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Research Article

Facilitating Strategic Operations by Making Use of a Model Incorporating a Stochastic Integral

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Abstract. Formulation, investigation, and practical interpretation of stochastic models are generally recognized as extremely useful research activities for a wide class of scientific disciplines. The paper makes use of a stochastic integral and a product of two positive random variables for formulating a stochastic model. A characterization and an interpretation in strategic operations arising in financial economics of the formulated stochastic model are also established by the paper.

Keywords. Model; Discounting; Characteristic function

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1. Introduction

In general, stochastic models are recognized as very strong tools of probability theory with extremely useful applications in a particularly wide variety of practical disciplines [2]. More precisely, stochastic models are becoming particularly important for making sufficiently clear the structure and evolution of complex systems arising in operational research, informatics, telecommunication, engineering, and many significant topics of economics and management. It is generally accepted that the stochastic models constitute the most significant class of models with valuable applications to practical situations. It is also very well known that many of the basic problems concerning stochastic models can be described in terms of distributions functions. The classical analysis incorporates a powerful method for solving problems of this kind. In many cases it is quite suitable to investigate the properties of a stochastic model by incorporating the corresponding characteristic function instead of the distribution function.

It is of some particular theoretical and practical importance to make a comment on stochastic discounting models. More clearly, stochastic discounting models are generally considered as the most reliable analytical tools for supporting and improving operations for making proactive decisions in many disciplines affected by the presence of complex systems. It is quite understood that the presence of proactivity in decision making substantially facilitates strategic thinking and strategic management. The paper mainly concentrates on substantially extending the strategic applicability of a new stochastic discounting model incorporating a stochastic integral [4] and a random contraction [1].

The paper concentrates on the implementation of the following three purposes. The first purpose is the formulation of a stochastic model by the incorporation of a stochastic integral [4] and the incorporation of the product of two positive random variables [3]. The second purpose is the establishment of a characterization of the distribution of the formulated stochastic model by making use of characteristic functions [5]. The third purpose is the interpretation of the formulated stochastic model as a stochastic discounting model arising in the discipline of financial economics.

In conclusion, it can be said that the purposes of the paper constitute very valuable factors of very useful stochastic discounting modeling activities strongly supporting several very significant strategic operations.

2. Formulation of a Stochastic Model

We suppose that L is a positive random variable with characteristic function $\varphi_L(u)$, V is a positive random variable with distribution function $F_V(v)$, δ is a positive real number and we consider the random variable

$$C = Le^{-\delta V}.$$

Let $\{X(t), t \geq 0\}$ be a stochastic process with stationary, independent, positive increments and $E(X(t)) = \mu$, $V(X(t)) = \sigma^2 t$. We also suppose that $\{X(t), t \geq 0\}$ is continuous in probability and that its sample paths are right continuous and have left limits. Moreover, we suppose that the increment $J = X(t+1) - X(t)$ has characteristic function $\varphi_J(u)$. The stochastic integral

$$S = \int_0^\infty e^{-rt} dX(t), \quad r > 0$$

exists in the sense of convergence in probability and finite almost surely. In addition, the distribution function of S is continuous and

$$\varphi_S(u) = \exp\left(\frac{1}{r}\int_0^u \frac{\log \varphi_J(w)}{w} dw\right)$$

is the characteristic function of the stochastic integral S [4].

We consider the stochastic model

$$Y = \int_0^\infty e^{-rt} dX(t) + Le^{-\delta V}.$$

The theoretical investigation and the practical interpretation of the above stochastic model constitutes the purposes of the paper.

3. Characterizing the Distribution of a Stochastic Model

This section of the paper establishes a characterization of the distribution of the stochastic model formulated by the previous section.

Theorem 3.1. *We suppose that the stochastic integral S, the random variable L, the random variable V are independent and the random variable V follows the exponential distribution with distribution function $F_V(v) = 1 - e^{-\lambda v}$, $\lambda > 0$. The characteristic function of the stochastic model Y has the form*

$$\varphi_Y(u) = \exp\left(\frac{1}{r}\int_0^u \frac{\log\varphi_J(w)}{w}dw\right)\exp\left[a\int_0^u \frac{\exp\left(\frac{1}{r}\int_0^w \frac{\log\varphi_J(\theta)}{\theta}d\theta\right)-1}{w}dw\right], \quad a = \frac{\lambda}{\delta}$$

if and only if

$$Y \overset{d}{=} L \tag{1}$$

where $\overset{d}{=}$ means equality in distribution.

Proof. Only the sufficiency condition will be proved since the necessity can be proved by reversing the argument. From the independence of S, L, V it follows that L, $\exp(-\delta V)$ are independent and it can be readily proved that C is independent of S. Hence the characteristic function of the stochastic model $Y = S + C$ has the form

$$\varphi_Y(u) = \exp\left(\frac{1}{r}\int_0^u \frac{\log\varphi_J(w)}{w}dw\right)\int_0^1 \varphi_L(uw)\,d\left(1 - F_V\left(-\frac{1}{\delta}\log w\right)\right). \tag{2}$$

Since the random variable V follows the exponential distribution with parameter λ then (2) can be written in the form

$$\varphi_Y(u) = \exp\left(\frac{1}{r}\int_0^u \frac{\log\varphi_J(w)}{w}dw\right)a\int_0^1 \varphi_L(uw)w^{a-1}dw, \quad a = \frac{\lambda}{\delta}$$

or equivalently

$$\varphi_Y(u) = \exp\left(\frac{1}{r}\int_0^u \frac{\log\varphi_J(w)}{w}dw\right)\frac{a}{u^a}\int_0^u \varphi_L(w)w^{a-1}dw. \tag{3}$$

If we make use of (1) in (3) we get the integral equation

$$\varphi_Y(u) = \exp\left(\frac{1}{r}\int_0^u \frac{\log\varphi_J(w)}{w}dw\right)\frac{a}{u^a}\int_0^u \varphi_Y(w)w^{a-1}dw$$

which can be written in the form

$$u^a\varphi_Y(u)\exp\left(-\frac{1}{r}\int_0^u \frac{\log\varphi_J(w)}{w}dw\right) = a\int_0^u \varphi_Y(w)w^{a-1}dw. \tag{4}$$

From the integral equation (4) we get the differential equation

$$au^{a-1}\varphi_Y(u)\exp\left(-\frac{1}{r}\int_0^u \frac{\log\varphi_J(w)}{w}dw\right) + u^a\exp\left(-\frac{1}{r}\int_0^u \frac{\log\varphi_J(w)}{w}dw\right)\frac{d\varphi_Y(u)}{du}$$
$$+ u^a\varphi_Y(u)\left(-\frac{1}{r}\frac{\log\varphi_J(u)}{u}\right)\exp\left(-\frac{1}{r}\int_0^u \frac{\log\varphi_J(w)}{w}dw\right) = a\varphi_Y(u)u^{a-1}. \tag{5}$$

It is easily seen that for $u \neq 0$ the differential equation (5) can be written in the form

$$a\varphi_Y(u)\exp\left(-\frac{1}{r}\int_0^u \frac{\log\varphi_J(w)}{w}dw\right) + u\exp\left(-\frac{1}{r}\int_0^u \frac{\log\varphi_J(w)}{w}dw\right)\frac{d\varphi_Y(u)}{du}$$
$$+u\varphi_Y(u)\left(-\frac{1}{r}\frac{\log\varphi_J(u)}{u}\right)\exp\left(-\frac{1}{r}\int_0^u \frac{\log\varphi_J(w)}{w}dw\right) = a\varphi_Y(u)$$

or equivalently in the form

$$a\varphi_Y(u) + u\frac{d\varphi_Y(u)}{du} - u\varphi_Y(u)\frac{1}{r}\frac{\log\varphi_J(u)}{u} = a\varphi_Y(u)\exp\left(\frac{1}{r}\int_0^u \frac{\log\varphi_J(w)}{w}dw\right). \tag{6}$$

It is readily seen that (6) can be written in the form

$$u\frac{d\varphi_Y(u)}{du} = a\varphi_Y(u)\exp\left(\frac{1}{r}\int_0^u \frac{\log\varphi_J(w)}{w}dw\right) - a\varphi_Y(u) + u\varphi_Y(u)\frac{1}{r}\frac{\log\varphi_J(u)}{u}$$

or equivalently

$$\frac{d\varphi_Y(u)}{du} = \left\{a\left[\frac{\exp\left(\frac{1}{r}\int_0^u \frac{\log\varphi_J(w)}{w}dw\right)-1}{u}\right] + \frac{1}{r}\frac{\log\varphi_J(u)}{u}\right\}\varphi_Y(u). \tag{7}$$

Integrating in (7) with due regard to the conditions

$$\varphi_Y(0) = 1, \quad \varphi_J(0) = 1$$

we get that

$$\varphi_Y(u) = \exp\left(\frac{1}{r}\int_0^u \frac{\log\varphi_J(w)}{w}dw\right)\exp\left[a\int_0^u \frac{\exp\left(\frac{1}{r}\int_0^w \frac{\log\varphi_J(\theta)}{\theta}d\theta\right)-1}{w}dw\right]. \qquad \square$$

4. Application

This section of the paper concentrates on the interpretation of the formulated stochastic model as a useful tool in the area of stochastic discounting operations. We consider an economic asset with indefinite life and let the random variable $X(t)$ denotes the income generated by the economic asset in the time interval $[0,t]$ then the stochastic integral

$$S = \int_0^\infty e^{-rt}dX(t)$$

denotes the present value of the income generated by the economic asset during its indefinite life, where r denotes force of interest [4].

We also suppose that the economic asset generates an additional random income L at the random time V. Hence the random variable

$$C = Le^{-\delta V}$$

denotes the present value of the additional random income L generated by the economic asset at the random time V, where δ denotes force of interest [6].

In conclusion, the stochastic discounting model

$$Y = \int_0^\infty e^{-rt}dX(t) + Le^{-\delta V}$$

denotes the present value of the total income generated by the economic asset.

5. Conclusion

It is readily seen that the incorporation of two very well known stochastic discounting models, as principal components, of the stochastic model formulated by the paper constitute a particularly useful analytical operation. Moreover, the structural elements and the mathematical form of each principal component of the formulated stochastic model make quite clear the suitability of such a stochastic model for solving significant problems arising in the disciplines of strategic thinking and strategic decision making. More precisely, the formulated stochastic model is the sum of two stochastic discounting models. In other words, such a sum significantly supports the predictability of decision makers.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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