}(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\epsilon \in C$, $H(\epsilon)$ may be considered as the set of ϵ-elements of the soft set (H,C) or as the set of ϵ-approximate elements of the soft set.

Definition 2.2 (Soft graph [1])**.** A 4-tuple $G^* = (G,S,T,A)$ is called a soft graph if it satisfies the following conditions:

(1) $G = (V,E)$ is a simple graph.

(2) A is non empty set of parameters.

(3) (S,A) is a soft set over V.

(4) (T,A) is soft set over E.

(5) $(S(a),T(a))$ is a subgraph of G $\forall$ $a \in A$.

The subgraph $(S(a),T(a))$ is denoted by $F(a)$ for convenience. A soft graph can also be represented by

$$G^* = (G,S,T,A) = \{F(x), x \in A\}.$$

In this paper we denote this soft graph as (F,A).

Definition 2.3 (Degree of a vertex in soft graph [10])**.** Let $G = (V,E)$ be a simple connected graph, C be any non-empty subset of V. If set valued function $S : C \to \mathcal{P}(V)$ is defined as $S(x) = \{y \in V \mid d(x,y) \le 1\}$ and set valued function $T : C \to \mathcal{P}(E)$ is defined as $T(x)$=$\{xu \in E \mid u \in S(x)\}$, then (F,C) is a soft graph of G where $F(x) = (S(x),T(x))$. The degree of a vertex $v \in V$ is defined as $\max\{\deg_{F(v_i)}(v), \forall v_i \in C\}$. It is denoted by $\deg_{(F,C)}(v)$ or $d_{(F,C)}(v)$.

3. Adjacency and Incidence Matrix of Soft Graph

Definition 3.1 (Adjacent vertices in soft graph)**.** Let $G = (V,E)$ be a simple connected graph such that $C \subseteq V$, a set valued function $S : C \to \mathcal{P}(V)$ is defined as $S(x) = \{y \in V \mid d(x,y) \le 1\}$ and a set valued function $T : C \to \mathcal{P}(E)$ is defined as $T(x) = \{xu \in E \mid u \in S(x)\}$. Thus (F,C) be a soft graph of G where $F(x) = (S(x),T(x))$. Any two vertices v_i and v_j in V are said to be adjacent with respect to soft graph (F,C) if

(1) $\{v_i, v_j\} \subseteq F(v_i) \cap F(v_j)$; if $v_i, v_j \in A$ and $i \neq j$

(2) $v_i \in F(v_j)$; if $v_i \notin A$ and $v_j \in A$.

If both the vertices v_i, v_j are not in C then are said to be not adjacent.

Example 3.2. Consider the graph $G=(V,E)$ as shown in Figure 1.

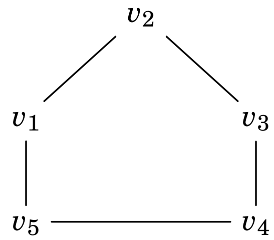

Figure 1. $G=(V,E)$

Let $C=\{v_1,v_4\}$ and define $S(x)=\{z\in V\mid d(x,z)\le 1\}$, $T(x)=\{xu\in E\mid u\in S(x)\}$.
Then $S(v_1)=\{v_1,v_2,v_5\}$, $S(v_4)=\{v_3,v_4,v_5\}$, $T(v_1)=\{v_1v_2,v_1v_5\}$, $T(v_4)=\{v_4v_3,v_4v_5\}$.
Denote $F(x)=(S(x),T(x))\ \forall\ x\in C$.

By above definition we find the adjacent vertices with respect to soft graph

(1) The vertices v_1 and v_2 are adjacent since $v_2\in F(v_1)$ as $v_1\in C$ and $v_2\notin C$.

(2) The vertices v_1 and v_5 are adjacent since $v_5\in F(v_1)$ as $v_1\in C$ and $v_5\notin C$.

(3) The vertices v_3 and v_4 are adjacent since $v_3\in F(v_4)$ as $v_4\in C$ and $v_3\notin C$.

(4) The vertices v_1 and v_4 are not adjacent since $\{v_1,v_4\}\notin F(v_1)\cap F(v_4)$ and $v_1,v_4\in C$.

(5) The vertices v_2 and v_3 are not adjacent since $v_2\notin C$ and $v_3\notin C$.

Definition 3.3 (Adjacency matrix of a soft graph)**.** Let $G=(V,E)$ be a simple connected graph, $C\subseteq V$ and (F,C) be a soft graph of G where set valued function $S:C\to\mathcal{P}(V)$ is defined as $S(x)=\{y\in V\mid d(x,y)\le 1\}$, a set valued function $T:C\to\mathcal{P}(E)$ is defined as $T(x)=\{xu\in E\mid u\in S(x)\}$ and $F(x)=(S(x),T(x))$. Let $A=\bigcup_{v\in C}S(v)=\{v_1,v_2,\dots,v_n\}$. The adjacency matrix of the soft graph (F,C) is a square matrix of order $n\times n$ denoted as $\mathcal{A}(F,C)=(c_{ij})$, (i,j)th entry c_{ij} is given by

$$c_{ij}=\begin{cases}1, & \text{if } v_i \text{ is adjacent to } v_j\\ 0, & \text{if } v_i \text{ is not adjacent to } v_j,\ \ i,j=1,2,3,\dots,n.\end{cases}$$

Definition 3.4 (Incidence matrix of a soft graph)**.** Let $G=(V,E)$ be a simple connected graph, $C\subseteq V$ and (F,C) be a soft graph of G where set valued function $S:C\to\mathcal{P}(V)$ is defined as $S(x)=\{y\in V\mid d(x,y)\le 1\}$, a set valued function $T:C\to\mathcal{P}(E)$ is defined as $T(x)=\{xu\in E\mid u\in S(x)\}$ and $F(x)=(S(x),T(x))$. Let $A=\bigcup_{v\in C}S(v)=\{v_1,v_2,\dots,v_n\}$ and $B=\bigcup_{v\in C}T(v)=\{e_1,e_2,\dots,e_m\}$. The incidence matrix of a soft graph (F,C) is a matrix $\mathcal{I}(F,C)=(b_{ij})$ of order $n\times m$ where (i,j)th entry b_{ij} is given by

$$b_{ij}=\begin{cases}1, & \text{if } e_j\in F(v_i)\\ 0, & \text{otherwise}\end{cases}$$

Example 3.5. Consider the graph $G = (V,E)$ as shown in Figure 2.

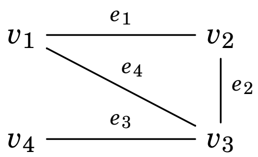

Figure 2. $G = (V,E)$

Let $C = \{v_1, v_2\}$ and $S(x) = \{z \in V | d(x,z) \leq 1\}$, $T(x) = \{xu \in E | u \in S(x)\}$.
Then, $S(v_1) = \{v_1, v_2, v_3\}$, $S(v_2) = \{v_1, v_2, v_3\}$, $T(v_1) = \{v_1v_2, v_1v_3\} = \{e_1, e_4\}$, $T(v_2) = \{v_2v_1, v_2v_3\} = \{e_1, e_2\}$.
Here $A = \{v_1, v_2, v_3\}$ and $B = \{e_1, e_2, e_4\}$.
The adjacency matrix of soft graph (F,C) is given by

$$\mathcal{A}(F,C) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \end{array} \begin{array}{c} \begin{array}{ccc} v_1 & v_2 & v_3 \end{array} \\ \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \end{array}$$

The incidence matrix of soft graph (F,C) is given by

$$\mathcal{I}(F,C) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \end{array} \begin{array}{c} \begin{array}{ccc} e_1 & e_2 & e_4 \end{array} \\ \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array}$$

Theorem 3.6. *Let $G = (V,E)$ be a simple connected graph, $C \subseteq V$. If (F,C) be a soft graph given by $F(x) = (S(x), T(x))$ where $S(x) = \{z \in V \mid d(x,z) \leq 1\}$, $T(x) = \{xu \in E \mid u \in S(x)\}$, then for $v \in C$, the sum of corresponding row elements in a adjacency matrix $\mathcal{A}(F,C)$ is* $\deg(v)$ *with respect to soft graph (F,C).*

Proof. Here $G = (V,E)$ be a simple connected graph, (F,C) be a soft graph for $C \subseteq V$ where $S(x) = \{z \in V \mid d(x,z) \leq 1\}$, $T(x) = \{xu \in E \mid u \in S(x)\}$, $F(x) = (S(x), T(x))$.
Let

$$\bigcup_{v_i \in C} S(v_i) = \{v_1, v_2, \dots, v_m\}.$$

Surely,

$$C \subseteq \bigcup_{v_i \in C} S(v_i).$$

Now, we write the adjacency matrix of soft graph (F,C) as follows:

$$\mathcal{A}(F,C) = \begin{array}{c} \\ v_1 \\ v_2 \\ \cdots \\ v_p \\ \cdots \\ v_m \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & \cdots & v_m \end{array} \\ \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ a_{p1} & a_{p2} & \cdots & a_{pm} \\ \cdots & \cdots & \ddots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mm} \end{pmatrix} \end{array}$$

Claim: $\sum_{q=1}^{m} a_{pq} = \deg(v_p)\ \forall\ p = 1,2,3,\dots,m$.
By definition of degree of a vertex with respect to soft graph

$$\deg(v_p) = \max\{\deg_{F(v_i)}(v_p),\ \forall\ v_i \in C\}.$$

As $v_p \in C$ for $p = 1,2,3,\dots,m$ it implies $S(v_p)$ is the set of all vertices which are adjacent to v_p. Hence $\deg(v_p)$ with respect $F(v_p)$ is maximum, since $S(v_p)$ collects all the vertices of v_p which are adjacent to v_p.
Thus, we can easily observe that $\deg(v_p) = \deg_{F(v_p)}(v_p) = |S(v_p)| - 1$
as $v_p \in S(v_p)$ and v_p is not adjacent to itself.
If v_p is adjacent to v_q then $a_{pq} = 1$ otherwise 0.
But the number of adjacent vertices to v_p is $|S(v_p)| - 1$. i.e.

$$\sum_{q=1}^{m} a_{pq} = |S(v_p)| - 1.$$

It implies

$$\sum_{q=1}^{m} a_{pq} = \deg(v_p).$$

Hence for $v \in C$ the sum of corresponding row elements in a adjacency matrix $\mathcal{A}(F,C)$ is $\deg(v)$ with respect to soft graph (F,C). □

Remark 3.7. In the above theorem, if $v \notin C$, then the sum of elements corresponding to v in a adjacency matrix $\mathcal{A}(F,C)$ is not equal to $\deg(v)$ with respect to soft graph (F,C).

Theorem 3.8. *Let $G = (V,E)$ be a Petersen graph. If (F,A), (F,B) be soft graphs of G for $A \subseteq V$, $B \subseteq V$ given by $A = \{v_1,v_2,\dots,v_5\}$ and $B = \{v_6,v_7,\dots,v_{10}\}$ where $F(x) = (S(x),T(x))$, $S(x) = \{z \in V \mid d(x,z) \le 1\}$, $T(x) = \{xu \in E \mid u \in S(x)\}$, $\forall\ x \in C$, then for any $v \in V$, row total of adjacency matrix of soft graph is equal to degree of v with respect to soft graph.*

Proof. Consider the Petersen graph $G = (V,E)$ as shown in Figure 3.

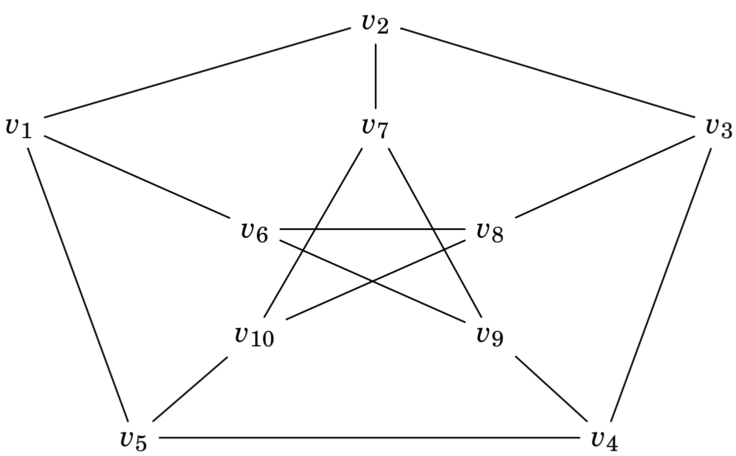

Figure 3. Petersen graph

1. Let $A = \{v_1, v_2, v_3, v_4, v_5\}$, $S(x) = \{z \in V \mid d(x,z) \leq 1\}$, $T(x) = \{xu \in E \mid u \in S(x)\}$.

   Denote $F(x) = (S(x), T(x))\ \forall\ x \in A$.

   Then $S(v_1)=\{v_1,v_2,v_5,v_6\}$, $S(v_2)=\{v_1,v_2,v_3,v_7\}$, $S(v_3)=\{v_2,v_3,v_4,v_8\}$, $S(v_4)=\{v_3,v_4,v_5,v_9\}$, $S(v_5) = \{v_1,v_4,v_5,v_{10}\}$, $T(v_1) = \{v_1v_2,v_1v_6,v_1v_5\}$, $T(v_2) = \{v_2v_1,v_2v_3,v_2v_7\}$, $T(v_3) = \{v_3v_2,v_3v_4,v_3v_8\}$, $T(v_4)=\{v_4v_3,v_4v_5,v_4v_9\}$, $T(v_5)=\{v_5v_1,v_5v_4,v_5v_{10}\}$.

   The adjacency matrix of soft graph (F,A) with the row total is given by

$$\begin{array}{c} \\ v_1\\ v_2\\ v_3\\ v_4\\ v_5\\ v_6\\ v_7\\ v_8\\ v_9\\ v_{10}\end{array}\begin{array}{c}\begin{array}{ccccccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & \text{Row total}\end{array}\\ \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 3\\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 3\\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 3\\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 3\\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 3\\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1\end{pmatrix}\end{array}$$

   Hence the degree of the vertices with respect to soft graph are given by,

   (i) $\deg_{F(v_1)}(v_1) = 3$, $\deg_{F(v_2)}(v_1) = 1$, $\deg_{F(v_3)}(v_1) = 0$, $\deg_{F(v_4)}(v_1) = 0$, $\deg_{F(v_5)}(v_1) = 1$, $\deg_{(F,A)}(v_1) = \max\{3,1,0,0,1\} = 3$.

   Similarly, we calculate degrees of remaining vertices.

   (ii) $\deg_{(F,A)}(v_2) = \max\{1,3,1,0,0\} = 3$

   (iii) $\deg_{(F,A)}(v_3) = \max\{0,1,3,1,0\} = 3$

   (iv) $\deg_{(F,A)}(v_4) = \max\{0,0,1,3,1\} = 3$

   (v) $\deg_{(F,A)}(v_5) = \max\{1,0,0,1,3\} = 3$

   (vi) $\deg_{(F,A)}(v_6) = \max\{1,0,0,0,0\} = 1$

   (vii) $\deg_{(F,A)}(v_7) = \max\{0,1,0,0,0\} = 1$

   (viii) $\deg_{(F,A)}(v_8) = \max\{0,0,1,0,0\} = 1$

   (ix) $\deg_{(F,A)}(v_9) = \max\{0,0,0,1,0\} = 1$

   (x) $\deg_{(F,A)}(v_{10}) = \max\{0,0,0,0,1\} = 1$

   Hence row total of corresponding elements in adjacency matrix is equal to degree of a vertex with respect to soft graph (F,A) for all $v \in V$.

2. Similar proof can be produced for $B = \{v_6, v_7, v_8, v_9, v_{10}\}$. □

Remark 3.9. In above theorem, if Q is an arbitrary subset of V such that $Q \neq A$ and $Q \neq B$ then row total of adjacency matrix for corresponding elements not in Q with respect to soft graph need not be equal to degree of a corresponding vertex with respect to graph (F,Q).

Theorem 3.10. *Let $G = (V,E)$ be a simple connected graph, $C \subseteq V$ and (F,C) be soft graph of G where $F(x) = (S(x), T(x))$, $S(x) = \{z \in V \mid d(x,z) \leq 1\}$, $T(x) = \{xu \in E \mid u \in S(x)\}$, $\forall\ x \in C$. If $\mathcal{A}$ be* *adjacency matrix of soft graph (F,C), for any positive integer k, $\mathcal{A}^k$ denote k times matrix multiplication of $\mathcal{A}$, then the number of walks from v_i to v_j in (F,C) of length k is atmost b_{ij}^k, (i,j)th entry in $\mathcal{A}^k$.*

Proof. We use mathematical induction to prove the result.
Let $k=1$, by definition of adjacency matrix, (i,j)th entry of $\mathcal{A}$ is a walk of length 1, since given graph $G=(V,E)$ be a simple connected graph there are no loops and parallel edges.
Now, assume the result is true for $k-1$. Let $\mathcal{A}=(a_{ij})$, $\mathcal{A}^{k-1}=(c_{ij})$, $\mathcal{A}^k=(b_{ij})$.
We know that $\mathcal{A}^k=\mathcal{A}^{k-1}\times\mathcal{A}$, i.e., $b_{ij}=\sum\limits_{t=1}^{n} c_{it}\cdot a_{tj}$.
Every v_i-v_j walk of length k consists of v_i to v_t walk of length $k-1$ and v_t to v_j walk of length 1 for every v_t adjacent to v_j.
There are atmost c_{it} walks of length $k-1$ from v_i to v_t and a_{tj} walks of length 1 from v_t to v_j for each vertex v_t.
But in soft graph, if there exist a walk from v_i to v_t and v_t to v_j then there may or may not exist a walk from v_i to v_j via v_t.
Thus, the number of walks of length k from v_i to $v_j\le\sum\limits_{t=1}^{n} c_{it}\cdot a_{tj}=b_{ij}$.
Thus, number of walks of length k from v_i to v_j is atmost b_{ij} which is the (i,j)th entry in $\mathcal{A}^K$. □

Remark 3.11. Many results from classical graph theory may not hold good in soft graph theory. Here we present one of the such result.
Let $G=(V,E)$ be a simple connected graph and $B\subseteq V$. If (F,B) is a soft graph of G and $\mathcal{A}$, $\mathcal{I}$, $\mathcal{D}$ are adjacency matrix, incidence matrix and degree matrix of soft graph (F,S), then $\mathcal{A}+\mathcal{D}$ need not be equal to $\mathcal{I}\cdot\mathcal{I}^t$.

4. Conclusion

Soft graph is a new area of research in mathematics. In the present paper we have defined the notion of adjacency matrix and incidence matrix of soft graph. Also, we have proved that some of the classical results of graph theory are not true for soft graph theory.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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