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Research Article

Approximate Solutions for the Projects Revenues Assignment Problem

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Abstract. The aim of this research work is to find algorithms solving an NP-hard problem by elaborating several heuristics. This problem is to find an appropriate schedule to assign different projects, which will be expected to generate fixed revenues, to several cities. For this work, we assume that all cities have the same socio-economic and strategic characteristics. The problem is as follow. Given a set of projects which represented by its expected revenues. The objective is to distribute on several cities all projects with a minimum expected revenues gap between cities. Thus, our objective is to minimize the expected revenue gap. The suitable assignment is searching equity between cities. In this paper, we formulate mathematically the studied problem to find an approximate solutions and apply some methods to search resolution of the studied problem.

Keywords. Scheduling; Approximate solution; Heuristic; Optimization; Mathematic model

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1. Introduction

In several countries, the development must be done for all cities in the country. Indeed, to guarantee the regional development and to give the same chances for all the cities of the country, it is essential to seek a fair distribution of the new projects which are expected to create well-defined incomes (revenues). The impact of the distribution of projects can directly resulted on the socio-economic cycle.

The expected revenue distribution of projects on several cities is the main of this research. Thus, we have to find a good schedule to guarantee equity between cities in term of expected revenues. The result of a distribution, which does not seek the minimization of gap between the cities, can lead to a failure decision-making.

Given a set of projects to be distributed on several cities. Each project has its corresponding expected revenue and will be totally assigned to a chosen city. Seeking the equity, we formulate the mathematic model of the studied problem and we apply some algorithms to solve the problem. The mathematic model is based on the maximization of the minimum total expected revenues.

The study of literature is not rich in this kind of subject. Few works can be presented for the studied problem. In [5], authors present the problem of maximization the minimum completion time on identical parallel machines. The authors solve the problem by applying a branch and bound method to give to optimal solution. A more developed solutions to enhance the results given in [5], the authors in [8] gives a more performed optimal solutions to solve the problem.

The problem of selection of the best alternative from a finite set can be solved using the exact computing budget allocation method and expected value of information (EVI) approaches [3].

The maximization is treated in literature review for revenue. In [6], authors analyzed the case of two goods to have the optimal revenue. The authors showed that in the case of one-dimensional mechanisms at 73% of the exact revenue when the valuations of the two goods are independent and distributed identically. However, the ratio become 50% when they are independent.

Several researches can be cited to revenue management [4], [1], [7] and [2].

The paper is structured as follows. The section 2 is reserved for the problem definition. Section 3 consist the presentation of the approximate solutions by presentation of the different heuristics.

2. Problem Definition

Let P is the set of a given n_P projects to be scheduled on a fixed number of cities n_c.

Each project p with $p = \{1,\dots,n_p\}$ is characterized by its own expected revenue denoted by er_p. We denoted by C_{er_p} the cumulative expected revenue when project p is scheduled. The total expected revenue for each city after ending the assignment is denoted by T_{er_c} with $c = \{1,\dots,n_c\}$. The maximum (minimum) given expected revenue after finishing distribution on all cities is denoted by $T_{er_{\max}}$ ($T_{er_{\min}}$).

The expected revenue of each city is represented as follows $T_{er_1} \le T_{er_2} \le \dots \le T_{er_{n_c}}$. We can illustrate the studied problem in the following figure.

Example 1. Let $n_p = 7$ and $n_c = 2$. Table 1 represent the expected revenue er_p for each project p. The fixed unit for er_p is 103$.

Table 1. Instance for Example 1

p	1	2	3	4	5	6	7
er_p	120	60	45	156	160	190	87

We search to schedule the seven projects described above on the 2 cities. Applying the (SPT) algorithm, the result is given in Figure 1.

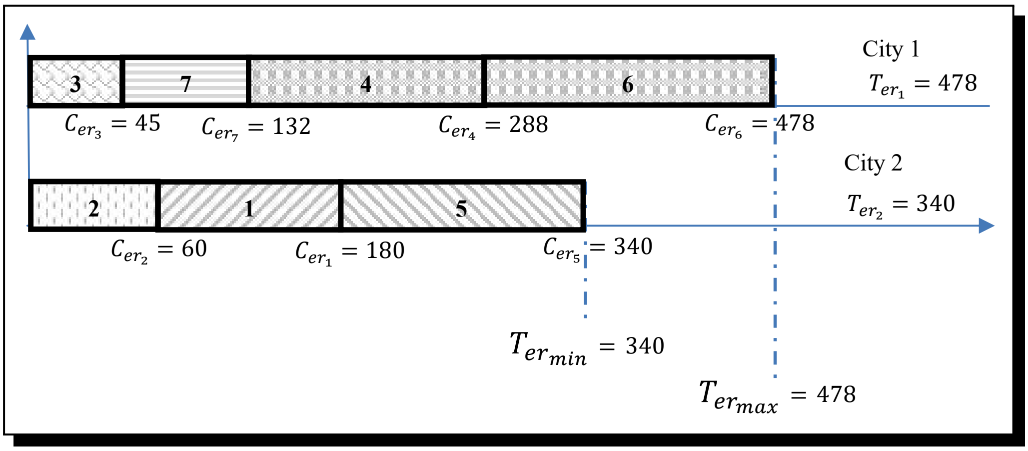

Figure 1. Expected revenue dispatching for Example 1

The schedule resulted to SPT is as follows. For city 1, we assign projects $\{3,7,4,6\}$, however, for city 2 we assign projects $\{2,1,5\}$.

Based on Figure 1, city 1 has a total expected revenue 478. However, city 2 has a total expected revenue of 340. The expected revenue gap between city 1 and city 2 is equal to $T_{er_{\max}} - T_{er_{\min}} = 138\$$. In this study our main goal is to reduce the obtained gap. Therefore, we have to try to find a schedule more efficient with gap less than 138.

To calculate the gap between cities, we can fix different indicators. For this work, the indicator that we propose for each city to reduce budget will be $T_{er_{\max}} - T_{er_{\min}}$. Therefore, considering the n_c cities the total expected revenue gap is given in Equation (1) below:

$$\text{Minimize} \sum_{c=1}^{n_c} [T_{er_c} - T_{er_{\min}}]. \tag{1}$$

Proposition 1. *The objective of the studied problem can be written as follows.*

$$\text{Min}\left(\sum_{p=1}^{n_p} er_p - n_c \times T_{er_{\min}}\right). \tag{2}$$

Proof. It is clearly to observe that $\sum_{p=1}^{n_p} er_p = \sum_{c=1}^{n_c} T_{er_c}$. Using this equality, you can formulate equation (1) as equation (2). □

We denoted by the total expected revenue gap between cities $ER_{\max} = \sum_{p=1}^{n_p} er_p - n_c \times T_{er_{\min}}$.

The optimal solution will be denoted by $ER^*_{\max}$. Using the notation cited in [5], we can denoted the studied problem as $P\|ER_{\max}$.

3. Approximate Solutions

In this section we present several approximate solutions for the studied problem based on different resolution method.

3.1 Probabilistic and Randomized heuristic (RP)

For this heuristic an adopt the assignment of the projects using a probabilistic method. First we order all projects according to their non-increasing order expected revenue.

The selection of the project to be assign to city is based on the choice among the 10 largest projects (having the largest expected revenue) of the one project applying a probability α.

In practice, we fix a probability α. Now, we try to extend this chosen probability to the 10 largest projects. We generate a number r randomly between $[1,100]$.

Theorem 1. *The probability of the choice of the project can be clustered as following:*

- *if* $r \in [1, \alpha \times 100]$*, then we assign the first largest project.*
- *if* $r \in [\alpha \times 100 + (i-1) \times st, \alpha \times 100 + i \times st]$*, for all* $i \in \{1,\dots,9\}$*, then we assign the* i^{th} *project, with* $st = \frac{1-\alpha}{9} \times 100$.

Proof. The idea is the follows. If r is between 1 and $\alpha \times 100$ we choose the first largest project. However, we subdivide the rest of interval into 9 sub-intervals. The step of the subdivision is $st = \frac{1-\alpha}{9} \times 100$. Therefore, we have 9 intervals between $[\alpha \times 100, 100]$ as follows. The first interval is $[\alpha \times 100, \alpha \times 100 + st]$. The second one is $[\alpha \times 100 + st, \alpha \times 100 + st + st] = [\alpha \times 100 + st, \alpha \times 100 + 2st]$. So on, until we reach the 9^{th} interval which is $[\alpha \times 100 + 8st, \alpha \times 100 + 9st]$.

Thus, we cluster the interval $[1,100]$ as follows:

- if $r \in [1, \alpha \times 100]$, then we assign the first largest project.
- if $r \in [\alpha \times 100, \alpha \times 100 + st]$, then we assign the second largest project.
- if $r \in [\alpha \times 100 + st, \alpha \times 100 + 2st]$, then we assign the third largest project.

And so on until reach the final sub-interval: If $r \in [\alpha \times 100 + 8st, \alpha \times 100 + 9st]$, then we assign the 10^{th} largest project.

In general, we have:

- if $r \in [1, \alpha \times 100]$, then we assign the first largest project.
- if $r \in [\alpha \times 100 + (i-1) \times st, \alpha \times 100 + i \times st]$, for all $i \in \{1,\dots,9\}$, then we assign the i^{th} project.

Assume that the function that schedule the project p on the city that having the least expected revenue is denoted by $Schedule(p)$.

The randomized probabilistic algorithm is as follows.

	Randomized probabilistic algorithm: $RP(\alpha)$
0	Order all projects as non-increasing order ER_p
1	Fixed a probability α
2	$p = 1$
3	**While**$(p \leq n_p)$**do**
4	Given r random in $[1,100]$
5	$st = \frac{1-\alpha}{9} \times 100$
6	**IF**$r \in [1, \alpha \times 100]$**then** Picked the first project and $Schedule(1)$
	Else

7	**For** ($i = 1$ to 9)
	If$r \in [\alpha \times 100 + (i-1) \times st, \alpha \times 100 + i \times st]$**then**
	$Interval = i$
	End IF
	End For

8	Picked the project $interval$ and $Schedule(interval)$
	End IF
9	$p = p + 1$
	End While
10	Calculate ER_{max}
End	Return ER_{max}

3.2 Iterative Randomized Heuristic (IR)

For this more enhanced heuristic we repeat more times the above algorithm. Indeed, we fixed a number of iteration denoted by *limit* and we iterate the randomly algorithm *limit* times. In experimental results we can fix $limit = 500$.

3.3 Iterative Randomized Probabilistic heuristic (IRP)

Now, we repeat iteratively the IR algorithm by modified the probability α. We can iterate the algorithm as the fixed values of α in $\{0.1;0.2;0.3;0.4;0.5;0.6;0.7;0.8;0.9\}$. After the iteration we save the best solution.

4. Conclusion

In this paper, we formulate mathematical model for the resolution of the problem that study the assignment of the expected revenue of the projects which will be distributed for several cities. Three heuristics are developed in this work by mathematical modeling and by giving the corresponding algorithms. These heuristics based essentially on the probabilistic and randomization of the selection of the largest projects that have the largest expected revenues. The impact of the good schedule has an impressive impact on the socio-economic point of view. Indeed, a good distribution guarantee the equity between cities which will be considered as an excellent economic indicator.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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